Stockholm University

Mathematical Statistics
Stockholm University
Bachelor Thesis **2011:7**
http://www.math.su.se

The interest volatility surface

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June 2011

Abstract

Pricing financial instruments are important for all financial institutions. To obtain a price financial institutions use theoretical pricing models. These models require parameters which describes the uncertainty of the price movements, the parameters used for this is the implied volatilities. The usual way to use these volatilities in the models is through volatility surfaces, these are surfaces where there exist a volatility for every combination of expiry and exercise date for the underlying asset. Today many financial institutions use values from Bloomberg, there are however many advantages in being able to obtain them within the institution without relying on Bloomberg. In this paper we will cover how these volatility surfaces are obtained using mathematical methods. We have focused on Black's model and piece-wise linear interpolation in the process of obtaining the implied volatilities. We will show that using these standard methods we can come close to the volatilities provided by Bloomberg, but there is evidence of Bloomberg having a non standard undisclosed step in their calculations.

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Preface

This paper constitutes a thesis in mathematical statistics and is written at Swedish Export Credit Corporation. The extent of the thesis is 15 ECTS and leads to a Bachelors' degree in Mathematical Statistics at the Department of Mathematics, Stockholm University.

I would like to extend my thanks to Swedish Export Credit Corporation and especially my mentor Martin Arnér. I would also like to thank my mentor at Stockholm University, Thomas Höglund. Thank you both for the support and guidance I have received while working on this paper.

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1 Introduction

1.1 The Importance of Volatility

For all financial institutions it is important to be able to price the financial instruments or securities included in their portfolios. The current portfolio value will change over time and for many reasons it is important to always know the current value of your portfolios.

To obtain the value of the portfolios there are different theoretical pricing models that can be used to price the various instruments which comprise the portfolio. These exist in many configurations and with a varying complexity. Because the theoretical pricing models are based on stochastic processes the implied volatility is used as a measurement of the uncertainty about the value and how the value will change over time in these models. The implied volatility is usually taken from a volatility surface, a surface where there is an implied volatility for ever combination of exercise and expiry date. The implied volatility is being calculated using interest rate swaptions because these swaptions are based on interest, and not based on stock prices.

The interest rate volatility surface shows implied volatilities for different expercise and expiry dates. These volatilities are used when pricing financial instruments. Because these will change over time as time to maturity amongst other factors changes, it is important to have a model to calculate these implied volatilities. These values are often obtained from some sort of financial information system e.g. Bloomberg. Alternatively, they are computed by the financial institutions themselves.

1.2 What We Wish to Accomplish

The purpose of this paper is to explain the mathematics behind the implied volatilities and some of the theoretical pricing models they may be used in. In the end we wish to have a working model to obtain a volatility surface when inputing our own data. This model will later be implemented with Swedish Export Credit Corporations (SEK's) systems to be used in for example sensitivity analysis to investigate how the value of an entire portfolio may change with the volatilities.

We will also use the model we obtain to confirm the implied volatilities provided by Bloomberg. We wish to investigate if Bloomberg use non standard methods when calculating the volatilities.

2 Some Important Financial Instruments

In this section we will describe and explain some of the most important financial instruments. Understanding these are important to be able to understand and use the theoretical pricing models described in section 3.

2.1 Interest-Rate Caps and Floors

An interest rate cap is a financial derivative where the buyer receives a payment at the end of every period, in which the interest rate exceeds an pre-agreed level.

The interest rate cap can also be expressed as a series of European call options which exists for every period, in which the cap agreement is in existence. These call options are in this case called caplets.

An interest rate floor is the opposite case. The buyer of a floor contract receives payments at the end of every period, in which the interest rate is below an pre-agreed level.

Analogously to the case of a cap, the interest rate floor contract, can be expressed as a series of European put options. These put options are in this case called floorlets.

2.2 LIBOR

LIBOR stands for London Interbank Offered Rate and it is the interest rate at which banks borrow money from each other. The LIBOR is widely used as a reference point for short term interest rates. Short term usually refers to periods of overnight up to 1 year. New LIBOR rates are published by the British Banking Association each London business day at 11 a.m. London time. The fixings are calculated by gathering quotes from a number of participating banks. Since the banks who participate have high credit rating the LIBOR is regarded as close to being a risk free rate. Figure 1 show an example on how a LIBOR curve may look for different currencies.

The LIBOR is offered in ten major currencies: EUR, USD, GBP, CHF, JPY, CAD, DKK, AUD, NZD and SEK.

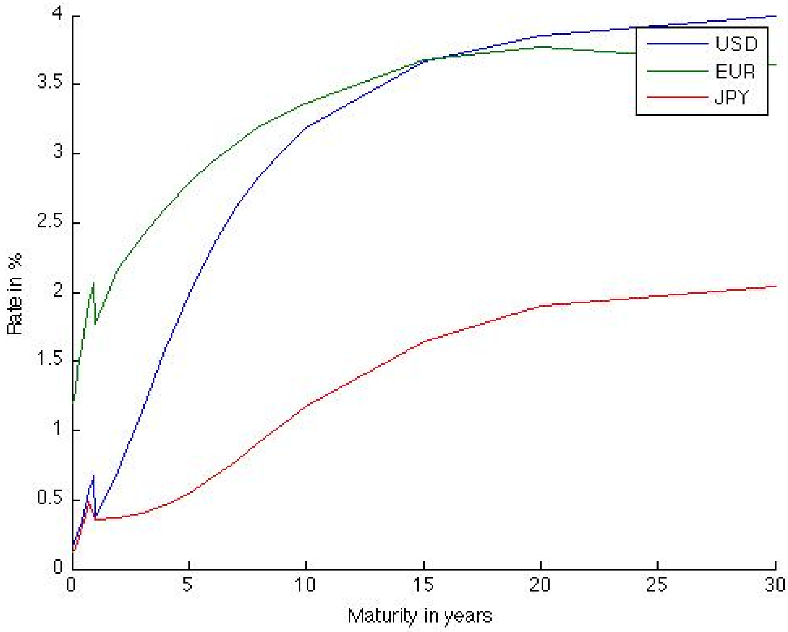

Figure 1: LIBOR curve

2.3 Exchange

An exchange is a marketplace where securities, derivatives and other financial instruments are traded. The main function of an exchange is to ensure that trades are done in a fair and orderly way. Also, an exchange provides price information on any securities traded on the exchange.

Exchanges are located all around the world and they give companies, governments and other groups a place to sell securities to the public in order to raise funds. Each exchange has requirements for anyone who wishes to list securities to be traded. Some exchanges have more stern requirements than others, but basic requirements usually include regular financial reports.

2.4 Over-the-Counter

Over-the-counter, or OTC, means a type of security that is traded outside of the stock exchange. It can refer to stocks traded by a dealer network and many other financial instruments such as derivatives, which is also traded by a dealer network.

These trades take place between dealers at different participants in the over-the-counter market. They are negotiated via telephone or computer. Participants are free to negotiate any deal which they mutually agree on. This means that the terms of a contract do not have to be those specified by an exchange. Financial institutions often act as market makers for the more commonly traded instruments. This means they are always ready to quote the price at which they are prepared to buy, the bid price, and also the price at which they are prepared to sell, the offer price.

2.5 Swaps and Interest Rate Swap

A swap is an agreement between two counterparties to exchange future cash flows. In the agreement the dates on which the cash flows are to be paid are defined, and it is also defined in which way they are to be calculated. A swap is a type of OTC transaction.

A future contract is a simple example on a swap. Consider e.g. one party might agree to that in three months they will pay \$12,000 and receive $500X$ where X is the market price of for example one ounce of silver on the same day.

However, a future contract only leads to an exchange on one date in the future whereas a swap usually leads to cash flow exchanges on several dates [6].

The value of a swap where floating is received and fixed is paid is given by

$$V_{swap} = B_{fl} - B_{fix}$$

where B_{fl} is the value of a floating-rate bond underlying the swap and B_{fix} is the value of a fixed-rate bond underlying the swap.

In an interest rate swap each counterparty agrees to pay either a fixed or a floating rate in an agreed currency to the other counterpart. The rate is calculated on a notional principal amount which is normally not exchanged between parties and only used to calculate the size of the cash flows.

The most common interest rate swap is one where a counterparty pays a fixed rate while receiving a floating rate, which is normally pegged to a reference rate such as the LIBOR. The market convention is that the payer of the fixed rate is called the "payer" and the receiver of the fixed rate is called the "receiver", even though each party pays and receives money.

Swaps can exist in many other kinds of configurations and can involve paying in one currency and receiving another, a so called currency swap. Consider e.g. an IRS where you receive the one year LIBOR once a year on a principal value of $10 million and pay a fixed rate of 3% per annum on the principal amount of 1,200 million yen.

Since a swap is a type of OTC transaction almost anything can be swapped [6].

2.6 Swaption

A swaption is an option which gives the buyer the right, but not the obligation, to enter an underlying swap. The term swaption typically refers to options on interest rate swaps.

Swaptions exist both as payer and receiver swaptions. A payer swaption gives the owner the right to enter into a swap where they pay the fixed leg and receive the floating leg. A receiver swaption on the other hand gives the owner the right to enter into a swap where they receive the fixed leg and pay the floating leg [5].

Consider e.g. a swaption which gives the holder the right to enter the swap on a future date $T, t < T < T_0$ at a given rate k. Here T_0 is the maturity date of the swap. Its value at T is given by

$$Swaption(T) = b(T) \max(k(T) - k, 0)$$

where

$$b(T) = \delta \sum_{i=1}^{n} P(T, T_i),$$

and δ is a parameter and $P(T, T_i)$ is a discount factor [9].

T is also called the exercise date, the date on which you can exercise the right to enter into a swap. The maturity date is also called the expiry date. Consider e.g. a swaption with exercise 20 years and expiry one month, this means that in 20 years we have the right, but not the obligation to enter into a swap that will mature one month after we enter it.

2.7 Yield Curve

Simply put the yield curve is the relation between the interest rate and the time to maturity of the debt for a borrower in a given currency. It can be described as investing for t years gives, a yield $Y(t)$.

The function Y is called the yield curve and is usually, but not always an increasing function of t. This function is only known with certainty for a few maturity dates but the other dates can be obtained by numerical techniques such as interpolation, or by bootstrapping.

3 Theoretical Pricing of Financial Instruments

Here we will explain why theoretical pricing exists and why it is important to financial institutions. We will also explain the most common theoretical pricing models and how they are used.

3.1 Theoretical Pricing

In the financial markets there exist a problem with pricing the instruments traded there. Because there are no universally set prices for the instruments it is hard to value your portfolio.

The solution to this problem is different models that will in theory value these instruments. These models have been shown not to reflect the market prices exactly, but in most cases it will give an approximation.

Since there are no model that is considered to be the only right one, financial institutions use different kinds of models. Different combination of interpolation and pricing can come closer to the market prices. But it is not only coming closer to market prices which is important, in many cases you may want to trade some exactness for time saving.

More complex pricing models with complex interpolation may require a lot more time when used to value a portfolio. Since it is important that these values are up to date and are usually updated every day it is necessary to take the computational time aspect into account.

For a model to be considered a good model for pricing it need to be both accurate and practical enough to be used on a daily basis. Therefore many financial institutions use less complex interpolation and pricing models. These models are usually considered to be a good enough approximate of the market price. A less complex model also helps with understanding how the model works.

However, a very important aspect of a pricing model is that it does not allow arbitrage. This will be covered more closely in the following section.

3.2 Arbitrage

Arbitrage is the way in which you can start with zero capital and at some later time can be sure not to have lost money and have a positive probability of having made money. As a formal definition one might use the following

Definition 1 *An arbitrage is a portfolio value process X(t) satisfying X(0)=0 and also satisfying for some time* $t > 0$

$$P\{X(T) \geq 0\} = 1, \quad P\{X(T) > 0\} > 0.$$

In other words, there exists an arbitrage if and only if there is a way to start with $X(0)$ and at a later time T have a portfolio value satisfying

$$P\{X(T) \geq \frac{X(0)}{D(T)}\} = 1, \quad P\{X(T) > \frac{X(0)}{D(T)}\} > 0.$$

where $D(T)$ is a discount process.

It is essential that any theoretical pricing model is free of arbitrage [8].

3.3 Geometric Brownian Motion

In mathematical finance the Brownian Motion process is fundamental to describe the evolution over time of a risky asset.

The process got it's name from R. Brown, a botanist. Brown used the process to describe the motion of a pollen particle suspended in fluid in the early 19th century. Brownian motion was not used to predict movements of stock prices until the early 20th century when L. Bachelier used Brownian motion as a model for stock price movements in his mathematical theory of speculation. The mathematical foundation for using Brownian motion as a stochastic process was done by N. Wiener a few decades later. Therefore, the process is sometimes called a Wiener process, and denoted $W(t)$.

A Brownian Motion $B(t)$ is a stochastic process with the following properties

1. Normal increments. $B(t) - B(s)$ has Normal distribution with mean 0 and variance $t - s$. This implies with $s = 0$ that $B(t) - B(0)$ has $N(0, t)$ distribution.

2. Independence of increments. $B(t) - B(s)$ is independent of the past, that is it does not depend on $B(u)$ where $0 < u < s$.

3. Continuity of paths. $B(t), t \geq 0$ are continuous functions of t.

A Geometric Brownian Motion or GBM is a stochastic process where the logarithm of the randomly varying quantity follows a Brownian Motion [9].

3.4 Black-Scholes

The Black-Scholes model is a mathematical model of a financial market containing certain derivatives. The model was developed by Fischer Black, Myron Scholes and Robert Merton [1].

Black-Scholes model is based on the following assumptions

1. The underlying asset price follows a GBM with μ and σ constant.
2. The short selling of securities with full use of proceeds is permitted.
3. There are no transactions cost or taxes. All securities are perfectly divisible.
4. There are no dividends during the life of the derivative.
5. There are no risk-less arbitrage opportunities.
6. Security trading is continuous.
7. The risk-free rate of interest, r, is constant and the same for all maturities. [6]

We will return to these assumptions shortly, let us first note that in Black-Scholes, the movement of the underlying asset's value is described by the following stochastic differential equation or SDE

$$dS(t) = \mu S(t)dt + \sigma S(t)dW(t) \tag{1}$$

where $S(t)$ is the value at time t and $W(t)$ is a Wiener process. This process describes the Black-Scholes model.

Using the assumptions above together with (1) it is possible with well known steps to obtain Black-Scholes partial differential equation (PDE). The solution to this PDE is well known and called the Black-Scholes formula.

In the Black-Scholes formula we define $T - t$ as the time to maturity, $d_{\pm}$ as the discount factor and $S(t)$ as the price of the asset at time t.

We also need

$$\bar{K} = e^{-r(T-t)}K, \quad l = \ln\left(\frac{S(t)}{\bar{K}}\right), \quad \bar{\sigma} = \sigma\sqrt{T-t} \;\; \textit{and} \;\; d_{\pm} = \frac{l}{\bar{\sigma}} \pm \frac{\bar{\sigma}}{2}.$$

Where $\bar{K}$ stands for the value of the strike price at time t and $\bar{\sigma}$ stands for the volatility under the remaining time to maturity.

We also need the density and distribution function of a standardized normal random variable and will define these as $\phi(x)$ and $\Phi(x)$ respectively. These functions are

$$\phi(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}} \text{ and } \Phi(x) = \int_{-\infty}^{\infty} \phi(y)\,\mathrm{d}y.$$

From this it is possible to get the Black-Scholes European type call option formula

$$C(t, S, K) = S(t)\Phi(d_{+}) - \bar{K}\Phi(d_{-}) \tag{2}$$

which is the solution to the Black-Scholes PDE. Here $C(t, S, K)$ is the value of the call at time t. So C is not a function only of type, but instead a function of t, S and K [4].

Using Black-Scholes pricing formula we assume that the value of the option follows a GBM. In Figure 2 we show a simulation of the value assuming that it follows a GBM, where $S(t)$ is the value of the asset at time t.

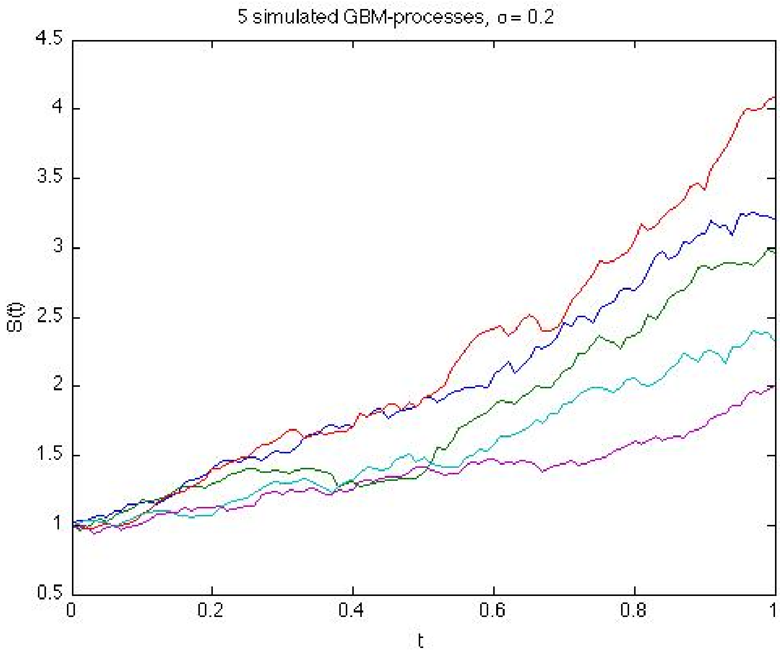

Figure 2: Value of asset assuming a geometric brownian motion

3.5 Black's Model

The Black-Scholes formula can be used to value call options on futures.

Assume $t < T < T_1$ and think of a derivative which at the time T both give the owner a new future contract in the asset with delivery at the time T_1 and an amount of the size

$$Y = max\left(0, S_{term}^{T_1}(T) - K\right)$$

where

$$S_{term}^{T_1}(t) = S(t)e^{r(T_1 - t)}. \tag{3}$$

You also need to remember that a future contract is worthless at the start and using this together with (3) we get

$$Y = e^{r(T_1-T)} \max\left(0, S(T) - Ke^{-r(T_1-T)}\right).$$

The contract at time $t < T$ then have the value

$$\begin{aligned}
& e^{r(T_1-T)} C\left(t, S(t), Ke^{-r(T_1-T)}; T\right) = \\
= \; & e^{r(T_1-T)} \left\{ S(t)\Phi\left(\frac{\ln\frac{S(t)}{Ke^{-r(T_1-T)}} + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}\right)\right. \\
- \; & \left. Ke^{-r(T_1-T)} e^{-r(T_1-T)} \Phi\left(\frac{\ln\frac{S(t)}{Ke^{-r(T_1-T)}} + \left(r - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}\right)\right\} \\
= \; & e^{-r(T-t)} \left\{ S_{term}^{T_1}(t)\Phi\left(\frac{\ln\frac{S_{term}^{T_1}(t)}{K} + \frac{\sigma^2}{2}(T-t)}{\sigma\sqrt{T-t}}\right) - K\Phi\left(\frac{\ln\frac{S_{term}^{T_1}}{K} - \frac{\sigma^2}{2}(T-t)}{\sigma\sqrt{T-t}}\right)\right\}
\end{aligned} \tag{4}$$

This is what is referred to as Black's formula [2].

As can be seen from the above, Black's formula comes from Black-Scholes model. It is mostly used to price bond options, interest rate caps / floors and swaptions.

Black's model is based on by the following process for the underlying asset

$$dF(t) = \sigma F(t) dW(t) \tag{5}$$

where $W(t)$ is a Wiener process and σ is the lognormal volatility [7].

The main difference between Black-Scholes model and Black's model is the fact that in Black-Scholes model we assume that the value of the underlying asset follows a GBM whereas in the case of Black's we only assume that the value at time T follows a lognormal distribution.

In Figure 3 we show a simulation of the value assuming that it follows a lognormal distribution. As can be seen in the figure, the difference between the lognormal case of Black's compared to the GBM case in Black-Scholes is that the lognormal case is drift-less, whereas the GBM case has drift.

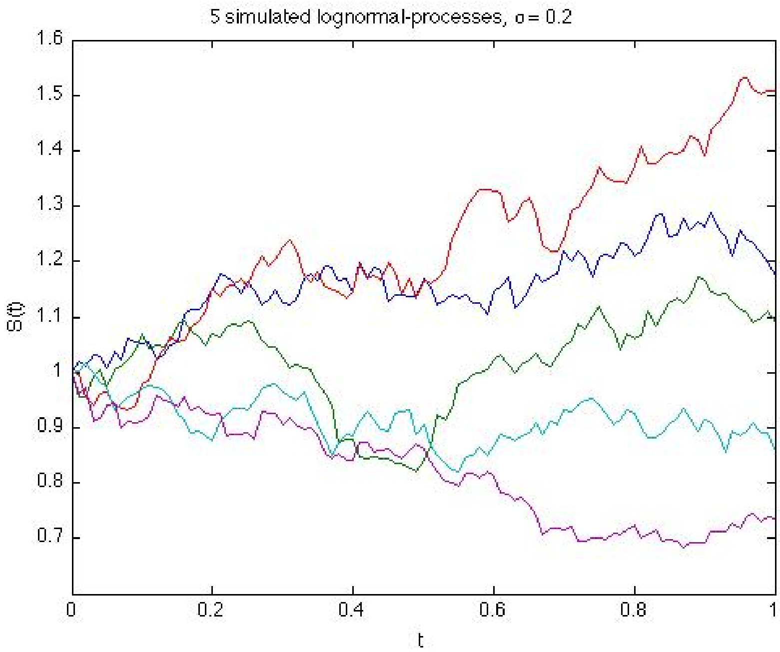

Figure 3: Value of asset assuming a lognormal distribution.

Another difference between these models is that in Black's formula we can replace the spot price with the forward price at time zero.

For a European call option with the underlying strike K, expiring T years in the future, (4) can be written as

$$C_0 = e^{-rT}[F_0\Phi(d_1) - K\Phi(d_2)] \tag{6}$$

where C_0 is the value of the call at time zero and

$$d_1 = \frac{\ln(F_0/K) + (\sigma^2/2)T}{\sigma\sqrt{T}} \text{ and } d_2 = \frac{\ln(F_0/K) - (\sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}.$$

3.6 At-the-Money

An at-the-money option is when the strike price equals the price of the underlying asset.

4 Numerical Methods

In this section we will describe some numerical methods used when calculating implied volatilities and volatility surfaces. We will also cover a few methods of interpolation.

4.1 Piece-Wise Linear Method

In some cases you might not want to calibrate your model against the prices on your swaptions which you already have. These prices is not always considered to be the best match the market prices. It might then be required to interpolate your implied volatilities, a so called smile interpolation. Another reason for interpolating might be because there might not exist prices for the strikes and maturities you are looking for.

Something that must be considered when interpolating is that some interpolation methods may give rise to arbitrage in the interpolated volatilities even if there is none in the original data. Arbitrage is when you make a profit without taking any risk at all, which we covered more closely in section 3.2.

The piece-wise linear method is the simplest method of smile interpolation. It is also the only one that fits the original data exactly. The formula to compute these interpolated values for a given $x \in [x_i, x_{i+1}]$, given vales y_i and y_{i+1} at x_i and x_{i+1} respectively is

$$y = \frac{y_i(x_{i+1} - x) + y_{i+1}(x - x_i)}{x_{i+1} - x_i}$$

where in our case x's are strikes and y's are prices [3].

4.2 Stochastic Interpolation

There are two main methods of stochastic interpolation, the CEV and SABR method. Something to remember is that many of the stochastic interpolation methods are given by stochastic differential equations and do not have an exact solution.

The constant elasticity of variance or CEV method is a stochastic interpolation method that uses the CEV model to model Libor forward rates. The method is fitted using non linear least squares. The model is given by the SDE

$$dF(t) = \sigma F(t)^{\beta} dW(t)$$

where $0 < \beta < 1$, $F(t)$ is the forward rate and $W(t)$ is a Wiener process. When $\beta = 1$ we get (5), the same process as in Black's model. So Black's model and the CEV model are closely related in a sense. The CEV model is said to be an improvement of Black's model[3].

The SABR method is a stochastic interpolation method that uses the SABR model to model LIBOR forward rates. In the SABR model we assume that volatility parameter itself follows a stochastic process.

This model is an extension of the CEV model in which the volatility parameter σ is assumed to follow a stochastic process. The dynamics of the model is given by

$$\begin{aligned} dF(t) &= \sigma(t) F(t)^{\beta} dW(t) \\ d\sigma(t) &= \alpha \sigma(t) dZ(t) \end{aligned}$$

where $F(t)$ is the forward rate process, α is a governing parameter to be estimated, this parameter is not stable over time and $W(t)$ and $Z(t)$ are Wiener processes with

$$E[dW(t)dZ(t)] = \rho dt$$

which is a well known result from stochastic calculus (see e.g. [8] or [9]), where the correlation ρ is assumed to be constant [3].

As with many other stochastic volatility models the SABR model do not have an explicit solution for most values. The SABR model only have an explicit solution for the special case where $\beta = 0$. For other values of β it can only be solved approximately [7].

4.3 Implied Volatility, Volatility Smile and Volatility Surface

In section 3.5 we listed a few assumptions used to derive the Black-Scholes differential equation. In real life the assumption that the volatility σ is constant does not hold. The volatility changes over time, it is needed to reflect this change in any theoretical pricing model.

This is where the implied volatilities becomes interesting. We can as an example solve the Black-Scholes European type call option formula for the volatility when the value of the call $C(t, S, K)$ is given and the only unknown variable is the volatility. Then we obtain the so called implied volatility, it is the volatility that together with the given input data would result in the given value of the call. In other words, it is implied by the model. The same logic work for puts or other assets. These volatilities can later be used in the theoretical pricing model.

Although Black's model is used by financial institutions it is not used as the model was first intended as mentioned above. The difference is that in practice, users allow the volatility to depend on the strike price K and time to maturity T, so we get $\sigma(K, T)$. As noted before the implied volatility is not constant as assumed in the theoretical model.

If you plot the implied volatility as a function of the strike price you create what is called a volatility smile. It's name comes from the curving shape. It is in this step that you might use one of the previously described interpolation methods. You also use your interpolation between the volatility smile curves to obtain a volatility surface.

5 Results

5.1 Obtaining Implied Volatilities

To obtain the implied volatilities we can use Black's formula. Because all parameters except the volatility in Black's formula is known to us it is easy to use this formula to solve for the volatility. Each combination of expiry and exercise date have a certain strike price and value of the call linked to them that we need to solve for. Remember the notation explained in section 2.6, where the exercise date is the date on which we have the right, but not the obligation to enter a swap that will expire or mature or the expiry date. If the expiry for example is said to be 1 year we will mean that the underlying swap matures one year after it is entered. This means it matures one year after the exercise date.

We can start by reducing Black's formula into something easier to work with. As we are only interested in the case of at-the-money, volatilities we can assume that $F_0 = K$. Solving for other cases than at-the-money is not covered in this thesis but the theory is the same. This would make it possible to reduce d_1 and d_2 to

$$d_1 = \frac{(\sigma^2/2)T}{\sigma\sqrt{T}} = \frac{\sigma}{2}\sqrt{T} \text{ and } d_2 = d_1 - \sigma\sqrt{T} = -\frac{\sigma}{2}\sqrt{T} = -d_1.$$

Knowing this and that $F_0 = K$ makes it possible to reduce (6) in these steps

$$\begin{aligned} C_0 &= e^{-rT}\left[F_0\Phi(d_1) - K\Phi(d_2)\right] \\ &= e^{-rT}\left[K\Phi\left(\frac{\sigma}{2}\sqrt{T}\right) - K\Phi\left(-\frac{\sigma}{2}\sqrt{T}\right)\right] \\ &= e^{-rT}K\left[\Phi\left(\frac{\sigma}{2}\sqrt{T}\right) - \left(1 - \Phi\left(\frac{\sigma}{2}\sqrt{T}\right)\right)\right] \\ &= e^{-rT}K\left[2\Phi\left(\frac{\sigma}{2}\sqrt{T}\right) - 1\right] \qquad (7) \end{aligned}$$

From (7) we can solve for $\Phi\left(\frac{\sigma}{2}\sqrt{T}\right)$ and we get

$$\Phi\left(\frac{\sigma}{2}\sqrt{T}\right) = \frac{C_0}{2e^{-rT}K} + \frac{1}{2}.$$

This means that the implied volatility can be written as

$$\sigma = \frac{2\Phi^{-1}\left(\frac{C_0}{2e^{-rT}K} + \frac{1}{2}\right)}{\sqrt{T}}. \tag{8}$$

Where Φ^{-1} is the inverse of the standardized normal distribution function.

This is the mean volatility per year until maturity, for this application this is not the interesting value. We are interested in knowing the total volatility until maturity. This will be obtained by multiplying (8) with $\sqrt{T}$. This leaves us with

$$\sigma = 2\Phi^{-1}\left(\frac{C_0}{2e^{-rT}K} + \frac{1}{2}\right). \tag{9}$$

We can use one of several numerical software e.g. Matlab to receive a numerical value. For this paer we have used Matlab and solved (9) for three major currencies, JPY, USD and AUD. This gives us a matrix with one implied volatility for every combination of exercise and expiry date[1].

5.2 Using Interpolation

Now that we have obtained all of our implied volatilities we can start building our volatility smiles and then volatility surfaces. To obtain the volatility smiles we need to use smile interpolation.

As mentioned before the more complex interpolation methods require more time when used and the complexity makes it hard to follow the actual calculations.

Because of this we have chosen to use the piece-wise linear interpolation method for our smile interpolation. Even though the stochastic interpolation methods would provide a few advantages, many financial institutions use the piece-wise linear interpolation method due to the simplicity of it. So using this method is not something that is considered unusual for financial

[1] Data collected from ICAP and Bloomberg.

institutions to do.

So we use this method to obtain our volatility smiles for all different exercise dates. From this we get many volatility smile curves. In Figure 4 we have shown one of the smiles.

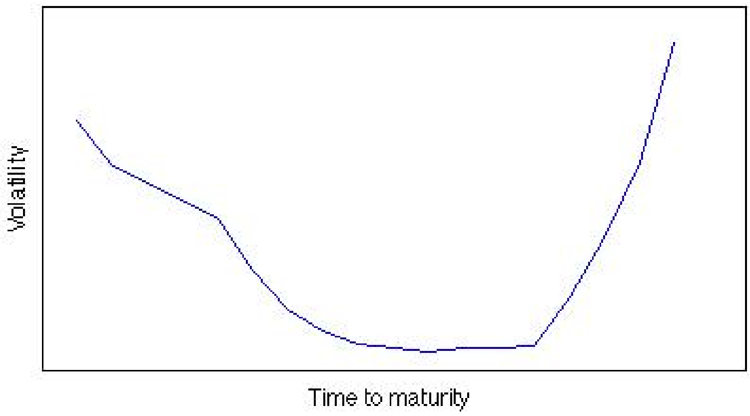

Figure 4: Volatility smile for one exercise date of our volatilities for USD.

5.3 Volatility Surface

So now that we have our volatility smiles, then we again can use piece-wise linear interpolation between the curves to bind them together into a surface. In this surface it is possible to obtain the implied volatility for any combination of expiry and exercise date. This means we no longer have values for only the data points we started with, due to the interpolation we can obtain the volatility at any level within the surface. In Figure 5 we have shown the volatility surface we obtained when using data for USD.

The values we have obtained does not exactly match those of Bloomberg, but we had anticipated that. To obtain the exact same figures all of the input data needs to be exactly the same. Also, Bloomberg does not disclose their calculation methods, so it is possible that they use non standard methods in their calculations. As there exist many different conventions for obtaining interest rates, strike prices, swap prices and such, there will always be a difference in the volatilities we obtain to those which Bloomberg publishes.

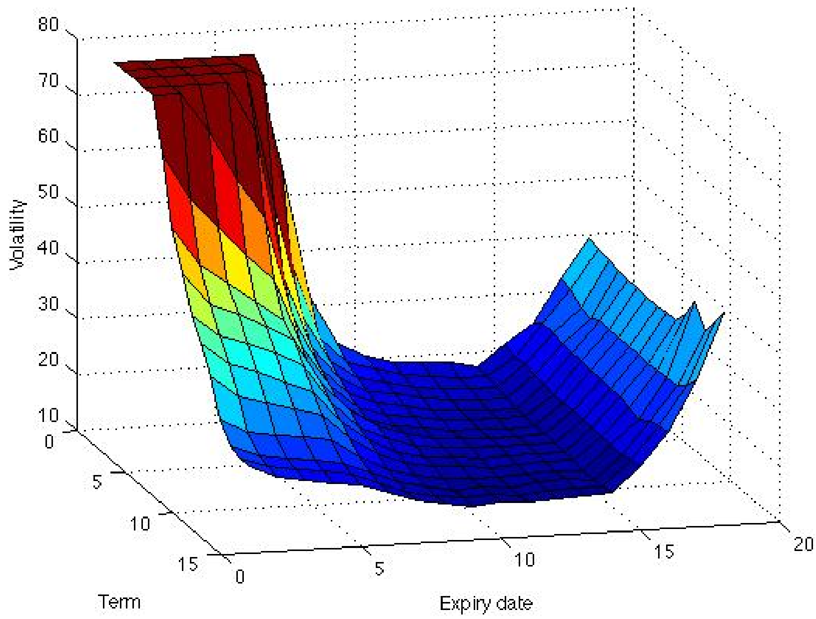

Figure 5: Volatility surface for our volatilities for USD.

At this time the system for pricing portfolios at AB SEK is not set up in such a way that you can easily plug in your own implied volatilities, this have prevented us from using the implied volatilities obtained from our model when AB SEK values their portfolios. The model and code built for this paper will remain at AB SEK to later be used in such a valuation, when it has been integrated with the systems.

To illustrate how much and where our implied volatilities differs from the ones Bloomberg provides we can take the difference between the volatilities for each combination of exercise and expiry date. After this is done we can do the same interpolation as before to get a surface showing the difference. In Figure 6 we show such a surface. We can note the differences are low for most volatilities, but they get bigger as we get closer to the extreme points, which are the points where both the exercise and expiry date are close to time zero.

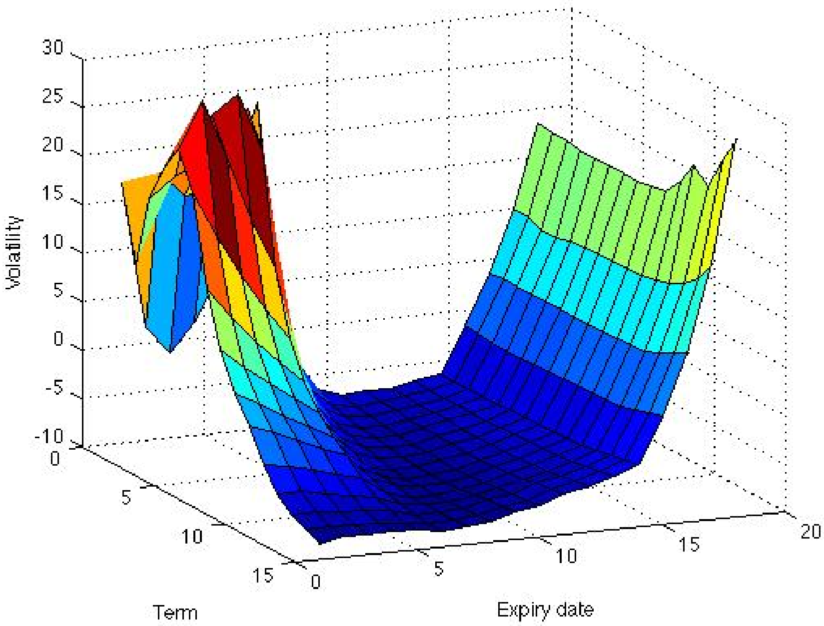

Figure 6: Surface showing the difference between our volatilities and Bloomberg's.

6 Discussion

6.1 Interpreting the Implied Volatilities

Even though the implied volatilities we obtained does not exactly match those from Bloomberg the pattern is the same. The differences between them are at acceptable levels given that we do not know the precise way in which Bloomberg does their calculations. At the extreme expiry points our implied volatilities become very large, this is supported by the math. This does not occur in the volatilities from Bloomberg though.

As we showed in Figure 6 the differences are bigger close to the extreme points, the points when the time to expiry is close to time zero. At these points you would anticipate the volatilities to be higher. Bloomberg's volatilities does not raise much near the extreme points. We can note that our volatility surface has a more bent shape.

We interpret this as Bloomberg using a more complex non-standard model to obtain the implied volatilities. It is possible that they have calibrated their model with help from empirical data to make the surface smooth even at the extreme points. This is nothing which can be fit into the extent of this paper, but it would be something very interesting for later studies.

6.2 Areas for Further Studies

Something worth noting is that when the exercise and expiry comes close to zero the implied volatility grows larger, as shown in Figure 5 we can see them being constant in the start and later drop drastically. If this poses a problem when using the volatility surface in a theoretical pricing model it is possible to remove the extreme values and extrapolate from the rest of the surface. Taking these steps would require to use large quantities of historical market data. This is nothing that is required for us to do in this paper, but it is an option.

For further research it would be interesting to plug these implied volatilities into a pricing model for entire portfolios and this is something that could be the focus in a valuation sensitivity analysis. If this is done it would be possible to see how much the entire portfolio value would shift when using the implied volatilities we have obtained here instead of those provided by Bloomberg. This is something that in the future will be performed at Swedish Export Credit Corporation using the model built for this paper.

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