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Abstract

This thesis describes an Italian internal valuation model consistent with the future Solvency II directive. Applied to an example policy it is used to calculate the mark-to-market reserve, the value of business in force, the cost of the embedded minimum guarantee and the required risk capital for the contract. Moreover, the new solvency capital requirement is calculated and compared to the current solvency margin imposed under the Solvency I directive. The model is proved to be consistent with the future Solvency II regulations as it provides a market-based valuation of assets and liabilities that accurately takes into account the different risk factors associated with a contract. Is is also able to price the minimum guarantees embedded in participating policies; something that is of great importance in Italy, where virtually all insurance policies are of this type.

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Contents

1 Introduction

The new Solvency II regulation that will substitute the current solvency directive for insurance companies is expected to be fully implemented at the end of 2012, but many companies have already started to adopt the new methods proposed and are developing internal models for the market-based valuation of their assets and liabilities. In Italy, professors Massimo De Felice at Università di Roma "La Sapienza" and Franco Moriconi at Università di Perugia have developed a system that is currently being used by several large Italian insurance companies such as Fondiaria-SAI Group, Gruppo Reale Mutua Assicurazioni, RAS Group and Alleanza Assicurazioni. This method, which was first applied in the insurance business and taught in actuarial courses in the early 1990's and thus by now is well-established in Italy, will be at the center of this thesis. Applied to an example policy it will be used to calculate the mark-to-market reserve, the value of business in force, the cost of the embedded minimum guarantee and the required risk capital for the contract. Moreover, the new solvency capital requirement will be calculated and compared to the current solvency margin imposed under the Solvency I directive.

The outline of this thesis is as follows: First some background information will be presented regarding participating policies, which are very common in Italy and also will have a central role in the following calculations, and the Solvency II regulation, which constitutes the foundation in the development of internal models. Next an example policy will be introduced demonstrating the participation mechanism and the minimum guarantee. It will be used to illustrate the De Felice-Moriconi (DFM) model in the chapter that follows. The stochastic processes underlying the valuation model will be described, leading up to a valuation equation and its solution through parameter calibration and Monte Carlo simulation. The procedure for pricing the options embedded in the contract will then be derived and subsequently definitions of value of business in force and risk capital will be made. The latter will be used in the calculation of the solvency capital requirement. Finally, having laid the theoretical foundation the DFM model will be applied to real data provided by an Italian mutual insurance company.

2 Background

2.1 Participating policies

Participating life policies were introduced on the Italian market in the early 1980's as a way to protect the insured benefits from inflation. Today almost every contract offered by Italian life insurance companies is of this kind. The participating policies provide an annual revaluation of the benefits, and sometimes also of the premia, based on the return of a fund that the insurer manages separately from the rest of their activities and in which the policy reserves are invested. For this reason they are called *separate funds*[1]. The Italian companies' separate funds are mainly composed of bonds with low credit risk[2].

The revaluation mechanism is specific of the policy type. Conceptually it is analogous to an indexation[3] but with a *minimum guarantee*. It is based on the idea of letting the insured in on a part of the financial profit that arises if the fund return is higher than the technical rate. The minimum guarantee ensures that the policy holder gets at least a specified minimum rate even if the fund return should drop below this level. From a financial point of view this type of policy is a derivative contract, with the separate fund as the underlying asset.

Before the introduction of participating policies many Italian insurance companies, particularly the mutual ones, still offered their customers forms of profit participation but the conditions were often discretionary of the insurer. In today's participating policies the profit participation is contractualized in a precise and binding "revaluation rule". Through this rule the amount of the benefits is linked to the capital market and therefore any valutaion method used for pricing the policy has to be consistent with the valuation method used in capital markets.

2.2 Solvency II

The *Solvency II* framework is a review of the current insurance directives (*Solvency I*). One of its aims is to update the approach to determining capital requirements in an insurance company, i.e. the amount of capital needed to be held against unforseen losses. Another aim is to enhance supervisory control and transparency within the companies. This will lead to both higher protection for the insurance holders and an increased competition between EU companies. Solvency II introduces a common European approach to asset-liability management that is based on economic principles instead of on the simple factor-based models developed in the early 1970's under Solvency I. It is a risk-based approach, which means that risk is measured on consistent principles and the capital is allocated accurately to where the risks are.

[1]Throughout the thesis the term "reference fund", and also simply "the fund", will often be used to indicate the separate fund. The three terms are considered equivalent.

[2]A bond's credit risk is the risk that the face value and/or interest will not be repaid by the issuer.

[3]Generally, an indexation is a periodic adjustment of the value of some regular scheduled payment based on the movement of a price index.

2.2.1 Capital requirements

The traditional solvency margin, i.e. the capital requirement under Solvency I, is based on prudential technical assumptions and does not account for market risk. Since it is also more or less arbitrary the level of required capital often differs between countries.

Solvency II has two thresholds:

- the *solvency capital requirement* (SCR), which is the target level of capital enabling the insurer to meet their obligations while taking into consideration adverse scenarios connected to various sources of risk (e.g. underwriting risk, market risk and operational risk). If an insurer is not able to meet the SCR, supervisory action will be triggered and the company will be required to restore the lacking capital;

- the *minimum capital requirement* (MCR), which is the lowest level of capital required. A breach of this level will trigger severe supervisory actions, including closure of the company to new business.

Since the SCR is calculated under a risk-based approach the amount of capital needed will be proportional to the risks involved, which leads to a more efficient use of finances. It prevents the companies from having to hold too much capital, which would impede them in making investments and increase the cost of insurance for their customers. In the same way, a risk-based SCR also prevents the companies from holding to little capital, which would lead to a higher risk of failure. Solvency II provides a standard model for calculating the SCR, but companies can also use their own internal models or a combination of both. Since the standard model cannot reflect company-specific characteristics like the focusing on particular business niches or strategies or the use of reinsurance programs with different features, the development of internal models is strongly encouraged. However, these models have to be consistent with the Solvency II directive[4].

2.2.2 Mark-to-market valuation vs. traditional valuation

Under Solvency II the valuation of assets and liabilities should be made using market data. For many assets and some liabilities market data is readily available, but when this is not the case the valuation has to be performed with market-consistent techniques. One such technique is the use of stochastic processes to model the courses of the various components on the

[4]Before an internal model can be used it needs to be approved by Solvency II regulators. The insurer will have to show that the model is fully embedded in their business, that it is calibrated according to Solvency II definitions and that it is based on adequate actuarial and statistical techniques. They will also have to provide a detailed and up-to-date documentation of the model.

market (interest rates, stock market indices etc.). The process of revaluating a security to reflect its current market value instead of its aquisition price is called *marking to market*. Mark-to-market valuation is also often referred to as *fair valuation*. In [4] fair value is defined as "the market value, if a sufficiently active market exists, OR an estimated market value, otherwise". With the traditional valuation approach the reserve is statutory, and this means that even if the policy is participating it will be valued as if it weren't. As a consequence there is no way of pricing the minimum guarantee option embedded in the contract, nor the excess-return, and the financial risks associated with the contract cannot be accounted for. As with the SCR, the mark-to-market valuation can be done using either the Solvency II standard approach or an approved internal model.

3 An example contract

To illustrate the DFM valuation model we use the case of an endowment insurance contract with annual premium payments. If the insured is alive at maturity time T they will receive the benefit C_T. If they should die at time $t < T$ the benefit C_t will be payed out. The policy is a participating one, which means that the initial sum insured C_0 increases every year by a fraction β of the return I_t earned by the fund in which the premium is invested. The policy also has a minimum guarantee $i^{\min}$, which means that even if $I_t < i$, where i is the technical rate and $i^{\min} \geq i$, the sum insured will not decrease. The benefit C_t at the end of year t, for $t = 1, 2, \ldots, T$, is thus readjusted as follows:

$$C_t = C_{t-1}(1 + \rho_t), \tag{1}$$

where ρ_t is the readjustment rate, defined as:

$$\rho_t = \frac{\max\{\beta I_t, i^{\min}\} - i}{1 + i}.$$

The rate of return I_t is defined as:

$$I_t = \frac{F_t}{F_{t-1}} - 1, \tag{2}$$

where F_t is the market value of the fund at time t. β and i are specified by the contract and are fixed at time 0. For integers t and n such that $0 \leq t \leq n \leq T$ expression (1) can be written as:

$$C_n = C_t \Phi(t, n), \tag{3}$$

where

$$\Phi(t, n) = \prod_{k=t+1}^{n} (1 + \rho_k)$$

is the readjustment factor and $\Phi(t, t) = 1$. In this example policy the annual premium A_n is revalued in the same way as the benefit C_n, and for simplicity's sake we use the same readjustment factor. Analogous to (3) we can therefore write the year n premium as:

$$A_n = A_t \Phi(t, n).$$

Now let Y_n denote the liability of the insurer at time n, $n = t+1, t+2, \ldots, T$. This amount is defined through the following "probability stream":

$$Y_n = \begin{cases} C_n & \text{with probability } \mathbf{P}_t(C_n; n) \\ 0 & \text{with probability } 1 - \mathbf{P}_t(C_n; n) \end{cases}$$

The probability measure $\mathbf{P}$ is contractually specified and identified through given mortality tables. The expression $\mathbf{P}_t(C_n; n)$ is to be interpreted as the probability at time t that the amount C_n will be payed at time n (for $t < n$). For the time n premium X_n we analogously get:

$$X_n = \begin{cases} A_n & \text{with probability } \mathbf{P}_t(A_n; n) \\ 0 & \text{with probability } 1 - \mathbf{P}_t(A_n; n) \end{cases}$$

We see that Y_n and X_n are affected by both financial and technical (actuarial) uncertainty. The financial uncertainty has to do with the course of the market in which the reserves are invested and is thus linked to the readjustment factor Φ. The technical uncertainty depends on the life of the insured and is therefore relative to the probability measure $\mathbf{P}$. This means that both (C_n, A_n) and (Y_n, X_n) are random variables. In this valuation model it is assumed that the two uncertainties are independent, which lets us compute them separately.

Here we introduce the valuation functional $V(t; Z)$ that assigns a value at time t to the random variable Z. We can now write:

$$V(t; Y_n) = V(t; C_n)\mathbf{P}_t(C_n; n) = C_t V(t; \Phi(t, n))\mathbf{P}_t(C_n; n).$$

Analogously for the premia we have:

$$V(t; X_n) = V(t; A_n)\mathbf{P}_t(A_n; n) = A_t V(t; \Phi(t, n))\mathbf{P}_t(A_n; n).$$

If this were a non-participating policy we would have $V(t; \Phi(t, n)) = V(t; 1) = (1+i(t, n))^{-(n-t)}$ and the financial uncertainity would disappear: the valuation functional would be the market discount factor $v(t, n) = (1+i(t, n))^{-(n-t)}$, being $i(t, n)$ the market rate in t with maturity n. Unfortunately things get more complicated when dealing with participating policies. Premia and benefits are exposed to not only technical but also financial uncertainity and the valuation functional is expressed by the *stocastic valuation factor*

$$u(t, n) = V(t; \Phi(t, n)).$$

We define the expected values of the benefits and the premia, respectively $\bar{Y}_t(n)$ and $\bar{X}_t(n)$, as:

$$\bar{Y}_t(n) = C_t \mathbf{P}_t(C_n; n) \tag{4}$$

$$\bar{X}_t(n) = A_t \mathbf{P}_t(A_n; n) \tag{5}$$

for the streams of benefits $\mathbf{Y} = \{Y_n; n = t+1, t+2, \ldots, T\}$ and premia $\mathbf{X} = \{X_n; n = t+1, t+2, \ldots, T\}$. The *traditional reserve* R_t is defined as:

$$R_t = \sum_{n=t+1}^{T} \bar{Y}_t(n)(1+i)^{-(n-t)} - \sum_{n=t+1}^{T} \bar{X}_t(n)(1+i)^{-(n-t)}, \tag{6}$$

where $v(t) = (1+i)^{-(n-t)}$ is the *contractual discount factor* calculated using the technical interest rate i. The *stochastic reserve* V_t is defined as:

$$V_t = V(t, \mathbf{Y}) - V(t, \mathbf{X}) = \sum_{n=t+1}^{T} \bar{Y}_t(n) u(t,n) - \sum_{n=t+1}^{T} \bar{X}_t(n) u(t,n). \tag{7}$$

The factor $u(t,n)$ can be interpreted as the time t price of an indexed zero-coupon bond (ZCB) with maturity n and face value 1. The benefits are then valued as a portfolio consisting of T stochastic ZCB with maturity $n = t+1, t+2, \ldots, T$ and face value $\bar{Y}_t(n)$. The discussion is of course the same for the premia.

By calculating $u(t,n)$ the functional $V(t; Y_n)$ gives us a marked-based (fair) valuation of the outstanding liabilities generated by the policy, and to do this we need to use a stochastic pricing model. In the following section the DFM approach to calculating the stochastic reserve V_t will be presented, using the assumption of independence between the financial and the technical uncertainties to divide expression (7) into two parts that are computed separately. Since the financial part involving the factor $u(t,n)$ is the most complicated it will be discussed more thoroughly. The calculation of the technical part is more straightforward and will be dealt with later in section 4.6.

4 Specification of the valuation model

The following discussions will be based on the assumption of a *perfect market*, i.e. a "theoretical free-market situation where

1. buyers and sellers are too numerous and too small to have any degree of individual control over prices,

2. all buyers and sellers seek to maximize their profit (income),

3. buyers and sellers can freely enter or leave the market,

4. all buyers and sellers have access to information regarding availability, prices, and quality of goods being traded, and

5. all goods of a particular nature are homogeneous, hence substitutable for one another." (cited from [3])

Typically the reference funds contain both bonds and stock (at least)[5]. To be able to price a contract we therefore need to determine the characteristics of the stochastic process F_t representing the market value of the reference fund. We thus assume:

$$F_t = \alpha S_t + (1-\alpha) W_t, \qquad 0 \leq \alpha \leq 1,$$

where S_t is a stock index, W_t is a bond index and α is a fixed constant. Since the yearly return I_t is linked to F_t through equation (2) it too will depend on these variables. I_t is thus affected by interest rate risk through the spot rate r_t and stock market risk through the stock index S_t[6]. In the DFM approach the interest rate risk and the stock market risk are modeled using the Cox-Ingersoll-Ross (CIR) model and the Black-Scholes (BS) model respectively. The two sources of uncertainty r_t and S_t are correlated, and by combining the CIR and the BS models the complete valuation model is obtained.

4.1 Interest rate uncertainty

The spot rate r_t can be seen as a diffusion process described by the stochastic differential equation

$$dr_t = f^r(r_t, t)dt + g^r(r_t, t)dZ_t^r,$$

where Z_t^r is a standard Brownian motion, f^r is the drift function and g^r is the diffusion function. In the CIR model these last two functions are of the forms:

$$f^r(r_t, t) = \alpha(\gamma - r_t), \qquad \alpha, \gamma > 0$$
$$g^r(r_t, t) = \rho\sqrt{r_t}, \qquad \rho > 0$$

This model is characterized by the dynamics of "return", or *revertion* to the mean. In the long run the rate will become constant, settling on the level γ. The parameter α is the speed of adjustment to this level (also called mean-reversion coefficient). With ρ being the volatility parameter, the CIR model is a mean-reverting square-root process having a non-centered chi-square transition density. It is mathematically more complicated to deal with than for example the Vasicek model described in [8], pp. 318–320. Even so it is

[5] Other components could be government bonds, real estate etc.

[6] If there are more sources of risk the model has to be extended; for example, if a policy also provides inflation protection a factor for real interest risk must be added.

considered more suitable for describing r_t because it doesn't allow negative values of the spot rate which for long maturities could produce discount factors greater than 1.

To prevent arbitrage, the market price of interest risk in the CIR model is described by the function:

$$h^r(r_t, t) = \pi \frac{\sqrt{r_t}}{\rho},$$

where $\pi \in \mathbb{R}$ is a constant of arbitrary sign (see also [8] p. 301).

4.2 Stock price uncertainty

The diffusion process for the stock index S_t is given by:

$$dS_t = f^S(S_t, t)dt + g^S(S_t, t)dZ_t^S,$$

where Z_t^S is a standard Brownian motion. In the BS model the functions f^S and g^S have the forms:

$$f^S(S_t, t) = \mu S_t, \qquad \mu \in \mathbb{R}$$
$$g^S(S_t, t) = \sigma S_t, \qquad \sigma > 0$$

The process thus describes a geometrical Brownian motion with instantaneous expected return μ and volatility σ, which implies that S_t has a log-normal transition density.

The market price of stock market risk in the BS model is described by the function:

$$h^S(S_t, t) = \frac{\mu - r_t}{\sigma}.$$

4.3 The valuation equation

Since the price at time t of our indexed ZCB is a stochastic diffusion process it is by the Markovian property not only a function of time but also of the state variables r_t and S_t:

$$u(t, T) = u(r_t, S_t, t; T), \qquad 0 \le t \le T.$$

It can be described by the following stochastic differential equation:

$$du = a(r, S, t)dt + b^r(r, S, t)dZ^r + b^S(r, S, t)dZ^S.$$

The coefficients a, b^r and b^S are specified using Itô's lemma[7] (in two variables):

$$a(r, S, t) = \frac{1}{2}(g^r)^2 \frac{\partial^2 u}{\partial r^2} + \frac{1}{2}(g^S)^2 \frac{\partial^2 u}{\partial S^2} + \eta g^r g^S \frac{\partial^2 u}{\partial r \partial S} + f^r \frac{\partial u}{\partial r} + f^S \frac{\partial u}{\partial S} + \frac{\partial u}{\partial t} \quad (8)$$

[7] Itô's lemma is described in appendix A.

$$b^r(r, S, t) = g^r \frac{\partial u}{\partial r} \tag{9}$$

$$b^S(r, S, t) = g^S \frac{\partial u}{\partial S} \tag{10}$$

Here η is the instantaneous correlation coefficient between the two sources of risk:

$$\mathrm{Cov}_t(dZ_t^r, dZ_t^S) = \eta dt, \qquad \eta \in \mathbb{R},$$

where $\mathrm{Cov}_t(\cdot)$ is the covariance conditional of the information available in t. In coherence with the arbitrage pricing theory[8], u can be described as satisfying the relation:

$$a - ru = h^r b^r + h^S b^S. \tag{11}$$

By substituting the functions a, b^r and b^S in (11) with the expressions (8), (9) and (10) respectively, we get:

$$\begin{aligned} &\frac{1}{2}(g^r)^2 \frac{\partial^2 u}{\partial r^2} + \frac{1}{2}(g^S)^2 \frac{\partial^2 u}{\partial S^2} + \eta g^r g^S \frac{\partial^2 u}{\partial r \partial S} \\ &\quad + (f^r - h^r g^r) \frac{\partial u}{\partial r} + (f^S - h^S g^S) \frac{\partial u}{\partial S} + \frac{\partial u}{\partial t} = ru \end{aligned}$$

This is the *valuation equation*, and it has the terminal condition

$$u(T, T) = \Phi(0, T). \tag{12}$$

It follows from the fundamental theorem of asset pricing[9] that if a risk-neutral measure Q is constructed then the prices of all derivatives can be computed using discounted expectations under this measure, ruling out arbitrage. This means that the discounted price process

$$u(t, T) e^{-\int_0^t r_\tau d\tau}, \qquad 0 \le t \le T,$$

is a martingale with respect to Q, and thus that:

$$V(t; \Phi(t, T)) = u(t, T) = \mathrm{E}_t^Q[e^{-\int_t^T r_\tau d\tau} \Phi(t, T)], \tag{13}$$

where $\mathrm{E}_t^Q[\cdot]$ is the risk-neutral expectation conditional of the information available in t. This is the solution to the valuation equation under condition (12).

Under the natural probability measure P the vector containing the unknown

[8] According to the arbitrage pricing theory the expected return of a financial asset can be modeled as a linear function of various factors, where sensitivity to changes in each factor is represented by a factor-specific coefficient.

[9] The theorem states in short that if there exists an equivalent martingale measure (i.e. a risk-neutral probability measure) then there is no arbitrage.

parameters is $\mathbf{p} = \{\alpha, \gamma, \rho, \mu, \sigma, \eta, \pi\}$. But if we look at the valuation equation we see that the coefficients in front of $\frac{\partial u}{\partial r}$ and $\frac{\partial u}{\partial S}$ are really given by the functions:

$$\hat{f^r} = f^r - h^r g^r = \alpha(\gamma - r_t) - \pi r_t = \alpha\gamma - (\alpha + \pi) r_t,$$

$$\hat{f^S} = f^S - h^S g^S = \mu S_t - (\mu - r_t) S_t = r_t S_t.$$

These new coefficients are risk-adjusted ([8], pp. 301–302) and if we let $\hat{\alpha} = \alpha + \pi$ and $\hat{\gamma} = \alpha\gamma$ we get a new vector $\hat{\mathbf{p}} = \{\hat{\alpha}, \hat{\gamma}, \rho, \sigma, \eta\}$ under the risk-neutral measure Q containing a reduced set of parameters. These are sufficient for pricing purposes, but if we want to use percentile methods based on the underlying density functions (e.g. for calculating risk capitals) we will need the whole set of natural parameters. This can however easily be solved by subjectively specifying the level of the long-run rate γ. Then the remaining parameters α and π are directly derived through:

$$\alpha = \frac{\hat{\alpha}\hat{\gamma}}{\gamma}, \qquad \pi = \alpha - \hat{\alpha}.$$

4.4 Parameter estimation

We start by making the following definition:

The term structure for the CIR model of the time t in-force market prices, i.e. the set of ZCB prices $\{v(t,T), T \geq t\}$ for all maturities T subsequent to t, is:

$$v(t,T) = A_{T-t} e^{-r_t B_{T-t}},$$

where A and B are deterministic functions depending only on the time to maturity $\tau = T - t$ through

$$A_\tau = \left(\frac{2d e^{(\hat{\alpha}+d)/2}}{(\hat{\alpha}+d)(e^{d\tau} - 1) + 2d} \right)^{\nu}$$

and

$$B_\tau = \frac{2e^{d\tau} - 1}{(\hat{\alpha}+d)(e^{d\tau} - 1) + 2d}$$

with

$$d = \sqrt{\hat{\alpha}^2 + 2\rho^2}$$

and

$$\nu = 2\frac{\hat{\alpha}\hat{\gamma}}{\rho^2}.$$

A somewhat simplified approach to estimating the parameters $\hat{\alpha}$, $\hat{\gamma}$ and ρ of the CIR model is by calibration on the interest-sensitive securities market.

Swap rates[10] ω_T with different maturities are quoted daily on the interest rate swap market (e.g. Euribor), and since

$$\omega_T = \frac{1 - v^*(0,T)}{\sum_{t=1}^{T} v^*(0,t)},$$

where $v^*(0,T)$ is the time 0 price of a ZCB with maturity T, by observing the values of ω_T for different values of T we can solve for $v^*(0,T) = v^*(\tau_k)$, $k = 1, 2, \ldots, n$, and thus obtain the ZCB term structure. This is then compared to the CIR term structure $v(\tau_k; \hat{\alpha}, \hat{\gamma}, \rho)$ and the unknown parameters are estimated by minimizing the sum of squared errors between model price and market price, i.e. by solving the minimization problem

$$\min \sum_{k=1}^{n} (v(\tau_k; \hat{\alpha}, \hat{\gamma}, \rho) - v^*(\tau_k))^2,$$

where the errors are assumed to be normally distributed and uncorrelated.

The remaining two parameters of the vector $\hat{\mathbf{p}}$, σ and η, can be externally specified. The value of σ is usually set to the same value as the historical volatility of the reference fund's stock component. The correlation coefficient η is considered to have little effect on the valuation and its value is usually derived through econometric studies of the Italian market.

As mentioned in the beginning of this section, what is described here is a simplified calibration technique, but it is sufficient for illustrating purposes. The "standard" calibration procedure is more complicated as it uses cross sections[11] and historical time series[12] from a large quantity of data.

4.5 Monte Carlo simulation

Having estimated the parameters of the CIR and BS models we can compute the price $u(t,T)$ by solving equation (13). This has to be done numerically, for example through Monte Carlo simulation. For $n = t+1, t+2, \ldots, T$ the procedure is as follows:

1. A descrete time equivalent to the risk-neutral stochastic differential equations $dr_t = \hat{\alpha}(\hat{\gamma} - r_t)dt + \rho\sqrt{r_t}dZ_t^r$ and $dS_t = r_t S_t dt + \sigma S_t dZ_t^S$ is defined.

[10] An interest rate swap is a contract in which two counterparties agree to exchange interest payments of differing character. The most common interest rate swap is one where one counterparty pays a fixed rate (the *swap rate*) while receiving a floating rate.

[11] Data collected by observing many subjects at the same point of time, or without regard to differences in time.

[12] Data collected by following one subject's changes over the course of time.

2. A descrete sample path for r and S is generated based on given starting values r_0 and S_0.

3. The annual values I_n of the fund return are calculated along the paths and the corresponding values of the readjustment factor $\Phi(t, n)$ are then derived.

4. The descrete equivalents of the discount factors $e^{-\int_t^n r_\tau d\tau}$ are calculated along the paths.

5. The discounted values $\Phi(t, n)e^{-\int_t^n r_\tau d\tau}$ are computed for each n.

This process is iterated N times (N is usually a large number such as 1000) and the stochastic valuation factor $u(t, n)$ is derived as the average of the N discounted values from step 5.

4.6 Technical uncertainty

We now consider the technical part of equation (7) for the stochastic reserve.

For the benefits we have:

$$Y_n = \begin{cases} C_n \cdot \mathbb{I}_{n-1<T_x\leq n}, & n = t+1, t+2, \ldots, T-1 \\ C_n, & n = T, \end{cases}$$

where $\mathbb{I}_{\mathcal{E}}$ is the indicator function for the event $\mathcal{E}$ and T_x is the remaining lifetime of the insured aged x. For the premia we have:

$$X_n = \begin{cases} A_n \cdot \mathbb{I}_{T_x\geq n}, & n = t+1, t+2, \ldots, T-1 \\ 0, & n = T. \end{cases}$$

Using actuarial notation we can now express $\bar{Y}_t(n)$ and $\bar{X}_t(n)$ as:

$$\bar{Y}_t(n) = \begin{cases} C_t \cdot {}_{n-1}p_x \cdot q_{x+n-1}, & n = t+1, t+2, \ldots, T-1 \\ C_T, & n = T \end{cases}$$

$$\bar{X}_t(n) = \begin{cases} A_t \cdot {}_{n}p_x, & n = t+1, t+2, \ldots, T-1 \\ 0, & n = T, \end{cases}$$

where the probabilities are defined according to:

$$\begin{aligned} {}_tp_x &= \frac{l(x+t)}{l(x)} = P(T_x > t) \\ {}_tq_x &= 1 - \frac{l(x+t)}{l(x)} = P(T_x \leq t) \\ {}_tp_x \cdot {}_{s-t}q_{x+t} &= \frac{l(x+t) - l(x+s)}{l(x)} = P(t < T_x \leq s) \end{aligned}$$

Expression (7) now becomes:

$$V_t = C_t \sum_{n=t+1}^{T-1} {}_{n-1}p_x \cdot q_{x+n-1} \cdot u(t,n) + C_T \cdot u(t,T)$$
$$- A_t \sum_{n=t+1}^{T-1} {}_{n}p_x \cdot u(t,n)$$

Since the actuarial probabilities are easily computed using standard mortality tables and the value of the financial factor $u(t,n)$ is given by (13), the time t stochastic reserve V_t is now completely determined.

5 Embedded options

The stochastic reserve can be decomposed with respect to the minimum guarantee and to the excess-return, enabling the insurer to better survey the costs of their provided benefits. In order to make the calculations as simple as possible we will here assume that our example policy has a single premium and that there is no technical uncertainty, i.e. that $\mathbf{P}_t(C_T;T) = 1$. This means that the expressions for the traditional and the stochastic reserves reduce to

$$R_t = C_t(1+i)^{-(T-t)} \tag{14}$$

and

$$V_t = V(t; Y_T) = C_t V(t; \Phi(t,T)) = C_t V\left(t; \prod_{n=t+1}^{T} (1+\rho_n)\right)$$

respectively.

5.1 Put decomposition

In this section we will compare the value V_t of the stochastic reserve with a "base value" B_t defined as:

$$B_t = C_t V(t; \Phi^B(t,T)),$$

where

$$\Phi^B(t,T) = \prod_{n=t+1}^{T} (1+\rho_n^B)$$

is the "base readjustment factor" and

$$\rho_t^B = \frac{\beta I_t - i}{1+i}$$

is the "base readjustment rate". B_t can thus be seen as a policy analogous to V_t but without the minimum guarantee.

By considering for a moment only the payoffs $\Phi(t,T)$ and $\Phi^B(t,T)$ and using definition (14) of the traditional reserve we can write:

$$\Phi(t,T) = \frac{R_t}{C_t} \prod_{n=t+1}^{T} (1 + \max\{\beta I_n, i^{\min}\}),$$

$$\Phi^B(t,T) = \frac{R_t}{C_t} \prod_{n=t+1}^{T} (1 + \beta I_n).$$

To simplify things even more we now consider the case of one time step $[t, t+1]$ and make the following decomposition of $\Phi(t, t+1)$:

$$\begin{aligned}
\Phi(t,t+1) &= \frac{R_t}{C_t}(1 + \max\{\beta I_{t+1}, i^{\min}\}) \\
&= \frac{R_t}{C_t}(1 + \beta I_{t+1} + \max\{i^{\min} - \beta I_{t+1}, 0\}) \\
&= \frac{R_t}{C_t}\left(1 + \beta I_{t+1} + \beta \max\left\{\frac{i^{\min}}{\beta} - I_{t+1}, 0\right\}\right) \\
&= \frac{R_t}{C_t}(1 + \beta I_{t+1}) + \frac{R_t}{C_t}\beta \max\left\{\frac{i^{\min}}{\beta} - I_{t+1}, 0\right\} \\
&= \Phi^B(t,t+1) + \frac{R_t}{C_t}\beta\left[\frac{i^{\min}}{\beta} - I_{t+1}\right]^+.
\end{aligned}$$

We see that if the fund return $I_{t+1} < \frac{i^{\min}}{\beta}$, that is if it turns out to be lower than the minimum guaranteed return, then $\Phi - \Phi^B > 0$ and the policy holder exercises their right to exchange the payoff of an unguaranteed contract for that of a guaranteed one. These are the characteristics of a *European put option*[13] having $\frac{i^{\min}}{\beta}$ as the strike price. Returning to V_t and B_t we now can write:

$$Put_t = V_t - B_t = C_t\left[V(t; \Phi(t,T)) - V(t; \Phi^B(t,T))\right]^+.$$

This is the price of the put option embedded in the contract and thus the time t fair value of the minimum guarantee. Because the option is part of the contract, unlike ordinary put options it is basically given to the policy holder without them having to pay for it, so if it is exercised the insurance company loses money. It is therefore an important part of a company's risk management to be able to correctly price these options. If we for illustration purposes consider the one-year interval $[t-1, t]$ and for simplicity let $i = 0$ then according to the participation rule the amount $C_{t-1}\max\{\beta I_t, i^{\min}\}$ will

[13] A European put option is a contract giving the owner the right, but not the obligation, to sell a specified amount of an underlying security at a specified strike price on a given date.

be credited to the policy and the amount $C_{t-1}(I_t - \max\{\beta I_t, i^{\min}\})$ will be retained by the insurance company. Concentrating only on the rate of return we first make the following rewritings:

$$\begin{aligned} \max\{\beta I_t, i^{\min}\} &= \beta I_t + \max\{i^{\min} - \beta I_t, 0\} \\ &= \beta I_t + [i^{\min} - \beta I_t]^+, \end{aligned} \tag{policy}$$

$$\begin{aligned} I_t - \max\{\beta I_t, i^{\min}\} &= (1-\beta) I_t - \max\{i^{\min} - \beta I_t, 0\} \\ &= (1-\beta) I_t - [i^{\min} - \beta I_t]^+. \end{aligned} \tag{company}$$

The allocation of the annual return can now be illustrated in a payoff diagram (figure 1). Each year the insurer thus makes an investment gain

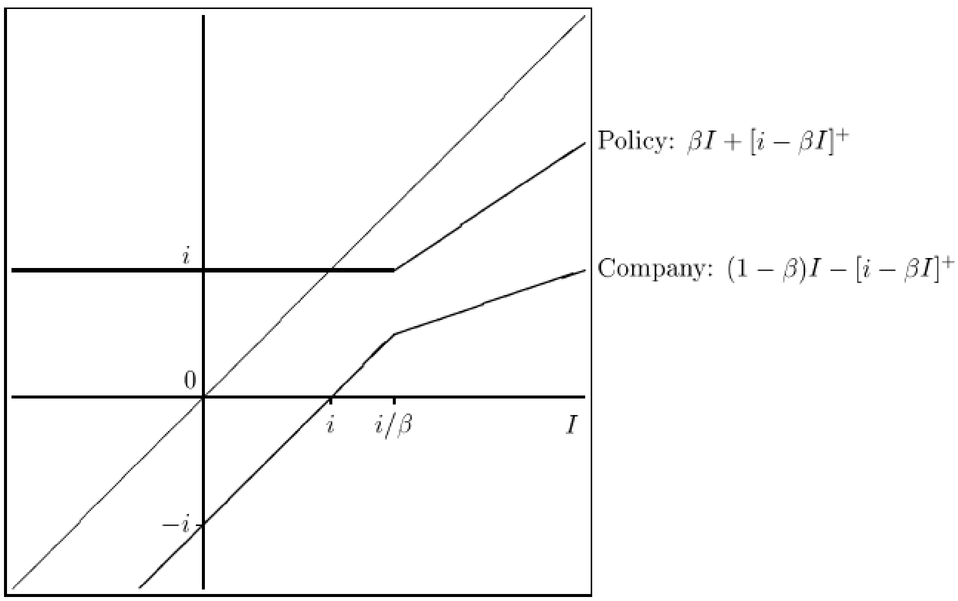

Figure 1: Allocation of the annual return

$(1-\beta)I_t$ and shortens[14] a put option written on the reference fund. It is clear that the minimum guarantee embedded in the contract could potentially threaten the company's solvency. When the fund return is greater than the minimum guarantee the put option is worthless, so for high returns the put price will be lower. But as soon as the return hit the level $\frac{i^{\min}}{\beta}$ the put option is exercised, so with falling returns the put price will rise.

We also define

$$V_t = B_t + Put_t$$

[14]Shortening a put option means selling the right to sell the underlying asset at a particular strike price to an option holder. In this case the insurer is "selling" the right to exchange the payoff of an unguaranteed contract for that of a guaranteed one to the insured, only without getting payed.

as the *put decomposition* of the stochastic reserve. This decomposition is very useful because it allows us to compare the contract with guaranteed return to a "pure investment" contract without any guarantees.

5.2 Call decomposition

Here we compare V_t with the value of a minimum guaranteed terminal benefit G_t defined as:

$$G_t = C_t V(t; \Phi^G(t,T)),$$

where the "guaranteed readjustment factor" is given by

$$\Phi^G(t,T) = (1+i)^{-(T-t)} \prod_{n=t+1}^{T} (1+i^{\min}) = \frac{R_t}{C_t} \prod_{n=t+1}^{T} (1+i^{\min}),$$

where the last equality follows from (14). Analogously to the case of the put we now make the following decomposition of $\Phi(t,t+1)$ for the time interval $[t,t+1]$:

$$\begin{aligned}
\Phi(t,t+1) &= \frac{R_t}{C_t}(1+\max\{\beta I_{t+1}, i^{\min}\}) \\
&= \frac{R_t}{C_t}(1+i^{\min}+\max\{\beta I_{t+1}-i^{\min}, 0\}) \\
&= \frac{R_t}{C_t}(1+i^{\min}) + \frac{R_t}{C_t}\beta \max\left\{I_{t+1}-\frac{i^{\min}}{\beta}, 0\right\} \\
&= \Phi^G(t,t+1) + \frac{R_t}{C_t}\beta\left[I_{t+1}-\frac{i^{\min}}{\beta}\right]^+ .
\end{aligned}$$

If $I_{t+1} > \frac{i^{\min}}{\beta}$, that is if the return of the fund turns out to be higher than the minimum guaranteed return, then $\Phi - \Phi^G > 0$ and the policy holder exercises their right to participate to the excess-return. This can be recognized as a *European call option*[15], again with $\frac{i^{\min}}{\beta}$ as the strike price. We thus have:

$$Call_t = V_t - G_t = C_t\left[V(t;\Phi(t,T)) - V(t;\Phi^G(t,T))\right]^+$$

which is the price of the call option embedded in the contract and thus the time t fair value of the excess-return. We return to the amounts $C_{t-1}\max\{\beta I_t, i^{\min}\}$ and $C_{t-1}(I_t - \max\{\beta I_t, i^{\min}\})$ from the previous section credited to the policy and the company respectively, but this time we make a different rewriting:

$$\begin{aligned}
\max\{\beta I_t, i^{\min}\} &= i^{\min} + \max\{\beta I_t - i^{\min}, 0\} \\
&= i^{\min} + [\beta I_t - i^{\min}]^+,
\end{aligned} \qquad \text{(policy)}$$

[15] A European call option is a contract giving the owner the right, but not the obligation, to buy a specified amount of an underlying security at a specified strike price on a given date.

$$
\begin{aligned}
I_t - \max\{\beta I_t, i^{\min}\} &= (I_t - i^{\min}) - \max\{\beta I_t - i^{\min}, 0\} \\
&= (I_t - i^{\min}) - [\beta I_t - i^{\min}]^+.
\end{aligned}
\qquad \text{(company)}
$$

The functions are the same as before but can be interpreted in a different way: each year the company collects the payoff $I_t - i^{\min}$, i.e. the fund return in excess of the minimum guaranteed return, (or pays it, if negative) and shortens a call option written on the fund. By exercising the call the policy holder "buys" in on a piece of this excess-return, but without having to pay.

We define

$$V_t = G_t + Call_t$$

as the *call decomposition* of the stochastic reserve. This decomposition lets us compare the contract with guarantee to one that has a deterministic yield (i.e. a known terminal benefit).

5.3 The surrender option

Another option that needs mentioning is the one that the insured has to surrender the contract. This can be done at any time before maturity and the surrender value at time t is of the form

$$\Sigma_t = C_t \gamma_t,$$

where γ_t, $0 < \gamma_t \leq 1$, is a contractually specified redemption coefficient. The surrender option can thus be considered as an *American put option* embedded in the contract: the insured has the right to at any time sell back the contract to the insurer at the strike price Σ_t. But in order to price it as such we would have to make the assumption that the insured *rationally* exercises this right, which is often not the case. Firstly, the insured's reason for surrender may not have anything to do with the course of the market. It could instead depend on the development of their personal finances and consumptions. Secondly, though Italian insurance companies publish quarterly reports on the separate funds the information on the returns is usually provided with a delay of one or two months, so even if the insured had "rational" reasons for surrender they would not have all the information needed to rationally exercise the option. The surrenders will therefore be treated as purely technical events, i.e. by modeling them using experience-based probability tables and considering them together with mortality to be independent of market events.

6 Value of business in force

Up until now we have only considered the pure premium stochastic reserve V_t calculated upon *first order*, i.e. prudential, or pessimistic, technical bases.

There are however other ways to calculate the stochastic reserve, and we therefore make the following definitions:

$V_t^{(2)}$: pure premium stochastic reserve calculated upon *second order* technical bases (realistic mortality probabilities)

$V_t^{(3)}$: pure premium stochastic reserve calculated upon *third order* technical bases (realistic mortality and surrender probabilities)

$\hat{V}_t^{(3)}$: *office premium* stochastic reserve calculated upon third order technical bases (realistic mortality and surrender probabilities and premium loading Π)

The difference between the traditional reserve R_t and the reserve $\hat{V}_t^{(3)}$,

$$E_t = R_t - \hat{V}_t^{(3)},$$

is called the *value of business in force* (VBIF) of the policy and represents the value at time t, at current market prices, of the total gross profit generated by the insurance contract.

The VBIF can be decomposed with respect to the different reserves, with each component expressing a specific contribution to the total profit:

$$\begin{array}{ll} E_t^F = R_t - V_t : & \text{financial profit} \\ E_t^D = V_t - V_t^{(2)} : & \text{mortality profit} \\ E_t^S = V_t^{(2)} - V_t^{(3)} : & \text{surrender profit} \\ E_t^L = V_t^{(3)} - \hat{V}_t^{(3)} : & \text{premium loading profit} \end{array}$$

Obviously we have

$$E_t = E_t^F + E_t^D + E_t^S + E_t^L.$$

7 Risk capital

The definition of *risk capital*[16] is quite similar to that of Value-at-Risk (VaR). Given a portfolio, a fixed probability p and a period θ, the VaR is the maximum loss in the portfolio's value during θ with probability p, caused by an adverse movement of some risk factor. Here θ is usually a very short period (a few days) and the impact on the VaR of the deterministic price variation during this time can therefore be neglected. But if we extend

[16] Also called risk based capital or economic capital.

the VaR definition to longer time horizons, e.g. where $\theta = 1$ year, these price variations can no longer be ignored. Moreover, an insurance company that wants to be able to maintain a given credit rating may want to set p at a very low level, giving it the signification of a one-year default probability[17]. The risk capital can be defined as an extended version of VaR with respect to θ and p.

7.1 Financial risk capital

In our market model there are two sources of uncertainty: that of bonds and that of stock, and they are described by an interest rate process r_t and a stock index process S_t respectively. If we at the valuation time t consider a contract that at time $s > t$ will pay a random amount Z_s that is only affected by financial uncertainty, then the value loss in $[t, T]$, where $T > t$ is fixed, is defined by

$$L(t, T, r_t, S_t, Z_s) = -(V(T, r_T, S_T, Z_s) - V(t, r_t, S_t, Z_s)).$$

This is a random variable that is affected by both uncertainties mentioned above. We note however that L has a deterministic value variation caused only by the passing of time: if the market is static we still get a variation in value when the maturity s is shortened by $T - t$ years. To avoid this *time decay* we make the following redefinition:

$$L'(t, T, r_t, S_t, Z_s) = -(V(t, r_T, S_T, Z_s) - V(t, r_t, S_t, Z_s)). \qquad (15)$$

This lets us calculate the loss L' with respect to the value of the contract at time t, but using the random market conditions of time T.

We now let $K^{intr}(t, T, p, Z_s)$ and $K^{stock}(t, T, p, Z_s)$ represent the interest rate risk capital and the stock market risk capital respectively. They both depend on the time interval $[t, T]$ and a predetermined probability p. If we consider the variables $L'(t, T, r_T, S_t, Z_s)$ (random only with respect to the interest rate uncertainty) and $L'(t, T, r_t, S_T, Z_s)$ (random only with respect to the stock market uncertainty) we can define the two risk capitals implicitly through

$$\begin{aligned} \text{Prob}_t(L'(t, T, r_T, S_t, Z_s) < K^{intr}(t, T, p, Z_s)) &= 1 - p \\ \text{Prob}_t(L'(t, T, r_t, S_T, Z_s) < K^{stock}(t, T, p, Z_s)) &= 1 - p, \end{aligned}$$

where Prob_t is linked to the distributions of r_t and S_T respectively. The calculation of the two risk capitals is then done using the underlying percentile method:

[17] A *credit rating* is an estimate, based on previous dealings, of a company's ability to fulfill its financial commitments. *Default* is the failure to fulfill such a commitment.

If we assume that the function $V(t,r,S,Z_s)$ is strictly monotone with respect to r and S we can write:

$$\begin{aligned} K^{intr}(t,T,p,Z_s) &= \max\{L'(t,T,r_T^+,S_t,Z_s), L'(t,T,r_T^-,S_t,Z_s)\} \\ K^{stock}(t,T,p,Z_s) &= \max\{L'(t,T,r_t,S_T^+,Z_s), L'(t,T,r_t,S_T^-,Z_s)\}, \end{aligned}$$

where the percentiles r_T^+, r_T^-, S_T^+ and S_T^- are defined through

$$\begin{aligned} \text{Prob}_t(r_T < r_T^+) &= 1-p \\ \text{Prob}_t(r_T < r_T^-) &= p \\ \text{Prob}_t(S_T < S_T^+) &= 1-p \\ \text{Prob}_t(S_T < S_T^-) &= p \end{aligned}$$

and are calculated using the distribution functions of r_T and S_T which as seen in sections 4.1–4.2 are non-centered chi-squared and lognormal respectively. In this way we only need to consider the quantities $L'(t,T,r_T^+,S_t,Z_s)$, $L'(t,T,r_T^-,S_t,Z_s)$, $L'(t,T,r_t,S_T^+,Z_s)$ and $L'(t,T,r_t,S_T^-,Z_s)$ which in turn are obtained by calculating $V(t,r_t,S_t,Z_s)$, $V(t,r_T^+,S_t,Z_s)$, $V(t,r_T^-,S_t,Z_s)$, $V(t,r_t,S_T^+,Z_s)$ and $V(t,r_t,S_T^-,Z_s)$ and subtracting according to (15).

7.2 Technical risk capital

The technical risk capital is defined separately for mortality and surrender and like the financial risk capital it is calculated using the underlying percentile method.

For the mortality risk capital we get:

$$K^{mort}(t,T,p,Z_s) = \max\{L'(t,T,q_x^+,Z_s), L'(t,T,q_x^-,Z_s)\}.$$

The "shock" levels q_x^+ and q_x^-, where x is the age of the insured at the valuation time, can be determined using different models. The hypothesis used here is that of a lognormal one-year mortality rate. We have:

$$q_x^{\pm} = q_x e^{\pm\ \eta\sigma}, \tag{16}$$

where q_x is obtained from second-order mortality tables, σ is the standard deviation of the disturbance process of the logarithm of the mortality rate and η is a factor that depends on the desired confidence level.

The surrender risk capital is calculated in a similar way, i.e.:

$$K^{surr}(t,T,p,Z_s) = \max\{L'(t,T,s^+,Z_s), L'(t,T,s^-,Z_s)\}.$$

The determination of the "shock" levels s^+ and s^- follows the indications in QIS3[18]: in the "up" scenario the central surrender rate s is increased according to the formula

$$s^+ = \max\{s + 3\%, s \cdot 50\%\}$$

and in the "down" scenario it is decreased by 50%.

7.3 Solvency capital requirement

The standard formula for calculating the SCR is divided into different modules forming the tree structure in figure 2. At the root of the tree is the

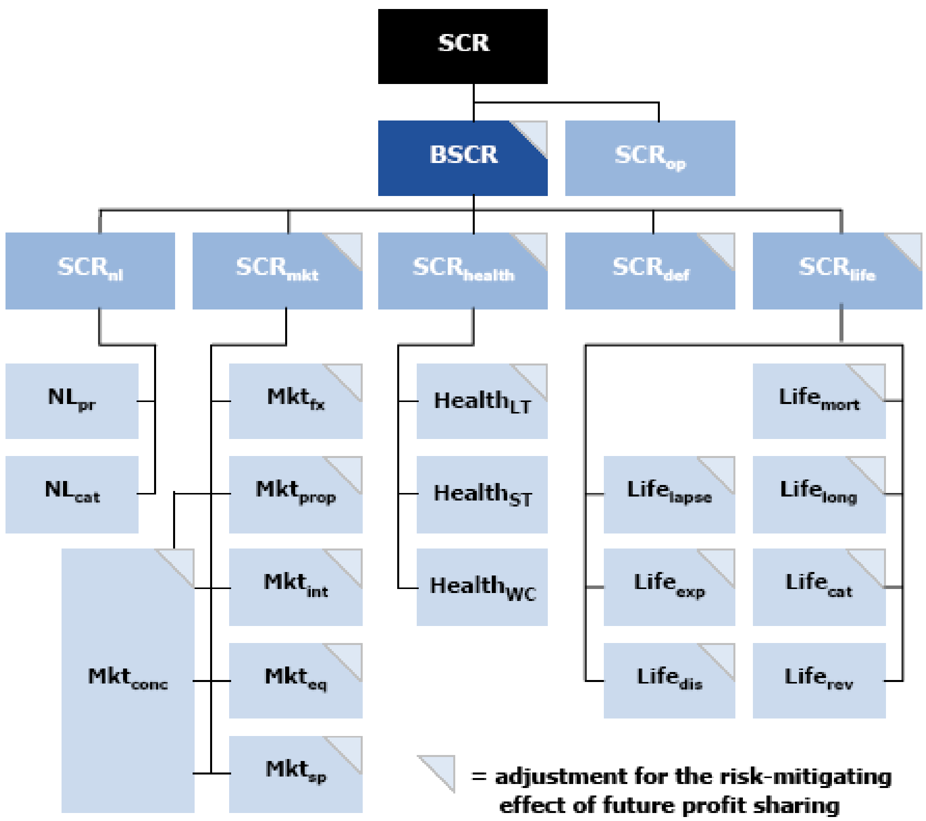

Figure 2: SCR structure

overall formula

$$\text{SCR} = \text{BSCR} + \text{SCR}_{\text{op}}.$$

[18] As a preparation for Solvency II, the Committee of European Insurance and Occupational Pensions Supervisors (CEIOPS) has launched a number of Quantitative Impact Studies (QIS) in order to investigate the effect on the proposals on company and industry levels. Companies participating to QIS fill out extensive pre-defined spreadsheets and the results are then collected by CEIOPS in an overall report.

$\mathrm{SCR_{op}}$ is the capital charge for operational risk, which is defined in [7] as "the risk of loss arising from inadequate or failed internal processes, people, systems or external events". BSCR is the basic solvency capital requirement before any adjustments and it combines the charges from five major risk categories: market risk ($\mathrm{SCR_{mkt}}$), counterparty default risk ($\mathrm{SCR_{def}}$), life underwriting risk ($\mathrm{SCR_{life}}$), non-life underwriting risk ($\mathrm{SCR_{nl}}$) and health underwriting risk ($\mathrm{SCR_{health}}$). A somewhat simplified[19] formula for calculating the BSCR is:

$$\mathrm{BSCR} = \sqrt{\sum_{r\times c} \mathrm{CorrSCR_{r,c}} \cdot \mathrm{SCR_r} \cdot \mathrm{SCR_c}},$$

where $\mathrm{CorrSCR_{r,c}}$ is the cell in row r, column c of the correlation matrix CorrSCR in figure 3 and $\mathrm{SCR_r}$ and $\mathrm{SCR_c}$ are the capital charges for the individual SCR risks according to the rows and columns of CorrSCR. With

CorrSCR=	SCR_{mkt}	SCR_{def}	SCR_{life}	SCR_{health}	SCR_{nl}
SCR_{mkt}	1				
SCR_{def}	0.25	1			
SCR_{life}	0.25	0.25	1		
SCR_{health}	0.25	0.25	0.25	1	
SCR_{nl}	0.25	0.5	0	0,25	1

Figure 3: SCR correlation matrix

this correlation matrix the individual risk modules are linearly combined to form the overall SCR, with a confidence level of 99.5%. For the purpose of this thesis we will deal only with the $\mathrm{SCR_{mkt}}$ and the $\mathrm{SCR_{life}}$, which are associated with the financial risk capital and the technical risk capital respectively.

7.3.1 The SCR market risk module

In QIS4 the market risk is defined as the risk arising "from the level or volatility of market prices of financial instruments. Exposure to market risk is measured by the impact of movements in the level of financial variables such as stock prices, interest rates, real estate prices and exchange rates" ([7] p. 131). The $\mathrm{SCR_{mkt}}$ is composed of the capital charges of six sub-risks: interest rate risk ($\mathrm{Mkt_{int}}$), equity risk ($\mathrm{Mkt_{eq}}$), property risk ($\mathrm{Mkt_{prop}}$), spread risk ($\mathrm{Mkt_{sp}}$), risk concentrations ($\mathrm{Mkt_{conc}}$) and currency risk ($\mathrm{Mkt_{fx}}$). These are combined in the same way as with the BSCR but

[19] The complete formula also includes the effects of risk mitigating, i.e. the action to reduce the severity of a risk.

using the market correlation matrix CorrMkt in figure 4. The calculation formula is:

$$\mathrm{SCR_{mkt}} = \sqrt{\sum_{r\times c} \mathrm{CorrMkt_{r,c}} \cdot \mathrm{Mkt_r} \cdot \mathrm{Mkt_c}}$$

We will here only deal with the $\mathrm{Mkt_{int}}$ and the $\mathrm{Mkt_{eq}}$ which we define

CorrMkt	Mkt_{int}	Mkt_{eq}	Mkt_{prop}	Mkt_{sp}	Mkt_{conc}	Mkt_{fx}
Mkt_{int}	1					
Mkt_{eq}	0	1				
Mkt_{prop}	0.5	0.75	1			
Mkt_{sp}	0.25	0.25	0.25	1		
Mkt_{conc}	0	0	0	0	1	
Mkt_{fx}	0.25	0.25	0.25	0.25	0	1

Figure 4: Mkt correlation matrix

to equal the interest rate risk capital and the stock market risk capital respectively from section 7.1.

7.3.2 The SCR life underwriting risk module

This risk arises from the underwriting of life insurance contracts and is composed of the capital charges of the following sub-risks: mortality risk ($\mathrm{Life_{mort}}$), longevity risk ($\mathrm{Life_{long}}$), disability/morbidity risk ($\mathrm{Life_{dis}}$), lapse risk ($\mathrm{Life_{lapse}}$), expense risk ($\mathrm{Life_{exp}}$), revision risk ($\mathrm{Life_{rev}}$) and catastrophe risk ($\mathrm{Life_{CAT}}$). These are combined using the life correlation matrix in figure 5, the calculation formula being

$$\mathrm{SCR_{life}} = \sqrt{\sum_{r\times c} \mathrm{CorrLife_{r,c}} \cdot \mathrm{Life_r} \cdot \mathrm{Life_c}}$$

We will here only deal with the $\mathrm{Life_{mort}}$ and the $\mathrm{Life_{lapse}}$ which we define to

CorrLife =	$Life_{mort}$	$Life_{long}$	$Life_{dis}$	$Life_{lapse}$	$Life_{exp}$	$Life_{rev}$	$Life_{CAT}$
$Life_{mort}$	1						
$Life_{long}$	0	1					
$Life_{dis}$	0.5	0	1				
$Life_{lapse}$	0	0.25	0	1			
$Life_{exp}$	0.25	0.25	0.5	0.5	1		
$Life_{rev}$	0	0.25	0	0	0.25	1	
$Life_{CAT}$	0	0	0	0	0	0	1

Figure 5: Life correlation matrix

equal the mortality risk capital and the surrender risk capital respectively from section 7.2.

7.4 The traditional solvency margin

The Solvency I margin used in Italy is calculated according to [12] as 4% of the traditional reserve +0.3% of the positive sums at risk. This is without considering reinsurance. For term life insurance policies with durations non longer than 3 years the share of positive sums at risk is instead 0.1% and for durations between 3 and 5 years it is 0.15%. In the case of our example policy the formula for calculating the solvency margin (SM) is:

$$\mathrm{SM} = 0.04R_t + 0.003[C_t - R_t]^+. \tag{17}$$

8 Valuation of the contract

The DFM model will now be used to valuate a contract that has been provided by an Italian mutual insurance company. Both the value of the traditional reserve and the stochastic reserve under various assumptions will be calculated, along with the VBIF and the price of the embedded options. Finally the SCR will be computed and compared to the traditional solvency margin.

8.1 Contract description

The contract is of the type decribed in section 3. When we start the valuation ten years have passed since the issue date and five years remain until maturity. We call this time 0 and thus have $t = 0, 1, 2, \ldots, T$ with $T = 5$.

Characteristics

of the tariff	Tariff type	Participating endowment
	Premium type	Annual readjustable
	Demographical bases	SI81
	Technical rate i	4.00%
	Participation coefficient β	0.8
of the contract	Sex	M
	Policy duration	15 years
	Number of premia	15
	Anti-duration (time from issue)	10 years
	Age reached	52

We can see that the premium is payed until maturity if the insured should not die before that time. The mortality tables SI81 are based on demographical data from 1981 collected by Istituto Nazionale di Statistica (ISTAT) and can be found in appendix C.1.

Financial characteristics of the contract

at time 0	Assured capital - maturity	23 403.08
	Assured capital - death	23 403.08
	Surrender value	14 482.76
	Pure premium (-)	-1 184.42
	Office premium (-)	-1 355.94

These figures have been provided by the insurance company and will be our time 0 "starting values" in the valuation. All amounts are given in EUR.

Management strategy for the fund

Bond component	100.00%
Stock component	0.00%

As we can see, the fund where the premium is invested is made up entirely by bonds which means that the financial part of the valuation only will be affected by interest rate uncertainty.

8.2 Valuation assumptions

In section 6 four different ways of calculating the stochastic reserve were presented, each one with a different set of mortality and surrender assumptions and with pure or office premium.

Valuation assumptions for the three orders

Order	I	II	III
Mortality assumption	central	central	central
Surrender assumption	central	central	central
Mortality	100.00% SI81	66.00% SI92	66.00% SI92
Surrender	0.00%	0.00%	4.20%

Our example contract uses the SI81 mortality tables for the first order calculations, but for the second and third orders the more recent SI92 tables are used (see appendix C.2). We are however still dealing with prudential figures, so to get more realistic assumptions the probabilities are reduced to 66% of their original values.

Valuation assumptions for the technical risk capital

Order	III	III	III	III
Mortality assumption	up	down	central	central
Surrender assumption	central	central	up	down
Mortality	72.26% SI92	61.20% SI92	66.00% SI92	66.00% SI92
Surrender	4.20%	4.20%	7.20%	2.10%

The mortality "shock" levels are calculated using formula (16) with $\sigma = 2.934\%$ and $\eta = 2.575834$. The surrender "shock" levels are set as described in section 7.2 according to QIS3 and with a central surrender rate of 4.20%.

8.3 Calculation of reserves

All the calculations in this section will be carried out using first order pure premium assumptions. For the second and third orders and for the office premium the approach is analogous so the details will be omitted (they can however be found in appendix B). All the final results will be presented in the next section.

Values

Year	0	1	2	3	4	5
Death benefit		23 403.08	23 403.08	23 403.08	23 403.08	23 403.08
Maturity benefit		0.00	0.00	0.00	0.00	23 403.08
Surrender value		16 170.01	17 904.61	19.687.61	21 520.07	23 403.08
Pure premium (-)		-1 184.42	-1 184.42	-1 184.42	-1 184.42	-1 184.42

The values of the death and maturity benefits and the pure premium are those provided by the insurance company from the financial characteristics table in section 8.1. The time t surrender value S_t is calculated using the formula

$$\Sigma_t = \frac{C_t(a+t)}{d(1+r)^{-(T-t)}}$$

where $a = 10$ is the antiduration of the policy, $d = 15$ is the duration of the policy and the rate $r = 1.50\%$ is contractually specified.

Probabilities at beginning of year (conditional)

Year	0	1	2	3	4	5
Death		0.00809	0.00900	0.01001	0.01107	0.01228
Life		0.99191	0.99100	0.98999	0.98893	0.98772
Surrender		0.00000	0.00000	0.00000	0.00000	0.00000
Premium payment		1.00000	1.00000	1.00000	1.00000	1.00000

The conditional death probability, i.e. the probability of death in year t if alive in year $t-1$ for an x-year-old individual (here we have $x = 52+t-1$), is given directly by the SI81 mortality tables. The conditional probability of remaining alive is of course one minus that of death. For surrender the probability is 0 since we are dealing with first order assumptions. The conditional probability that the premium will be payed is 1 because it is payed at the beginning of the year when the insured definitely is alive.

Probabilities at time 0 (non conditional)

Year	0	1	2	3	4	5
Death		0.00809	0.00893	0.00984	0.01078	0.01181
Life	1.00000	0.99191	0.98298	0.97314	0.96236	0.95055
Surrender		0.00000	0.00000	0.00000	0.00000	0.00000
Premium payment		0.99191	0.98298	0.97314	0.96236	0.95055

The unconditional death probability is calculated using the law of total probability:

$$P(\text{death in } t) = P(\text{death in } t|\text{alive in } t-1)P(\text{alive in } t-1).$$

The unconditional probability of remaining alive in t is the probability of being alive in $t-1$ minus the probability of dying in t. The unconditional premium probability is the same as that of remaining alive.

Projected cash flows

Year	0	1	2	3	4	5
Death		189.40	209.03	230.22	252.18	276.47
Life		0.00	0.00	0.00	0.00	22 245.76
Surrender		0.00	0.00	0.00	0.00	0.00
Premium payment		-1 174.83	-1 164.26	-1 152.60	-1 139.84	0.00

We get the projected cash flows by multiplying the unconditional transition probabilities by the corresponding amounts from the "Values" table above.

Rates

Year	0	1	2	3	4	5
Technical rate		4.00%	4.00%	4.00%	4.00%	4.00%
Guaranteed rate besides technical		0.00%	0.00%	0.00%	0.00%	0.00%
Market spot rate		4.68%	4.53%	4.51%	4.52%	4.53%

The technical rate i is contractually specified and serves at the same time as the minimum guaranteed rate since the "guaranteed rate besides technical" is 0.00%, i.e. $i^{\min} = i$. The insured is thus not entitled to more than a 4.00%

rate of return but neither any less should I_t drop below 4.00%.

Discount factors

Year	0	1	2	3	4	5
Technical rate	1.00000	0.96154	0.92456	0.88900	0.85480	0.82193
Market rate	1.00000	0.95526	0.91525	0.87602	0.83801	0.80115

The technical ($v(t)$) and market ($v(0,t)$) discount factors are obtained from the corresponding rates as follows:

$$v(t) = (1+i)^{-t}, \qquad v(0,t) = (1+i(0,t))^{-t}.$$

Valuation factors

Year	0	1	2	3	4	5
Premium	1.00000	0.95873	0.91929	0.88137	0.84518	0.81039
Benefits	1.00000	0.95873	0.91929	0.88137	0.84518	0.81039
Surrender	1.00000	0.95873	0.91929	0.88137	0.84518	0.81039

The same valuation factors are used for premium, benefits and surrender. The risk-neutral parameters $\hat{\alpha}$, $\hat{\gamma}$ and ρ of the CIR model have been estimated through calibration (we don't have to consider the parameter σ belonging to the BS model nor the instantaneous correlation coefficient η because the fund contains no stock). The values are:

$$\hat{\alpha} = 0.215451168$$
$$\hat{\gamma} = 0.049246370$$
$$\rho = 0.045732693$$

Equation (13) has then been solved through Monte Carlo simulation producing the stochastic valuation factor $u(0,n)$ for $n = 0, 1, \ldots, 5$.

Valuation factors (contract without guarantee)

Year	0	1	2	3	4	5
Premium	1.00000	0.95295	0.90810	0.86528	0.82469	0.78590
Benefits	1.00000	0.95295	0.90810	0.86528	0.82469	0.78590
Surrender	1.00000	0.95295	0.90810	0.86528	0.82469	0.78590

This is the "base valuation factor" $u^B(t,T) = V(t;\Phi^B(t,T))$ corresponding to the base value B_t used in the put decomposition. It is calculated in the same way as the valuation factor with guarantee but using the base readjustment factor $\Phi^B(t,T)$ defined in section 5.1.

Traditional valuation with technical rate

Year	0	1	2	3	4	5
Pure premium (-)		-1 129.65	-1 076.42	-1 024.66	-974.34	0.00
Death benefit		182.11	193.26	204.67	215.57	227.24
Maturity benefit		0.00	0.00	0.00	0.00	18 284.40
Surrender benefit		0.00	0.00	0.00	0.00	0.00
Reserve	15 102.18					

The previously calculated projected cash flows for premia and benefits are discounted to time 0 using the technical discount factor, and their sum gives the time 0 traditional reserve R_0 defined by (6) in section 3.

Traditional valuation with market rate

Year	0	1	2	3	4	5
Pure premium (-)		-1 122.27	-1 065.58	-1 009.71	-955.20	0.00
Death benefit		180.92	191.32	201.68	211.33	221.50
Maturity benefit		0.00	0.00	0.00	0.00	17 822.25
Surrender benefit		0.00	0.00	0.00	0.00	0.00
Reserve	14 676.25					

In the same way as with the technical rate we here get the traditional market rate reserve by discounting the cash flows using the market spot rate. This reserve will not be used in the following calculations but it is nevertheless interesting to look at since it shows the evolution of the valuation from traditional to stochastic reserve. In other words, it is an approach half way between the traditional and the stochastic approach: it uses the assumption of constancy for the future benefits typical of the former and the market structure typical of the latter.

Fair valuation

Year	0	1	2	3	4	5
Pure premium (-)		-1 126.35	-1 070.28	-1 015.87	-963.37	0.00
Death benefit		181.58	192.16	202.91	213.14	224.05
Maturity benefit		0.00	0.00	0.00	0.00	18 027.72
Surrender benefit		0.00	0.00	0.00	0.00	0.00
Reserve	14 865.69					

This is the time 0 stochastic reserve V_0 defined by (7) in section 3 or in other words, the contract's *fair value*. The cash flows are thus discounted using the market based valuation factor $u(0, n)$.

Fair valuation of only the guaranteed benefits

Year	0	1	2	3	4	5
Pure premium (-)		-1 122.27	-1 065.58	-1 009.71	-955.20	0.00
Death benefit		180.92	191.32	201.68	211.33	221.50
Maturity benefit		0.00	0.00	0.00	0.00	17 822.25
Surrender benefit		0.00	0.00	0.00	0.00	0.00
Reserve	14 676.25					

This is the reserve regarding only the guarantees denoted by G_t in section 5.2 (here of course we have the time 0 value G_0). Since the guaranteed rate besides technical is 0 the cash flows are discounted by only the market rate and G_0 is thus equal to the traditional market rate reserve. We will use this reserve in the calculation of the embedded call option.

Fair valuation of the benefits without guarantee

Year	0	1	2	3	4	5
Pure premium (-)		-1 128.70	-1 075.57	-1 024.86	-976.78	0.00
Death benefit		180.49	189.82	199.21	207.97	217.28
Maturity benefit		0.00	0.00	0.00	0.00	17 483.04
Surrender benefit		0.00	0.00	0.00	0.00	0.00
Reserve	14 271.91					

The stochastic reserve without guarantees is the base value B_t (here B_0) defined in section 5.1. The cash flows are thus discounted using the "contract without guarantee" valuation factor $u^B(0, n)$. We will use this reserve in the calculation of the embedded put option.

8.4 Embedded options

Components of the stochastic reserve

Components	Value	% of the stochastic reserve	% of the traditional reserve
Base component	14 271.91	96.01%	94.50%
Put component	593.78	3.99%	3.93%
Guarantee component	14 676.25	98.73%	97.18%
Call component	189.45	1.27%	1.25%

The time 0 put value is given by the formula

$$Put_0 = V_0 - B_0 = 14865.69 - 14271.91 = 593.78$$

derived in section 5.1. This is the cost of the minimum guarantee, and dividing by V_0 and R_0 we get a view of the proportion of the stochastic and traditional reserves that the guarantee represents. Figure 6 shows the put price in percent of the stochastic reserve for different variations of the interest rate. The put price corresponding to the central interest rate is compared to those corresponding to the rates used as “up” and “down” shocks in the risk capital calculations. We see that the price of the put option increases as

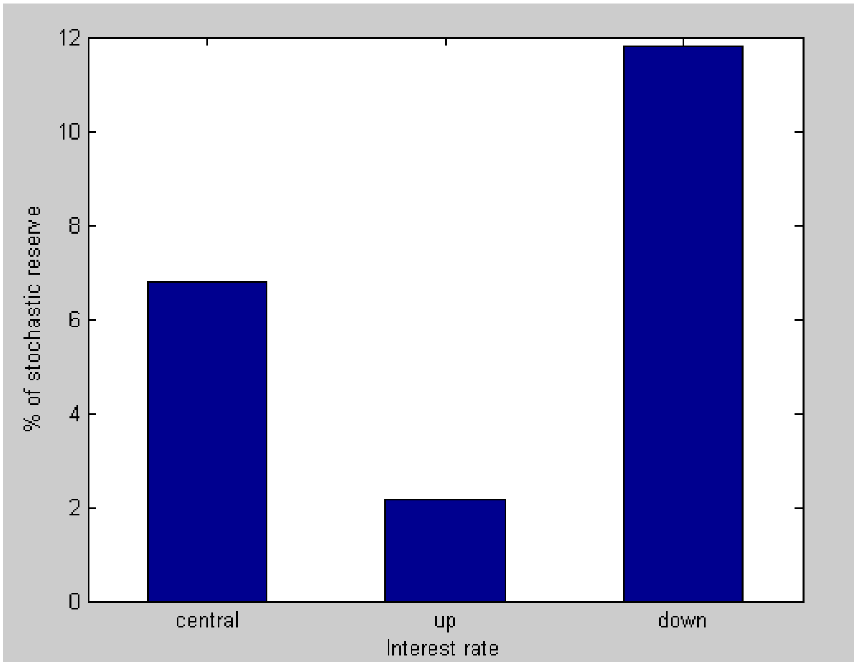

Figure 6: Put prices in percent of the stochastic reserve under different interest rate scenarios.

the interest rate, and thus the fund return, decreases and vice versa. This is consistent with the characteristics of the minimum guarantee discussed in section 5.1.

The calculations of the time 0 call value from section 5.2 are analougus:

$$Call_0 = V_0 - G_0 = 14865.69 - 14676.25 = 189.45,$$

and we get the time 0 fair value of the excess-return generated by the guaranteed contract.

8.5 VBIF

Calculation of the VBIF

Order	I	II	III	III
Premium	pure	pure	pure	office
Interest rate assumption	central	central	central	central
Mortality assumption	central	central	central	central
Surrender assumption	central	central	central	central
Traditional reserve	15 102.18			
Stochastic reserve	14 865.69	14 773.30	14 853.22	14 301.38

This table shows a summary of the stochastic reserve calculations made with the first, second and third order pure premium assumptions and the third order office premium assumption. From section 6 we see that they correspond to V_0, $V_0^{(2)}$, $V_0^{(3)}$ and $\hat{V}_0^{(3)}$ respectively. We also have the value of the traditional reserve R_0. This is all the information we need for calculating the time 0 VBIF:

$$E_0 = R_0 - \hat{V}_0^{(3)} = 15102.18 - 14301.38 = 800.80.$$

VBIF decomposition

Financial	236.49
Mortality	92.39
Surrender	-79.91
Loading	551.83
Total	800.80

The various components of the VBIF are calculated using the formulae given in section 6:

$$\begin{aligned}
E_0^F &= R_0 - V_0 &&= 15102.18 - 14865.69 = 236.49 \\
E_0^D &= V_0 - V_0^{(2)} &&= 14865.69 - 14773.30 = 92.39 \\
E_0^S &= V_0^{(2)} - V_0^{(3)} &&= 14773.30 - 14853.22 = -79.91 \\
E_0^L &= V_0^{(3)} - \hat{V}_0^{(3)} &&= 14853.22 - 14301.38 = 551.83
\end{aligned}$$

We see that the surrender "profit" is actually a loss. It is however covered by the premium loading which makes the biggest positive contribution to the VBIF. The second largest profit comes from the financial component and this shows how a profit can arise just by changing from a statutory to a stochastic valuation approach.

8.6 Risk capital

For calculating the risk capital we need the natural probability distribution of the CIR model. From the previous risk-neutral calibration we already have the value of ρ, and by assigning a value to γ we can derive the remaining parameters α and π as described in section 4.3:

$$\gamma = 0.042859341$$
$$\alpha = \frac{\hat{\alpha}\hat{\gamma}}{\gamma} = \frac{0.215451168 \cdot 0.049246370}{0.042859341} = 0.247558355$$
$$\pi = \alpha - \hat{\alpha} = 0.247558355 - 0.215451168 = 0.032107187$$

The CIR density function for these parameter values is illustrated in figure 7. The time interval $[t, T]$ is 1 year.

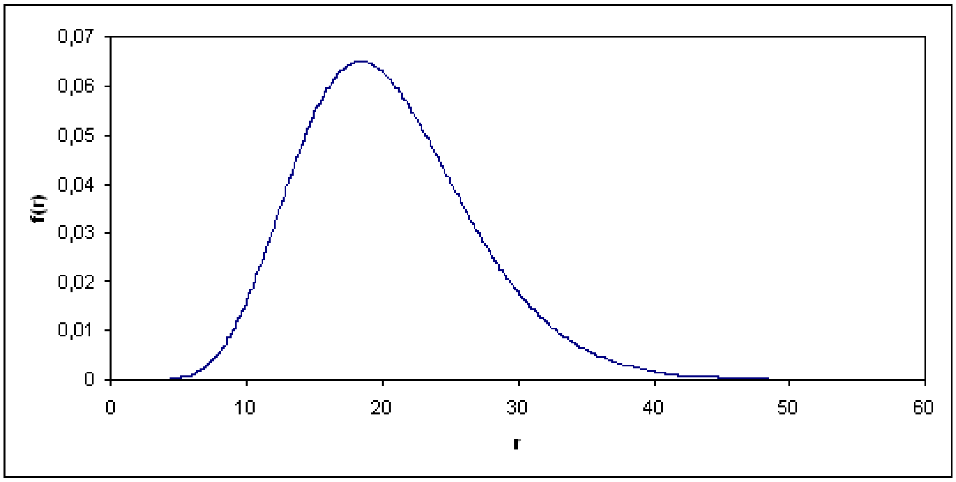

Figure 7: The CIR non-central chi-square density function

Interest rate risk capital

Order Premium	III office	III office
Interest rate Mortality Surrender	up central central	down central central
Stochastic reserve $\hat{V}_0^{(3)*}$ Difference $-(\hat{V}_0^{(3)} - \hat{V}_0^{(3)*})$ Adverse market move	13 624.57 -676.81	15 115.44 814.06 a.m.m.
Risk capital % of stochastic reserve % of traditional reserve		814.06 5.69% 5.39%

Since the reference fund contains only bonds there is no stock market risk and therefore the interest rate risk capital alone represents the financial risk capital. The value loss is calculated based on formula (15) for r^+ and r^- with $p = 0.005$:

$$\begin{aligned} L'(t,T,r_T^+) &= -(V(t,r_T) - V(t,r_T^+)) \\ &= -(14301.38 - 13624.57) \\ &= -676.81 \end{aligned}$$

In this case the "loss" turns out to be negative which means that the company instead makes a profit if the interest rate is high.

$$\begin{aligned} L'(t,T,r_T^-) &= -(V(t,r_T) - V(t,r_T^-)) \\ &= -(14301.38 - 15115.44) \\ &= 814.06 \end{aligned}$$

Since this result is positive we have an *adverse market move*, i.e. a move (in this case an interest rate fall) that causes a loss for the risk in question.

The interest rate risk capital is now obtained by taking the maximum of the two losses, or in this case choosing the only loss. Should both scenarios lead to a profit the risk capital will be superfluous and therefore be set to 0.

$$\begin{aligned} K^{intr}(t,T,p) &= \max\{L'(t,T,r_T^+), L'(t,T,r_T^-)\} \\ &= \max\{-676.81, 814.06\} \\ &= 814.06 \end{aligned}$$

Mortality risk capital

Order Premium	III office	III office
Interest rate Mortality Surrender	central up central	central down central
Stochastic reserve $\hat{V}_0^{(3)*}$ Difference $-(\hat{V}_0^{(3)} - \hat{V}_0^{(3)*})$ Adverse market move	14 307.51 6.12 a.m.m.	14 295.70 -5.68
Risk capital % of stochastic reserve % of traditional reserve	6.12 0.04% 0.04%	

First the value loss is calculated for the two "shock" levels q_x^+ and q_x^-:

$$\begin{aligned} L'(t,T,q_x^+) &= -(V(t,q_x) - V(t,q_x^+)) \\ &= -(14301.38 - 14307.51) \\ &= 6.12 \end{aligned}$$

$$
\begin{aligned}
L'(t,T,q_x^-) &= -(V(t,q_x) - V(t,q_x^-)) \\
&= -(14301.38 - 14295.70) \\
&= -5.68
\end{aligned}
$$

We then get the mortality risk capital through:

$$
\begin{aligned}
K^{mort}(t,T,p) &= \max\{L'(t,T,q_x^+), L'(t,T,q_x^-)\} \\
&= \max\{6.12, -5.68\} \\
&= 6.12
\end{aligned}
$$

Surrender risk capital

Order Premium	III office	III office
Interest rate Mortality Surrender	central central up	central central down
Stochastic reserve $\hat{V}_0^{(3)*}$	14 394.30	14 232.81
Difference $-(\hat{V}_0^{(3)} - \hat{V}_0^{(3)*})$	92.92	-68.58
Adverse market move	a.m.m.	
Risk capital	92.92	
% of stochastic reserve	0.65%	
% of traditional reserve	0.62%	

Here the value loss for s^+ and s^- becomes:

$$
\begin{aligned}
L'(t,T,s^+) &= -(V(t,s) - V(t,s^+)) \\
&= -(14301.38 - 14394.30) \\
&= 92.92
\end{aligned}
$$

$$
\begin{aligned}
L'(t,T,s^-) &= -(V(t,s) - V(t,s^-)) \\
&= -(14301.38 - 14232.81) \\
&= -68.58
\end{aligned}
$$

We then get the surrender risk capital through:

$$
\begin{aligned}
K^{surr}(t,T,p) &= \max\{L'(t,T,s^+), L'(t,T,s^-)\} \\
&= \max\{92.92, -68.58\} \\
&= 92.92
\end{aligned}
$$

8.7 SCR

Having computed the risk capitals we can now calculate the solvency capital requirement as described in section 7.3. For the market risk we get:

$$\begin{aligned}
\mathrm{SCR_{mkt}} &= \sqrt{\sum_{r\times c} \mathrm{CorrMkt_{r,c}} \cdot \mathrm{Mkt_r} \cdot \mathrm{Mkt_c}} \\
&= \sqrt{\mathrm{CorrMkt_{int,int}} \cdot (\mathrm{Mkt_{int}})^2} \\
&= \sqrt{1 \cdot (K^{intr})^2} \\
&= \sqrt{1 \cdot 814.06^2} \\
&= 814.06
\end{aligned}$$

Because the fund contains only bonds the market risk SCR equals the interest rate risk capital.

For the life underwriting risk we get:

$$\begin{aligned}
\mathrm{SCR_{life}} &= \sqrt{\sum_{r\times c} \mathrm{CorrLife_{r,c}} \cdot \mathrm{Life_r} \cdot \mathrm{Life_c}} \\
&= \sqrt{1 \cdot (K^{mort})^2 + 0 \cdot K^{mort} \cdot K^{surr} + 1 \cdot (K^{surr})^2} \\
&= \sqrt{6.12^2 + 92.92^2} \\
&= 93.12
\end{aligned}$$

We can now compute the BSCR:

$$\begin{aligned}
\mathrm{BSCR} &= \sqrt{\sum_{r\times c} \mathrm{CorrSCR_{r,c}} \cdot \mathrm{SCR_r} \cdot \mathrm{SCR_c}} \\
&= \sqrt{1 \cdot (\mathrm{SCR_{mkt}})^2 + 0.25 \cdot \mathrm{SCR_{mkt}} \cdot \mathrm{SCR_{life}} + 1 \cdot (\mathrm{SCR_{life}})^2} \\
&= \sqrt{814.06^2 + 0.25 \cdot 814.06 \cdot 93.12 + 93.12^2} \\
&= 830.85
\end{aligned}$$

Due to the limited information available about the example contract we cannot calculate the $\mathrm{SCR_{op}}$ so our final result will be a first approximation to the overall SCR. We thus get:

$$\mathrm{SCR} \approx \mathrm{BSCR} = 830.85.$$

We define as *required capital* the amount of capital that the insurance company would need to hold to be able to meet all its commitments. Under the DFM approach the required capital for this example contract is:

$$\text{fair value} + \mathrm{SCR} = 14865.69 + 830.85 = 15696.54.$$

8.8 SM

The traditional Solvency I margin is calculated according to formula (17) as:

$$\text{SM} = 0.04 \cdot 15102.18 + 0.003 \cdot (23403.08 - 15102.18) = 628.99.$$

Under the traditional approach the required capital for this contract is thus:

$$\text{traditional reserve} + \text{SM} = 15102.18 + 628.99 = 15731.17.$$

9 Conclusions

The purpose of this thesis has been to describe the theoretical structure of the DFM internal model and then putting it into practical use by performing a valuation of a participating policy provided by an Italian insurance company. The DFM model has proved to be consistent with the future Solvency II regulations as it provides a market-based valuation of assets and liabilities that accurately takes into account the different risk factors associated with a contract. It is also able to price the minimum guarantees embedded in participating policies; something that is of great importance in Italy, where virtually all insurance policies are of this type.

The practical application of the model has produced the values of the stochastic reserve and of the embedded put and call options. In addition, the financial and technical risk capitals have been computed and integrated in the standard QIS3 model for the calculation of the solvency capital requirement. The traditional reserve has also been calculated according to current Solvency I principles, and by comparing it to the stochastic reserve the value of business in force has been derived. As a measure of the difference between the two valuation approaches the traditional solvency margin has been computed and compared to the SCR. This last result is worth discussing more deeply:

Looking at the required capitals computed in sections 8.7–8.8 the results are very similar, but the margins and their proportions of the respective reserves reveal a greater difference between the two valuation approaches. In this example, the fact that the SCR is higher than the SM indicates that according to the DFM model and the assumptions made the Solvency I margin is not sufficient for covering the risks that the contract is exposed to. However, the traditional reserve is greater than the stochastic reserve, or rather it is overabundant with respect to the fair valued commitments. Thus on the whole the required capital under the traditional approach is sufficient to cover both the commitments and the risks. It is important to note that the valuation performed here generally is extended to a company's whole portfolio of contracts, why the results obtained can only be seen as indicative. In any case, calculation of the required capital using a market-consistent model gives a better measure of a contract or portfolio's risk profile. Since the traditional solvency margin is calibrated solely on the statutory reserve and the sums of risk it is not able to take into account the actual riskiness of a contract.

The DFM model involves more aspects than the ones described in this thesis. One of them concerns the investment strategy for the reference fund. Since this to some degree is discretionary of the insurer and since the fund represents the underlying of the embedded options it follows that the prices of the minimum guarantees can be partly controlled. Finding the most advantageous trading strategy is therefore an important aspect of the DFM approach that together with its more detailed risk analyses would be an interesting issue to study further.

A Itô's lemma

The "total differential" rule states that if $y = f(x,t)$ is a function of the variables x and t and if it has first derivatives, then it's differential can be expressed as the sum of the differentials of the independent variables multiplied by the relative partial derivatives:

$$dy = \frac{\partial f}{\partial t}dt + \frac{\partial f}{\partial x}dx.$$

This rule can be modified to suit the stochastic differential $dZ(t)$:

If $Y = f(Z,t)$ is a function of $Z(t)$ and of time, then the stochastic differential of the process $Y(t)$ is expressed by:

$$dY = \left(\frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial Z^2}\right)dt + \frac{\partial f}{\partial Z}dZ.$$

More generally, if $X(t)$ is a process described by the stochastic differential equation

$$dX = a(X,t)dt + b(X,t)dZ \tag{18}$$

and if we consider the function $Y = f(X,t)$, then we have:

$$dY = \left(\frac{\partial f}{\partial t} + \frac{1}{2}b^2(X,t)\frac{\partial^2 f}{\partial X^2}\right)dt + \frac{\partial f}{\partial X}dX. \tag{19}$$

By inserting expression (18) into expression (19) and regrouping the terms by "dt" and by "dZ", (19) can be written as:

$$dY = \left(\frac{\partial f}{\partial t} + a(X,t)\frac{\partial f}{\partial X} + \frac{1}{2}b^2(X,t)\frac{\partial^2 f}{\partial X^2}\right)dt + b(X,t)\frac{\partial f}{\partial X}dZ.$$

This result is known as *Itô's lemma*. Based on this rule it is possible to describe the "probabilistic dynamics" of a diffusion processes. For example, if the dynamics of the process $Y(t)$ is described by the stochastic differential equation

$$dY = a_Y(Y,t)dt + b_Y(Y,t)dZ,$$

then the dependence $Y = f(X,t)$ implies that the coefficients a_Y and b_Y too are functions of X and t and Itô's lemma supplies the forms of these functions:

$$a_Y(X,t) = \frac{\partial f}{\partial t} + a(X,t)\frac{\partial f}{\partial X} + \frac{1}{2}b^2(X,t)\frac{\partial^2 f}{\partial X^2}$$

and

$$b_Y(X,t) = b(X,t)\frac{\partial f}{\partial X}.$$

B Additional valuation results

B.1 Second order pure premium valuation

Time						
Year	0	1	2	3	4	5
Values						
Death benefit	-	23403.08	23403.08	23403.08	23403.08	23403.08
Life benefit	-	0.00	0.00	0.00	0.00	23403.08
Surrender value	-	16170.01	17904.61	19687.61	21520.07	23403.08
Pure premium (-)	-	14482.76	14482.76	14482.76	14482.76	0.00
Probabilities						
at beginning of year (conditional)						
Death	-	0.00362	0.00408	0.00455	0.00513	0.00573
Life	-	0.99638	0.99592	0.99545	0.99487	0.99427
Surrender	-	0.00000	0.00000	0.00000	0.00000	0.00000
Premium payment	-	1.00000	1.00000	1.00000	1.00000	1.00000
at time 0 (non conditional)						
Death	-	0.00362	0.00406	0.00452	0.00507	0.00563
Life	1.00000	0.99638	0.99232	0.98780	0.98273	0.97710
Surrender	-	0.00000	0.00000	0.00000	0.00000	0.00000
Premium payment	-	0.99638	0.99232	0.98780	0.98273	0.97710
Projected cash flows						
Death	-	84.67	95.07	105.71	118.61	131.78
Life	-	0.00	0.00	0.00	0.00	22867.24
Surrender	-	0.00	0.00	0.00	0.00	0.00
Premium payment	-	14430.37	14371.53	14306.12	14232.71	0.00
Rates						
Technical rate		4.00%	4.00%	4.00%	4.00%	4.00%
Guaranteed rate besides technical		0.00%	0.00%	0.00%	0.00%	0.00%
Market spot rate		4.68%	4.53%	4.51%	4.52%	4.53%
Factors						
of total amount						
Guaranteed rate besides technical	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
discount						
Technical rate	1.00000	0.96154	0.92456	0.88900	0.85480	0.82193
Market rate	1.00000	0.95526	0.91525	0.87602	0.83801	0.80115

valuation						
Premium	1.00000	0.95873	0.91929	0.88137	0.84518	0.81039
Benefits	1.00000	0.95873	0.91929	0.88137	0.84518	0.81039
Surrender	1.00000	0.95873	0.91929	0.88137	0.84518	0.81039
valuation (contract without guarantee)						
Premium	1.00000	0.95295	0.90810	0.86528	0.82469	0.78590
Benefits	1.00000	0.95295	0.90810	0.86528	0.82469	0.78590
Surrender	1.00000	0.95295	0.90810	0.86528	0.82469	0.78590
Valuations						
traditional with technical rate						
Pure premium (-)		13875.35	13287.29	12718.09	12166.18	0.00
Death benefit		81.41	87.90	93.97	101.39	108.32
Life benefit		0.00	0.00	0.00	0.00	18795.20
Surrender benefit		0.00	0.00	0.00	0.00	0.00
Reserve	71315.10					
traditional with market rate						
Pure premium (-)		13784.69	13153.51	12532.47	11927.22	0.00
Death benefit		80.88	87.01	92.60	99.40	105.58
Life benefit		0.00	0.00	0.00	0.00	18320.15
Surrender benefit		0.00	0.00	0.00	0.00	0.00
Reserve	70183.51					
fair value						
Pure premium (-)		13834.83	13211.56	12608.98	12029.20	0.00
Death benefit		81.17	87.40	93.17	100.25	106.80
Life benefit		0.00	0.00	0.00	0.00	18531.36
Surrender benefit		0.00	0.00	0.00	0.00	0.00
Reserve	70684.70					
fair value only guaranteed benefits						
Pure premium (-)		13784.69	13153.51	12532.47	11927.22	0.00
Death benefit		80.88	87.01	92.60	99.40	105.58
Life benefit		0.00	0.00	0.00	0.00	18320.15
Surrender benefit		0.00	0.00	0.00	0.00	0.00
Reserve	70183.51					
fair value without guarantee						
Pure premium (-)		13801.40	13151.73	12531.64	11943.81	0.00
Death benefit		80.68	86.33	91.47	97.82	103.57

Life benefit		0.00	0.00	0.00	0.00	17971.46
Surrender benefit		0.00	0.00	0.00	0.00	0.00
Reserve	69859.90					

B.2 Third order pure premium valuation

Time						
Year	0	1	2	3	4	5
Values						
Death benefit	-	23403.08	23403.08	23403.08	23403.08	23403.08
Life benefit	-	0.00	0.00	0.00	0.00	23403.08
Surrender value	-	15836.08	17680.98	19577.67	21527.28	23530.98
Pure premium (-)	-	-1184.42	-1184.42	-1184.42	-1184.42	0.00
Probabilities						
at beginning of year (conditional)						
Death	-	0.00362	0.00408	0.00455	0.00513	0.00573
Life	-	0.99638	0.99592	0.99545	0.99487	0.99427
Surrender	-	0.04200	0.04200	0.04200	0.04200	0.04200
Premium payment	-	1.00000	1.00000	1.00000	1.00000	1.00000
at time 0 (non conditional)						
Death	-	0.00362	0.00389	0.00415	0.00446	0.00474
Life	1.00000	0.95453	0.91071	0.86847	0.82770	0.78836
Surrender	-	0.04185	0.03993	0.03809	0.03631	0.03460
Premium payment	-	0.95453	0.91071	0.86847	0.82770	0.78836
Projected cash flows						
Death	-	84.67	91.08	97.01	104.28	110.99
Life	-	0.00	0.00	0.00	0.00	18450.07
Surrender	-	662.71	706.08	745.74	781.73	814.14
Premium payment	-	-1130.57	-1078.66	-1028.64	-980.35	0.00
Rates						
Technical rate		4.00%	4.00%	4.00%	4.00%	4.00%
Guaranteed rate besides technical		0.00%	0.00%	0.00%	0.00%	0.00%
Market spot rate		4.68%	4.53%	4.51%	4.52%	4.53%
Factors						
of total amount						
Guaranteed rate besides technical	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
discount						

Technical rate	1.00000	0.96154	0.92456	0.88900	0.85480	0.82193
Market rate	1.00000	0.95526	0.91525	0.87602	0.83801	0.80115
valuation						
Premium	1.00000	0.95873	0.91929	0.88137	0.84518	0.81039
Benefits	1.00000	0.95873	0.91929	0.88137	0.84518	0.81039
Surrender	1.00000	0.95873	0.91929	0.88137	0.84518	0.81039
valuation (contract without guarantee)						
Premium	1.00000	0.95295	0.90810	0.86528	0.82469	0.78590
Benefits	1.00000	0.95295	0.90810	0.86528	0.82469	0.78590
Surrender	1.00000	0.95295	0.90810	0.86528	0.82469	0.78590
Valuations						
traditional with technical rate						
Pure premium (-)		-1087.09	-997.28	-914.45	-838.00	0.00
Death benefit		81.41	84.21	86.24	89.14	91.23
Life benefit		0.00	0.00	0.00	0.00	15164.62
Surrender benefit		637.22	652.81	662.96	668.22	669.16
Reserve	15050.39					
traditional with market rate						
Pure premium (-)		-1079.98	-987.24	-901.11	-821.55	0.00
Death benefit		80.88	83.36	84.99	87.39	88.92
Life benefit		0.00	0.00	0.00	0.00	14781.33
Surrender benefit		633.06	646.24	653.28	655.10	652.25
Reserve	14656.91					
fair value						
Pure premium (-)		-1083.91	-991.60	-906.61	-828.57	0.00
Death benefit		81.17	83.73	85.50	88.14	89.95
Life benefit		0.00	0.00	0.00	0.00	14951.74
Surrender benefit		635.36	649.09	657.27	660.70	659.77
Reserve	14831.73					
fair value only guaranteed benefits						
Pure premium (-)		-1079.98	-987.24	-901.11	-821.55	0.00
Death benefit		80.88	83.36	84.99	87.39	88.92
Life benefit		0.00	0.00	0.00	0.00	14781.33
Surrender benefit		633.06	646.24	653.28	655.10	652.25
Reserve	14656.91					
fair value without guarantee						

Pure premium (-)		-1128.70	-1075.57	-1024.86	-976.78	0.00
Death benefit		80.68	82.71	83.94	86.00	87.23
Life benefit		0.00	0.00	0.00	0.00	14499.99
Surrender benefit		631.53	641.19	645.27	644.68	639.84
Reserve	13917.16					

B.3 Third order office premium valuation

Time Year	0	1	2	3	4	5
Values						
Death benefit	-	23403.08	23403.08	23403.08	23403.08	23403.08
Life benefit	-	0.00	0.00	0.00	0.00	23403.08
Surrender value	-	15836.08	17680.98	19577.67	21527.28	23530.98
Office premium (-)	-	-1355.94	-1355.94	-1355.94	-1355.94	0.00
Probabilities						
at beginning of year (conditional)						
Death	-	0.00362	0.00408	0.00455	0.00513	0.00573
Life	-	0.99638	0.99592	0.99545	0.99487	0.99427
Surrender	-	0.04200	0.04200	0.04200	0.04200	0.04200
Premium payment	-	1.00000	1.00000	1.00000	1.00000	1.00000
at time 0 (non conditional)						
Death	-	0.00362	0.00389	0.00415	0.00446	0.00474
Life	1.00000	0.95453	0.91071	0.86847	0.82770	0.78836
Surrender	-	0.04185	0.03993	0.03809	0.03631	0.03460
Premium payment	-	0.95453	0.91071	0.86847	0.82770	0.78836
Projected cash flows						
Death	-	84.67	91.08	97.01	104.28	110.99
Life	-	0.00	0.00	0.00	0.00	18450.07
Surrender	-	662.71	706.08	745.74	781.73	814.14
Premium payment	-	-1294.29	-1234.86	-1177.59	-1122.31	0.00
Rates						
Technical rate		4.00%	4.00%	4.00%	4.00%	4.00%
Guaranteed rate besides technical		0.00%	0.00%	0.00%	0.00%	0.00%
Market spot rate		4.68%	4.53%	4.51%	4.52%	4.53%
Factors						
of total amount						
Guaranteed rate						

besides technical	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
discount						
Technical rate	1.00000	0.96154	0.92456	0.88900	0.85480	0.82193
Market rate	1.00000	0.95526	0.91525	0.87602	0.83801	0.80115
valuation						
Premium	1.00000	0.95873	0.91929	0.88137	0.84518	0.81039
Benefits	1.00000	0.95873	0.91929	0.88137	0.84518	0.81039
Surrender	1.00000	0.95873	0.91929	0.88137	0.84518	0.81039
valuation (contract without guarantee)						
Premium	1.00000	0.95295	0.90810	0.86528	0.82469	0.78590
Benefits	1.00000	0.95295	0.90810	0.86528	0.82469	0.78590
Surrender	1.00000	0.95295	0.90810	0.86528	0.82469	0.78590
Valuations						
traditional with technical rate						
Office premium (-)		-1244.51	-1141.70	-1046.88	-959.36	0.00
Death benefit		81.41	84.21	86.24	89.14	91.23
Life benefit		0.00	0.00	0.00	0.00	15164.62
Surrender benefit		637.22	652.81	662.96	668.22	669.16
Reserve	14494.78					
traditional with market rate						
Office premium (-)		-1236.38	-1130.21	-1031.60	-940.51	0.00
Death benefit		80.88	83.36	84.99	87.39	88.92
Life benefit		0.00	0.00	0.00	0.00	14781.33
Surrender benefit		633.06	646.24	653.28	655.10	652.25
Reserve	14108.09					
fair value						
Office premium (-)		-1240.87	-1135.19	-1037.90	-948.56	0.00
Death benefit		81.17	83.73	85.50	88.14	89.95
Life benefit		0.00	0.00	0.00	0.00	14951.74
Surrender benefit		635.36	649.09	657.27	660.70	659.77
Reserve	14279.89					
fair value only guaranteed benefits						
Office premium (-)		-1236.38	-1130.21	-1031.60	-940.51	0.00
Death benefit		80.88	83.36	84.99	87.39	88.92
Life benefit		0.00	0.00	0.00	0.00	14781.33
Surrender benefit		633.06	646.24	653.28	655.10	652.25
Reserve	14108.09					

fair value without guarantee						
Office premium (-)		-1292.15	-1231.32	-1173.27	-1118.23	0.00
Death benefit		80.68	82.71	83.94	86.00	87.23
Life benefit		0.00	0.00	0.00	0.00	14499.99
Surrender benefit		631.53	641.19	645.27	644.68	639.84
Reserve	13308.10					

C Mortality tables

C.1 The SI81 tables

Male				**Female**		
Age	$\mathbf{L_x}$	$\mathbf{q_x}(\cdot\mathbf{1000})$		**Age**	$\mathbf{L_x}$	$\mathbf{q_x}(\cdot\mathbf{1000})$
0	100000	15,33		0	100000	12,04
1	98467	0,77		1	98796	0,71
2	98391	0,53		2	98726	0,49
3	98339	0,40		3	98678	0,32
4	98300	0,34		4	98646	0,25
5	98267	0,33		5	98621	0,23
6	98235	0,31		6	98598	0,21
7	98205	0,30		7	98577	0,22
8	98176	0,30		8	98555	0,20
9	98147	0,28		9	98535	0,17
10	98120	0,28		10	98518	0,17
11	98093	0,27		11	98501	0,18
12	98067	0,31		12	98483	0,18
13	98037	0,40		13	98465	0,22
14	97998	0,52		14	98443	0,25
15	97947	0,69		15	98418	0,26
16	97879	0,90		16	98392	0,28
17	97791	1,03		17	98364	0,30
18	97690	1,14		18	98334	0,34
19	97579	1,15		19	98301	0,37
20	97467	1,10		20	98265	0,39
21	97360	1,09		21	98227	0,39
22	97254	1,09		22	98189	0,35
23	97148	1,05		23	98155	0,35
24	97046	1,04		24	98121	0,35
25	96945	1,01		25	98087	0,39
26	96847	0,98		26	98049	0,42
27	96752	0,98		27	98008	0,42
28	96657	0,97		28	97967	0,44

Male				**Female**		
Age	$\mathbf{L_x}$	$\mathbf{q_x}$(·**1000**)		**Age**	$\mathbf{L_x}$	$\mathbf{q_x}$(·**1000**)
29	96563	0,98		29	97924	0,45
30	96468	0,98		30	97880	0,49
31	96373	1,04		31	97832	0,52
32	96273	1,07		32	97781	0,54
33	96170	1,08		33	97728	0,56
34	96066	1,17		34	97673	0,65
35	95954	1,22		35	97610	0,71
36	95837	1,34		36	97541	0,80
37	95709	1,50		37	97463	0,90
38	95565	1,70		38	97375	0,96
39	95403	1,88		39	97282	1,05
40	95224	2,09		40	97180	1,15
41	95025	2,29		41	97068	1,23
42	94807	2,53		42	96949	1,34
43	94567	2,79		43	96819	1,49
44	94303	3,18		44	96675	1,61
45	94003	3,63		45	96519	1,86
46	93662	4,14		46	96339	2,06
47	93274	4,69		47	96141	2,27
48	92837	5,22		48	95923	2,49
49	92352	5,74		49	95684	2,71
50	91822	6,43		50	95425	2,86
51	91232	7,21		51	95152	3,18
52	90574	8,09		52	94849	3,48
53	89841	9,00		53	94519	3,89
54	89032	10,01		54	94151	4,31
55	88141	11,07		55	93745	4,68
56	87165	12,28		56	93306	5,10
57	86095	13,42		57	92830	5,59
58	84940	14,54		58	92311	6,06
59	83705	16,25		59	91752	6,81
60	82345	17,56		60	91127	7,53
61	80899	19,05		61	90441	8,35
62	79358	20,51		62	89686	9,14
63	77730	22,02		63	88866	10,05
64	76018	23,98		64	87973	10,96
65	74195	26,57		65	87009	12,08
66	72224	28,99		66	85958	13,45
67	70130	31,74		67	84802	14,94
68	67904	34,55		68	83535	16,56
69	65558	37,87		69	82152	18,54
70	63075	42,14		70	80629	21,11

Male				Female		
Age	**L_x**	**$q_x(\cdot 1000)$**		**Age**	**L_x**	**$q_x(\cdot 1000)$**
71	60417	46,63		71	78927	23,96
72	57600	51,77		72	77036	27,25
73	54618	57,16		73	74937	30,87
74	51496	62,84		74	72624	34,95
75	48260	68,88		75	70086	39,42
76	44936	76,29		76	67323	44,86
77	41508	83,36		77	64303	50,73
78	38048	90,75		78	61041	57,13
79	34595	98,77		79	57554	63,97
80	31178	107,58		80	53872	71,39
81	27824	117,67		81	50026	79,50
82	24550	127,86		82	46049	88,49
83	21411	138,85		83	41974	98,30
84	18438	150,61		84	37848	109,02
85	15661	163,21		85	33722	120,75
86	13105	176,73		86	29650	133,49
87	10789	191,03		87	25692	147,36
88	8728	206,35		88	21906	162,42
89	6927	222,75		89	18348	178,77
90	5384	240,16		90	15068	196,44
91	4091	258,37		91	12108	215,56
92	3034	277,85		92	9498	236,05
93	2191	298,49		93	7256	258,13
94	1537	320,10		94	5383	281,81
95	1045	343,54		95	3866	307,04
96	686	367,35		96	2679	334,08
97	434	391,71		97	1784	362,11
98	264	416,67		98	1138	392,79
99	154	448,05		99	691	424,02
100	85	470,59		100	398	457,29
101	45	511,11		101	216	490,74
102	22	500,00		102	110	527,27
103	11	545,45		103	52	557,69
104	5			104	23	

C.2 The SI92 tables

Male				Female		
Age	**L_x**	**$q_x(\cdot 1000)$**		**Age**	**L_x**	**$q_x(\cdot 1000)$**
0	100000	8,79		0	100000	6,91
1	99121	0,45		1	99309	0,44
2	99076	0,33		2	99265	0,30

Male				**Female**		
Age	$\mathbf{L_x}$	$\mathbf{q_x}(\cdot\mathbf{1000})$		**Age**	$\mathbf{L_x}$	$\mathbf{q_x}(\cdot\mathbf{1000})$
3	99043	0,25		3	99235	0,22
4	99018	0,21		4	99213	0,18
5	98997	0,20		5	99195	0,15
6	98977	0,20		6	99180	0,13
7	98957	0,20		7	99167	0,13
8	98937	0,19		8	99154	0,11
9	98918	0,19		9	99143	0,12
10	98899	0,18		10	99131	0,13
11	98881	0,17		11	99118	0,14
12	98864	0,21		12	99104	0,15
13	98843	0,25		13	99089	0,17
14	98818	0,37		14	99072	0,19
15	98781	0,55		15	99053	0,22
16	98727	0,74		16	99031	0,25
17	98654	0,88		17	99006	0,28
18	98567	0,98		18	98978	0,29
19	98470	1,05		19	98949	0,31
20	98367	1,08		20	98918	0,31
21	98261	1,13		21	98887	0,30
22	98150	1,18		22	98857	0,29
23	98034	1,18		23	98828	0,31
24	97918	1,22		24	98797	0,33
25	97799	1,25		25	98764	0,38
26	97677	1,31		26	98726	0,42
27	97549	1,36		27	98685	0,45
28	97416	1,44		28	98641	0,47
29	97276	1,51		29	98595	0,50
30	97129	1,54		30	98546	0,53
31	96979	1,59		31	98494	0,55
32	96825	1,57		32	98440	0,57
33	96673	1,53		33	98384	0,59
34	96525	1,51		34	98326	0,60
35	96379	1,50		35	98267	0,63
36	96234	1,50		36	98205	0,67
37	96090	1,53		37	98139	0,72
38	95943	1,61		38	98068	0,77
39	95789	1,65		39	97992	0,84
40	95631	1,75		40	97910	0,89
41	95464	1,89		41	97823	0,97
42	95284	2,07		42	97728	1,06
43	95087	2,25		43	97624	1,17
44	94873	2,48		44	97510	1,29

Male				**Female**		
Age	$\mathbf{L_x}$	$\mathbf{q_x}(\cdot\mathbf{1000})$		**Age**	$\mathbf{L_x}$	$\mathbf{q_x}(\cdot\mathbf{1000})$
45	94638	2,69		45	97384	1,43
46	94383	3,03		46	97245	1,60
47	94097	3,45		47	97089	1,79
48	93772	3,85		48	96915	1,97
49	93411	4,23		49	96724	2,13
50	93016	4,58		50	96518	2,27
51	92590	4,97		51	96299	2,46
52	92130	5,48		52	96062	2,70
53	91625	6,18		53	95803	3,00
54	91059	6,90		54	95516	3,28
55	90431	7,77		55	95203	3,57
56	89728	8,68		56	94863	3,92
57	88949	9,68		57	94491	4,34
58	88088	10,81		58	94081	4,79
59	87136	12,06		59	93630	5,27
60	86085	13,41		60	93137	5,74
61	84931	14,86		61	92602	6,26
62	83669	16,42		62	92022	6,90
63	82295	18,12		63	91387	7,65
64	80804	19,99		64	90688	8,50
65	79189	22,04		65	89917	9,45
66	77444	24,20		66	89067	10,46
67	75570	26,28		67	88135	11,57
68	73584	28,59		68	87115	12,99
69	71480	31,03		69	85983	14,60
70	69262	34,23		70	84728	16,49
71	66891	36,99		71	83331	18,37
72	64417	40,30		72	81800	20,43
73	61821	43,76		73	80129	22,70
74	59116	47,87		74	78310	25,54
75	56286	52,62		75	76310	28,87
76	53324	58,15		76	74107	32,78
77	50223	63,34		77	71678	37,15
78	47042	69,47		78	69015	42,02
79	43774	76,69		79	66115	47,74
80	40417	84,84		80	62959	54,24
81	36988	94,84		81	59544	62,24
82	33480	105,05		82	55838	71,03
83	29963	115,98		83	51872	80,87
84	26488	127,64		84	47677	91,34
85	23107	140,52		85	43322	103,13
86	19860	153,78		86	38854	116,31

Male				**Female**		
Age	$\mathbf{L_x}$	$\mathbf{q_x}(\cdot\mathbf{1000})$		**Age**	$\mathbf{L_x}$	$\mathbf{q_x}(\cdot\mathbf{1000})$
87	16806	166,55		87	34335	129,87
88	14007	179,62		88	29876	144,23
89	11491	193,19		89	25567	159,70
90	9271	207,96		90	21484	176,60
91	7343	227,70		91	17690	200,62
92	5671	244,58		92	14141	221,41
93	4284	262,61		93	11010	243,87
94	3159	281,42		94	8325	268,23
95	2270	301,32		95	6092	294,32
96	1586	321,56		96	4299	322,40
97	1076	343,87		97	2913	352,56
98	706	366,86		98	1886	383,88
99	447	391,50		99	1162	418,24
100	272	415,44		100	676	452,66
101	159	440,25		101	370	489,19
102	89	471,91		102	189	529,10
103	47	489,36		103	89	561,80
104	24	541,67		104	39	615,38
105	11	545,45		105	15	666,67
106	5	600,00		106	5	600,00
107	2	500,00		107	2	500,00
108	1	1000,00		108	1	1000,00
109	0			109		

References

[1] Alm, E., Andersson, G., von Bahr, B. & Martin-Löf, A. (2006), *Livförsäkringsmatematik II*. Stockholm: Svenska försäkringsföreningen.

[2] Björk, T. (1998), *Arbitrage theory in continuous time*. Oxford: Oxford University Press.

[3] BusinessDictionary.com, "perfect competition". Retrieved August 23, 2008, from BusinessDictionary.com website: http://www.businessdictionary.com/definition/perfect-competition.html.

[4] CAS Task Force (2000), "White paper on fair valuing property/casualty insurance liabilities". *Casualty Actuarial Society*.

[5] CEA & Towers Perrin Tillinghast (2006), "Solvency II introductory guide". *European Insurance Federation*.

[6] CEIOPS (2007a), "QIS3 technical specifications". *Committee of European Insurance and Occupational Pensions Supervisors*.

[7] CEIOPS (2007b), "QIS4 technical specifications". *Committee of European Insurance and Occupational Pensions Supervisors*.

[8] Castellani, G., De Felice, M. & Moriconi, F. (2006), *Manuale di finanza III. Modelli stocastici e contratti derivati*. Bologna: il Mulino.

[9] Cox, J.C., Ingersoll, J.E. & Ross, S.A. (1985), "A theory of the term structure of interest rates". *Econometrica* **53**, pp.384–406.

[10] De Felice, M. & Moriconi, F. (2005), "Market based tools for managing the life insurance company". *Astin Bulletin* **35**, 1.

[11] Duffie, D. (1992), *Dynamic asset pricing theory*. Princeton: Princeton University Press.

[12] ISVAP (2008), *Regolamento n. 19 del 14 marzo 2008*.