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The Pricing of Catastrophe Cover In Life Reinsurance

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Abstract

What is the correct price of a catastrophe cover in life reinsurance? During a review of the current standard model due to Strickler (1960) we find that this model has some serious shortcomings. We therefore present a new model for the pricing of catastrophe excess of loss cover(Cat XL). The new model for annual claim cost C is based on a compound Poisson process of catastrophe costs. To evaluate the distribution of the cost of each catastrophe we use the Peaks Over Threshold model for the total number of lost lives in each catastrophe and the beta binomial model for the proportion of these corresponding to customers of the insurance company. To be able to estimate the parameters of the model, two data sets with catastrophe data were collected and compiled, one international set with catastrophes claiming at least 20 lives and one Swedish list of accidents claiming at least 4 lives. Fitting the new model to data, we find the fit to be good. We also present how to extrapolate data, thus being able to draw conclusions about catastrophes claiming less than 20 lives. Finally we give the price of a Cat XL contract and perform a sensitivity analysis of how the parameters affect $E[C]$ and $\mathrm{SD}(C)$ and thus the price.

KEY WORDS: Catastrophe excess of loss, life reinsurance, catastrophe model, catastrophe data, CAT XL.

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1 Introduction

How should you price catastrophe cover in life reinsurance? In this paper we will present the current pricing model by Strickler (1960) and then introduce a new model. For the reader who is unfamiliar with reinsurance we start with a brief overview of the subject and then present the concept of catastrophe cover in more detail.

1.1 Reinsurance

Insurance companies sell risk protection, i.e. for a premium they take over the financial risk of unknown events that might happen to their clients. Some risks are so large that an insurance company does not want the carry the whole risk for itself. A part of the risk can then be *ceded*, i.e. passed on, to another insurance company who is willing to share the risk for a premium. The part of the risk that is kept is called the *retention*. The insurance company ceding the risk is called the *cedent* or *direct company* and the company that takes over the risk is called the *reinsurer*. For this purpose there are specialized reinsurance companies, often recognized by having 'Re' in their name.

Traditionally the three main purposes with reinsurance are:

1. To protect the cedent from catastropic events:
   For example the huge claim that can be the result of a storm.
2. To even out the business results over the years:
   Minimizing the effects of the random variation in claim experience.
3. To gain capacity for writing new business:
   In every sold policy there is a potential claim, so every policy in force will be treated as a debt on the balance sheet, thus requiring an asset to balance. Writing new business can be costly, with commissions to brokers and work by underwriters taking much of the first premium. Expanding the business will therefore demand a lot of capital and by ceding away a part of the risk the demand for capital on the balance sheet is lessened to the same extent.

Modern life reinsurance companies will also provide life companies with services like medical underwriting/risk evaluation, product development and financing of new business.

There are two main forms or principles for reinsurance business, proportional and non-proportional. The following rely to a large extent on A practical guide to reassurance and Heidenfors (1989).

1.2 Proportional reinsurance

The principle of proportional reinsurance is that the reinsurer takes a proportion of the risk premium and then is liable to pay the same proportion of all claims arising from the risk in question. Proportional reinsurance is typical for life reinsurance, but less common on the non-life side. The different proportional reinsurance types are as follows.

1.2.1 Facultative reinsurance

Facultative means optional, i.e. the ceding company chooses to reinsure a particular (large) risk. The reinsurer is not obliged to offer terms and if it does, there is no obligation for the direct company to accept them. This can be contrasted with treaty arrangements where there are obligations assumed by both the ceding company and the reinsurer.

1.2.2 Automatic treaty reinsurance

A treaty is a binding agreement between a ceding company and a reinsurer for the automatic reinsurance of a defined category of business. This is advantageous to the ceding company in that it streamlines the processing of new business, provides security of reinsurance cover, and reduces reinsurance costs. The streamlining of new business processing is important on simple cost grounds as the less work that is required to administer reinsurance the lower the costs and the more competitive the cedent can set its premiums. A further element of competitiveness can be obtained by the cedent having the ability to accept many cases above its own retention (insurance companies have their own retention, the maximal sum of risk per contract they are allowed to keep for themselves) without the need to refer to the reinsurer.

1.2.3 Quota share

The simplest treaty form of proportional reinsurance is called a quota share (QS) arrangement. Under this method, the ceding company retains a fixed proportion of every policy and the rest goes to the reinsurer. The split may be for example 70/30 where the cedent retains 70% of the business and the reinsurer takes 30%. Quota share reinsurance is the most straightforward for administration purposes. Usually the ceding company will have a fixed overall maximum retention and will need to reassure all of the excess above this limit i.e. a surplus reassurance arrangement will sit above the quota share treaty.

1.2.4 Surplus

For individual life business, most treaties are constructed to provide surplus reinsurance. This means that the ceding company retains fully all policies which fall below its retention level and reinsures those amounts which exceed this limit. In this manner the reinsured proportion for different policies under the same treaty can vary between 0-99%. For example, under a treaty with retention 10 000 three different policies with sum of risk 5 000, 20 000 and 40 000 will be reinsured to 0%, 50% and 75% respectivley.

1.3 Non-proportional reinsurance

While proportional reinsurance focus on the policies, non-proportional reinsurance is focused on claim amounts. Non-proportional reinsurance offers, as we will see, a cost effective protection against large claims.

1.3.1 Excess of loss

With an excess of loss (XL) contract the reinsurer will cover claim costs exceeding an excess point, and up to a certain limit. If a claim exceeds also this limit, the excess spills over, back to the cedent. (Unlimited cover is not an attractive prospect.) The XL contract can cover a whole portfolio or only an individual risk.

For example, a cover might be 400 000 excess of 100 000 (henceforth denoted in units of thousands, as 400 xs 100). Under this contract a claim of x on the cedent will result in the payment of $\min(\max(x - 100, 0), 400)$ by the reinsurer.

For large covers, it is common to divide the cover in different layers, so that diferent reinsurers can take different layers. A cedent who wants a cover of 4 500 excess of 500 might have trouble finding a reinsurer willing to take such a large exposure. By dividing the cover into three layers, 1^{st} 500 xs 500, 2^{nd} 1500 xs 1000 and 3^{rd} 2500 xs 2500 the problem may be solved by writing contracts with three different reinsurers, and the total cover will still be 4500 xs 500.

Consider for example a cedent with a policy that might result in a claim of 200, but the cedent wants to limit its maximum liability to 100. This can be accomplished with a 50% QS but that will mean giving away 50% of the premium and splitting an possible smaller claim as well. A 100 xs 100 XL contract will give the desired protection, and maybe cost only 10% of the premium.

Note the conceptual difference between life and non-life insurance; a life policy has a predetermined, limited risksum, while in non-life, the claim distribution is often open-ended, i.e. there is no known limit to the possible claim size. So if in the above example the policy is a life policy, paying 200 in the event of the insured's death, the XL contract would not have been cheaper than the QS, since a claim always would cost 200.

The price of an XL contract is a matter of negotiation between cedent and reinsurer. Depending on what type of business is covered, enough data for dependable statistics can be hard to get, so the actuary can have a hard time finding an actuarially motivated price. A good stomach feeling, as well a sense for the reinsuranse market, is advised.

1.3.2 Stop-loss

This type of reinsurance, also sometimes called aggregate excess of loss, is to provide a protection against random fluctuations in aggregated claims experience over a time period, which would cause a loss to be reported.

Under a stop loss protection, it is necessary to define an expected level of claims. This means a predicted figure based on assumptions about the nature of the risks being carried by the direct company or based on its past claims experience. The next figure to be decided is the priority level. This is the proportion of the expected claims which will be paid by the ceding company. Claims above the priority level will be paid by the reinsurer up to an agreed limit.

In some cases, the ceding company will continue to participate in claims above the priority level. This is favored by reinsurers as it gives the ceding company a continuing financial interest in a good claim administration. Note that a stop loss cover nearly always relates to amounts of claims in total and not to numbers of claims. For this reason, there will be a limit on the amount of any one claim which can be taken into account in the stop loss calculation. For protection above that amount, the ceding company must turn to other reinsurance i.e. XL cover.

Stop-loss is uncommon in practice within the life reinsurance business.

1.4 Catastrophe cover

Since catastrophe cover is the main topic of this article it will get a more detailed presentation.

1.4.1 Motivation

A basic assumption in life insurance is that lives are independent. That is also often the case, but by considering events like aircraft accidents, buses plunging into rivers, floods and earthquakes, it is easy to see that there are exceptions to this rule. Such *catastrophic events* claiming many lives can have a severe impact on a life company. To protect itself from the consequenses of a catastrophe, the life company can buy a *catastrophe excess of loss cover* (Cat XL) for its portfolio.

One example of a catastrophe, in 1989 when a Norwegian charter aircraft from Partnair disapeared over Skagerack, an accident that claimed 55 lives. All passengers were from the same company, and the life company that had the group life policy as well collective pension plan faced a claim of 36MNOK (4MEUR). The total (life) claim amount was 47MNOK. Thanks to catastrophe covers the Norwegian life companies received 35MNOK from the reinsurers.

1.4.2 The Cat XL contract

The catastrophe excess of loss cover provides the ceding company with protection against the consequences of a catastrophic event. To be more precise:

If M or more persons insured by the ceding company lose their lives as a result of a single event and if the corresponding aggregate net retention (the part that is not ceded under another reinsurance contract) payable by the ceding company exceeds the amount S, the excess will be paid by the reinsurer, with the understanding that the maximum amount payable by the reinsurer in respect of each such event does not exceed a specified amount L, i.e. we have a L xs S XL contract.

The definition of a claim arising from an event clearly needs careful wording. Usually there is an ‘hours clause’ which states that claims are deemed to arise from the same event only if they occur within a period of 72 hours of the event taking place. In the case of a storm or an earthquake, where it may be difficult to define the exact time of occurrence of the event, the life company is usually allowed to choose its own starting point for the agreed time limit, so long as it bears a reasonable comparison with the actual occurrences. Often the time period will be longer for natural occurrences than for accidents.

How many deaths constitute a catastrophe? The lives of a couple living and traveling together, insured by same company, are not independent. Given that there is a car accident or fire, there is a relatively high risk that they both get killed. This situation is so common that it is something a life company has to expect. However, more than two lives lost in a single event is often considered to be catastrophic so M is typically chosen between 3 and 5. The retention S in a Cat XL contract is by the same argument often chosen to be at least twice the retention the cedent has in its individual life surplus contract. In this way the reinsurer starts to pay after a minimum of two 'full' claims. The choice of M and S ultimately depends on the cedent's attitude to risk. Observe that for a life company with a high retention for the individual life cover, the loss of four or five insured lives, with small sum insureds, in a single accident might not result in a total claim amount that exceeds twice the own retention, and therefore will not trigger a $M = 3$, $S = 2$ Cat XL contract for example.

2 Strickler's model

2.1 Overwiew

The first paper that addressed the issue of pricing catastrophe life reinsurance was an article by Strickler (1960). His results, with some minor changes and additions, are still used today. We therefore present his article.

2.2 The model

Consider a Cat XL contract as defined above. The reinsurer's liability will depend on two factors, (a) the frequency distribution of events causing the loss of M or more lives and (b) the distribution of retained sums at risk in in the portfolio of the ceding company.

2.2.1 Distribution of sum at risk

Turning to (b), in practice it is often not possible to get the exact distribution from the cedent, but the average sum at risk is known. Let Z denote the sum at risk of one single catastrophe, expressed as a percentage of the average sum at risk. Let further w_n be the probability density function of Z conditional on the event n lives are lost. Strickler states that the exponential distribution with mean one,

$$w_1(z) = e^{-z} \quad z > 0$$

is a good approximation when $n = 1$.

The function w_n is obtained by means of convolution, reiterated $(n-1)$ times of w_1. This yields a gamma distribution $\Gamma(n, 1)$ for Z, in other words w_n satisfies the equation

$$w_n(z) = \int_0^z w_{(n-1)}(\xi)\, w_1(1)(z-\xi)\, dz \;=\; e^{-z}\frac{z^{n-1}}{(n-1)!} \tag{1}$$

2.2.2 Frequency of large accidents

Turning to the frequency distribution of catastrophic events, the first problem is to find data. Strickler found that the Statistical Bulletin of the Metropolitan Life Insurance Company in New York had supplied summaries of the accidents in the US which had claimed five or more lives for the period 1946-1950.

The annual number of deaths for each million of population resulting from accidents claiming n or more lives could then be approximated by the graduating function

$$A(n) = 8 \cdot 100^{1/n} \cdot n^{-1/3} \tag{2}$$

$A(1)$ shows the normal rate of accident mortality, i.e. 800 per annum per million population or 0.08%, a value slightly in excess of the actual accident rate experienced during the period of observation. Similarly $A(5) = 11.75$ somewhat exceeds the observed value of 8.8. According to the 'Statistical Bulletin', accidents claiming 5 to 10 lives probably occur more frequently than is shown in the statistics; a certain margin was, therefore justified. Finally, it will be noted that the graduating function $A(n)$ slightly overstates the number of lives lost as a result of big catastrophes for the observed period.

Strickler notes that the conditions in Western Europe probably are similar, so that the results may be used there as well. There is no motivation for the explicit choice of $A(n)$, it is just a functions whose curve fits the data.

2.2.3 The basis for calculation

Next we obtain the function

$$H(n) = \frac{A(n) - A(n+1)}{n}$$

which indicates the annual number of accidents, per million lives insured, claiming exactly n victims.

The use of the function $H(n)$ as a basis for the calculation of the premium would involve certain difficulties. With increasing values of n, this function tends to zero so slowly that formula (4) which is used in the calculation of the standard deviation would not converge at all. In order to overcome this difficulty Strickler's solution is to equate $H(n)$ to zero for sufficiently large values of n. Looking at historical data, and remembering that we are only concerned with insured lives, Strickler thinks it is a sufficiently conservative assumption to put $H(n) = 0$ when $n > 1500$.

Let further

$$h(n) = \frac{H(n)}{\sum_{i=0}^{\infty} H(i)}$$

$h(n)$ is equal to the probability of a single accident, if and when it occurs, claiming exactly n victims. It will be the basis of all succeeding calculations.

2.2.4 Strickler's calculations

Since the reinsurer covers only accidents claiming M or more lives, the unconditional probability density function w of Z is a mixture of all w_n according to

$$w(z) = \sum_{n=M}^{\infty} h(n)\, w_n(z)$$

2.2.5 Expected value

If the reinsurer assumes full liability for claims in excess of the limit $z = S$, the net reinsurance premium for one accident takes the form of

$$\Pi_{M,S} = E\left[(Z-S)_+\right] = \int_S^{\infty} (z-S)\, w(z)\, dz \;=\; \sum_{n=M}^{\infty} h(n) \int_S^{\infty} w_n(z)(z-S)\, dz$$

where $x_+ = \max(0, x)$

Using (1) we obtain

$$\Pi_{M,S} = e^{-S} \sum_{n=M}^{\infty} h(n) \left(\frac{S^n}{(n-1)!} + (n-S) \sum_{j=0}^{n-1} \frac{S^j}{j!} \right). \tag{3}$$

The quantity S, i.e. the retention of the ceding company in the event of a catastrophe, is expressed as a multiple of the average amount at risk retained. The following rates of net premium per accident are obtained for various values of M and S:

	$S = 0$	$S = 5$	$S = 10$	$S = 20$
$M = 1$	1.0505	.0164	.0042	.0025
$M = 2$	.0825	.0099	.0042	.0025
$M = 3$	.0329	.0087	.0042	.0025
$M = 4$	.0200	.0080	.0042	.0025
$M = 5$	.0145	.0074	.0041	.0025
$M = 6$	.0116	.0069	.0041	.0025

Table 1: Net reinsurance premium

When M is constant the net reinsurance premium rapidly decreases with increasing S; when S remains constant the premium decreases slowly with increasing M; for large values of S, this decrease becomes almost imperceptible.

2.2.6 Standard deviation

The standard deviation of the reinsurance claims can be calculated on similar lines:

$$\sigma_{M,S}^2 = \mathrm{Var}\left((Z-S)_+\right) = \int_S^\infty (z-S)^2 w(z)\, dz - \Pi_{M,S}^2.$$

The evaluation of this formula leads to the following equation:

$$\sigma_{M,S}^2 + \Pi_{M,S}^2 = e^{-S} \sum_{n=M}^{\infty} h(n) \left[(z-S)\frac{S^{n+1}}{n!} + [n+(n-S)^2] \sum_{j=0}^{n} \frac{S^j}{j!} \right]. \quad (4)$$

We thus obtain the following values for σ:

	$S=0$	$S=5$	$S=10$	$S=20$
$M=1$	1.3493	.8199	.7737	.7323
$M=2$	.9907	.8121	.7737	.7323
$M=3$	.9157	.8104	.7736	.7323
$M=4$	.8874	.8092	.7736	.7323
$M=5$	.8719	.8081	.7736	.7323
$M=6$	.8617	.8070	.7735	.7323

Table 2: Standard deviation σ of reinsurance claims

Apart from the case where $M = 1$ and $S = 0$ i.e. where the entire accident risk is reinsured, the standard deviation is a very large multiple of the net premium; the proportion of the standard deviation to the net premium increases as S increases.

2.2.7 Pricing principles

The values shown in Tables 1 and 2 represent net premiums and standard deviations for a portfolio in which exactly one single or multiple accident is expected to occur per annum. Such a portfolio includes $\frac{10^6}{\sum_1^\infty H(n)} = 1314$ lives. When the portfolio includes N lives, the net premium of table 1 must be multiplied by $\frac{N}{1314}$ and the standard deviation of table 2 by $\sqrt{\frac{N}{1314}}$. If, as is usual and appropriate in such cases, the gross premium is determined as the net premium plus a specified proportion α of the standard deviation of net premium, for a portfolio of N lives, it will be calculated as

$$\Pi_{M,S} \cdot \frac{N}{1314} + \alpha \cdot \sigma_{M,S} \cdot \sqrt{\frac{N}{1314}}. \tag{5}$$

The gross premium calculated in accordance with formula (5) is expressed as a multiple of the average sum at risk and must therefore be transformed to an expression in terms of monetary units. If R is the total sum at risk of a portfolio we obtain, by multiplication with the average sum at risk $\frac{R}{N}$, the gross reassurance premium

$$P_{M,S} = R\left[\frac{\Pi_{M,S}}{1314} + \alpha \cdot \sigma_{M,S}\sqrt{\frac{1}{1314 \cdot N}}\right]$$

The magnitude of α will depend on the reinsurer's total volume of business under catastrophe reinsurance agreements and also on the degree of safety which he considers necessary.

For $\alpha = 0.5$ and a portfolio of $N = 10\ 000$ and $N = 100\ 000$ we obtain in Table 3 presented annual gross rates of reinsurance premium per million units of sum at risk R retained by the cedent.

		$S=0$	$S=5$	$S=10$	$S=20$
$N = 10\ 000$	$M=1$	986	126	110	103
	$M=2$	199	120	110	103
	$M=3$	151	118	110	103
	$M=4$	138	118	110	103
	$M=5$	131	118	110	103
	$M=6$	128	117	110	103
$N = 100\ 000$	$M=1$	858	48	37	34
	$M=2$	106	43	37	34
	$M=3$	65	42	37	34
	$M=4$	54	41	37	34
	$M=5$	49	41	37	34
	$M=6$	46	40	37	34

Table 3: Annual gross rates of reinsurance premium

When M=1 and S=0 the accident risk of the whole portfolio will be reinsured. If the net premium is $A(1) = 0.08\%$ the gross reinsurance premium includes a safety margin of barely$(0.0986-0.08)/0.08 \approx 25\%$ (for $N = 10\,000$) or as little as $(0.0858 - 0.08)/0.08 \approx 7\%(N = 100\ 000)$ of the net premium. However, the safety margin, expressed as a percentage of the net premium, rapidly increases as M and S increase. When S equals 20 or more, the gross premium decreases very slowly with increasing S; for example, when $S = 50$ the gross premium is only 10% less than the gross premium for $S = 20$.

The fact that, for $S \geq 20$, the reinsurance premium hardly decreases with increasing S, suggests that the reinsurance premium must decrease substantially when the risk assumed by the reinsurer is limited.

2.2.8 Pricing with limited cover

As stated above, a reinsurer is not willing to assume unlimited responsibility for a claim but only up to an amount L. Hence the net premium will be reduced to

$$\Pi_{M,S,L} = E\left[\min((Z-S)_+, L)\right] = \Pi_{M,S} - \Pi_{M,L+S}.$$

The standard deviation of the reinsurance claims is then calculated to

$$\begin{aligned}
\sigma^2_{M,S,L} &= \operatorname{Var}\left(\min((Z-S)_+, L)\right) \\
&= \int_S^L (z - S - \Pi_{M,S,L})^2 w(z)\,dz + \Pi^2_{M,S,L} \cdot \int_0^S w(z)\,dz + \\
&\quad +(L - \Pi_{M,S,L})^2 \cdot \int_{L+S}^\infty w(z)\,dz
\end{aligned}$$

Transformation of this expression leads to

$$\sigma^2_{M,S,L} + \Pi^2_{M,S,L} = \int_S^\infty (z-S)^2 w(z)\,dz - \int_{L+S}^\infty (z-L)^2 w(z)\,dz - 2L \cdot \Pi_{M,L} \quad (6)$$

The two integrals will be equivalent in accordance with formula (4) and $\Pi_{M,L}$ is determined by equation (3).

In Table 4 rates of gross reassurance premiums per million sum at risk are given for $M = 3$, $S = 5$ and various values of L, using $\alpha = 0.5$ as previously.

2.3 Additions to Strickler

Several authors have dealt with Strickler's theory. Next we present some of the suggested modifications.

2.3.1 Modification of the claim distribution

Erik Alm suggested a generalization of the distribution for individual claims to a general exponential distribution with expected value a, setting

$$w_1(z) = \frac{1}{a} \cdot e^{-\frac{z}{a}} \quad z > 0.$$

Repeated convolution of w_1 $n-1$ times gives the density function of a $\Gamma(n, a)$-distribution, i.e.

$$w_n(x) = \frac{z^{n-1}}{a^n(n-1)!} \cdot e^{-\frac{z}{a}} \quad z > 0$$

2.3.2 Modified parameters

Function 2, $A(n)$ is derived from American data from the 1950s. Things might have changed since that time, so it would by nice to be able to update the parameters with new data at hand. Morten Harbitz pointed out that the general form of $A(n)$ should be

$$A(n) = a \cdot 100^{\frac{1}{n}} \cdot n^{-b}, \quad a, b \in \Re$$

and based on newer data (FT business report 1988/89, World loss report) he estimated for Europe $a = 2.75, b = 1/7$ and for USA/Canada $a = 2, b = 1/6$. He pointed out that it was difficult to get a good fit of $A(n)$ at both ends of the data. See Harbitz (1992???)

2.4 Conclusion - Strickler's model

Strickler's model is elegant in certain ways, and it is easy to use. It provides a simple price list. But there are some flaws in the model as well.

	Gross premium for:	
L	$N = 10\ 000$	$N = 100\ 000$
10	21	9
20	34	14
50	50	20
∞	118	42

Table 4: Gross premiums with limit L

From a probabilistic point of view, Strickler's approach is somewhat backwards, starting with the ad-hoc $A(n)$ function and from that defining probabilities $h(n)$. There is no obvious way to adjust $A(n)$ to new data, we can not use our statistical standard methods like the ML-method.

Setting 1500 as the maximum of insured lives that can be lost in a single event is of course a real drawback.If there is something we know about catastrophes in general, it is that there is no upper limit for how severe they can be.

Assuming a constant (deterministic) rate of accidents i.e. one accident per 1314 insured lives and year is simply not realistic.

We have to remember that Strickler's article was written in 1960, long before the computing power of today. Some simplifications are always necessary in a model. But with the recent development in computers, as well as in statistical theory, today we should be able to do better. And in the next chapter we try to build a more realistic model!

3 A new model for life Cat XL pricing

3.1 The approach

Let us approach the problem of determining the price of a Cat XL in the following manner: We will deal with the problem by breaking it down into four parts and then model each part separately:

1. The number of catastrophes that happens during the contract period denoted K.
2. The number X_k of deaths from the k:th catastrophe.
3. The number Y_k of claims resulting from the X_k deaths.
4. The cost Z_k of the Y_k claims from the k:th catastrophe.

1. and 2. address all catastrophes that actually happens, 3. and 4. the eventual costs they will inflict on a Cat XL contract. We can express the total cost due to catastrophes during the contract period as

$$C = \sum_{k=1}^{K} Z_k \tag{7}$$

Our goal is to calculate the expected value and variance of C, the total claim on the Cat XL contract during its duration, and hence it's price. Next we will specify how to model each part.

3.2 Catastrophe rate

It is not an especially bold statement to say that the number of catastrophes during a time period is stochastic. (As opposed to Strickler's assumption of a deterministic catastrophe rate.) To be more precise, we make the following assumptions:

1. The number of catastrophes in disjoint time intervals are independent.
2. Only one catastrophe occurs at a time.
3. The probability of a catastrophe occurring at a specific moment in time is zero.

This implies, see Johansson (2005, Chapter 4.1), that the number of catastrophes claiming at least m lives is a Poisson process with intensity λ_m per year and that K_m, the number of catastrophes claiming at least m lives during one year is

$$K_m \sim Po(\lambda_m). \tag{8}$$

We know that some areas of the world are more prone to catastrophes than others. There are a number of reasons for this, the geography can have a influence in many ways, the local climate determines the occurrences of storms and floods, earthquakes happen in seismic active regions, mountains are a danger for aircrafts etc. The catastrophe intensity should therefore vary between different places. A geographical partition of the world into regions with possibly different λ_m:s is therefore advised when doing statistical analysis of catastrophe intensities.

In life insurance people are insured, not property, and people move around. When collecting catastrophe statistics, we try to record the origin of the people who lost their lives rather than just where the catastrophe occurred.

The first assumption may be questioned, since it is known that the conditions giving phenomenas like storms/typhoons/cyclones vary in a way that makes storms cluster together and not be independent of each other. One great rainfall might saturate the earth, greatly increasing the risk of a landslide during subsequent rains. So in regions were catastrophes caused by the weather are common, you might find an *overdispersion*, i.e. a greater variance than what is expected from the Poisson distribution. A way to model this is to use a mixed Poisson model, in which the intensity λ is seen as a random variable instead of a constant. See for example Johansson (2005, Chapter 4.2) for more on mixed Poisson models.

3.3 Number of deaths

What is the distribution of X_k, the number of deaths in the k:th catastrophe?

First, let X denote the number of dead in an arbitrary accident. Let $P_m(n) = P(X = n|X \geq m)$ and $F_m(n) = P(X \leq n|X \geq m)$. In accordance with Strickler's notation we have $P_1(n) = h(n)$. Our main interest here are 'catastrophes', accidents were several persons have died i.e. we are really interested in the tail of the P_1 distribution. When studying outcomes that exceed a certain threshold it is suitable to use the *peaks over thresholds* (POT) model, see Rootzén & Tajvidi (1995). Given a threshold m (thus only studying catastrophes claiming at least m lives), we assume

1. $X_1, X_2 \ldots X_K$ are independent, identically distributed (i.i.d) $\sim X$.

2. $\tilde{X} \sim \text{GPD}(m - \frac{1}{2}, \sigma_m, \xi_m)$ i.e. $\tilde{X}$ has a *Generalized Pareto Distribution* (GPD).

3. $X = \text{round}(\tilde{X})$, where round$(x)$ is the integer closest to x. This is since $X \in N$ and $\tilde{X} \in \Re$. We say that X has a *Discrete Generalized Pareto Distribution* (DGPD), $X \sim \text{DGPD}(m, \sigma_m, \xi_m)$.

Recall that the generalized Pareto distribution has a CDF

$$G_{(m-\frac{1}{2},\sigma,\xi)}(x) = 1 - [1 + \xi(x - m + \frac{1}{2})/\sigma]^{-1/\xi}$$

were $m \in \Re$, $x \geq m - \frac{1}{2}$ and $\sigma > 0$.

If $\tilde{X} \sim \text{GPD}(m - \frac{1}{2}, \sigma, \xi)$ then

$$E[\tilde{X}] = m - \frac{1}{2} + \frac{\sigma}{1-\xi} \ (\xi < 1)$$

$$Var(\tilde{X}) = \frac{\sigma^2}{(1-\xi)^2(1-2\xi)} \ (\xi < 1/2)$$

If $\xi \geq 1/2$ the variance does not exist, and if $\xi \geq 1$ the same holds for the expected value.

With X only taking integer values we have

$$E[X] \approx E[\tilde{X}]$$

$$Var(X) \approx Var(\tilde{X}).$$

3.4 Number of claims

What is the number of claims that will hit a life office given a catastrophe? We model like this: Let Y be the number of claims resulting from a catastrophe with a death toll of X. We want to find the properties of the random variable Y. It is clear that $0 \leq Y \leq X$. The more policies sold, the likelier a claim. Let

$$q = \frac{\text{Nr of sold policies}}{\text{Size of total population}}$$

Then we assume $E[Y|X] = q \cdot X$ i.e. that the expected number of claims is proportional to the market penetration.

In life insurance the standard assumption is that lives are independent, the distribution of Y would then be $Y|X \sim \mathrm{Bin}(X, q)$. This holds as a first approximation, but as discussed above, the independence assumptions is highly questionable in the case of a catastrophe (or epidemic for that matter). (Think for example of miners working in a mine that collapses). For catastrophes we would like to take the possible dependence into account. Recall that Strickler assumes that when hit, all lost lives in the catastrophe will result in a claim. This assumption is not satisfying either since it obviously is wrong. Both approaches are however compatible with $E[Y|X] = q \cdot X$.

Y should have a character in between the two extremes, independence and total dependence, either no policyholder is hit, but given that one is, it will be likely that there are more than one. For very large catastrophes the number of claims should be close to the expected value i.e.

$$Y/X \overset{a.s.}{\to} q \text{ as } X \to \infty$$

A distribution that would reflect the above mentioned properties is the *Beta-binomial*:

$$\begin{aligned} Y|X &\sim \mathrm{Bin}(X, p), \\ p|X &\sim \mathrm{Beta}(d(X)q, d(X)(1-q)),\ 0 < d(X) < \infty \end{aligned}$$

This implies: $E[p] = q$ so $E[Y|X] = q \cdot X$ as it should.

The beta distribution and the function $d(X)$ adds the desired flexibility. We can think of it like this: For every catastrophe a $p \in [0; 1]$ is drawn from a beta distribution with mean q. This p is the probability that a life in this catastrophe was insured, and hence Y the total number of insured lives lost is Bin(X,p).

How does $d(X)$ affect the distribution? The limits for $d(X)$ gives:

$$\begin{aligned} \lim\ d(X) \to \infty &\Rightarrow Y|X \sim \mathrm{Bin}(X, q) \\ \lim\ d(X) \to 0 &\Rightarrow P(Y = 0|X) = 1 - q, P(Y = X|X) = q \end{aligned}$$

They correspond to the two extremes, independence and total dependence. The trick here is to choose $d(X)$ in a way so that $d(X) \to \infty$ as $X \to \infty$ and that $d(X)$ is small for small X. A candidate would be $d(X) = \theta \cdot \log(X)$, $\theta \in \Re^+$. By doing so, we get a certain degree of dependence for smaller catastrophes (think e.g of traffic accidents) and independence for the really large catastrophes.

We notice that the variance

$$\mathrm{Var}(Y|X) = q(1-q)\left(X + X(X-1)/(d(X)+1)\right),$$

is a decreasing function of $d(X)$,with $\text{Var}(Y|X) = Xq(1-q)$ for complete independence ($d(X) = \infty$) and $\text{Var}(Y|X) = X^2q(1-q)$ for complete dependence ($d(X) = 0$).

Remember that the Cat XL contract states that at least M insured lives have to die in order to be a valid claim. Let $Y'_k \sim \text{Beta bin}(X, q, d(X))$ be the number of insured lives lost in the k:th catastrophe, then

$$Y_k, = \begin{cases} Y'_k, & \text{if } \; Y'_k \geq M \\ 0, & \text{if } \; Y'_k < M \end{cases}$$

It could be noted that the beta distribution is known to be used in non-life catastrophe modeling in a similar manner, were the percentage of damage done to a building due to a natural peril (storm, flood, earthquake) is modeled as being beta distributed. See Woo (1999).

3.5 Distribution of claims

What is the size of a claim? Different policyholders can have different insured amounts and depending of type of policy and the time it has been in force the sum at risk varies. Strickler's assumption, that the distribution $P(Z \leq z|Y = 1) = 1 - e^{-z}$ of a single claim Z is exponential with expected value 1, is a good approximation, assuming that Z is expressed as a multiple of the average sum at risk. It also provides a gamma distribution $\Gamma(m, 1)$ for the total cost $Z|Y = m$ of m claims, cf. (1).

Alternatively, since in most cases risks over a certain retention R are already reinsured (so that there is an upper limit to the sum at risk), a truncated exponential may be used giving the CDF

$$P(Z \leq z|Y = 1) = \begin{cases} 1 - e^{-z}, & z < R \\ 1, & z \geq R \end{cases}$$

Here R is expressed as a multipel of the average sum at risk.

In the case where the Cat XL covers a group life policy where all sum assureds are the same, there is no randomness and $Z|Y = m = m$.

With the modern IT-technology, it is in fact often possible to get hold of all the individual risk sums. In that case, where you have the exact distribution, you can in fact numerically compute the distribution, but for large collectives the computations can be time consuming and for simplicity a approximated distribution may be used.

Whatever method we choose, we have to consider S and L, the retention and maximal liability of the Cat XL contract. If $Z'_k = \sum_{i=1}^{Y_k} Z_{ki}$ is the actual claim amount from the Y_k insured lives lost in catastrophe k we set

$$Z_k = \begin{cases} 0, & \text{if} \quad Z'_{Y_k} < S \\ Z'_k - S, & \text{if} \quad S \leq Z'_{Y_k} < L + S \\ L, & \text{if} \quad L + S \leq Z'_{Y_k} \end{cases}$$

3.6 Total annual claim

Now we are ready to address the question, what is the total annual cost C? Recall (7) that

$$C = \sum_{k=1}^{K} Z_k$$

As we have seen, C will depend on the contract parameters M,S and L as well as model parameters $\lambda_M, \sigma_M, \xi_M, q, \theta$ and the choice of claim distribution function.

Thus, for a given set of parameters, we can run simulations to compute the properties of C, expected value and variance. Knowing the expected value and variance of C we can set the price of the Cat XL contract.

4 Data

To be able to set the correct technical price on an insurance a theoretical model is not enough, one also need statistical data in order to estimate the parameters of the model. For many types of insurances the insurance company can rely on its own claim experience for the estimates. But since catastrophic events are, almost by definition, rare, even for a reinsurer with a large Cat XL portfolio the use of claim experience as the only source for pricing the contracts would be unsatisfactory. Ideally you would know about all accidents all over the world claiming at least three lives, but that is not possible since such a database does not exist. However, there are some data collected by various agents that can come to good use, as will be seen below.

Two sets of data were compiled for the article. An international set containing the catastrophes during the period 1984–2004 claiming at least 20 lives, and a Swedish one containing accidents during the period 1970–2004 claiming at least four lives.

4.1 International data

Swiss RE's yearly publication '*sigma*, Natural catastrophes and man-made disasters in xxxx' lists catastrophic events from all over the world that have 'at least 20 dead or missing'. Complete data sets was available from the years 1983–91, 1994–99 and 2002–04. Only data from those years was compiled. Some well known catastrophes (and a lot of unknown) such as 9/11 (2001) are therefore missing. Data was sorted after continent and region, as well as the cause of the disaster. Only events that fit the standard Cat XL contracts 72 hour rule were taken into account. Long lasting 'conditions' such as heat waves, cold spells and floods were therefore sometimes excluded, even if they took many lives. Acts of war and military accidents are not accounted for since they are excluded from the insurance contracts. In total there were 3055 observations.

4.1.1 Classification

There are two main types of catastrophes, natural, like storms, earthquakes and landslides, and man-made, like traffic accidents, collapsing buildings and acts of terror. The man-made are divided into seven groups, see Table 5 below. Geographically the world is divided into 16 regions. A comprehensive list of which countries that belong to which region is found in the appendix.

Abbreviation	Cause
AV	Aviation
BU	Buildings collapsing
FI	Fire and chemicals
MI	Mining accidents
NC	Natural catastrophes
OT	Other
TR	Traffic and railway
WA	Water, ships and ferrys
	Region
SAM	South America
NAM	North America
CAR	The Caribbean
CAM	Central America
WEU	Western Europe
EEU	Eastern Europe
SUN	Former republics of the Soviet Union
SAS	The Indian sub-continent
SEA	South East Asia
MIE	The Middle East
FAE	The Far East
CAS	Central Asia
OCE	Oceania
NAF	North Africa
MAF	Sub-Saharan Africa
SAF	Southern Africa

Table 5: Abbreviations

4.1.2 Descriptive statistics of data

Here we present a summary of the data. First, in Tables 6-7, some facts about the size of the catastrophes, sorted after type and region together with the observed number for each type.

We also present, in Table 8, the incidence rate for catastrophes, in absolute numbers as well as adjusted for the number of inhabitants in each region. Annual population figures are from the U.S. Census Bureau (2002). For reasons explained bellow, it is informative to present the figures for two periods of time, the first containing the years 1983–91 and the second containing the years 1994–99 and 2002–04.

Type	Nr of Obs	Mean	Median	Max	Cause of worst disaster
AV	266	72	46	520	Japan Airlines, Japan, -85
BU	82	44	30	577	Dept Store collapses, South Korea -95
FI	192	89	37	3 000	Chemical factory, India, -84
MI	160	47	35	200	Gold mine, Peru, -89
NC	884	890	52	220 000	Tsunami, South East Asia, -04
OT	147	74	39	1 426	Stampede in Mecca, Saudi Arabia, -90
TR	827	40	30	645	Gas explosion hit train, Soviet, -89
WA	511	85	42	4 300	Ferry sinks, Indonesia, -87

Table 6: Total number of catastrophes, sorted after type

Region	Obs	Mean	Median	Max	Cause of worst disaster
SAM	302	307	36	50 000	Landslides, Venezuela, -99
NAM	105	64	35	805	Heatwave, USA, -95
CAR	53	141	40	3 344	Flood, Haiti, -04
CAM	99	115	33	5 000	Earthquake, Mexico, -85
WEU	132	67	35	852	Ferry, Sweden, Estonia, -94
EEU	59	44	31	183	Aircraft, Poland, -87
SUN	122	516	40	55 000	Gas explosion hit train, Soviet, -89
SAS	748	390	40	138 868	Tropical storm, Bangladesh, -91
SEA	315	873	41	220 000	Tsunami, Indonesia, Sri Lanka..., -04
MIE	172	726	40	41 000	Earthquake, Iran, -03
FAE	94	169	39	6 425	Earthquake, Japan, -95
CAS	381	78	38	1 000	Earthquake, China, -88
OCE	16	186	36	2 183	Earthquake,Papua–New Guinea, -98
NAF	83	108	38	2 266	Earthquake, Algeria, -03
MAF	315	87	41	1 863	Ferry, Gambia, -02
SAF	59	43	29	176	Fire, South Africa, -86

Table 7: Total number of catastrophes, sorted after region

Region	Average nr per year		Per 100 million inhabitants	
	83–91	94–04	83–91	94–04
SAM	18,9	14,8	6,8	4,5
NAM	6,3	5,3	2,3	1,8
CAR	2,1	3,7	5,7	8,8
CAM	6,1	4,9	5,7	3,7
WEU	8,9	5,9	2,4	1,5
EEU	5,0	1,7	4,2	1,4
SUN	3,4	10,1	1,2	3,5
SAS	40,6	41,3	3,9	3,2
SEA	18,4	16,6	4,3	3,2
MIE	6,6	12,6	3,4	4,9
FAE	7,4	3,0	3,9	1,5
CAS	17,1	26,4	1,5	2,1
OCE	1,0	0,8	3,9	2,6
NAF	2,6	6,6	2,3	4,6
MAF	12,7	22,3	2,9	3,8
SAF	2,9	3,7	7,5	8,2

Table 8: Annual incidence rates λ_{20} of catastrophes for various regions

Victims	Frequency	$\hat{P}_4(n)$
4	88	0,47
5	42	0,22
6	21	0,11
7	7	0,037
8	8	0,042
9	6	0,032
10	1	0,005
11	3	0,016
12	0	0
13	1	0,005
14	1	0,005
15	3	0,016
16	1	0,005
17	0	0
18	0	0
19	0	0
20≤	6	0,032
Total	189	1

Table 9: Swedish accidents

4.1.3 Comments on data

Natural catastrophes are responsible for the worst catastrophes, and earthquakes can have the most devastating consequences. (The earthquake that caused the tsunami 2004 measures as one of the three largest on the Richter scale over the past century.) Naturally, they mainly appear in seismological active regions of the world.

The trend in the western world is that of fewer catastrophes. The explanation for this is probably technological advances, where safety has been a prioritized issue. For example, better monitoring of railway traffic, and safer vehicles.

4.2 Swedish data

The Swedish Rescue Services Agency, SRSA, (Räddningsverket) keeps a record over Swedish accidents claiming at least four lives. Data from 1970–2004, a total of 189 observations, was used. The frequencies and empirical probability $\hat{P}_4(n)$ are displayed in Table 9.

Victims	Type	Cause	Year
20	FI	Hotel fire in Borås.	1978
20	AV	A SAS plane collides in the fog at Lineata Airport, Italy.	2001
22	AV	Aircraft crashes due to ice on it's wings.	1977
63	FI	Fire at a discotheque in Gothenburg.	1998
501	WA	The ferry Estonia sinks in bad weather.	1994
543	NC	Swedish tourists in Thailand hit by the tsunami	2004

Table 10: Sweden's worst catastrophes

Period	$\# \geq 4$	Per 100 million	$\# \geq 20$	Per 100 million
1970–79	6,1	74,35	0,2	2,42
1980–89	6,6	78,67	0	0
1990–99	3,8	43,42	0,2	2,26
2000–04	4,8	53,68	0,4	4,46
1970–89	6,35	76,51	0,1	1,21
1990–2004	4,13	46,84	0,27	3,00
1970–2004	5,4	63,79	0,17	1,97
1983–91, 1994–99, 2002–04	4,77	55,13	0,18	1,87

Table 11: Annual Swedish incidence rates λ_4 and λ_{20}, for various periods of time.

The incidence in Sweden for catastrophes seems to be in line with the one in western Europe (WEU).

4.3 Sources of error

There are numerous sources of error in the data. The most important are:

- Uncertainty in the actual number of dead. For larger catastrophes and in less developed countries it is often not possible to get the exact death toll, rather the figures are an estimation.
- Incurred but not reported, (IBNR). Especially for smaller accidents in remote areas, there is a chance that it never will come to the attention of those who are collecting the data. Thanks to the IT revolution, things get public to an extent never experienced before, so the data quality is probably better for the late -90s than for the -80s.
- Some regimes are not willing to report accidents and when it is apparent to the outer world that a catastrophe has occurred they report figures that is below the actual ones. This can be seen in the lack of reported catastrophes from the Soviet Union (where the reported number has risen significantly after 1991), China and North Korea. Official data has been used in this article, but it is worth mentioning that for several of the catastrophes there are independent reports talking about figures twice as high as the official ones, for instance in Turkey and Iran.

5 Fitting the model to data

5.1 Cat intensity

According to the model presented in Section 3, the number of catastrophes per year of a given region has a Poisson distribution, equation (8):

$$K_m \sim \mathrm{Po}(\lambda_m)$$

and the number of catastrophes in different years are independent. Since $E(K) = Var(K) = \lambda$ if (8) holds, we can check the validity of the Poisson assumption for each region by comparing the sample mean and sample variance of the yearly number of accidents during the periods 1983–1991 and 1994–2004 respectively, as shown in Table 12.

Region	1983–91			1994–2004		
	mean	var	var/mean	mean	var	var/mean
SAM	18,9	48,9	2,59	14,8	17,2	1,16
NAM	6,3	10,3	1,62	5,3	8,0	1,5
CAR	2,1	2,9	1,36	3,7	4,5	1,23
CAM	6,1	10,1	1,65	4,9	5,6	1,15
WEU	8,9	9,4	1,05	5,9	13,9	2,35
EEU	5,0	12,8	2,55	1,7	1,0	0,6
SUN	3,4	6,5	1,9	10,1	9,9	0,98
SAS	40,6	314,0	7,74	41,3	161,5	3,91
SEA	18,4	150,8	8,17	16,6	35	2,12
MIE	6,6	15,8	2,41	12,6	33,5	2,67
FAE	7,4	4,3	0,57	3,0	3,3	1,08
CAS	17,1	93,4	5,46	26,4	25,3	0,96
OCE	1,0	0,5	0,5	0,8	0,7	0,89
NAF	2,6	2,3	0,89	6,6	3,5	0,54
MAF	12,7	54,8	4,32	22,3	90,3	4,04
SAF	2,9	3,4	1,16	3,7	8,3	2,25

Table 12: Catastrophe intensities $\hat{\lambda}_{20}$ and $Var(\hat{\lambda}_{20})$.

Looking in the table we see that generally, the variance is closer to the mean during the period 1994–2004 than 1983–1991. The reason is probably that, as discussed in Section 4.3, the data is more complete for later years. For the most regions, the mean and the variance are fairly close to each other, in accordance with the Poisson assumption. But for some regions such as SEA and SAS the variance is more than twice the mean, indicating an over dispersion. As discussed in Section 3.2 over dispersion is likely to occur due to weather phenomena, which both SEA and especially SAS are prone to. For MAF, the middle part of Africa, were the variance is four times the mean the cause is probably a combination of generally poor data and weather phenomenas.

5.2 Cat size

Is the GPD a good model for the number of lost lives in a catastrophe? The available data was used to estimate the parameters of the GPD with the maximum likelihood (ML) method. This was done for all regions. The estimated parameters are displayed in Table 13.

Region	$\hat{\sigma}_{20}$ (std err $\hat{\sigma}_{20}$)	$\hat{\xi}_{20}$ (std err $\hat{\xi}_{20}$)
SAM	15,5 (1,7)	0,83 (0,10)
NAM	17,2 (3,1)	0,68 (0,17)
CAR	18,4 (4,8)	0,98 (0,26)
CAM	13,2 (2,6)	0,98 (0,20)
WEU	14,8 (2,6)	0,84 (0,17)
EEU	13,6 (3,1)	0,63 (0,21)
SUN	20,2 (3,3)	0,79 (0,15)
SAS	20,2 (1,4)	1,00 (0,07)
SEA	19,8 (2,4)	1,15 (0,12)
MIE	18,3 (2,9)	1,38 (0,18)
FAE	17,9 (3,9)	1,12 (0,22)
CAS	20,6 (2,0)	0,76 (0,09)
OCE	17,1 (8,0)	1,13 (0,49)
NAF	15,7 (3,4)	1,02 (0,22)
MAF	25,6 (2,7)	0,66 (0,10)
SAF	10,0 (2,2)	0,61 (0,20)

Table 13: Estimated parameters for the $DGPD(20, \sigma_{20}, \xi_{20})$ model. Standard errors are in parenthesis.

Which inferences can be drawn? Remember that if $\xi \geq 1/2$ the variance does not exist. Since $\hat{\xi} > 1/2$ for all regions this indicates a heavy tail for the distribution, i.e. large catastrophes are to be expected. For some regions, were $\hat{\xi} > 1$, not even the expected value exists! Is a catastrophe claiming an infinite number of lives to be expected? Think of the scale, compared to 20, 200 000 is nearly infinite. The conclusion is that large catastrophes have happened in the past and that major catastrophes, claiming tens of thousands of lives, are to be expected in the future.

We now have the estimates, but is the GPD really a good choice? We plot the empirical probabilities (represented by bars) together with the fitted GPD (marked as circles and connected with lines for clarity). It can be hard to see if the fit is good, especially in the tail of the distribution. Therefore we also use quantile - quantile plots (QQ-plots) who can be very helpful to evaluate if a distribution describes a data set in satisfying way. Figure 1–4 show plots of various regions. The dashes of the QQ-plots mark a 95% confidence interval.

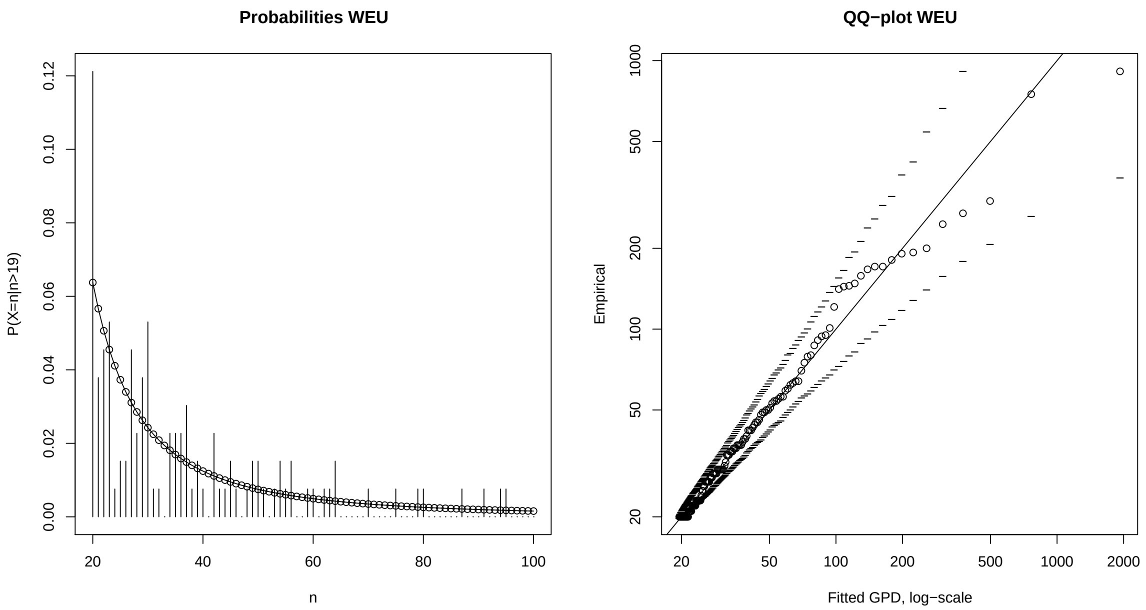

Figure 1: Empirical probabilities, fitted GPD and QQ-plot, WEU

5.2.1 Conclusion GPD

Indeed, judging from the plots, the GDP seems like a plausible distribution for modeling the size of catastrophes. This should not come as surprise, accidents claiming 20 or more lives belongs to the tail of the distribution of lives lost given a fatal accident and the GPD is *the* distribution for tails, see Hult & Lindskog (2004) What might be a greater surprise is the fact that the tail is so heavy that no variance exists and that for some regions there is no expected value.

The insecurity in data for the exact death toll are for some regions clearly visible, looking at SAS and MAF we see large spikes representing 100, 150, 200. . . dead.

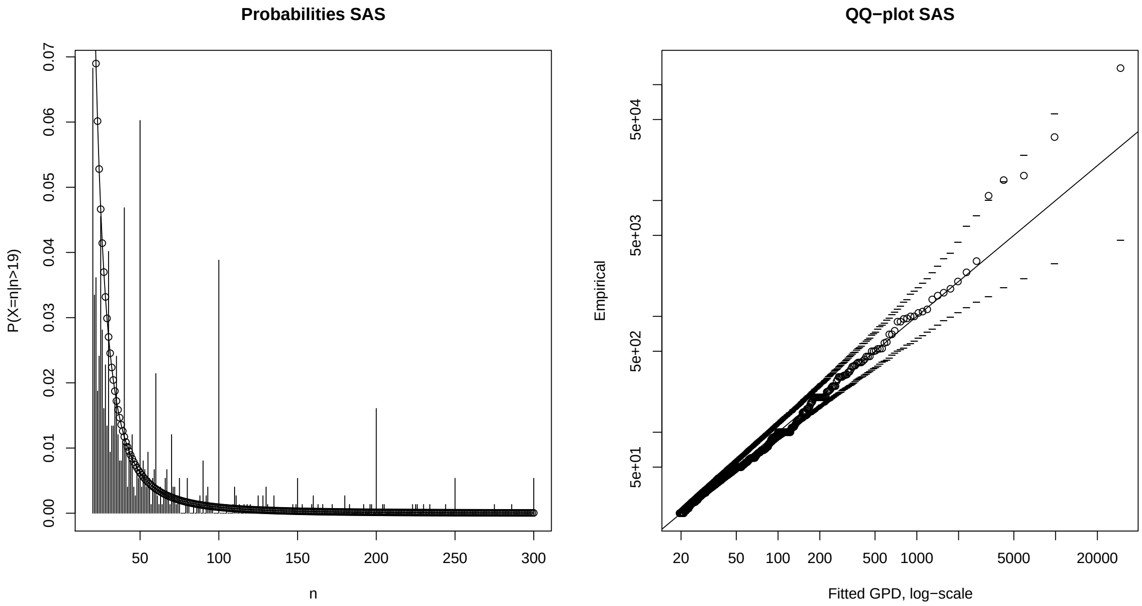

Figure 2: Empirical probabilities, fitted GPD and QQ-plot, SAS

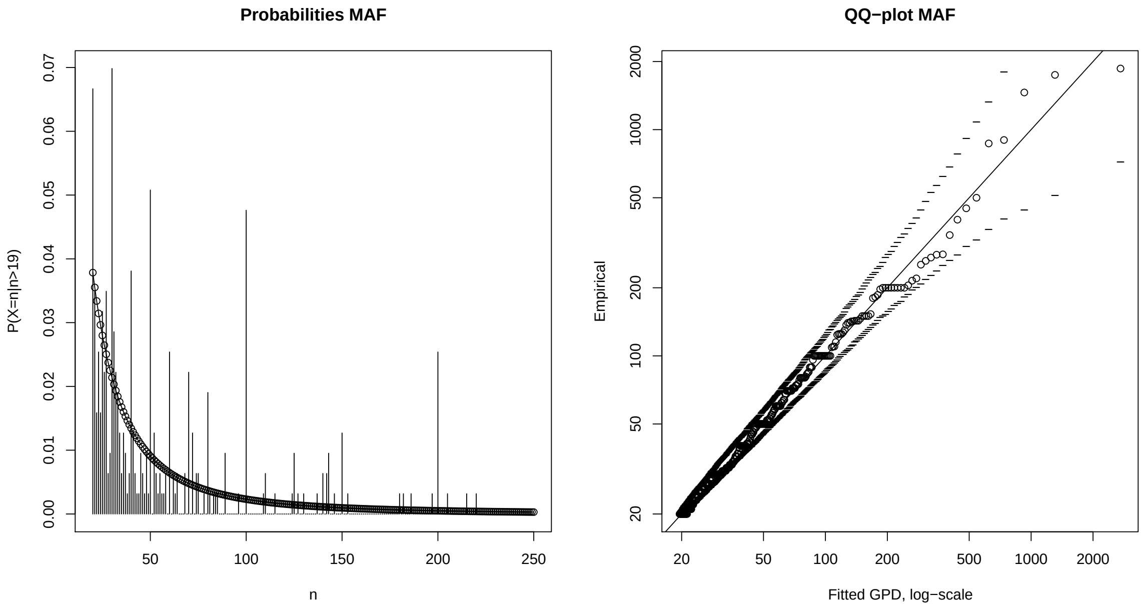

Figure 3: Empirical probabilities, fitted GPD and QQ-plot, MAF

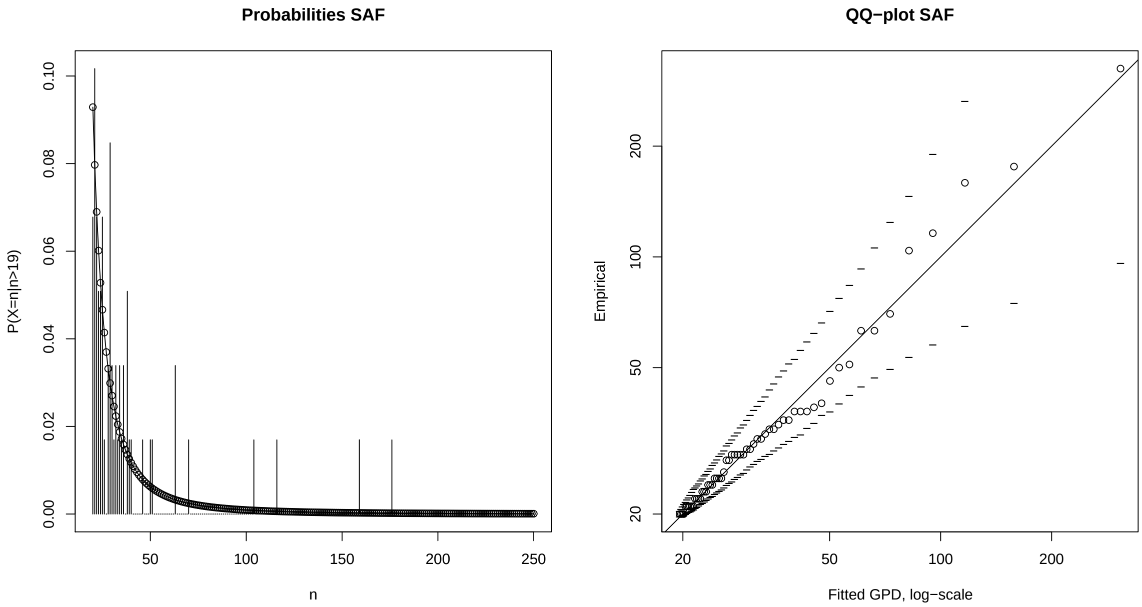

Figure 4: Empirical probabilities, fitted GPD and QQ-plot, SAF

5.3 Distribution of insured lives

Unfortunately, there is no available data for the distribution of insured lives in catastrophes. Hence, it is not possible for us to make inference about $d(X)$. In particular, we cannot estimate the parameter θ. What we can do is to let θ vary between two extremes, having either a binomially distributed number of insureds from the company of interest in each catastrophe ($\theta = \infty$), or all/no claims with probabilities q and $1 - q$ respectively ($\theta = 0$). Then we can assess which θ-value seems plausible to use.

5.4 Inference from Swedish data

We do the same analysis for the Swedish data. Remember that the dataset consists of accidents claiming four of more lives. First we analyze the catastrophe intensity, the result is found in Table 14. We find that the mean and the variance are close to each other, the data shows no sign of over dispersion. Also, we note that the number of accidents have decreased from 1990 and onward compared to the -70s and -80s. Technological development toward better safety is probably the cause.

Turning our attention to fitting a GPD to the cat size we should ask ourselves if it is possible. For the international data set we had catastrophes claiming

Year	mean	var	var/mean
1970–1979	6,1	7,7	1,26
1980–1989	6,6	9,4	1,42
1990–1999	3,8	4,2	1,10
2000–2004	4,8	1,7	0,35
1990–2004	4,1	3,4	0,82

Table 14: Swedish catastrophe intensities $\hat{\lambda}_4$ and $\mathrm{Var}(\hat{\lambda}_4)$.

20 or more lives, which certainly belong to the tail of the distribution. The question is if four dead can be considered to belong to the tail as well. The only way to find out is to run the parameter estimation and inspect the results. Doing so, σ_4 and ξ_4 are found to be 1.37(0.16) respectively 0.66(0.010). Looking at the corresponding plots we find that the fit indeed is good, even if the tail seems to be a bit underestimated judging from the QQ-plot. It is then worth to remember that the two largest catastrophes, Estonia and the tsunami, are extreme in the modern Swedish history. They would have been the largest catastrophes even if we had data from the whole 20th century. In light of this fact, we conclude that the GPD gives a good fit for the Swedish data.

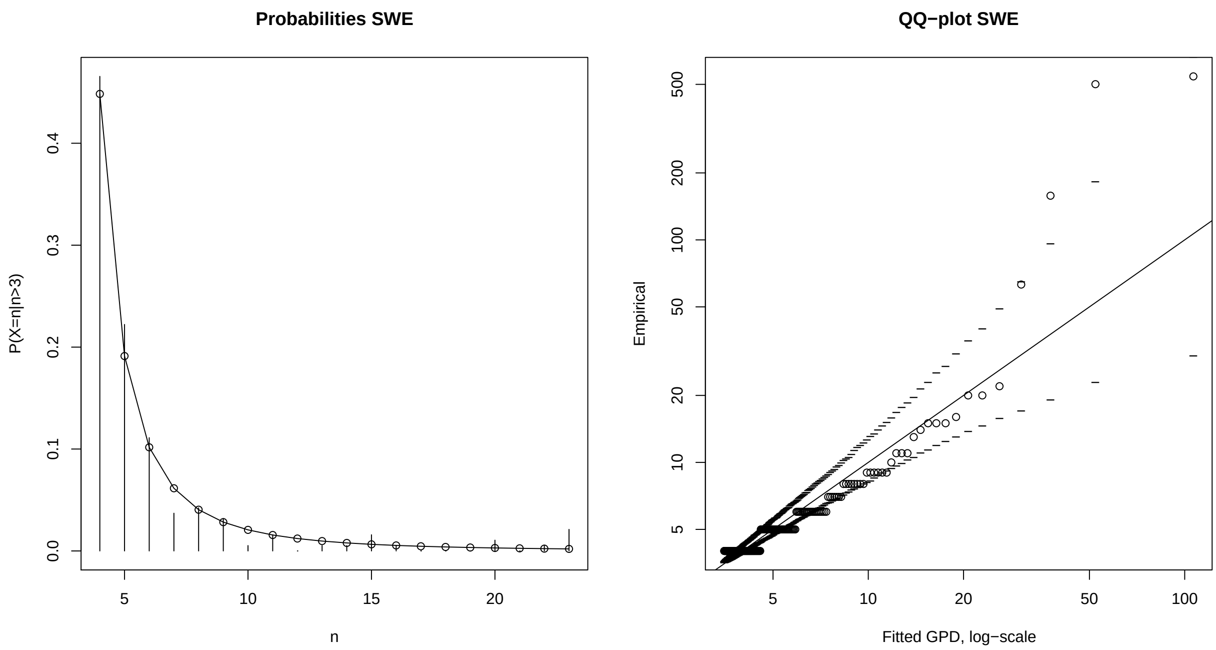

Figure 5: Empirical probabilities, fitted GPD and QQ-plot SWE, at least four dead.

5.5 Extrapolating parameter estimates

We have estimates $\hat{\lambda}_{20}$, $\hat{\sigma}_{20}$ and $\hat{\xi}_{20}$ for all regions but in order to do the pricing we would like to have estimates of λ_4, σ_4 and ξ_4. Is it possible to get such estimates without having data for accidents with at least four dead? Consider first the following proposition:

Proposition: Let m and $u \geq m$ be integers and $X \sim \text{DGPD}(m, \sigma, \xi)$. Recall from Section 3.3 that $X = \text{round}(\tilde{X})$, where $\tilde{X} \sim \text{GPD}(m - \frac{1}{2}, \sigma, \xi)$. Hence $X|X \geq u \sim \text{DGPD}(u, \sigma_u, \xi)$ were $\sigma_u = \sigma + \xi(u - m)$

Proof: Let x be an integer $\geq u$. Then

$$\begin{aligned}
P(X \geq x|X \geq u) &= \frac{P(\tilde{X} > x - \frac{1}{2})}{P(\tilde{X} > u - \frac{1}{2})} \\
&= \frac{[1 + \xi(x - \frac{1}{2} - m + \frac{1}{2})/\sigma]^{-1/\xi}}{[1 + \xi(u - \frac{1}{2} - m + \frac{1}{2})/\sigma]^{-1/\xi}} \\
&= \left[\frac{1 + \xi(x - m)/\sigma}{1 + \xi(u - m)/\sigma}\right]^{-1/\xi} \\
&= \left[\frac{1 + \xi(u - m)/\sigma}{1 + \xi(u - m)/\sigma} + \frac{\xi(x - u)/\sigma}{1 + \xi(u - m)/\sigma}\right]^{-1/\xi} \\
&= \left[1 + \frac{\xi(x - \frac{1}{2} - u + \frac{1}{2})}{\sigma + \xi(u - m)}\right]^{-1/\xi} \\
&\Rightarrow P(\tilde{X}|\tilde{X} \geq u - \frac{1}{2}) \sim \text{GPD}(u - \frac{1}{2}, \sigma + \xi(u - m), \xi) \\
&\Rightarrow P(X|X \geq u) \sim \text{DGPD}(u, \sigma + \xi(u - m), \xi)
\end{aligned}$$

□

This means that we have a simple relation between the threshold u and the other parameters. We will use the fact that this relation holds not only for an increasing threshold, but for a decreasing threshold as well, so that for a threshold $u \leq m$, $\sigma_u = \sigma_m - \xi_m(m - u)$ and $\xi_u = \xi_m$, given that $\sigma_u > 0$. If $\sigma_u \leq 0$ then the DGPD does not exist.

Confidence intervals for σ_u can be obtained by the following means: Let V be the 2×2 covariance matrix for $\hat{\sigma}_m$ and $\hat{\xi}$. Then

$$\text{Var}(\hat{\sigma}_u) = \text{Var}(\hat{\sigma}_m - \hat{\xi}(m - u)) = a * V * a', \tag{9}$$

where $a = (1, -(m - u))$ and a' is the transpose of a. Assuming that $\hat{\sigma}_u$ is an approximately normally distributed estimator we get a confidence interval for σ_u through (9) and quantiles from the normal distribution.

We are now ready to calculate $\hat{\sigma}_4$ and $\hat{\xi}_4$ for every region. We must however be careful, it is not certain that a GPD with threshold $u = 4$ will fit the actual data ranging from 4 and up, but the Swedish data suggest that DGPD can be successfully fitted with such a low threshold.

To calculate $\hat{\lambda}_u$ first note that given σ_u, ξ_u we can calculate $1 - F_u(m) = P(X > m|X \geq u)$, the proportion of catastrophes claiming more than m lives. Assuming λ_m is known, we get

$$\lambda_u = \lambda_m/(1 - F_u(m))$$

by means of well known results on thinning of Poisson processes.

5.5.1 Estimates of σ_4

We calculate estimates of σ_4 according to the formula $\hat{\sigma}_4 = \hat{\sigma}_{20} - \hat{\xi}_{20}(20 - 4)$.

Region	$\hat{\sigma}_4$	Confidence interval
SAM	2,22	(-3,32; 7,76)
NAM	6,32	(-3,67; 16,31)
CAR	2,72	(-12,02; 17,46)
CAM	-2,48	(-12,29; 7,33)
WEU	1,36	(-8,09; 10,81)
EEU	3,52	(-7,49; 14,53)
SUN	7,56	(-2,23; 17,35)
SAS	4,22	(-0,17; 8,57)
SEA	1,44	(-5,94; 8,74)
MIE	-3,78	(-13,21; 5,65)
FAE	-0,02	(-12,54; 12,50)
CAS	8,44	(2,37; 14,51)
OCE	-0,98	(-26,89; 24,93)
NAF	-0,62	(-12,35; 11,11)
MAF	15,04	(7,63; 22,45)
SAF	0,24	(-9,04; 9,52)

Table 15: Estimates of σ_4 together with 95% confidence intervals for various regions.

As we see in Table 15 $\hat{\sigma}_4 < 0$ for some regions, but all confidence intervals includes positive numbers. In fact the confidence intervals are very wide, which is a problem if we want to do an accurate calculation of C.

A way to pick a $\hat{\sigma}_4$ from the wide confidence interval is to use our knowledge of the proportion of catastrophes larger than 20. From the Swedish data $1-F_4(20) = 3.2\%$ and if we belive that this proportion holds in other regions, we choose $\hat{\sigma}_4$ so that $1 - F_4(20)$ is close to 3.2%.

6 Pricing

Finally, let us see some pricing of a Cat XL contract!

6.1 The pricing principle of a Cat XL contract

The pure premium of a usual insurance policy is $E[C]$, the expected cost of claims. In theory, according to the law of large numbers, by having a large portfolio of contracts the variance of the total cost of claims, and hence the risk, is small. This ensures that the total claim experience for the portfolio is close to the expected value. The pure premium (in practice also loaded for admin. expenses and profit) is therefore the price P of the insurance policy.

In non-proportional reinsurance it is often not possible to acquire a portfolio with a large number of independent contracts so the reinsurance portfolio will be subject to larger fluctuations, i.e. there is a lot of risk involved. The reinsurer wants to get paid for this risk and therefore adds a percentage of the standard deviation to the price. This gives the pricing formula

$$P = E[C] + \alpha * SD(C), \text{ where typically } \alpha \in [0.1; 0.5]$$

6.2 The rating factors and properties of C

As we have seen there are many factors that affect the price P of a Cat XL contract. In this model the parameters are: the catastrophe rate λ, the size of catastrophes determined by (σ, ξ), the market penetration q, the dependence parameter θ, the contract parameters M, S and L, and the extent α to which we take the standard deviation of the claim cost C into account. This gives a price $P = P(\lambda, \sigma, \xi, q, \theta, M, S, L, \alpha)$

Remember that we also have an assumption of the claim distribution, exponential, truncated exponential or deterministic in the group insurance case.

We will now investigate how each parameter affects the price P. What really interests us is how the claim cost C depends on the parameters, to make things clear, we present the effect which the parameters have on $E[C]$ and $SD(C)$.

As a starting point we use the following Cat XL contract: A Swedish insurance company that insures it's portfolio of 900 000 policies, Sweden's population being approximately 9 million people this yields $q = 0.1$. The other parameter values are according to our previous findings $\lambda = 4.13$, (table 11), $\sigma = 1.37, \xi = 0.66$ (Section 5.4). We assume that $\theta = 0.1$ and that the distribution of insured sums is such that they are all the same. Finally, we set $M = 3, S = 5$, $L = 100$ which is a realistic choice for a Cat XL contract. For this contract we find $E[C] = 0.93$ and $SD(C) = 5.29$ and with $\alpha = 0.2$ we get $P = 1.99$.

6.2.1 λ

How does C depend on λ? According to equation (7) we have

$$C = \sum_{k=1}^{K} Z_k.$$

Let $E[Z] = \mu, Var(Z) = \kappa^2$. Then (7) and (8) imply

$$E[C] = \mu \cdot E[K] = \mu \cdot \lambda$$

Thus $E[C]$ is linear in λ. What about $Var(C)$? Using a well known formula for the variance we find

$$\begin{aligned} Var(C) &= Var(E[C|K]) + E[Var(C|K)] \\ &= Var(\mu K) + E[\kappa^2 K] \\ &= \mu^2\lambda + \kappa^2\lambda \\ &= (\mu^2 + \kappa^2)\lambda \end{aligned}$$

so that $Var(C)$ is linear in λ as well.

Indeed, Figure 6 show that $E[C]$ and $Var(C)$ are linear in λ, just as predicted by theory.

6.2.2 σ and ξ

For a GPD a higher σ means a flatter curve, and hence a higher expected value of X, cf. Figures 3 and 4. They have the same ξ-value but MAF has $\sigma = 25.6$ compared to SAF having $\sigma = 10.0$.

ξ determines the weight of the tail of the GPD. Remember that for $Var(X)$ to exist it is required that $\xi < 1/2$ and for $E[X]$ to exist we need $\xi < 1$.

Thus $E[C]$ is expected to be increasing in σ and ξ. Judging from the plots, $E[C]$ is more sensitive to changes in ξ than in σ.

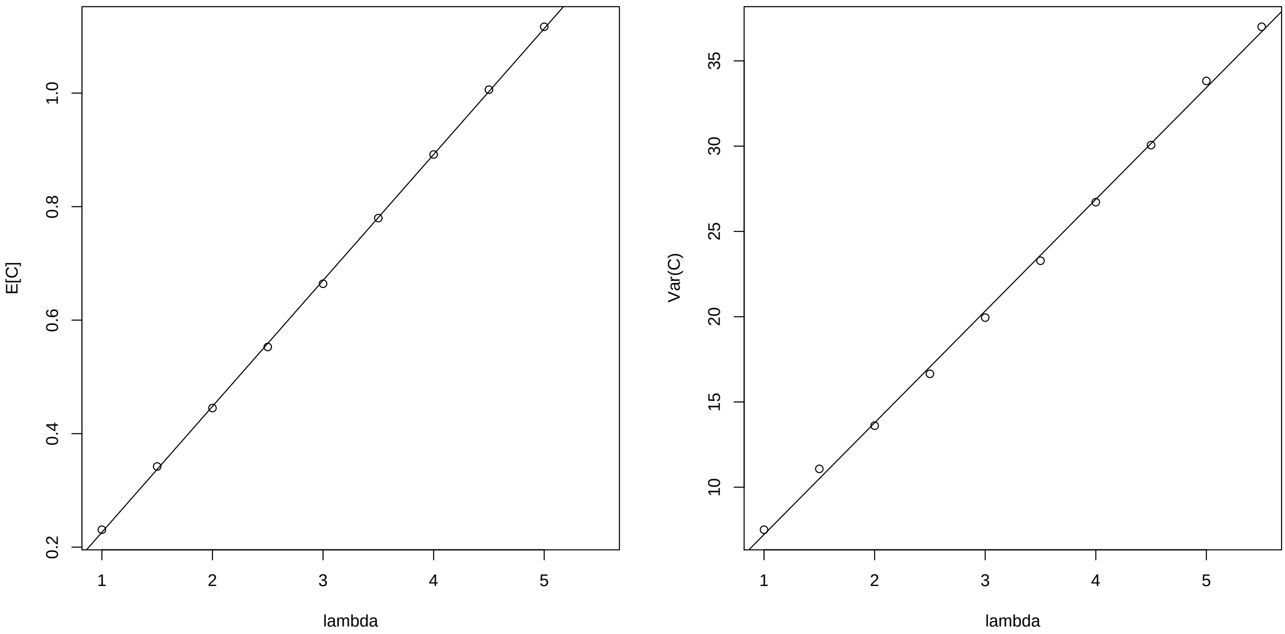

Figure 6: $E[C]$ and $Var(C)$ as functions of λ, SWE data

6.2.3 q, θ

Recall from Section 3.4 that in this model $E[Y|X] = q \cdot X$, hence we expect $E[C]$ to be linear in q. Figure 9 confirms this. We also find $\text{Var}(C)$ to be linear in q.

In Section 3.4 we also saw that a small θ implies large dependence and a large θ implies more of independence, but that $E[Y]$, the number of insured lives lost, does not depend on θ. So how does the dependency parameter θ affect $E[C]$ and $SD(C)$?

Consider the following simplified situation: $q = 0.1$, and a bus crashes, killing 20 people. If we have total independence of insured lives then $Y \sim \text{Bin}(20, 0.1)$. So with probability $P(Y < 4) = 95.7\%$ a a Cat XL contract will not be triggered! If we have total dependence, in nine out of ten such crashes there would be no insured lives lost, but in the tenth all twenty will have been insured, thus triggering a Cat XL contract!

We draw the conclusion that $E[C]$ and $SD(C)$ is decreasing in θ. Computing $E[C]$ and $SD(C)$ as functions of θ reveals that that is the case, and that going from $\theta = 10$ to $\theta = 0.1$ triples $E[C]$ and increases $SD(C)$ with a factor of 1.8, thus having a real significant effecton the price! See Figure 10.

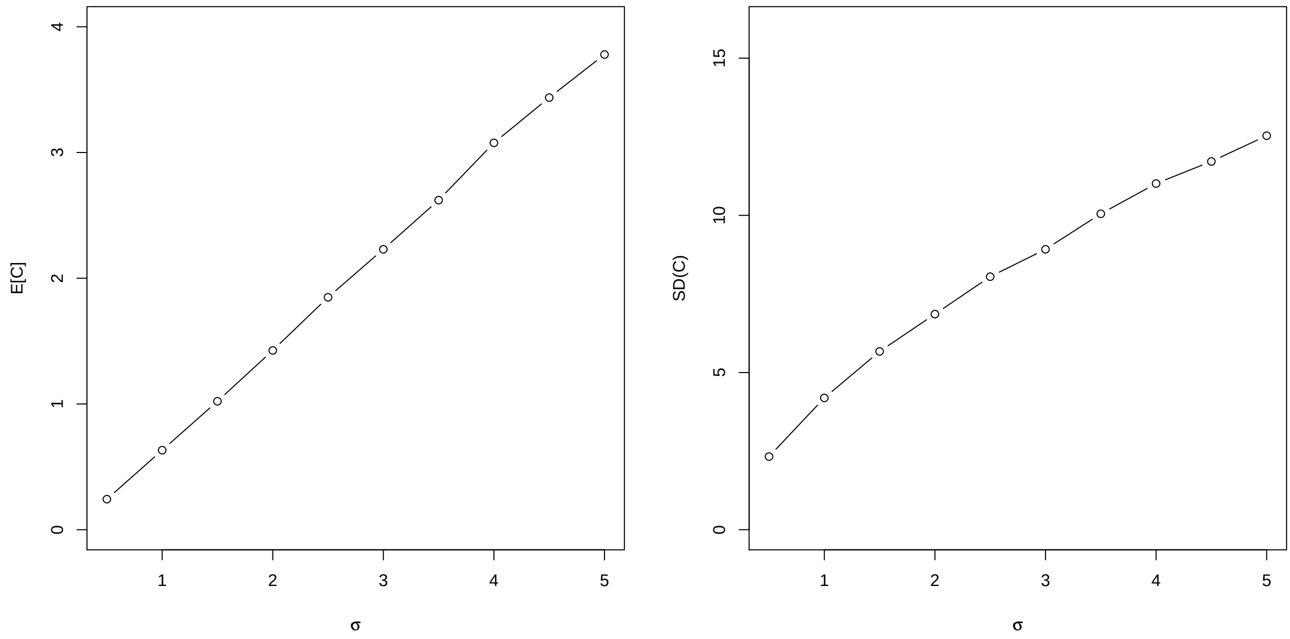

Figure 7: $E[C]$ and $SD(C)$ as functions of σ, SWE data

6.2.4 M, S and L

As a threshold M affects $E[C]$, but not as much as S witch is the deductible for each claim. The same holds for $SD(C)$. See Figures 11-12.

We see in Figure 13 that L, the maximal liability, has a real impact on the price, not so much trough $E[C]$ as trough $SD(C)$.

6.2.5 Claim distribution

How does the choice of claim distribution impact $E[C]$ and $SD(C)$? With a constant sum insured we got $E[C] = 0.93$ and $SD(C) = 5.29$. With the exponential we get $E[C] = 1.10$ and $SD(C) = 5.45$, while the at $Z = 5$ truncated exponential yields $E[C] = 1.08$ and $SD(C) = 5.41$. Hence, the effect of replacing constant insured sums by the (possibly truncated) exponential distribution is to increase $E[C]$ by 16–18% but $SD(C)$ by only 2–3%. The effect of using the more realistic truncated exponential seems to be neglible, at least for such a high truncation point as 5. For if $Z \sim \exp(1)$ then $P(Z > 5) = 0.67\%$.

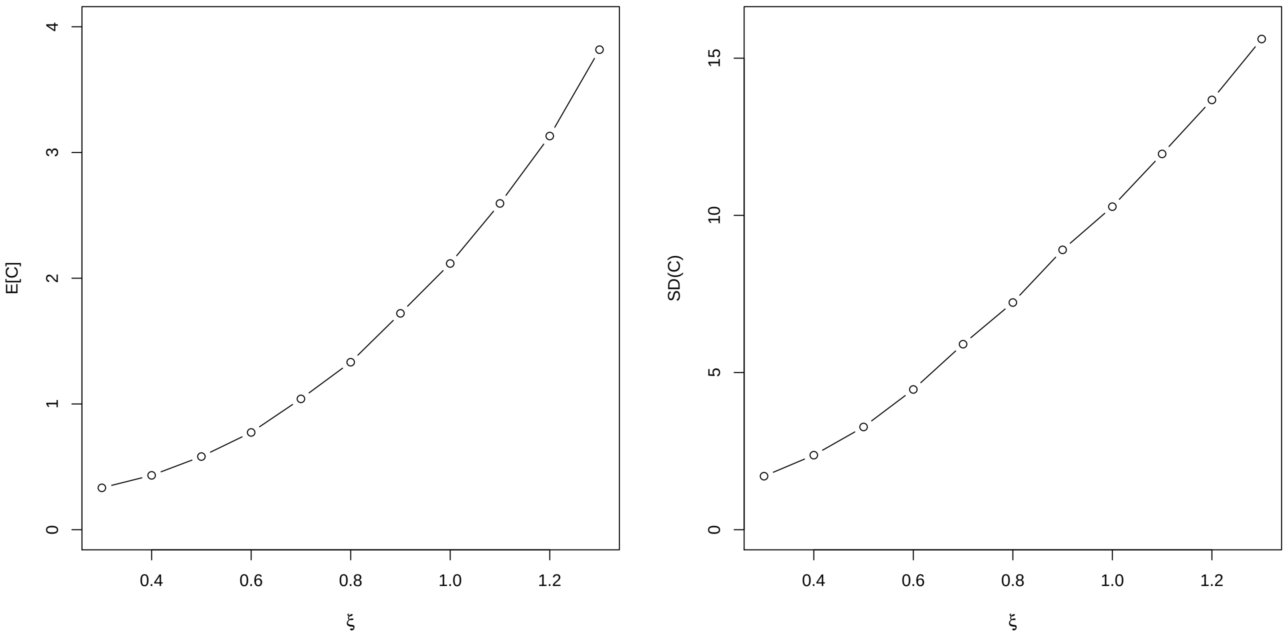

Figure 8: $E[C]$ and $SD(C)$ as functions of ξ, SWE data

6.2.6 Prices for some other regions

We also give the price of the above standard contract for some regions. λ_4, σ_4 and ξ_4 were estimated according to the methods in Section 5.5. We give two prices, one based on $\hat{\sigma}_4$ and one based on an adjusted $\hat{\sigma}_4$, were the adjustment is such that $1 - F_4(20) \approx 3.2\%$. The two ways of calculating the price yields

Region	Price original σ	Price adjusted σ
WEU	6.1	8.0
EEU	3.6	7.7
NAM	2.6	9.0

Table 16: Price for some regions.

quite different answers. The main reason is the differences in $\hat{\lambda}_4$. Further studies in accident intensities are needed in order to be able to give prices with a higher degree of certainty.

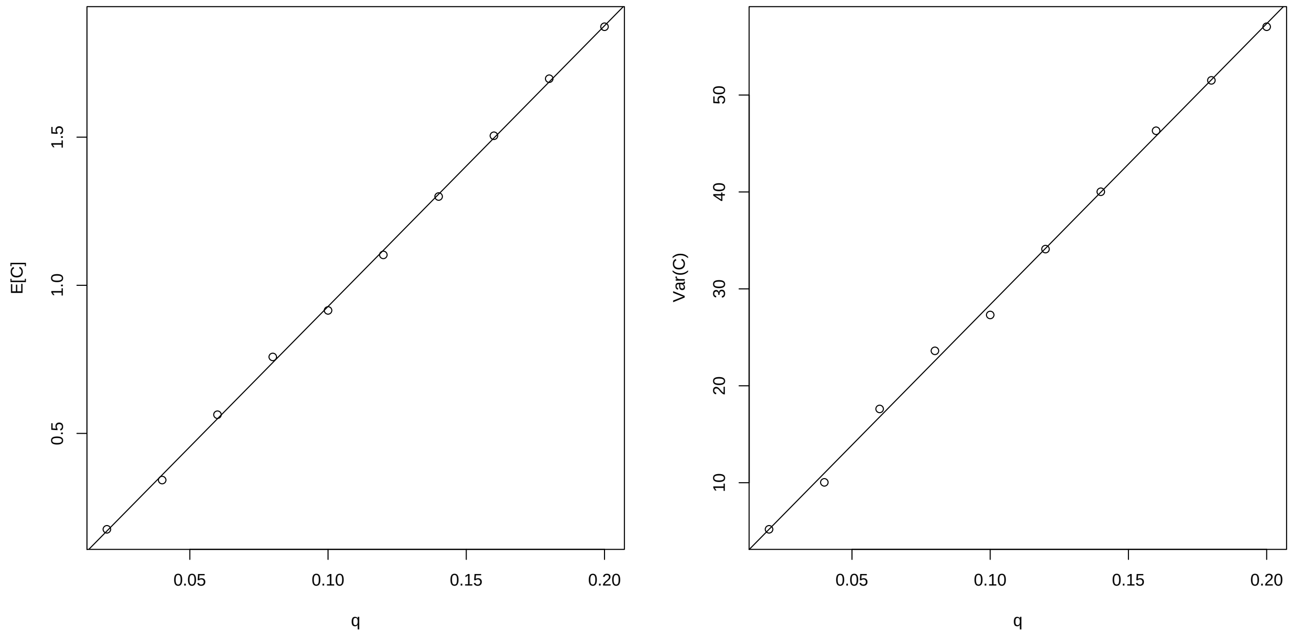

Figure 9: $E[C]$ and $\text{Var}(C)$ as functions of q, SWE data

6.3 Real world pricing

> There is a saying (Bostrom and Cirkovic 2008, page 177) in catastrophe reinsurance that "nothing is less than 1 on line", meaning that the vagaries of life are such that you should never price high-level risk at less than a chance of a total loss once in a hundred years (1%).

Prices of Cat-contracts are often related to the maximal liability of the reinsurer. They are given as a "rate on line",P/L, so for example for a contract with a maximal liability of 100 millions, a rate on line of 2% corresponds to a premium of $P = 2$ millions. For our standard contract, assuming $\alpha = 0.20$ the price is 1.99 and with $L = 100$ the rate on line is $1.99/100 = 1.99\%$.

There is always a fixed administrative expense associated with writing a contract and there is the cost of capital. However unlikely an event might seem, the reinsurer is at risk and wants to get paid for this. And there is also always the model risk, the humble actuary knows that there is a risk that the model used to calculate the price is flawed and thus have underestimated the risk of a catastrophic event. (Here you come to think of the so called "statistically impossible" events on the financial markets during the autumn of 2008. Independence can be a dangerous assumption.) Therefore reinsurers

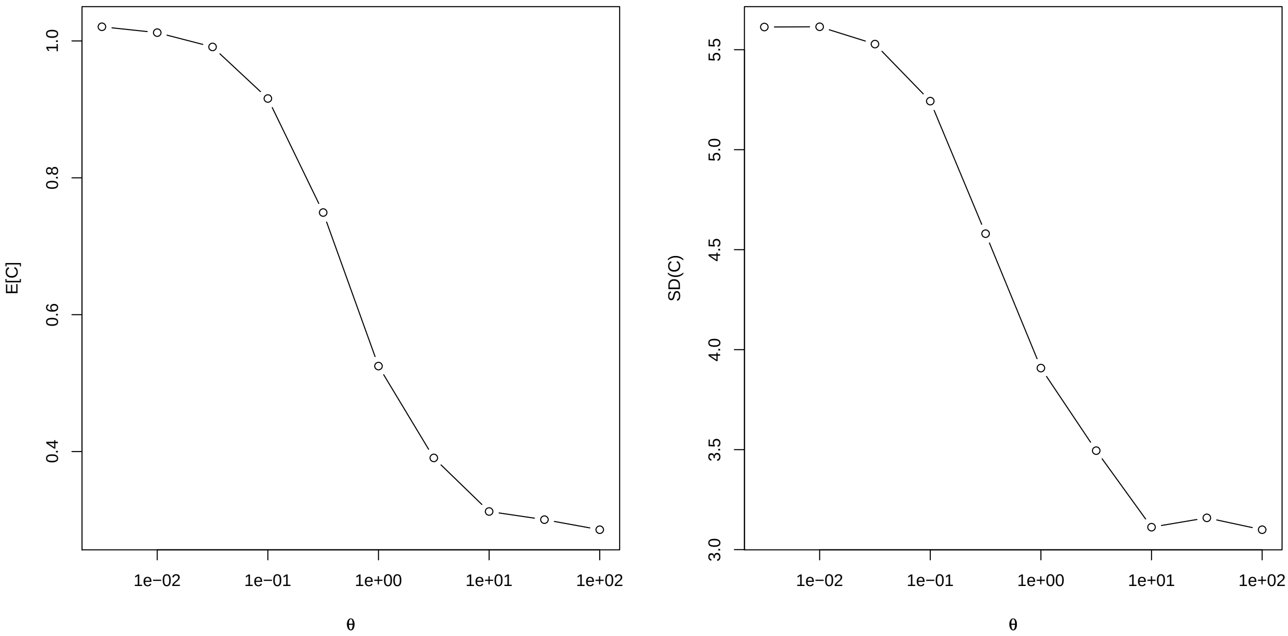

Figure 10: $E[C]$ and $SD(C)$ as functions of θ, SWE data

are reluctant to sell cover at a rate on line of less than about 1%.

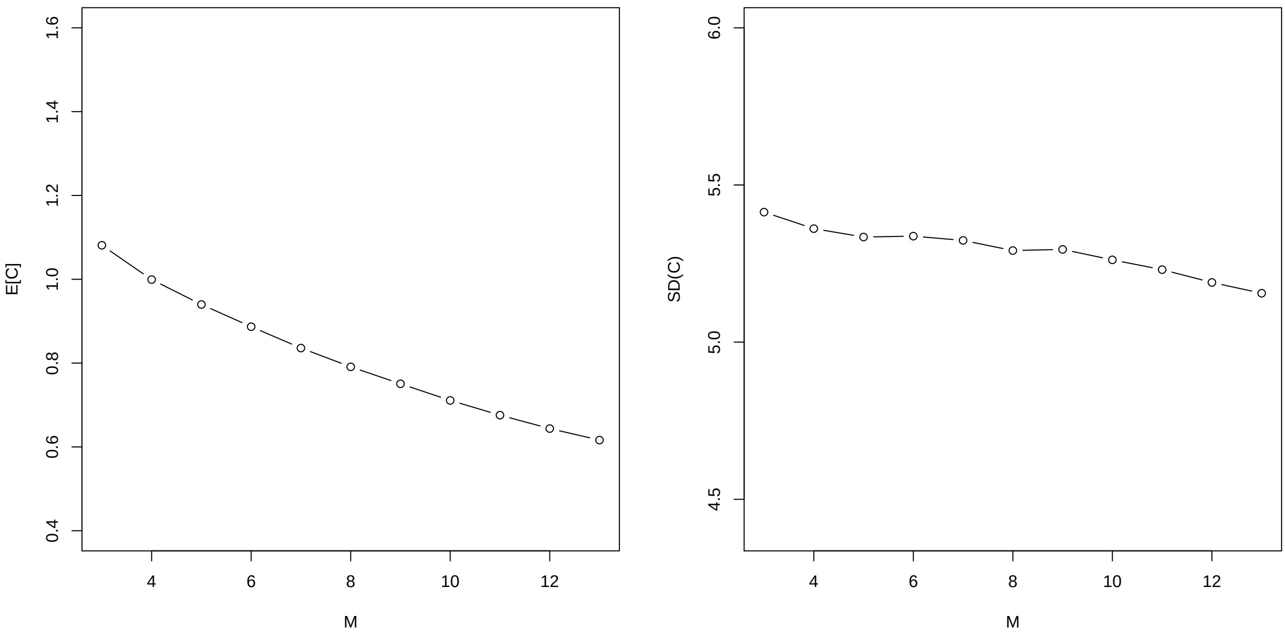

Figure 11: $E[C]$ and $SD(C)$ as functions of M, SWE data

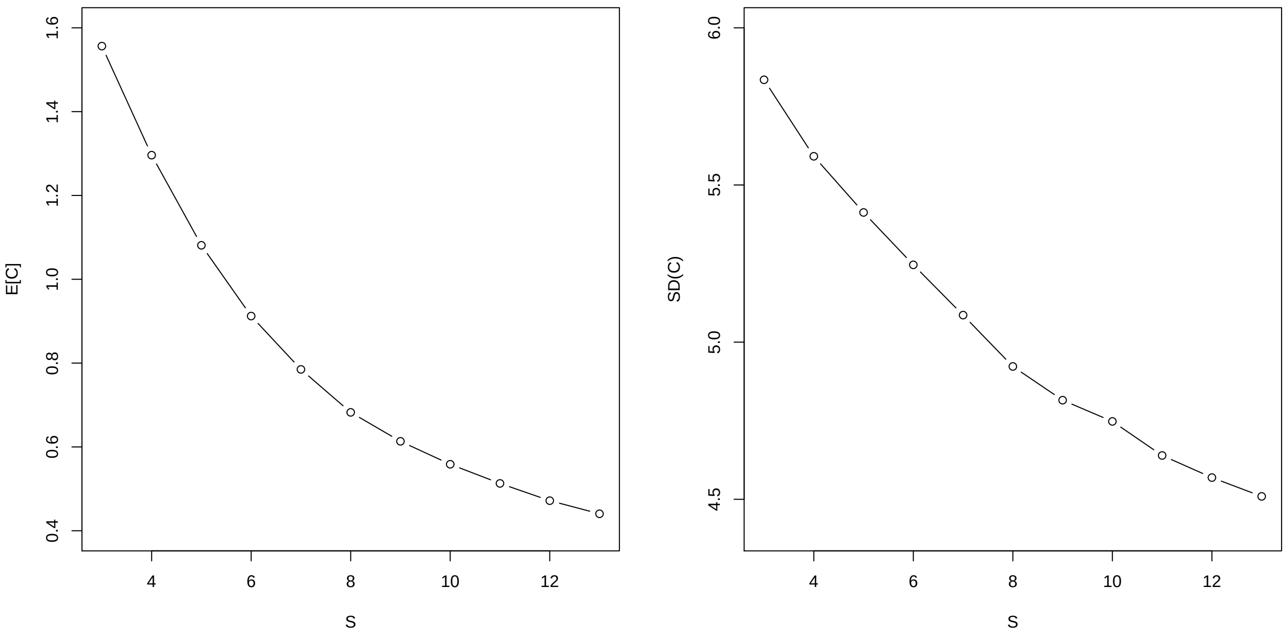

Figure 12: $E[C]$ and $SD(C)$ as functions of S, SWE data

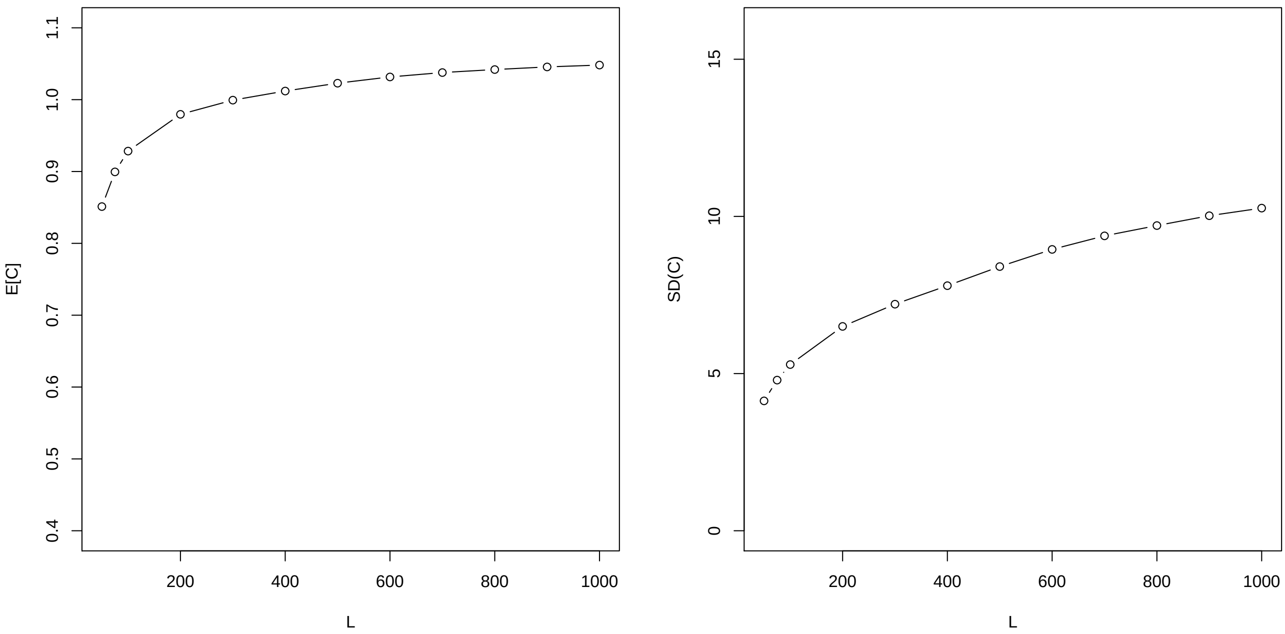

Figure 13: $E[C]$ and $SD(C)$ as functions of L, SWE data

7 Conclusion

In this paper, we have studied about how to price a catastrophe excess of loss contract in life reinsurance.

We first studied Strickler's old model for pricing. Although Strickler's model has it's merits, it is inflexible and to some extent unrealistic e.g. the deterministic catastrophe rate. Besides that, there is no statistically motivated way how to estimate the model parameters. Some more recent modifications of the model have made it more up to date but still not corrected these basic problems.

To get a mathematically-statistically satisfying way of pricing a Cat XL contract we were forced to develop a new model as follows: In equation (7) we expressed the total cost due to catastrophes during a contract period as $C = \sum_{k=1}^{K} Z_k$ were K, the number of catastrophes is assumed to be Poisson distributed and Z_k is the cost inflicted on the Cat XL contract by the k:th catastrophe. To obtain Z_k we start with X_k, the number of lost lives in the k:th catastrophe, assumed to have a generalized Pareto distribution. Then we turn to Y_k, the number of insured lives lost, assumed to follow a beta-binomial distribution conditional on X_k in order to reflect the possible dependence among lost insured lives. Then Z_k, the total loss, is the sum of the sums insured for each of the Y_k lost lives minus the retention stated in the Cat XL contract. The sum insured for each life can have a, possibly truncated, exponential distribution or be deterministic in case of a group policy. Finally the price P is calculated as $P = E[C] + \alpha \cdot SD(C)$.

In order to use the model for actual pricing we needed data for parameter estimation. We found two data sets, one international containing information about catastrophes, claiming at least 20 lives, from all over the world and a Swedish data set with data from accidents claiming at least four lives. With this data we were able to estimate parameters for both catastrophe intensity and size. Comparing the fitted model with the data, we found the fit to be good.

By using the estimated parameters together with data concerning the details of the contract, we could now get the price of a Cat XL contract by running computer simulations (in a Monte Carlo manner). We also conducted a sensitivity analysis for the price, varying one parameter at a time and observing how it affected $E[C]$ and $SD(C)$.

With the data at hand, we can now quote a price for any Cat XL cover in Sweden, or a high layer of a Cat XL contract in some other region of the world. We can quote a price for a low layer as well, but we would feel more confident if we had statistics from accidents claiming less than 20 lives as well. Even if we, with help from the Swedish data, can make an educated guess about parameter values for a low layer in other regions, the confidence intervals are quite wide.

Appendix A

Empirical probabilities, fitted GPD and QQ-plots for some regions

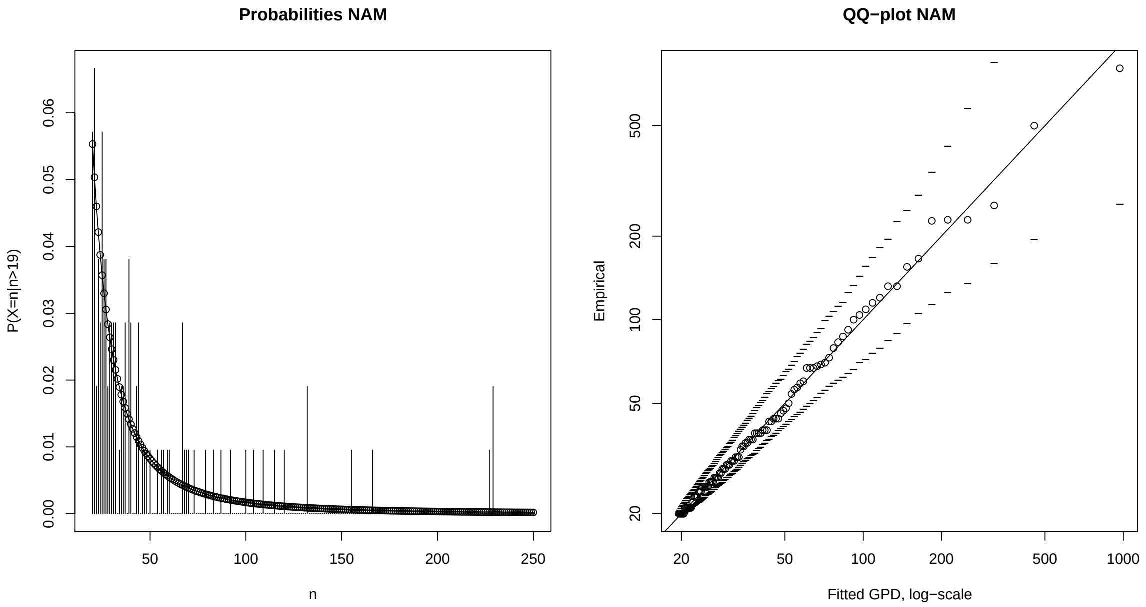

Figure 14: Empirical probabilities, fitted GPD and QQ-plot, NAM

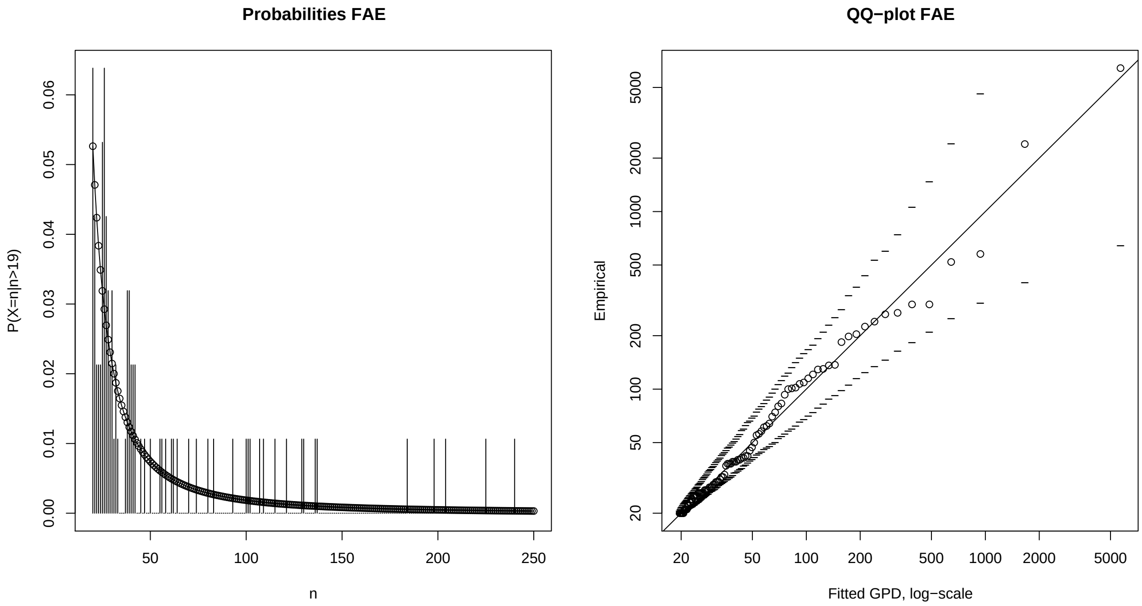

Figure 15: Empirical probabilities, fitted GPD and QQ-plot, FAE

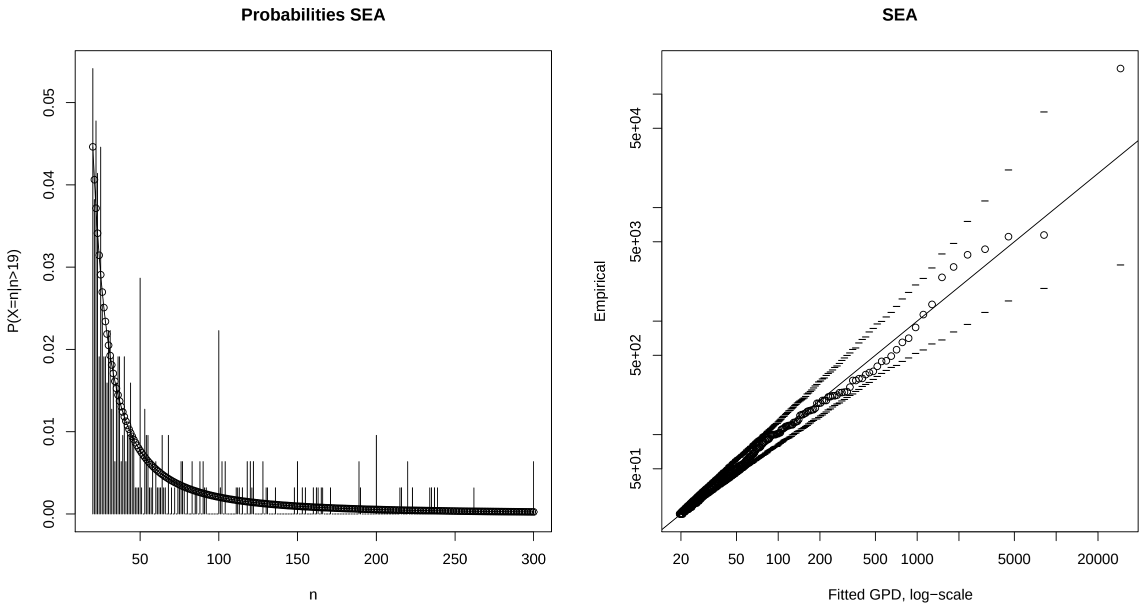

Figure 16: Empirical probabilities, fitted GPD and QQ-plot, SEA

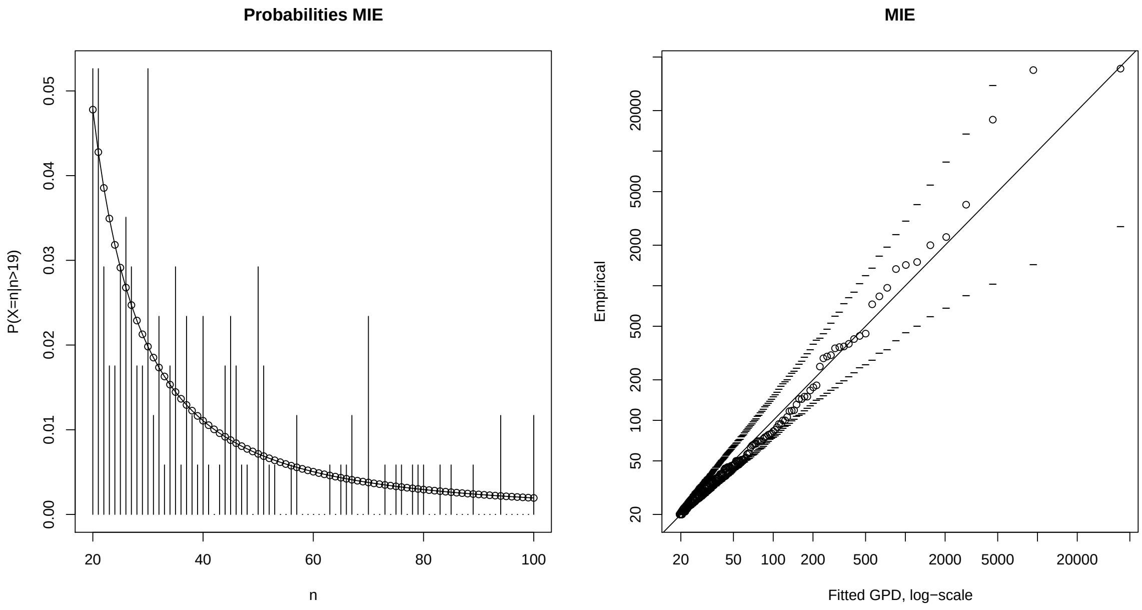

Figure 17: Empirical probabilities, fitted GPD and QQ-plot, MIE

Appendix B: List of countries included in various regions

CAM
Belize
Costa Rica
El Salvador
Guatemala
Honduras
Mexico
Nicaragua
Panama

CAR
Antigua
Aruba
Bahamas
Barbados
Bermuda
Cuba
Dom. Republic
Dominica
Grenada
Guadeloupe
Haiti
Jamaica
Martinique
Netherlands Antilles
Puerto Rico
St Lucia
St Vincent & Gr.
Trinidad & Tob.
Virgin Islands

CAS
China (PRC)
Mongolia
North Korea

EEU
Albania
Bulgaria
Czechoslovakia
Eastern Germany
Former Yugoslavia
Hungary
Poland
Roumania

FAE
Hong Kong
Japan
Singapore
South Korea
Taiwan

MAF
Angola
Benin
Botswana
Burkina
Burundi
Cameroun
Cape Verde
Central Africa
Chad
Comoros
Congo
Djibouti
Eq. Guinea
Ethiopia
Gabon
Gambia
Ghana
Guinea
Guinea-Bissau
Ivory Coast
Kenya
Liberia
Madagascar
Malawi
Mali
Mauretania
Mauritius
Mozambique
Niger
Nigeria
Réunion
Rwanda
Sao Tomé & P.
Senegal
Seychelles
Sierra Leone
Somalia
Sudan
Tanzania
Togo
Uganda
Zaire
Zambia
Zimbabwe

MIE
Afghanistan
Bahrain
Emirates
Iran
Iraq
Israel
Jordan
Kuwait
Lebanon
Oman
Qatar
Saudi Arabia
Syria
Turkey
Yemen North
Yemen South

NAF	SAF	SEA	Italy
Algeria	Lesotho	Brunei	Luxembourg
Egypt	Namibia	Burma	Malta
Libya	South Africa	Indonesia	Netherlands
Morocco	Swaziland	Kampuchea	Norway
Sahara		Laos	Portugal
Tunisia	SAM	Malaysia	Spain
	Argentine	Philippines	Sweden
NAM	Bangladesh	Thailand	Switzerland
Canada	Bhutan	Vietnam	(Western) Germany
Greenland	Bolivia		
USA	Brazil	SUN	
	Chile	Former republics of	
OCE	Colombia	the Soviet Union	
Australia	Ecuador		
Fiji	Guayana	WEU	
French Polynesia	Guayana(Fr)	Austria	
Guam	India	Belgium	
Kiribati	Macao	Channel Islands	
New Caledonia	Maldives	Cyprus	
New Zealand	Nepal	Denmark	
Papua	Pakistan	Finland	
Solomon Islands	Paraguay	France	
Tonga	Peru	Great Britain	
US Pacific Island	Sri Lanka	Greece	
Vanuatu	Surinam	Iceland	
Western Samoa	Uruguay	Ireland	
	Venezuela	Isle of Man	

Table 17: Regions - Countries

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