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A Fundamental Connection Between Credit Default Swaps and Equities

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Abstract

In this paper we aim to find a fundamental connection between Credit Default Swaps (CDS) and Equities, concentrating on the banking sector. To this end, we introduce factor model especially the Fama French three factor model. Moreover, we have also tried to find an explanatory distribution for the CDS spread. Here we introduce the Normal Inverse Gaussian (NIG) distribution.

The fundamental difference between the Fama French framework and the theories of traditional corporate valuation made the results difficult to interpret. While both theories agree that smaller companies are riskier, they do not agree on the riskiness of growth and value stocks. However, we could still see that the risk taken when investing in risky credit stocks could be explained by the Fama French factors, i.e. "value" and "size".

As an attempt to find an explanatory distribution for the CDS spread, we applied the "Chi-square goodness-of-fit-test". While the NIG distribution captured more of the heavy skewness and kurtosis compared to the Normal distribution, it could still not be fitted to the CDS spread.

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1 Introduction

The goal of this paper is to analyze credit default swap and to see how the credit derivative market interacts with the stock market. The credit derivative market has grown rapidly over the last few years. Credit Default Swaps (CDS) are the most common credit derivative products. In 2007, notional amount for credit default swaps were USD 45 trillion, compared to USD 630 billion in 2001.[1]

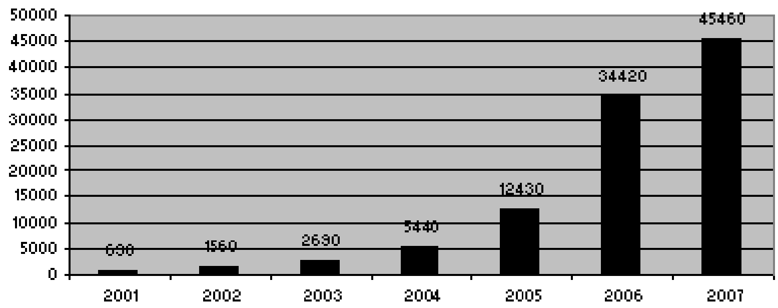


Figure 1: Outstanding CDS, notional amount in billion

A credit default swap is a legally enforced bilateral contract between two counterparts, under which one counterpart agrees to transfer the credit exposure for the compensation of periodical payments from the other counterpart. CDS's are traded over-the-counter (OTC) and are the most commonly traded credit derivative products. The main reason for this is that CDS gives investors the potential to manage credit risk in much the same way as market risk.

Furthermore, in this paper we will use CDS's as the representative of the credit derivative market. We will also focus on the banking sector, DJ Stoxx Banks index, as many banks actually have a CDS listening. Moreover, as an attempt to find a relationship between the credit derivative market and the stock market we will try to answer these questions:

- Is there a fundamental link between the movements in the stock price and the CDS spread?
- Can we find an explanatory distribution of the CDS spread?

In the first section of this paper we will give an introduction to credit default swaps. We will explain what the purpose of a credit default swap is and give some examples. Here, we will also introduce the concept of the CDS spread and discuss its link to the CDS contract.

In the second section, we will introduce some theories and models. We will start by discussing factor model as well as going into some general pricing theories. Here, we will implement the arbitrage pricing theory (APT), using the Fama French three factor model (FFT), to see how the CDS spread and the stock movements are linked together. The FFT factor model was created by Eugene F. Fama and Kenneth R. French (1992). They believed that there were

[1] www.isda.org ISDA market survey

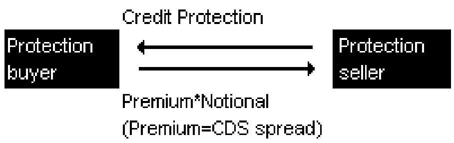
two other sources of risk that can describe the behavior of the market, aside from the market factor in the CAPM model. They discovered that "Size" and "Value" also had a link to the share price movements.

Furthermore, we will also introduce the normal inverse gaussian (NIG) distribution in this section. The NIG distribution belong to the family of hyperbolic distributions which was introduced by Barndorff-Nielsen (1977). One of the main reasons why this distribution is frequently used in finance and risk modeling, is because it allows one to take the third and fourth moment into account, i.e. skewness and kurtosis, as well as the mean and variance. First of all, we will go into some detailed mathematical and statistical properties. Secondly, we will discuss some benefits and drawbacks that the NIG distribution brings.

In the fourth and last section, we will go over how we intend to find a connection between the credit market and the stock market and explain the results. First of all, by using factor models and implementing the FFT factor model we will investigate if we can establish a fundamental link between share price movements and the movements in CDS spread. Secondly, we will try to implement the NIG distribution. The idea here is to investigate if the NIG distribution can be better fitted to the changes in CDS spread than the normal distribution. Furthermore, as the normal distribution is known to underestimate the tails of a stock's distribution we will test if the NIG distribution can be better fitted here as well.

2 Credit Default Swaps (CDS)

A credit default swap provides insurance against default, or in more general terms a credit event, by a particular company on a fixed income instrument. In this way the credit risk, or default risk, is transferred from the instrument owner to a third party.



For example, suppose that we own bonds issued by a company, the reference entity, for a notional amount of SEK 10 million. We are now exposed to credit risk, being the risk that the reference entity will default. This risk can be transferred to a third party in the form of a CDS contract. However, in compensation we would have to pay a risk premium, which is quoted in terms of a CDS spread. Now, suppose that a five-year CDS contract with a notional amount of SEK 10 million for a specific reference entity is traded at 200 basis points (what is known as the spread). This means that the CDS buyer, who in our example is the bond holder, will have to pay a fixed annual fee of SEK 200 000 to the CDS seller. This fee is often divided into quarterly payments. Now, in the case of a credit event the CDS buyer gets the right to sell the bonds with the notional amount of SEK 10 million to the CDS seller.

The CDS buyer is said to have a long position and the CDS seller a short position. The definition of a credit event varies a lot because credit default swaps are traded over-the-counter (OTC). This can of course also effect the risk premium when the contract is signed. The five-year CDS contract is the most liquid, which basically means that you have insurance over a five-year period. There are a few standard CDS maturities. However, the most traded are the five-year CDS contracts, which is why we have used the five-year CDS spread in this analysis.

2.1 CDS spread

The CDS spread is supposed to reflect instantaneous market perception of a company's credit worthiness. Roughly speaking, a spread of 200 basis points accounts for a default risk of 200/10000 or 2% on a yearly basis. Companies with high a CDS spreads require a greater compensation fee (risk premium) and are therefore more risky in terms of a CDS contract. Furthermore, a rise in the CDS spread should reflect that the market is challenging a company's credit worthiness.

The graph below illustrates the 5-year CDS spread for three European banks and the iTraxx. The iTraxx is one of the largest CDS indices and consists of the 125 most liquid 5-year CDS spreads in Europe. The recent turmoil in the credit market, the subprime crisis, has caused the CDS spread to increase. So, it seems that both Fortis bank and Deutsche bank were more exposed to the subprime crisis than Nordea. A rise in CDS spread should reflect that the company's credit worthiness is challenged by the market.

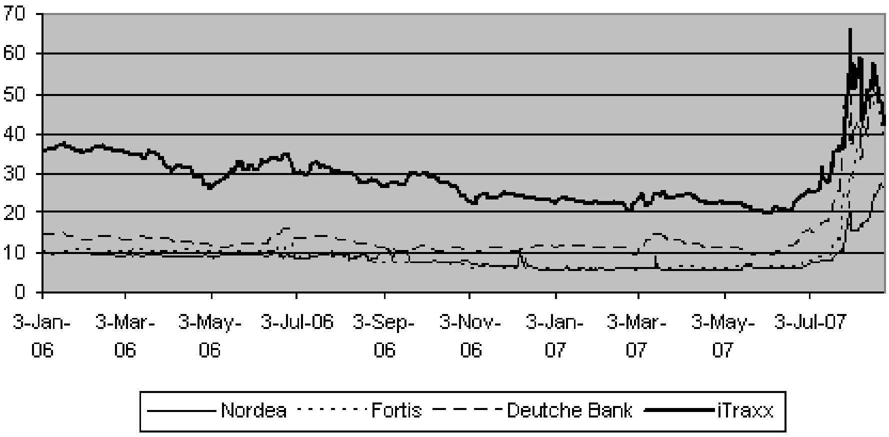


Figure 2: *CDS spread development for three European banks and the iTraxx.*

To get a better feeling of how a CDS contract and the underlying CDS spread are related we give another example. Let us assume that we enter a CDS contract with a notional amount of N and let the CDS spread at time $t_0 = 0$ be c_0 for a specific reference entity. Let T be the time of maturity for the contract and $t_k < T$ be a time in the future before maturity of the spread c_k. Let us assume further that we enter the contract with a long position, i.e. buying insurance.

At time $t = t_0$:
We enter a T-year CDS contract with a long position for the notional amount of N and CDS spread c_0. We now have to pay a risk premium of $N \cdot c_0$ to the CDS seller on an annual basis for the next T-years.

At time $t = t_k < T$:
The spread is now c_k. If we enter a CDS contract now, then we would have to pay $N \cdot c_k$ on an annual basis. Theoretically, if $c_k > c_0$ then we can make a profit of $N \cdot c_k - N \cdot c_0 = N \cdot (c_k - c_0)$ on an annual basis by having a short position in a CDS contract with the new spread.

Assuming that it is possible to go short and long at any time without any transaction cost then this can be realized. There are of course other restrictions that have to be taken into account. The contracts will have different delivery dates and if the long position is closed to maturity we would not experience this profit for a long time. Also, if the reference entity would default after the long position has matured, we would still have the obligation of compensating the CDS buyer and make a loss. However, in order not to complicate things we define the return of a CDS contract in this analysis as

$$R(t_k, t_{k+1}) = \frac{N \cdot (c_{k+1} - c_k)}{N \cdot c_k} = \frac{c_{k+1} - c_k}{c_k} = \frac{c_{k+1}}{c_k} - 1.$$

2.2 CDS Market vs. Stock Market

One of the main purposes of this study is to take a closer look at how the CDS market interacts with the stock market. We obtain the first link between the two markets by simple looking at the correlation, as you can see in the figures below.

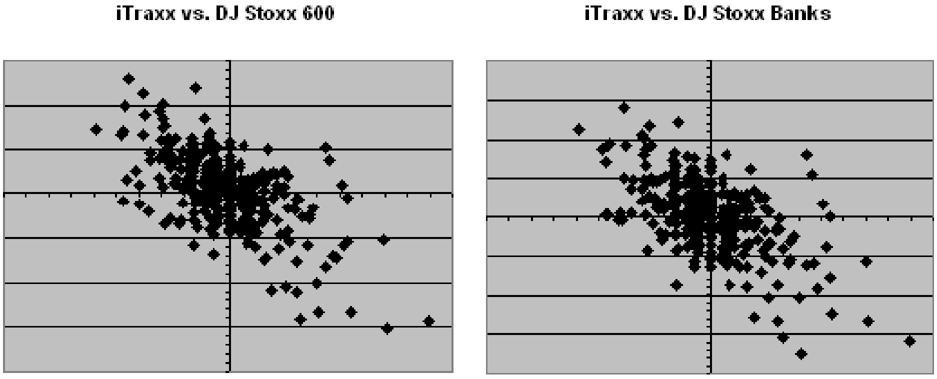


Figure 3: *iTraxx (x-axis) vs. DJ Stoxx 600 with -60% correlation and iTraxx vs. DJ Stoxx banks with -57% correlation. The DJ Stoxx 600 index is an equity index which contains 600 of the largest European stocks. Furthermore, the DJ Stoxx Banks index are composed by the bank stocks in the DJ Stoxx 600.*

We see that the CDS spread has a tendency of increasing as the stock prices fall and vice versa. More importantly, there seems to be information embedded in the stock price as well as in the CDS spread concerning each other. However, as an increasing CDS spread reflects that a company's credit worthiness is worsening, a fall in the stock does not necessarily reflect the same thing. Moreover, while the share price should reflect the value of the company there is no absolute link between a company's fundamentals and its CDS spread.

Nevertheless, it is relatively easy to understand why there should be negative correlation between the stock market and the CDS market. Deteriorating credit conditions for a company will challenge its credit worthiness. This should have a direct impact on both the CDS spread and the stock price, although not always in the expected way.

Credit risk is a major source of risk for most banks, which is why many banks are not only trading CDS but also hedged against in CDS contracts. Therefore, in this paper we will focus our analysis of the banking sector. We also choose this sector because many banks actually have a CDS spread listed in the market. Focusing on a sector like this makes it also easier to compare the results between companies because they are roughly exposed to the same underlying risk factors. We performed the analysis on the DJ Stoxx Banks index, which contains the largest European banks.

3 Theory and Models

3.1 Factor Models

A factor model assumes a linear relationship between an assets rate of return and basic sources of randomness, termed factors, that influence the assets. The factors that are used to explain randomness can be external factors or factors that are more linked to the assets. The Capital Asset Pricing model (CAPM) can be used as a factor model. It is one of the simplest factor models and has only one risk factor, the market factor.

Single-factor models are the simplest of factor models and they illustrates the concept quite well. Suppose that there are n assets with rates of return r_i, $i = 1, \ldots, n$. Assume further that there exists a single factor f which explains the rate of return r_i of all assets. We can then assume that the rate of return and the factor are related by following regression line:

$$r_i = \alpha_i + \beta_i f + \epsilon_i \qquad i = 1, \ldots, n$$

Here α_i is the intercept of the regression line and β_i is called factor loading and represents the slope, which measures the factor sensitivity.

The ϵ_i's are the error terms and without any loss of generality it can be assumed that the error terms have zero mean, $E[\epsilon_i] = 0$. Other assumptions are that the error terms are uncorrelated with the factor and with the error terms of other assets, i.e $Cov(f, \epsilon_i) = 0$ and $E[\epsilon_i \epsilon_j] = 0 \; \forall \; i \neq j$. These assumptions may of course not actually be true, but we will assume that they are in this analysis. It is also assumed that the variance of the error terms are known, and they are denoted by $\sigma^2_{\epsilon_i}$.

To complicate things, assume that we hold a portfolio of m assets, $m < n$. We define the weights in the portfolio for the j:th asset as w_j, $\sum_{j=1}^{m} w_j = 1$. If the rate of return follows a single-factor model, then the portfolio's rate of return r is explained by the following equation:

$$\begin{aligned} r &= \sum_{j=1}^{m} w_j \left(\alpha_j + \beta_j f + \epsilon_j\right) \\ &= \underbrace{\sum_{j=1}^{m} w_j \alpha_j}_{a} + \underbrace{\sum_{j=1}^{m} w_j \beta_j}_{b} f + \underbrace{\sum_{j=1}^{m} w_j \epsilon_j}_{e} \\ &= a + bf + e \end{aligned}$$

Under the assumptions that $E[\epsilon_i] = 0$, $Cov(f, \epsilon_i) = 0$ and $E[\epsilon_i \epsilon_j] = 0 \; \forall \; i \neq j$ we have that:

$$E[e] = E[\sum_{j=1}^{m} w_j \epsilon_j] = \sum_{j=1}^{m} w_j E[\epsilon_j] = 0$$

$$Cov(f, e) = Cov(f, \sum_{j=1}^{m} w_j \epsilon_j) = \sum_{j=1}^{m} w_j Cov(f, \epsilon_j) = 0$$

$$\sigma_e^2 = E\left[\sum_{j=1}^{m} w_j^2 \epsilon_j^2\right] = \sum_{j=1}^{m} w_j^2 \sigma^2_{\epsilon_j}$$

If a portfolio is equally-weighted, i.e $w_j = 1/m \ \forall\ j$, and the $\sigma^2_{\epsilon_i} = s^2$ are the same for all assets, then we can easily see the effects of diversification.

$$\sigma_e^2 = \sum_{j=1}^{m} w_j^2 \sigma^2_{\epsilon_j} = \frac{1}{m} s^2$$

Hence, as $m \to \infty$ we see that $\sigma_e^2 \to 0$. Therefore, in a well diversified portfolio the error term in the factor model will be small. The overall variance, volatility, of the portfolio is

$$\sigma_r^2 = b^2 \sigma_f^2 + \sigma_e^2.$$

If a portfolio is well-diversified the σ_e^2 will essentially be small. However, because b is an average of the assets β_j the $b^2\sigma_f^2$ term remains more or less constant. Even though the σ_e^2 goes to zero the portfolio variance does not. This is why a factor model carries two sources of risks: the risk caused by ϵ_i which is said to be diversifiable because it is essentially zero in a well-diversified portfolio; and the $\beta_i f$ term which is said to be the non-diversifiable or systematic, since it is present even in a well diversified portfolio.

One can proceed and extend the model to more than one factor. Assume now that we have a k-factor model explaining the rate of return for all assets as

$$r_i = \alpha_i + \beta_{1i} f_1 + \ldots + \beta_{ki} f_k + \epsilon_i \qquad i = 1, \ldots, n.$$

Again, the α_i's are the intercepts of the regression lines and β_{ti}'s, $t = 1, \ldots, k$, are the factor loadings. The factors $f_1, \ldots, f_k$ and ϵ_i are random variables. We have the same assumptions as before, i.e the expected value of the error term is zero and uncorrelated with the factors and with the error terms of other assets. However, it is not assumed that the factors are uncorrelated with each other. So the rate of return for a portfolio with m assets, $m < n$, can be defined as:

$$\begin{aligned} r &= \sum_{j=1}^{m} w_j \alpha_j + \sum_{j=1}^{m} w_j \beta_{1j} f_1 + \ldots + \sum_{j=1}^{m} w_j \beta_{kj} f_k + \sum_{j=1}^{m} w_j \epsilon_j \\ &= a + b_1 f_1 + \ldots b_k f_k + e \\ &= a + \mathbf{b}^t \mathbf{F} + \epsilon_i \end{aligned}$$

with variance

$$\begin{aligned} \sigma_r^2 &= \mathbf{b}^t \mathbf{C}_f \mathbf{b} + \sigma_e^2 \\ \sigma_e^2 &= E\left[\sum_{j=1}^{m} w_j^2 \epsilon_j^2\right] = \sum_{j=1}^{m} w_j^2 \sigma^2_{\epsilon_j} \end{aligned}$$

of the total return and error term, and with $\mathbf{b}^t = [b_1, \ldots, b_k]$, $\mathbf{F}^t = [f_1, \ldots, f_k]$ and $\mathbf{C}_f = Cov(\mathbf{F})$, which is the covariance matrix of the factors. The portfolio volatility has the same structure as before with the diversifiable, σ_e^2, and the non-diversifiable, $\mathbf{b}^t \mathbf{C}_f \mathbf{b}$, risks.

3.1.1 CAPM factor model

The CAPM factor model is a special case of a single-factor model. It assumes that there is only one factor that can explain the rate of return, the market r_M. CAPM also assumes that there exists a risk free rate r_f, which in the short run is constant. The CAPM factor model is expressed in terms of excess return, i.e $r_i - r_f$ and $r_M - r_f$.

$$r_i - r_f = \alpha_i + \beta_i(r_M - r_f) + \epsilon_i \qquad i = 1, \ldots, n$$

The expected value of the error term is zero, $E[\epsilon_i] = 0$. Taking the expected value of the equations gives the CAPM identity except for the present of α_i.[2] To understand the connection between the CAPM theory and the model described above, we use a result from linear regressions. It states that the best linear predictor of a random variable Y $(= r_i)$ given another random variable X $(= r_M)$ is given by the following regression line[3]:

$$\widehat{Y} = \mu_y + \frac{\sigma_{xy}}{\sigma_x^2}(X - \mu_x) = \underbrace{\mu_y - \mu_x \frac{\sigma_{xy}}{\sigma_x^2}}_{intercept} + \underbrace{\frac{\sigma_{xy}}{\sigma_x^2}}_{slope} X$$

Moreover, if r_f is constant then $\beta_i = \frac{\sigma_{r_i r_M}}{\sigma_M^2}$, which is exactly equal to the β_i in the CAPM identity. So the β_i coefficient basically measures the market exposure. However, the CAPM identity does not have an α_i.

From the CAPM point of view, α_i, can be regarded as a measure of the amount that an asset i is mispriced. A stock with a positive α_i is performing better then it should, and a stock with negative α_i is performing worse than it should. To this extent we can use the CAPM factor model to see if an asset or a portfolio is underperforming by comparing their α_i, if we assume that the market is the only factor that can explain an assets rate of return. The α_i is referred to as the risk adjusted excess return.

3.1.2 Arbitrage Pricing Theory (APT)

To interpret multi-factor models, we would have to resolve to a different asset pricing theory called the Arbitrage Pricing Theory (APT). APT is an extension of the CAPM theory with fever underling assumptions. The only assumption is that investors prefer greater to lesser returns. However, it is also assumed that the universe of assets is large and for APT to work exactly we must have an infinite number of assets, which differ from each other in a non-trivial way.

To see the connection between CAPM and APT, let us assume that there exists only one underling factor. To be more general we call this factor f and so we have, in absence of error term,

$$r_i = a_i + b_i f \qquad i = 1, \ldots, n.$$

Different assets have different a_i and b_i. So, the rate of return of a portfolio with two asset, i and j, having weights w and $1 - w$, can be written as

$$r = wa_i + (1 - w)a_j + [wb_i + (1 - w)b_j]f.$$

[2]The Capital Assets Pricing Model tells us that if the market is efficient, the expected return $E[r_i]$ of any asset satisfies: $E[r_i] - r_f = \beta_i(E[r_M] - r_f)$, where $\beta i = \frac{\sigma_{r_i r_m}}{\sigma_{r_M}^2}$. See Luenberger, D G. (1998), *Investment Science*, Chapter 7

[3]See Gut, Allan (1995): *An Intermediate Course in Probability*, Theorem 5.2 page 52.

We can hedge against the factor risk by selecting $w = b_j/(b_j - b_i)$. The portfolio's rate of return is then risk free and must be equal to the risk free rate if there are no arbitrage opportunities, i.e.

$$r = wa_i + (1-w)a_j = \frac{a_i b_j}{b_j - b_i} + \frac{a_j b_i}{b_i - b_j} = r_f.$$

This can be rearranged to

$$\frac{a_i - r_f}{b_i} = \frac{a_j - r_f}{b_j} = c,$$

where c is a constant. This is a general relation and must hold for all assets if there are no arbitrage opportunities. Continuing, this shows that a_i and b_i are not independent, $a_i = r_f + b_i c$. Furthermore, we can now get the expected rate of return of asset i as

$$E[r_i] = a_i + b_i E[f] = r_f + b_i c + b_i E[f] = r_f + b_i(E[f] + c) = r_f + b_i F.$$

Both r_f and F are constants and once they are known the expected return of an asset is completely determined by b_i. If the factor f is chosen to be the rate of return of the market, $F = E[r_M] - r_f$, and $b_i = \beta_i$ then the APT is identical to CAPM.

$$E[r_i] - r_f = \beta_i(E[r_M] - r_f)$$

However, instead of just having one market factor, like the CAPM theory, APT allows us to use several factors. Now, suppose that there are n assets whose rate of return are governed by $k < n$ factors according to the equations

$$r_i = a_i + \sum_{j=1}^{k} b_{ij} f_j \qquad i = 1, \ldots, n.$$

Then there are constants $F_1, \ldots, F_k$ such that

$$E[r_i] = r_f + \sum_{j=1}^{k} b_{ij} F_j \qquad i = 1, \ldots, n,$$

if we believe that the rate of return for an asset can be described by a multi-factor model.[4] Then, from an APT point of view, the intercept, α, in a multi-factor model can be regarded as a measure of the amount that an asset is mispriced. Basically, by using a factor model and comparing the α's of different assets or portfolios, we can see if an asset is underperforming or performing better than it should.

The problem with APT is that the theory itself does not tell us what these factors are. So, as a complement to the CAPM factor model, we will use the Fama French three factor model, described below, to measure α.

3.1.3 Fama French three Factor Model (FFT)

The Fama French three factor model was created by Eugene F. Fama and Kenneth R. French (1992) based on APT. They observed that there were two classes

[4]**Proof**: See Luenberger, D G. (1998), *Investment Science*, Page 209

of stocks that had a tendency of performing better than the market as a whole: stock with small market capitalization (small caps, opposites are called large caps) and stock with a high Book-to-Price ratio (value stocks, opposites are called growth stocks).

$$r_i - r_f = \alpha + \beta_M(r_M - r_f) + \beta_S SMB + \beta_V HML + \epsilon_i \qquad i = 1, \ldots, n$$

The model by Fama and French suggests that an assets rate of return is explained by the sensitivity of three factor. First of all, the excess return on a market portfolio (market risk). Secondly, the return on a portfolio of small caps vs. the return on a portfolio with large caps (SMB). Last, the return of a portfolio of stock with high Book-to-Price ratio vs. the stocks with low Book-to-Price ratio (HML).

The SMB factor, **S**mall **M**inus **B**ig market capitalization, captures the size effect. Fama and French discovered that small caps outperforms large caps. A high exposure to this factor, i.e. a high β_S, implies that the portfolio is relying more on stocks with small market capitalization to generate return.

The HML factor, **H**igh **M**inus **L**ow Book-to-Price ratio (Book value/Share Price), tells us something about value and growth.

- Growth stocks are generally fast-growing companies that have demonstrated above-average growth. Growth investors believe that the growth rates of these companies will allow them to outperform the stock market over time. Furthermore, what characterizes these stocks are that they tend to be more expensive relative to their current earnings. These stocks are therefore considered to be more risky because their book value seems to be relatively low compared to their stock price. Stock with low Book-to-Price ratio are called growth stocks.

- Value stocks are the opposites. Investing in value stocks is usually considered to be the more conservative and less risky investment strategy. This is because they tend to be cheap, or undervalued, relative to their earnings. Stocks with high a Book-to-Price ratio are called value stocks.

Moreover, a high exposure to this factor, i.e. a high β_V, suggest that the portfolio is relying more on value stocks to generate return.

However, value stocks are considered to be more risky under the FFT framework. Fama and French believed that higher return is reward for taking a high risk. The explanation for this lies in the efficient market assumptions. If the market is efficient, which is one of the underlying assumption in both CAPM and APT, then there can only be one reason to why the stock price looks cheap - it is more risky.

3.2 Normal Inverse Gaussian (NIG)

The Normal Inverse Gaussian distribution (NIG) belongs to the family of hyperbolic distributions and is a special case of the generalized hyperbolic distribution, introduced by Barndorff-Nielsen (1977). The NIG distribution is a mixture of the Inverse Gaussian distribution and the Normal distribution, which we will show below, and is often used in finance for risk modeling, since it also allows us to model kurtosis.

3.2.1 Properties

The density function is

$$f_x(\alpha,\beta,\delta,\mu) = \frac{\alpha\delta K_1\left(\alpha\sqrt{\delta^2+(x-\mu)^2}\right)}{\pi\sqrt{\delta^2+(x-\mu)^2}} e^{\delta\lambda+\beta(x-\mu)}$$

for all x $\in \Re$, where $\lambda = \sqrt{\alpha^2-\beta^2}$, $\alpha > |\beta| \geq 0$, $\delta > 0$ and K_v denotes a modified Bessel function of the third kind. The α parameter is related to the kurtosis and β to the skewness whereas δ and μ describe the scale and the location respectively.

3.2.2 Moment generating function

The Bessel function in the density function makes it hard to obtain the moments by integrating the density function. Luckily, the moment generating function can easily be computed as

$$\begin{aligned}
\Psi_X(t) = E[e^{tX}] &= \int_{-\infty}^{\infty} e^{tx} f_x(\alpha,\beta,\delta,\mu)dx \\
&= \int_{-\infty}^{\infty} e^{tx} \underbrace{\frac{\alpha\delta K_1\left(\alpha\sqrt{\delta^2+(x-\mu)^2}\right)}{\pi\sqrt{\delta^2+(x-\mu)^2}}}_{T(x)} e^{\delta\lambda+\beta(x-\mu)}dx \\
&= e^{t\mu}\int_{-\infty}^{\infty} T(x)e^{tx+\delta\lambda+\beta(x-\mu)-t\mu}dx \\
&= e^{\delta\sqrt{\alpha^2-\beta^2}+t\mu}\int_{-\infty}^{\infty} T(x)e^{(t+\beta)(x-\mu)}dx \\
&= e^{\delta\sqrt{\alpha^2-\beta^2}+t\mu-\delta\sqrt{\alpha^2-(t+\beta)^2}}\int_{-\infty}^{\infty} T(x)e^{(t+\beta)(x-\mu)+\delta\sqrt{\alpha^2-(t+\beta)^2}}dx \\
&= e^{\delta\left(\sqrt{\alpha^2-\beta^2}-\sqrt{\alpha^2-(t+\beta)^2}\right)+t\mu}\int_{-\infty}^{\infty} f_x(\alpha,t+\beta,\delta,\mu)dx \\
&= e^{\delta\left(\sqrt{\alpha^2-\beta^2}-\sqrt{\alpha^2-(t+\beta)^2}\right)+t\mu}.
\end{aligned}$$

The moment generating function can only exist if $\alpha > |\beta+t|$. We can now derive the first four moments; mean, variance, skewness and kurtosis. However, to avoid heavy computations one can instead use the cumulant generating function.

3.2.3 Cumulant generating function

The cumulant generating function $C_X(t)$ is defined as the logarithm of the moment generating function. This eliminates the exponential which makes it easier to differentiate, since

$$C_X(t) = \log(\Psi_X((t))) = \delta\left(\sqrt{\alpha^2-\beta^2}-\sqrt{\alpha^2-(t+\beta)^2}\right)+t\mu \qquad \alpha > |\beta+t|.$$

We will not go into any details about the cumulant generating function and instead just show how the moments are calculated. Differentiating $C_X(t)$ we find that,

$$
\begin{aligned}
C_X'(t) &= \mu + \frac{\delta(t+\beta)}{\sqrt{\alpha^2-(t-\beta)^2}} &\Rightarrow\quad \kappa_1 = C_X'(0) &= \mu + \frac{\delta\beta}{\sqrt{\alpha^2-\beta^2}} \\
C_X''(t) &= \frac{\delta\alpha^2}{\left(\sqrt{\alpha^2-(t+\beta)^2}\right)^3} &\Rightarrow\quad \kappa_2 = C_X''(0) &= \frac{\delta\alpha^2}{\left(\sqrt{\alpha^2-\beta^2}\right)^3} \\
C_X^{(3)}(t) &= \frac{3\delta\alpha^2(t+\beta)}{\left(\sqrt{\alpha^2-(t+\beta)^2}\right)^5} &\Rightarrow\quad \kappa_3 = C_X^{(3)}(0) &= \frac{3\delta\alpha^2\beta}{\left(\sqrt{\alpha^2-\beta^2}\right)^5} \\
C_X^{(4)}(t) &= \frac{3\delta\alpha^2(\alpha^2+4(t+\beta)^2)}{\left(\sqrt{\alpha^2-(t+\beta)^2}\right)^7} &\Rightarrow\quad \kappa_4 = C_X^{(4)}(0) &= \frac{3\delta\alpha^2(\alpha^2+4\beta^2)}{\left(\sqrt{\alpha^2-\beta^2}\right)^7}
\end{aligned}
$$

where κ_n is called the n:th cumulant. We can now calculate the first four moments of the NIG distribution by using these cumulants:

$$
\begin{aligned}
E[X] &= \kappa_1 = \mu + \delta\frac{\beta}{\lambda} \\
V[X] &= \kappa_2 = \delta\frac{\alpha^2}{\lambda^3} \\
S[X] &= \frac{\kappa_3}{\kappa_2^{3/2}} = \frac{3\beta}{\alpha\sqrt{\delta\lambda}} \\
K[X] &= \frac{\kappa_4}{\kappa_2^2} = \frac{3}{\delta\lambda}(1+4\frac{\beta^2}{\alpha^2})
\end{aligned}
$$

The kurtosis defined above, $K[X]$, is according to definition the excess kurtosis. A normally distributed random variable has a kurtosis of three and an excess kurtosis of zero. This is also important to take into account when programming. For instance, Matlab uses the kurtosis whereas Excel employs the excess kurtosis.

3.2.4 A Mixture Characterization

A random variable X follows a Normal Inverse Gaussian distribution with parameters α, β, δ and μ if

$$
\begin{aligned}
X|Y = y &\sim N(\mu+\beta y, y) \\
Y &\sim IG(\delta^2, \lambda^2)
\end{aligned}
$$

where $\lambda = \sqrt{\alpha^2-\beta^2}$ and IG is short for the Inverse Gaussian distribution, also know as the Wald distribution.

This can be proved as follows: Let Y follow an IG distribution with parameters δ^2 and λ^2, where $\lambda = \sqrt{\alpha^2-\beta^2}$. Then, the moment generating function of Y is:

$$
\Psi_Y(t) = e^{\delta\left(\sqrt{\alpha^2-\beta^2}-\sqrt{\alpha^2-\beta^2-2t}\right)}
$$

Since $X|Y = y \sim N(\mu + \beta y, y)$, we obtain by using the moment generating function that:

$$\Psi_X(t) = E[e^{tX}] = E[E[e^{tX}|Y]] = E[e^{\mu t + \beta Y t + \frac{Yt^2}{2}}] = e^{\mu t}\Psi_Y(\beta t + \frac{t^2}{2}) =$$
$$= e^{\delta\left(\sqrt{\alpha^2-\beta^2}-\sqrt{\alpha^2-(t+\beta)^2}\right)+t\mu}.$$

Since $\Psi_X(\cdot)$ uniquely characterizes the distribution of X it follows that $X \sim NIG(\alpha, \beta, \delta, \mu)$.

Another useful property is that if $X_1 \sim NIG(\alpha, \beta, \delta_1, \mu_1)$ and $X_2 \sim NIG(\alpha, \beta, \delta_2, \mu_2)$ are independent then $X_1 + X_2 \sim NIG(\alpha, \beta, \delta_1 + \delta_2, \mu_1 + \mu_2)$. This can be verified using moment generating functions as

$$\Psi_{X_1+X_2}(t) = E[e^{t(X_1+X_2)}] = E[e^{tX_1}]E[e^{tX_2}] = e^{(\delta_1+\delta_2)\left(\sqrt{\alpha^2-\beta^2}-\sqrt{\alpha^2-(t+\beta)^2}\right)+t(\mu_1+\mu_2)}$$
$$\Rightarrow X_1 + X_2 \sim NIG(\alpha, \beta, \delta_1 + \delta_2, \mu_1 + \mu_2)$$

and again using that $\Psi_{X_1+X_2}(\cdot)$ uniquely characterizes the distribution of $X_1 + X_2$.

3.2.5 Skewness and Kurtosis

Why the NIG distribution is so useful is because, as mentioned before, we are allowed to take the third and fourth moments into account, the skewness and

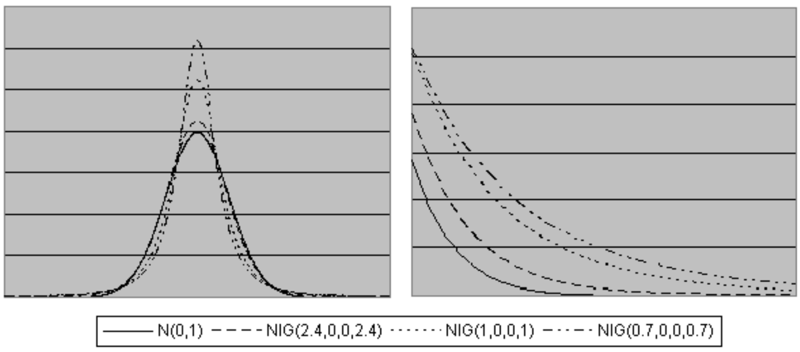


Figure 4: *Kurtosis*

kurtosis.

Figure 4 illustrates the effects of the kurtosis. The striped lines demonstrates the NIG distribution and the stroked line the Normal distribution. We can see that the tails become heavier and the peak higher as the kurtosis increases.

The skewness measures the asymmetry of the distribution. A positive skewness means that the right tail is longer and so the distribution's mass is shifted to the left, and the reverse is true for negative skewness. Figure 5 below illustrates a negative and a positive skewness for the NIG distribution compared to a Normal distribution which is completely symmetric.

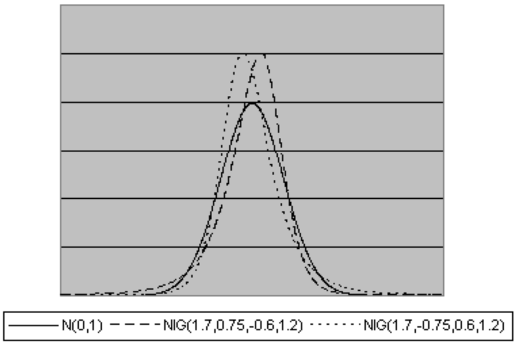


Figure 5: *Skewness*

It is a know fact that the normal distribution can underestimate the tails of a stock's distribution.[5] So kurtosis, together with skewness, can hopefully give new insights when we try to capture the distribution of a stock, as well as CDS spread. However, there is one problem with using the NIG distribution. The ML-estimates of the NIG parameters do not have a closed form and so we will estimate them using a slightly more primitive method.

3.2.6 Estimating the parameters

We will use a different approach to estimate the parameters of the NIG distribution. One can obtain the estimates for the NIG distribution simply by solving for α, β, δ and μ and use the moment estimates of the mean, variance, skewness and kurtosis. We illustrate below how we can solve the parameters with three simple steps:

Step (1);

$$S[X]^2 V[X] = \frac{9\beta^2}{\lambda^4} \Rightarrow \frac{9\beta^2}{S[X]^2 V[X]} = (\alpha^2 - \beta^2)^2$$

when $S[X] > 0 \Leftrightarrow \beta > 0$ and $S[X] < 0 \Leftrightarrow \beta < 0$, which gives us

$$\frac{9\beta^2}{S[X]^2 V[X]} = (\alpha^2 - \beta^2)^2 \Rightarrow \frac{3\beta}{S[X]\sqrt{V[X]}} = \alpha^2 - \beta^2 \Rightarrow \alpha^2 = \beta^2 + \frac{3\beta}{S[X]\sqrt{V[X]}}$$

Step (2);

$$\frac{K[X]}{S[X]^2} = \frac{\alpha^2}{3\beta^2} + \frac{4}{3} \Rightarrow \left[\alpha^2 = \beta^2 + \frac{3\beta}{S[X]\sqrt{V[X]}}\right] \Rightarrow \beta = \frac{3S[X]}{\sqrt{V[X]}(3K[X] - 5S[X]^2)}$$

Step (3); combining the last two displayed equations with the previously ob-

[5]See Höglund, T: *Mathematical asset management*, Chapter 2

tained expressions for $E[X]$, $V[X]$, $S[X]$ and $K[X]$, we obtain the final result

$$\alpha = \frac{3\sqrt{3K-4S^2}}{\sqrt{V}(3K-5S^2)}$$
$$\beta = \frac{3S}{\sqrt{V}(3K-5S^2)}$$
$$\mu = E - \frac{3S\sqrt{V}}{3K-4S^2}$$
$$\delta = \frac{3\sqrt{V(3K-5S^2)}}{3K-4S^2}$$

for all K $< 5S^2/3$ where $E = E[X]$, $V = V[X]$, $S = S[X]$ and $K = K[X]$. The parameters can now be estimated by replacing E, V, S and K with their sample estimate, which we have listed in the Appendix A.3. The ML-estimates can be obtain by using the so-called EM (Expectation Maximization)-algorithm. But unfortunately, these estimates does not have a closed form.[6]

3.2.7 Simulating a NIG random variable

We use the Rydberg-Monte Carlo method to simulate a NIG random variable which is based on the fact that if $Y \sim IG(\delta^2, \lambda^2)$ and $X|Y = y \sim N(\mu + \beta y, y)$ then $X \sim NIG(\alpha, \beta, \mu, \delta)$, where $\lambda = \sqrt{\alpha^2 - \beta^2}$ and $\alpha > |\beta| > 0, \delta > 0$ as found in subsection 4.1.3. So, assume that Y:s outcome is y then we can simulate X as a $N(\mu + \beta y, y)$ random variable.

$$X|Y = y \sim N(\underbrace{\mu + \beta y}_{\widehat{\mu}}, \underbrace{y}_{\widehat{\sigma^2}}) = Z\widehat{\sigma} + \widehat{\mu} = \mu + \beta y + Z\sqrt{y},$$

which gives $X = \mu + \beta Y + Z\sqrt{Y}$, where $Z \sim N(0,1)$ and $Y \sim IG(\delta^2, \lambda^2)$.

Most program packages can simulate a Normal random variable but we must still find a way to simulate an Inverse Gaussian random variable, $Y \sim IG(\delta^2, \lambda^2)$. This can be done with the following algorithm:

$$V = N(0,1)^2 \sim \chi_1^2,$$
$$W = \xi + \frac{\xi^2 V}{2\delta^2} - \frac{\xi}{2\delta^2}\sqrt{4\xi\delta^2 V + \xi^2 V^2}, \qquad \xi = \frac{\delta}{\lambda},$$
$$Y = W \cdot \mathbf{1}_{\{U \le \frac{\xi}{\xi + W}\}} + \frac{\xi^2}{W} \cdot \mathbf{1}_{\{U > \frac{\xi}{\xi + W}\}},$$

where $U \sim Uniform(0,1)$. We omit the proof that this algoritm indeed generates $Y \sim IG(\delta^2, \lambda^2)$, but refer the reader to Michael, Schucany and Haas (1976).

[6] See Dempster,Laird,Rubin (1977), *Maximum likelihood from incomplete data via the EM alogritm*, Series B, 39(1):1-38

4 Empirical results

4.1 Data

The data used in this study are provided by the quantitative research team at Fortis. The time period that is covered in this analysis is between January 2 2006 to July 29 2007.

Our CDS data consist of the 5-year CDS spread quotes for the banks in the DJ Stoxx banks index, see Appendix A.1. The spreads are the daily closing quotes and denominated in Euros. Furthermore, the stock price data are also quoted in Euros.

We used The DJ Stoxx 600 index as an approximation of the market, which is the parental index of the DJ Stoxx banks index. Furthermore, the FFT factors were also calculated with DJ Stoxx 600, see Appendix A.2 for more information.

4.2 Method

4.2.1 The idea behind the test

As an attempt to find a connection between the credit market and the stock market in terms of performance and risk, we divided the benchmark (DJ Stoxx Banks index) into three classes. The CDS spread was used to determine the classes; stocks with High, Mid and Low CDS spread. Moreover, this was done by ranking the stocks based on their CDS spread and then grouping them into the three groups. However, since the CDS spread varies over time, we updated the rankings daily, after which we calculate the equal-weighted return of the three portfolios.

The purpose of using the CAPM and FFT models is not only to measure the performance of the three portfolios in terms of excess return. We also use them to get a more fundamental explanation to why the three portfolios are performing the way they are. Moreover, this is where the SMB and HML factors in the FFT model come into to use.

4.2.2 Fitting the NIG distribution

We will use two methods to test how well the NIG distribution can be fitted to CDS spread and stock returns.

QQ-plots can be used to see if a sample fits a specific distribution. The Q stand for "quantiles" and so a QQ-plot is a scatter plot between the sample quantiles and the distribution's quantiles which is expected to fit the sample, in our case the NIG and the Normal distribution. By creating quantiles, typically as $k/(n+1)$ where n is the sample size and $k = 1, \ldots, n$, one plots the order statistics from the sample on the vertical axis and the quantiles of the comparison distribution on the horizontal axis. If the hypothesized distribution is correct, we approximately get a straight line.

These plots are very useful but they do not gives us a statistical measure like a p-value. For this one can use a "Chi-square goodness-of-fit-test". A Chi-square test basically measures the distance between the number of observed, O_i, and the expected values, E_i, which depends on the hypothesized distribution. In our case we have to estimate parameters of the hypothesized distribution, so

E_i has to be replaced by an estimate $\widehat{E_i}$. If

$$T = \sum_{i=1}^{m} \frac{(O_i - \widehat{E_i})^2}{\widehat{E_i}}$$

is large, we reject the hypothesized distribution, using the fact that $T \sim \chi^2_{m-p-1}$ when the distribution is indeed correct, where p is the number of parameters in the distribution and m the number of bins. The number of bins can of course differ a lot depending on the data but the rule of thumb is that each bin should at least have 5 observations. However, one of the requirements in performing this test is that we need the ML-estimates to calculate the expected values.[7] This can easily be done for the Normal distribution but not for the NIG distribution, as mention before. So we will ignore this detail and believe that it will not have a large impact on the results.

4.3 Result

We first investigate how the three portfolios performed, see figure 6. The stocks with a high CDS spread performs better than those with low a CDS spread. This suggests that the companies which are supposed to be more risky from the credit markets point of view tend to perform better on the stock market. However, this might be driven by some underlying characteristics and so we turn to the factor models.

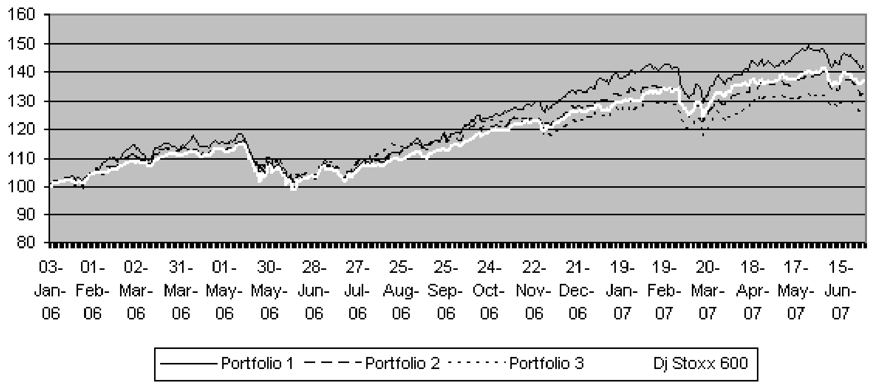


Figure 6: *Indexed development of the three portfolios (Portf. 1 = High CDS spread, Portf. 2 = Mid CDS spread, Portf. 3 = Low CDS spread), with 2006 as starting year.*

The parameters of the CAPM and FFT factor models are estimated using multiple linear regression, see the table below and Appendix A.4. We explained in the previous section that the intercept α of the factor models is a way of measuring the performance of an asset or portfolio after accounting for systematic risk. As we can see in the table below, both the CAPM and the FFT factor

[7] See Rao, R (1973): *Linear Statistical Inference and Its Applications (Second Edition)*, pages 391-393

models suggest that stocks with a high CDS spread give a greater return (the α in the table is annualized). However, the α's are not significantly different from zero, see Appendix A.4. We saw from the graph that the stocks with high CDS spread perform better. However, it seems that this is explained by the exposure to the risk factors. Furthermore, by studying the factor loadings, the β:s, in the FFT factor model we see that the portfolio with high CDS spread hold two important properties.

	CAPM: Annual α	β_M	R^2	FFT: Annual α	β_M	β_S	β_V	R^2
Portf. 1	2.41%	1.01	82.0%	2.12%	1.01	-0.18	0.24	82.7%
Portf. 2	-2.20%	1.00	84.1%	-2.57%	1.02	-0.45	0.31	86.5%
Portf. 3	-4.84%	1.00	82.4%	-5.47%	1.02	-0.63	0.55	87.8%

First of all, the portfolio with high CDS spread, portf. 1, has the smallest β_S. As the banks in the DJ Stoxx index contains the largest banks in Europe, we see that represented in the high CDS spread portfolio are the smallest banks in the index. Secondly, companies with a high CDS spread should also have a more pressured credit profile. Looking at the β_V, we see that the portfolio with high CDS spread is dominated by growth stock, which according to the definition have a low Book-to-Price value. Stock with a small book value relative to their share price should be considered to be more risky, which we discovered seems to be reflected in their CDS spread.

All in all, smaller companies are in general more risky. Investing in new and small growing companies is a risky business and is clearly reflected by their CDS spreads. Furthermore, as mentioned before, it is also know that small caps performed better than large caps. This can explain why the stocks with a high CDS spread are more exposed to the SMB factor.

However, the value vs. growth factor under the FFT framework implies that value stocks are more risky because they generally perform better. Furthermore, Fama and French believed that higher risk is rewarded in a higher return, which is also suggested by the CDS spread. But as we mentioned before, this depends in whether or not one believes in the efficient market theory. Moreover, the β_V factor suggested that the companies with a high CDS spread are growth stocks, which are often overvalued and expensive and thus more risky in the view of traditional investors. While this does not agree with the FFT framework, it does make some sense to see that the stocks with high CDS spread are less exposed to the HML factor.

4.3.1 NIG vs. Normal distribution

The next step was to see if we could fit the NIG distribution to a real asset. This can be done by randomly picking and using its historical data to compute method of moments estimates of parameters, draw QQ-plots and perform goodness-of-fit tests, as described above. Furthermore, in this section will not only see if it can be fitted to stock returns but also to the changes in the CDS spread (CDS return). We have used the stock price and the CDS spread of Nordea, which is a Swedish bank listed in the Swedish stock exchange OMX.

First of all, we try to fit the NIG distribution to daily stock returns, see figure 7. If the reader is familiar with QQ-plots then one can immediately see from the normal plot that the Nordea stock has a high kurtosis. This is also a general phenomenon and widely documented property amongst stocks. However, the fitted NIG distribution does not seem to miss this. The NIG distribution seems to fit perfectly while the normal distribution clearly underestimates the tail of the stock distribution.

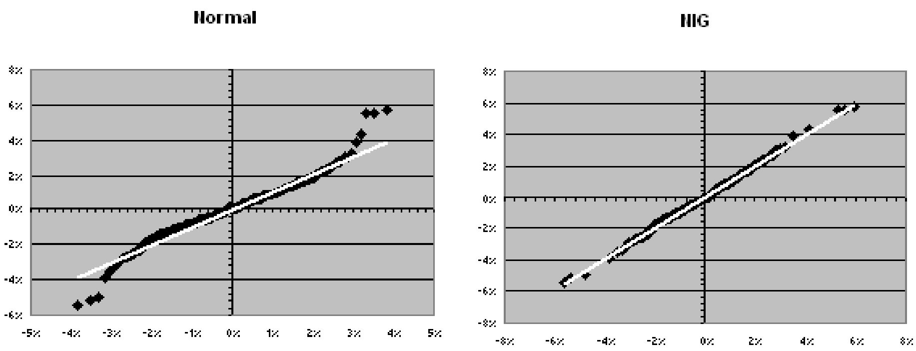


Figure 7: *QQ-plot: Using stock return of the Nordea share.*

Furthermore, this is also the conclusion from the Chi-square test. The Normal distribution can be rejected at a 1.0% significance level, while the NIG distribution is not be rejected.

	Chi-square	Bins	Parameters	d.f	p-value
Normal	26.6	13	2	10	0.30%
NIG	9.9	13	4	8	27.2%

Furthermore, we also performed the same analysis on Nordea's CDS spread. Here, we have used the notation explained in section 1 to calculate the changes in CDS spread. However, the result does not look that promising for CDS spread, see figure 8. We can see that the CDS spread has a high kurtosis by looking at the normal plot. Furthermore, NIG distribution seems to fit better then the Normal distribution but not good enough.

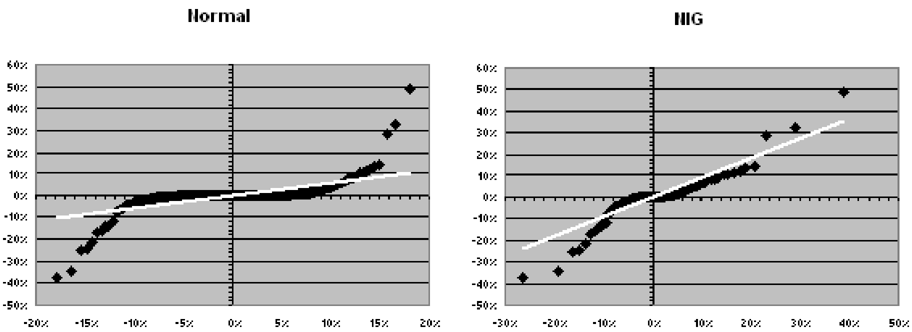


Figure 8: *QQ-plot: Using the CDS spread of Nordea.*

The chi-square test confirms the result. Both distributions are rejected at a 1.0% significant level even though NIG distribution fits marginally better. So, even though the NIG distribution can be fitted to stocks return it can not be fitted to the changes in CDS spread.

	Chi-square	Bins	Parameters	d.f	p-value
Normal	382.4	8	2	5	1.88e-78%
NIG	234.8	8	4	3	1.30e-48%

5 Conclusion

In this thesis, we have tried to find a connection between the fundamentals of the stock market and the credit market. Our main focus has been to investigate whether returns on stocks with different CDS spreads can be connected to its fundamental profile, "size" and "value". Furthermore, we have used the Fama French three factor model to find an explanation as to why the stocks with various CDS spreads are performing the way they are. Moreover, we have also tried to find an explanatory distribution for the movements in CDS spreads. Here, we focused on the NIG distribution as well as the Normal distribution.

The fundamental differences between the Fama French framework and the theories of traditional corporate valuation make the result difficult to interpret. While both of these theories assume that smaller companies are riskier, they do not agree on the riskiness of growth and value stocks. The FFT framework assumes that the market is efficient. This basically suggests that stocks that are delivering abnormal returns are doing so because they are more risky. Furthermore, as Fama and French discovered that value stocks perform better than growth stocks, this suggests that value stock are more risky. However, this does not agree with traditional corporate valuation, which finds growth stocks to be more risky because they are relatively expensive compared to their earnings.

Furthermore, as we performed the test on the bank sector we saw that stock with high CDS spreads perform better than stocks with low CDS spreads. While stocks with high CDS spreads are more risky, we see that these stocks consisted of predominantly the smaller banks and banks which fitted in to the growth stock criterium. Interesting enough, after accounting for the Fama French factors we could not draw the conclusion that the stocks with high CDS spreads actually deliver greater return, as the α's were not significant. This basically suggests that the risk taken when investing in stocks with high CDS spreads is explained by the risk factors in the FFT model, i.e size and value.

Furthermore, as an attempt to find an explanatory distribution for the CDS spread, we applied the "Chi-Square goodness-of-fit-test". This revealed that while the NIG distribution is applicable for stocks, it could not be fitted to CDS spreads. So, the NIG distribution could explain more but has still failed to capture the heavy skewness and kurtosis of the CDS spread.

A APPENDIX

A.1 DJ Stoxx Banks index

Name	CDS spread	Name	CDS spread
ABN amro		Dexia	
Alliance & Leicester		Deutsche Postbank	NA
Allied Irish Banks		DnB Nor	
Alpha Bank		EFG Eurobank Ergasias	
Anglo Irish Bank		Emporiki Bank of Greece	NA
Banca Antonveneta		Erste Bank Der Oesterreichi	
Banca Carige	NA	Fortis	
Banca Lombarda e Piemontese		Glitnir banki	NA
Banca Monte Dei Paschi		HBOS	
Banca Naz Lavoro		HSBC	
Banca Popolare Italiana		Intesa Sanpaolo	
Banca Popolare Milano		Jyske Bank	
Banco Comercial Portugues		Kaupthing Bank	
Banco de Valencia	NA	KBC Ancora	NA
Banco Espirito Santo		KBC Groupe	
Banco Pastor		Landsbanki island	
Banco Popolare	NA	Lloyds TSB	
Banco Popular Espanol		Mediobanca	
Banco Portugues De Inv.	NA	National Bank of Greece	NA
Banco Sabadell		Natixis	
Banco Santander		Nordea	
Bank of Greece	NA	Northern Rock	
Bank of Ireland	NA	Pohjola Bank	NA
Bank of Piraeus		Raiffeisen International Bank	
Bankinter	NA	Royal Bank Of Scotland	
Barclays		SEB	
Bayerische Hypo and Vereinsbank		San Paolo Imi	
BBVA		Societe Generale	
BNP Paribas		Standard Chartered	
Bradford & Bingley		Swedbank	
Capitalia	NA	Handelsbanken	
Commerzbank		Sydbank	NA
Credit Agricole		UBI Banca	NA
Credit Suisse		UBS	
Danske bank		Unicredit	
Depfa bank		Valiant	NA
Deutsche bank			

*NA= not available

A.2 Fama French Three Factor Model

To calculate the three factor in the Fama French three factor model we have used the DJ Stoxx 600 index. As the market factor we used the equal weighted DJ Stoxx 600. The SMB and HML factors are calculated as follows:

The universe, DJ Stoxx 600, is first divided into two groups based on the stocks market capitalization (Size). We then divided the stocks in the two groups into three subgroups based on their Book-to-Price ratio. This was done on a daily basis after which the total return in each subgroup was calculated.

	Small	Big
Value	Small Value	Big Value
Neutral	Small Neutral	Big Neutral
Growth	Small Growth	Big Growth

We then used the following formulas to obtain the SMB and HML factors:

$$SMB = \frac{1}{3}(S.Value + S.Neutral + S.Growth) - \frac{1}{3}(B.Value + B.Neutral + B.Growth)$$
$$HML = \frac{1}{2}(S.Value + B.Value) - \frac{1}{2}(S.Growth + B.Growth)$$

A.3 Moment estimates

For an independent sample $X_1, \ldots, X_n$ from the NIG distribution, the moment estimates that are used in excel are defined as follows,

$$\widehat{E} = \frac{1}{n}\sum_{i=1}^{n} X_i$$
$$\widehat{V} = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \widehat{E})^2$$
$$\widehat{S} = \frac{n}{(n-1)(n-2)}\sum_{i=1}^{n}\left(\frac{X_i - \widehat{E}}{\sqrt{\widehat{V}}}\right)^3$$
$$\widehat{K} = \frac{n(n+1)}{(n-1)(n-2)(n-3)}\sum_{i=1}^{n}\left(\frac{X_i - \widehat{E}}{\sqrt{\widehat{V}}}\right)^4 - \frac{3(n-1)^2}{(n-2)(n-3)}.$$

A.4 Regression tables

A.4.1 Portfolio 1

CAPM regression

Regression Statistics	
Multiple R	91.0%
R Square	82.1%
Adjusted R Square	82.0%
Standard Error	0.4%
Observations	389

ANOVA					
	d.f.	SS	MS	F	Significance F
Regression	1	0.027	0.027	1772.94	1.44e-146
Residual	387	0.006	1.52e-5		
Total	388	0.033			

	Coefficients	Standard Error	t-stat	P-Value	Lower 95%	Upper 95%
Alpha	9.5e-3%	0.02%	0.48	63.1%	-0.03%	0.05%
Market	1.01	2.4%	42.1	1.44e-144%	0.97	1.06

FFT regression

Regression Statistics	
Multiple R	91.0%
R Square	82.7%
Adjusted R Square	82.6%
Standard Error	0.4%
Observations	389

ANOVA					
	d.f.	SS	MS	F	Significance F
Regression	3	0.027	0.009	615.321	1.87e-146
Residual	385	0.005	1.47e-5		
Total	388	0.032			

	Coefficients	Standard Error	t-stat	P-Value	Lower 95%	Upper 95%
Alpha	0.01%	0.02%	0.43	66.8%	-0.03%	0.05%
SMB	-0.18	8.43%	-2.14	3.3%	-0.35	-0.01
HML	0.24	9.44%	2.57	1.1%	0.06	0.43
Market	1.01	2.61%	38.85	0.0%	0.96	1.06

A.4.2 Portfolio 2

CAPM regression

Regression Statistics	
Multiple R	91.7%
R Square	84.1%
Adjusted R Square	84.1%
Standard Error	0.4%
Observations	389

ANOVA					
	d.f.	SS	MS	F	Significance F
Regression	1	0.026	0.026	2050.70	9.69e-157
Residual	387	0.005	1.29e-5		
Total	388	0.031			

	Coefficients	Standard Error	t-stat	P-Value	Lower 95%	Upper 95%
Alpha	-8.9e-3%	0.02%	-0.49	62.7%	-0.03%	0.03%
Market	1.00	2.2%	45.28	9.69e-155%	0.96	1.05

FFT regression

Regression Statistics	
Multiple R	93.0%
R Square	86.5%
Adjusted R Square	86.4%
Standard Error	0.3%
Observations	389

ANOVA					
	d.f.	SS	MS	F	Significance F
Regression	3	0.027	0.009	821.761	6.3e-167
Residual	385	0.004	1.103e-5		
Total	388	0.031			

	Coefficients	Standard Error	t-stat	P-Value	Lower 95%	Upper 95%
Alpha	0.01%	0.02%	-0.62	53.8%	-0.04%	0.02%
SMB	-0.45	7.29%	-6.13	0.0%	-0.59	-0.30
HML	0.31	8.16%	3.81	0.0%	0.15	0.47
Market	1.02	2.25%	45.23	0.0%	0.97	1.06

A.4.3 Portfolio 3

CAPM regression

Regression Statistics	
Multiple R	90.8%
R Square	82.4%
Adjusted R Square	82.4%
Standard Error	0.3%
Observations	389

ANOVA					
	d.f.	SS	MS	F	Significance F
Regression	1	0.026	0.026	1812.20	4.40e-148
Residual	387	0.006	1.46e-5		
Total	388	0.032			

	Coefficients	Standard Error	t-stat	P-Value	Lower 95%	Upper 95%
Alpha	1.9e-2%	1.9e-2%	-1.02	30.9%	-0.06%	0.02%
Market	1.00	2.4%	42.57	4.40e-148%	0.96	1.05

FFT regression

Regression Statistics	
Multiple R	93.7%
R Square	87.9%
Adjusted R Square	87.8%
Standard Error	0.3%
Observations	389

ANOVA					
	d.f.	SS	MS	F	Significance F
Regression	3	0.028	0.009	615.321	6.44e-176
Residual	385	0.004	1.01e-5		
Total	388	0.032			

	Coefficients	Standard Error	t-stat	P-Value	Lower 95%	Upper 95%
Alpha	0.00%	0.02%	-1.39	16.6%	-0.05%	9.4e-3%
SMB	-0.63	6.98%	-9.08	5.60e-16%	-0.77	-0.50
HML	0.55	7.82%	7.04	9.13e-10%	0.40	0.70
Market	1.02	2.16%	47.20	3.62e-160%	0.98	1.06

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