Mathematical Statistics
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The Lithuanian Social Insurance Model

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Abstract

This Masters assignment presents the Lithuanian Social Insurance budget model, used by the Ministry of Social Security and Labour in the Republic of Lithuania for policy evaluation and to test different medium and long term assumptions. The model is deterministic, and combines actual data with assumptions, scenarios, about the future. This paper evaluates the introduction of the funded second pillar under the baseline assumptions set by the European Commission and analyses the consequences of this reform in this setting.

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Table of Contents

1. Introduction

The redistribution of resources over the lifecycle is a central economic theme in all societies. A pension scheme is the insurance and distribution tool that society uses in order to transfer these recourses from the young and able to the old in need, in order to counter old-age poverty. The changes now occurring in all post-industrialized societies due to the changing structure of demographics will without doubt change the conditions of this redistribution (The Economist 2002, 2004). This new era will put new demands on all pensions as well as all of social security. These new changes will not only be demographic but social and financial as well (Jollans 1997). The demographic outlook for many of the former communist countries of Eastern Europe looks even worse, due to extremely unfavourable demographics, this is the case in the Republic of Lithuania (se table 1:1 and 1:2 below).

Table 1:1: The cohort distribution for Lithuanian males by age in 2004

Figure A5. Population, Males, 2004. Baseline scenario.
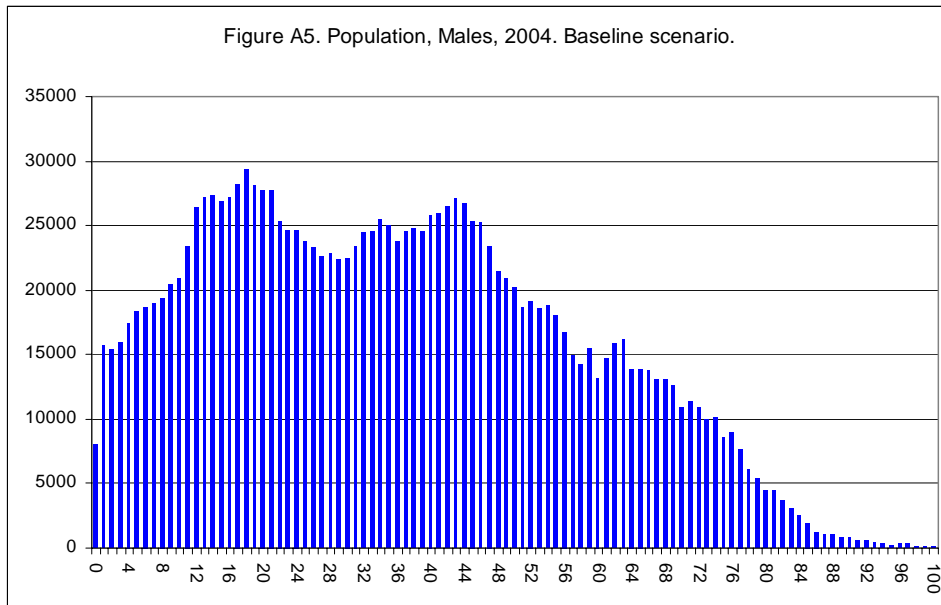
Source: Statistics Lithuania

Table 1:2: The cohort distribution for Lithuanian females by age in 2004

Figure A6. Population, Females, 2004. Baseline scenario.
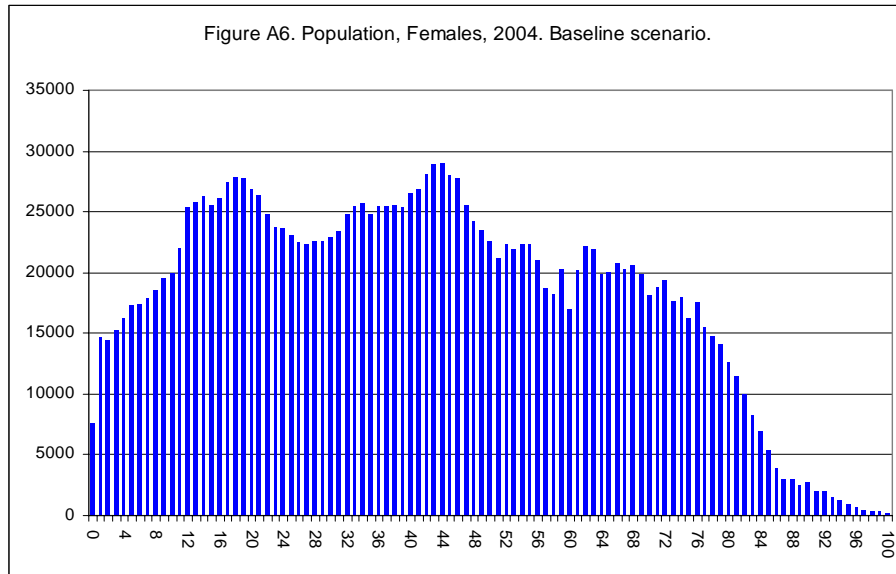
Source: Statistics Lithuania

A pension scheme is the instrument that society uses to transfer real resources from the young and able to the elderly. There are two ways to create a safe retirement. The first one is to save a part of the produce from the productive years in ones life and to consume it later. This is not only highly unrealistic but also quite unpractical and yields no interest, the other alternative is to trade produce from today into a claim on the produce of the future. This principle is the main purpose for all financial markets and this is done through a contract known as a pension scheme (Barr 2004).

Pay-as-you-go pensions

Social insurance (where public pension's schemes often are a large part) is almost always set as pay-as-you-go and financed by taxation. These schemes are often unfunded ($V(t)=0$) and where the sum contributions $C(t)$ often follow the sum benefits $B(t)$ (Iyer 1999).

$$C(t) = B(t) \qquad\qquad (1)$$

The basic principle behind the scheme is that contributions today are exchanged to the promise of benefits tomorrow. It is this promise that enables the contributors of today to continue contributing to the scheme, in a government scheme these number of taxpayer's (i.e. the labour force $L(t)$)

$$C(t)=c(t)\ \overline{w}(t)\ L(t) \qquad\qquad (2)$$

The sum of contributions $C(t)$ is thus equal to the product of the contribution rate $c(t)$, the average wage $\overline{w}(t)$ and the number of contributors (contributing insured) $L(t)$.

The labour force *L(t)* equals the product of the age and gender differentiated cohort $N_{s,t-a}(t)$ and its age (where the connection between birth year *x* and age *a* equals *x=t-a*) and gender specific labour participation rate $\rho_{s,a}(t)$.

$$L_{s,a}(t) = \rho_{s,a}(t) \, N_{s,t-a}(t) \qquad\qquad\qquad (3)$$

The total benefits on the other hand equal:

$$B(t) = \bar{b}(t)R(t) \qquad\qquad\qquad (4)$$

Where $\bar{b}(t)$ equals the average benefit and *R(t)* the number of retirees.

Thus it is obvious that if the cohort sizes vary, the contribution rate will also vary. This is usually handled by introducing a buffer fund which in turn smoothens these variations in practice this is achieved by letting *V(t)* differ from zero in the short run but not in the long run. But an aging society due to increased longevity leads to the fact that a larger number of aged are dependent on a relatively smaller number of active persons of working age and this puts another type of constraint on the public finances due not to varying cohort sizes but due to increasing benefits relative contributions.

Table 1:3: The Lithuanian demographic dependency ratio 2004-2100, population 65+/population 20-64

Figure 2. OA Demographic Dependency Ratio.
Population age 65– / Population age 20-64.
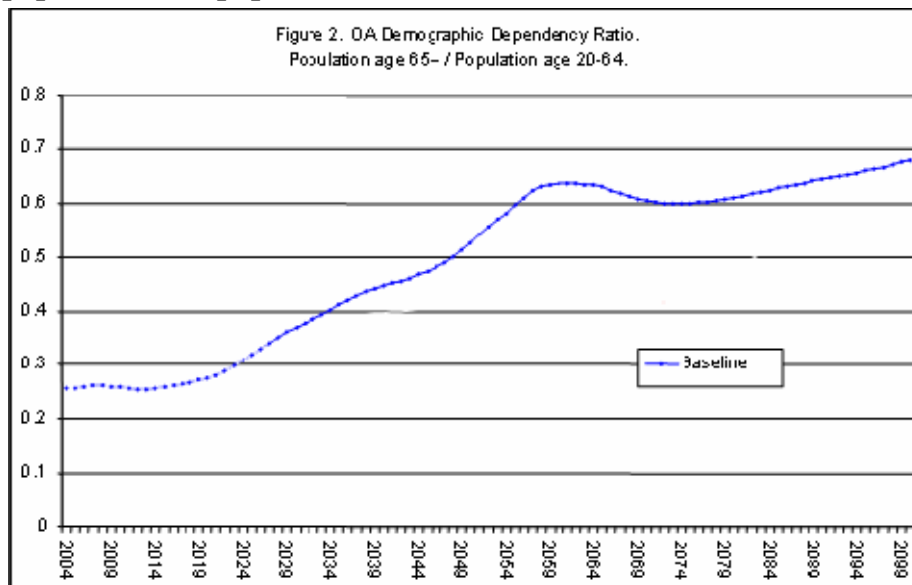
Source: Statistics Lithuania to 2050, own calculations

The table above shows the so called dependency ratio. The ratio is here defined as the sum of all above the age of 65 divided by the sum of all of ages between 20 and 64.

The problem with ageing societies is certainly well known but often not generally well understood. An underestimation of these effects will without doubt lead to the implementation of inappropriate policies that will hurt society and the current and future members of it. However when planning social security schemes or pensions in general, the planning period is often

at least 50 or perhaps 100 years. When looking back a century it is obvious that the demographic rates as well as the economic ones have been non-stable and highly unpredictable (Alho 2005). This said many countries throughout the world are trying to deal with this situation. Within the European Union this is a major concern of the European Commission (European Commission (2006c).

In 1958 the economist Paul Samuelson showed that in an economy where productivity grows at a rate $g(t)$ and the population grows at a rate $\lambda(t)$ the effective labour force growth $L(t)$ from generation to generation equals

$$L(t) = (1 + g(t))(1 + \lambda(t))L(t-1) \tag{5}$$

This equation is known as the Samuelson-Aaron condition.

$$(1 + g(t))(1 + \lambda(t)) \tag{6}$$

This means that in a pay-as-you-go scheme the benefits cannot be index by more than this, without increasing the burden of increased taxation on some generation.

But a small open economy can if the global market interest rate $\delta(t)$ is higher than growth of the tax base some gains can be made if a switch is made from a unfunded scheme to an funded scheme (if only partially). More formally the following inequality must hold:

$$\delta(t) > (1 + g(t))(1 + \lambda(t)) \tag{7}$$

Thus if the above is true then it will be a good idea to exchange produce from to day and thus trading it for more in the future. But this means that, *ceteris paribus*, contributions today have to be higher in order to ease future burdens.

Is this realistic? Well it might not altogether be unrealistic in a globalized world economy where financial markets are free to exploit these arbitrary conditions, and for a small country with a declining labour force it might an opportunity to hamper the negative effects but it is way beyond the scope of this paper to argue the one way or the other. However this strategy has been highly recommended by the World Bank and is also the reason for the increased funded element in the new Lithuanian pension's accumulation scheme.[2] This approach which is sometimes called a multi-pillar approach has become very influential in many pension reforms among them the one in Sweden.

In order for a pension scheme to be sustainable in the long run, it must be financially stable. This not the same thing as not allowing for budget deficits or budget surpluses, but in the long run it must be financially stable. The pension fund must be in the long run stable. Within the European Union the

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[2] The vastly influential World Bank publication "*Adverting the Old age Crisis*" was published 1991.

member countries are working on this issue (European Commission 2006a). In the Lithuanian Social Insurance model pensions are assumed to be financially stable if they run into an increasing deficit given the parameters set by the scenario.

These sometimes rapid changes increase the demands for policy analysis. The Lithuanian Social Insurance Budget Model is a tool created to counter and analyse different prognoses and the effect of their assumptions. The model was constructed by the members of the Swedish project group at the Swedish Insurance Agency and the entire project a part of a SIDA financed development program for the Lithuanian Ministry of Social Security and Labour. This essay presents the public pension module and the newly introduced pension accumulation scheme.

Due to the vast number of input-variables, output-variables, equations and aggregation levels most of these are gathered in the accompanying appendix named "*Model Description and Formulae*".

The structure of the paper is as follows, chapter two describes the model and its components and how it runs. Chapter three describes the Lithuanian public pension scheme and chapter four introduces the pension accumulation scheme. The baseline scenario is presented in chapter five and the final results regarding the answer to question if the increased levels of funding in the Lithuanian public pensions might be a way of decreasing the burdens of unfavourable demographics when running the baseline scenario are presenter in the last chapter.

2. The Model

The *Lithuanian Social Insurance Model* generates long-term projections of the expenditures and revenues of the social insurance budget. The model is built up around four groups of sub-models:

- Demographic Model
- Population Status and Labour-force Participation Model
- Income Model
- Pension Model

The Models in Brief

The demographic model enables the user to set scenarios for age-gender survival rates, the number and timing of births and immigration and emigration. All the population from 15 to the minimum pension age is assigned a status, e.g. working, unemployed, and disabled. The user specifies scenarios for all of these, usually by specifying age-gender rates (for unemployment, disability etc.) and the minimum pension age. The minimum pension age, set by the user, determines the normal age for retirement.

In the income model the user specifies the age-gender distribution of income and aggregate per capita wage growth. The model distributes income between *all earners* according to the *aggregate* per capita growth and the distribution scenario specified by the user. The user specifies the distribution between these. This means that the growth of the wage sum and GDP are also determined by a user-specified scenario for the development of the formal economy.

The model is presently designed to produce projections for many social insurance schemes but this paper is concerned only with old age. Although the full model calculates disability, short-term sickness, unemployment, maternity, survivor, orphans benefits and administration and other important outlays as well. The most elaborate modules are those that generate project disability and old-age benefits. The model can be expanded easily to generate projections of other transfers and revenues dependent on demography and/or earnings.

The main components of the models are key rates and distributions. In the scenario boxes it is possible to change both of these parameters and the changes between two consecutive values with any number of steps between them are linearly interpolated.
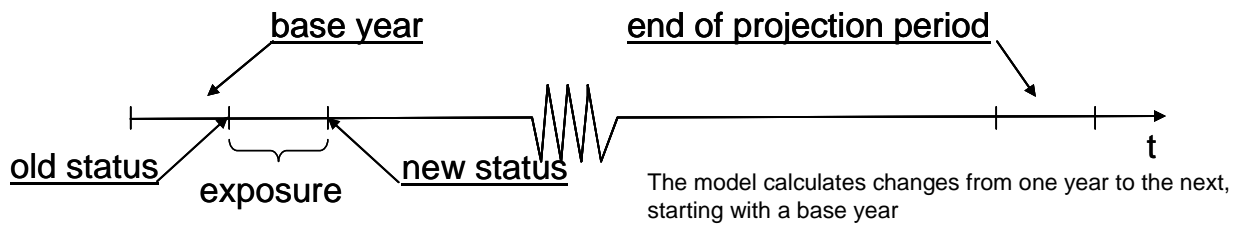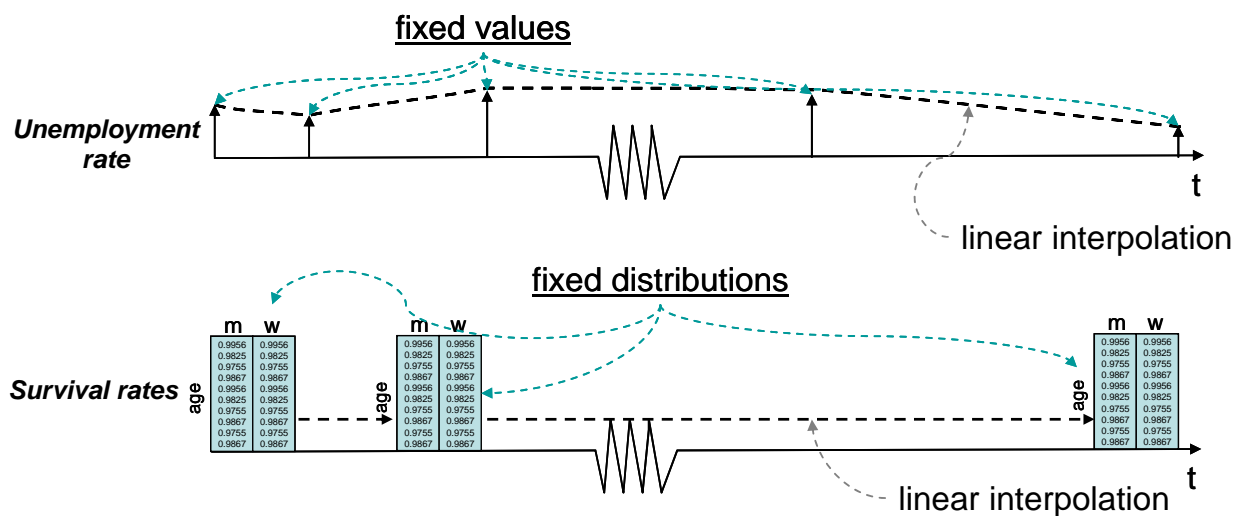
Figure 2:1 (linear interpolation)

base year

end of projection period

old status

new status

exposure

t

The model calculates changes from one year to the next, starting with a base year

Figure 2:2 (linear interpolation)

fixed values

Unemployment rate

t

linear interpolation

fixed distributions

Survival rates

m   w

age

m   w

age

m   w

age

t

linear interpolation

This approach gives the user a high degree of control and it is possible to construct many different scenarios. In the model it is possible to save these scenarios as well as the output data and then to compare them graphically in the compare module. To speed up the different calculations it is also possible to control the number of output variables.

2.1 What the User Does

For a specified set of rules for the calculation of benefits, the user steers the projections by choosing parameters that determine scenarios for the development of the population, participation in the labour force, the unemployment rate, the average wage and the degree of social insurance coverage in the formal economy.

Most of the key assumptions needed to run the model can be varied over the projection period. For example, birth, mortality, unemployment, disability, average wage, and rates of return on financial reserves can develop in various ways specified by the user. Some scenarios are specified by a vector, such as the rate of growth or rate(s) of interest earned by fund

reserves. Others are specified as changing age-gender distributions, e.g. survival rates, the distribution of income, and disability, work-injury and unemployment risks. This is done by specifying the desired development of parameters and in the Data module.

The model produces projections from the current base year 2005 up to the column limit in *Excel* which is 256.

Output
The model creates a vast amount of information, most of which is only needed in initial runs to check the reasonability of assumptions and scenarios. Typical information occurs in a matrix organized by age cohort and year, and separately for men and women. For example, the number of persons born a specific year and surviving at the end of each year covered by calculations in the model, the stock of disability pensioners (in a specific group) born in one year and still alive and under the age of 60 during each calculation year, etc. The user determines the level of the information to be saved.

Key assumptions and results are summarized in standard tables and charts. Other tables and charts may also be chosen at the discretion of the user. The model is programmed in *Visual Basic* and *Excel* using *Microsoft Office 2003*. The user can produce reports using the standard tools in *Microsoft Office*. The model includes several standard output tables and figures, including statements of revenues and expenditures for the four funds and a number of tables and figures used to examine the long-run development of the pension system(s).

2.2 The Demographic Model

The demographic module of the model generates the population from birth to 100 years of age on an annual basis for a period specified by the user.[3] The number of persons in each one-year age group is computed separately for men and women. This method of demographic analysis is called cohort analysis (Caswell 2001). The calculations start with an initial population specified by age and gender and are performed using assumptions about the number and timing of births per female cohort and age-gender specific survival rates. The user determines the total numbers and age distribution by gender for immigration and emigration. The population figures in the model refer to the numbers *at the end of the year*. Since most of these changes are very small from year to year it is obvious that they cannot change the overall population structure that much in a few years (Alho 2005).

Since survival rates, the number of children per woman as well as the timing of births (the current limits are set between ages 15 to 45) can be varied over time by specifying alternative scenarios, it is possible to study the impact of varying demographic assumptions of the development of benefits and revenues.

2.3 The Population Status and Labour-force Participation

The next step is to decide the status for each gender cohort. This is done in the status module, where all living persons are classified as either;

- Contract workers
- Self-employed
- Unemployed
- Disabled
- Old-age pensioner
- Others

Contract workers and Self employed person generate contributions where the contribution is equal to the average wage by age and gender. By definition each person between the initial-age, chosen in the model, presently 15 years of age, and retirement, which is specified by the user, belongs to one and only one of these status groups in any given year. The age-specific share of a certain status group is either specified exogenously on the basis of available statistics or determined endogenously in the model. The first is the case for the number of people marked as others i.e. persons in educational programs, military service etc. The remaining groups are determined by sub-models contained in specific modules. The user must

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[3] The basics for any public pension is demographics, the word originates from the Greek words *demos* (people) and *graphien* (to write).

provide statistics by age on the number of people in education and military service. The number of women on maternity leave is determined by the assumptions about the number of births per woman, the age distribution of births and an assumption about the average length of a maternity leave period.

The user provides at least one and typically more than one age-gender distribution for the groups below the minimum age for receiving an old-age pension. The user can then specify a scenario, starting with one distribution and ending with a different distribution, perhaps moving through one or more intermediate distribution states. The transformations between states (distributions) are calculated as smooth linear trends.

The numbers of persons with disability and unemployment status are determined by data and scenarios specified by the user. The user provides a set of starting frequencies (risks) by age and gender, and then selects a scenario for the future development of these.

Persons in the group "others" are not working – either in the formal or informal economy -but also fall outside the other groups, i.e. they are not mothers, unemployed, or disabled.

All registered unemployed are in the labour force, although they may not all be covered by unemployment insurance. The percentage covered by benefits can be specified in the model. The model distributes a total unemployment rate specified by the user in accordance with a user specified age-gender distribution. Both the distribution and the total unemployment rate can be made to vary over time.

The number of persons working (both inside and outside the formal economy) is determined as the residual when all the other status groups are subtracted from the total population in each age and gender group.

Persons working include the *contract workers* and the *self-employed* and data about them are obtained by the labour force survey.

The average retirement age is also a parameter set by the user.

2.4 The Income Model

The central economic variable in the model is the average real wage. Since we usually only have information about the formal economy, the average real wage rate represents the conditions in the formal economy alone. In the long run, the average real wage is also a measure of productivity. The average real wage times the number of persons working, derived from the population status module, and provides an estimate of total earnings in the economy.

The average wage is set as the total average wage for a specific age and sex, and all other wages weighted with a relative distribution in order to get each average wage for both sexes all ages.

The model enables the user to specify the *distribution of income* – i.e. the average wage - by gender and age for the three groups of persons working: employees and the self-employed.

Table 2:3: An example of what the data might look like
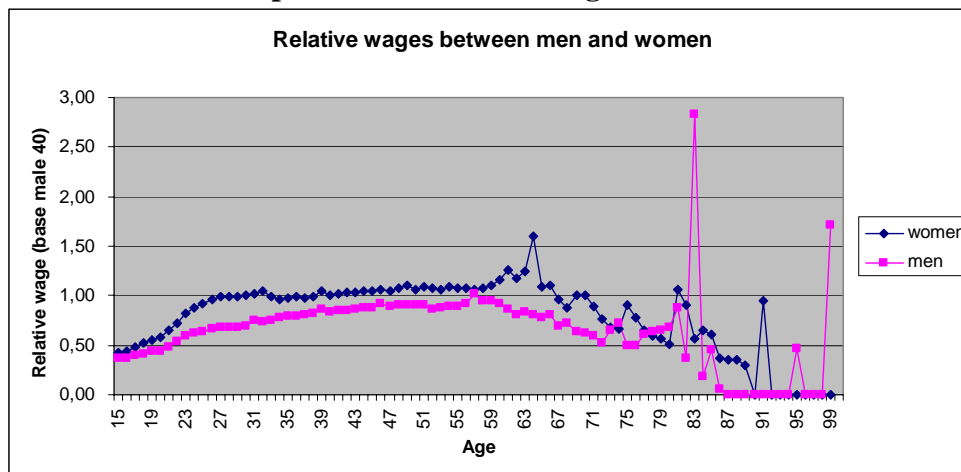
Source: Ministry of Social Security and Labour and own calculations

The figure above shows the relative wages in Lithuania 2004, the shakiness of the observations is due to few observations for higher ages.

Given wage costs and an assumption about the share of gross operating surplus (profits) in the economy a measure of GNP can be calculated. This is done in the model in order to provide an estimate of the contribution of the pension system(s) to the gross national saving rate. Since the share of wages in GNP is typically lower in developing economies, there is an option to enable the share to change over time.

All the values in the model are specified in real terms. This means that there is no need to specify a rate of inflation, although it is not difficult to modify the model to produce nominal value calculations.

3. The public old-age pensions scheme in Lithuania

The old-age social insurance operates on a pay as you go principle and is divided into two schemes a mandatory defined benefit scheme (the stat social insurance pension) and a voluntary funded scheme (pension accumulation) (Government of Lithuania 2005). The retirement age has recently been increased and is still increasing for women, and is currently at 62.5 years for men and 59 for women and will increase until it reaches 60 in 2006 (Government of Lithuania 2005).

A person is entitled if he or she meets one of the following conditions:
- The person reaches the age of retirement (2001 the retirement age was 62,5 years for men and 60 for women)
- Minimum state social pension insurance period (15 years)
  - The period during the compulsory state social insurance contributions are paid

Old age pensions consist of two parts:
- Basic pension
- Supplementary pension

The basic state social insurance *old age pensions* correspond to the state social insurance base pension is equal for all persons who have the qualify for the obligatory state social insurance pension period. In 2004 the average basic pension was 167 LTL which is approximately $ 65 or about € 50. For individuals without obligatory state social insurance period the basic part of the state social insurance pension period the state social security old age pension is commensurately reduced (Ministry of Social Security and Labour 2003).

The supplementary part of the state social insurance old age pension, represents the benefit related to the insurable income. Pension benefits are commensurable to the insurance contributions of the social insurance pension insurance paid by the individual (Ministry of Social Security and Labour 2003).

The formula for calculation of old age pension:

$$P_y = B_y + 0.005 \cdot (sk + SK)D_y \qquad (8)$$

P_y =total pension

B_y = the basic part of the pension (base pension)

s = experience before 1st of January 1995, this equals to number of working years before 1st of January 1995.

k = coefficient of insured wage divided by average insured wage before 1st of January 1995

S = experience after 1st of January 1995, this equals to the number of working years after 1st January 1995

K = coefficient of insured wage divided by average insured wage after 1st of January 1995

D_y = average insured wage

Cohort pension rights are calculated according to the Lithuanian pension formula.

4. The pension accumulation scheme

In 2003 the Seimas of the Republic of Lithuania passed the legislation of implementing the pension accumulation scheme from the first of January 2004. The purpose of the reform was to introduce a fully funded voluntary second tier. The goal of the reform is to provide a second tier of the first pillar pension system where the people of Lithuania are able to save a part of their compulsory social insurance contributions (Government of Lithuania 2005). Upon retirement the value of the account is used to purchase an annuity to complement the general pension's scheme. All insured individuals with full social insurance coverage will be able to participate in the scheme (Ministry of Social Security and Labour 2003).

Individuals who decide to participate continue to pay contributions to the Lithuanian State Social Insurance Fund Board "SoDra" and from there transfer funds to selected pension funds. Participants of the reform will receive pensions from Sodra as well as the private enterprises (Ministry of Social Security and Labour 2003).

The Lithuanian funded pension's scheme consists of two parts the active period when an insured participant accumulates funds and the retirement period when an insured participant withdraws the same funds in the form of an annuity.

Table 4:1: Pension contribution rates

	Basic pension	Supplementary pension	Pension Accumulation	Total old age pension
2004	10.5 %	10.5 %	2.5 %	21 %
2005	10.5 %	10.5 %	3.5 %	21 %
2006	10.5 %	10.5 %	4.5 %	21 %
>2006	10.5 %	10.5 %	5.5 %	21 %

There are four old-age pension modules. These produce projections of average benefits and costs for:

- The pre-reform defined-benefit system
- The defined-contribution, notional account pay-as-you-go (PAYG) system
- The defined-contribution funded (2nd Tier) funded system
- The defined-contribution funded (2nd Tier) funded system with refunding into the PAYG reserve

There is also a separate procedure for calculating a minimum income guarantee for old-age pensioners. In addition, it is possible:

- To index benefits fully or partially (with the degree determined by the user) to the growth of wages.

- To vary the contribution rates and sizes of the PAYG and funded tiers.
- To choose separate rates of return for the PAYG and funded reserves and vary these over time.
- To choose which age cohort is to be the first to participate in the 2nd Tier

5. The baseline scenario

Assumptions:

— All demographic parameters are the same as for the EuroStat Baseline scenario (European Commission 2005b).
— Full participation in the pension accumulation scheme after 2006
— Real rate of market return 5.5 %.
— Government bond rate 3.5 % (risk free interest rate) equals the market rate in the European Commission scenario.

It is assumed that all assets in the p-a-y-g buffer fund are used to purchase government bonds. It's also assumed that there is no difference between if the fund borrows or lends money to the government the bond the asset or liability interstate is the same.

Administration costs:
- Pension Accumulation fund
    o On fund 1.0 %
    o Premium inflow 3.0 %
- Annuity
    o Annuity loading 1.0 %

In the model we assume perfect foresight and use actual mortality to calculate the annuity; this is compensated by loading the real rate of return.

6. Results from the baseline scenario

This part of the paper sets the outline for the effects of the funded scheme and addresses the questions; under what conditions the funded scheme will succeed. With a demographic outlook like the one in Lithuania it is possible to obtain a perhaps higher standard of living for the elderly if the overall pensions are in fiscal balance; interest earned on the accumulated funded funds is higher than the growth of the contribution rate in the pay as you go scheme. The analysis in this paper is based upon the official baseline scenarios for the Republic of Lithuania made by EuroStat for the European Commission. The problem here is that the money diverted to the funded pension scheme has to be drawn from the general revenues of the pay as you and thus weakening this scheme.

Table 6:1:

Accumulated balance with pension accumulation scheme and a contribution rate of 21 %
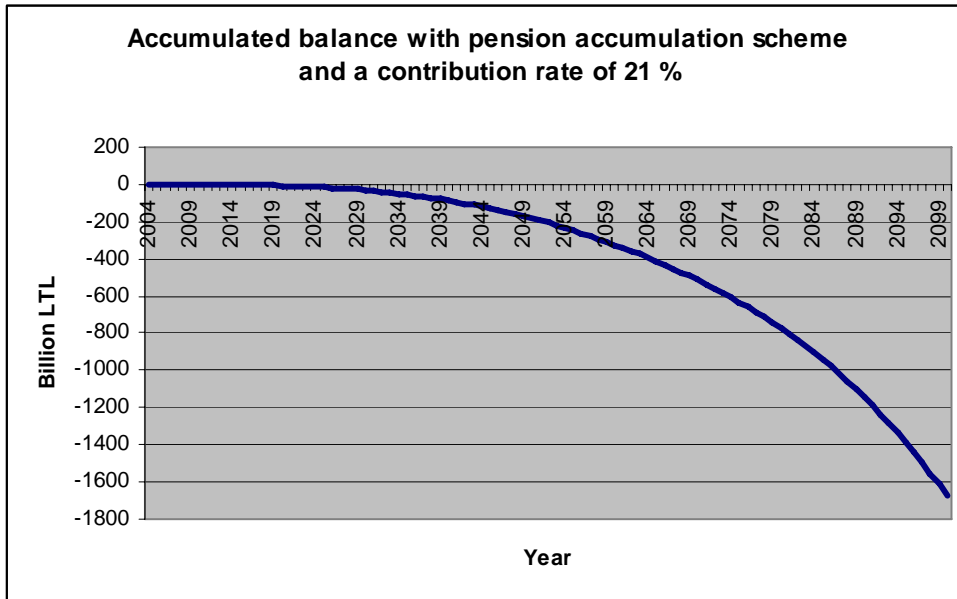
Source: Lundberg, Palmer, Svensson 2005 forthcoming

The initial run shows that the system is not financially stable in the long run *V(t)<0* for all years after 2007. *V(t)* in this respect is defined as the accumulated balance of the paygo buffer fund after transferring capital to the pension accumulation scheme. This means that the current situation is not stable. The first task is to make the system financially stable. The question is now what can be done.

Table 6:2:

Accumulated balance with the pension accumulation, increased pension age and a contribution rate of 24 %
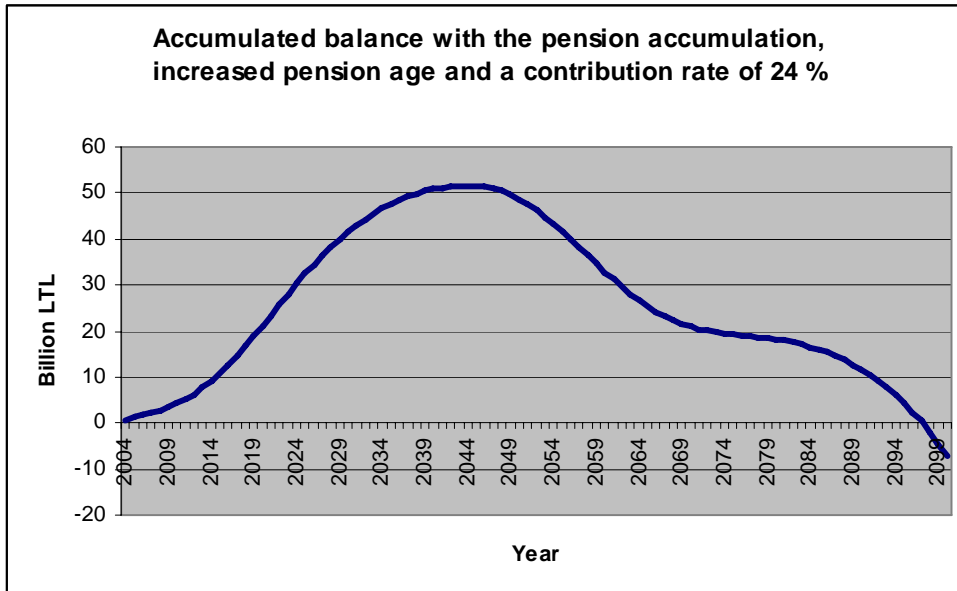
Source: Lundberg, Palmer, Svensson forthcoming

It is unreasonable to believe that the pension age will remain at 62 for the coming century, given marked advances in the health and working conditions of Lithuanians, and the resultant increase in longevity. In fact, the assumptions in this study are probably conservative in this respect. Because

of this, in a second set of calculations, we have raised the pension age to 65 for men born 1951 and for women born 1956, with a transition. In order to create a surplus in the p-a-y-g scheme and to free capital to the pension accumulation scheme we also have to increase the contribution rate to 24 %. When doing so the system becomes financially stable (se table 6:2).

Table 6:3: Compensation rates in the baseline scenario with high pension age scenario and 24% contribution rate.

	2010	**2030**	**2050**
Basic	0.171	0.167	0.166
Men Supplementary	0.270	0.209	0.120
Men Pension-Accumulation	0.019	0.116	0.296
Men Total	0.459	0.492	0.581
Basic	0.171	0.167	0.166
Women Supplementary	0.197	0.156	0.088
Women Pension-Accumulation	0.010	0.085	0.211
Women Total	0.379	0.408	0.465

When a system is financially stable it is meaningful to take a look at the pension levels and their distribution. In table 6:3 above we have the respective compensation rates for men and women for three different years 2010, 2030 and 2050. Compensation rates are in this case measured as average pension for the retiring cohort divided by average wage income for the working population during the year of retirement. The pension accumulation scheme will grow gradually, with a corresponding decline of the p-a-y-g supplementary scheme, during the coming 45-50 years. Individuals retiring around 2050 will, according to our scenario, have fully participated in the pension accumulation scheme during their whole working life.

In the baseline scenario and with our assumptions about rate of return within the pension accumulation scheme, total replacement rates will grow from 0.46 to 0.58 for men and from 0.38 to 0.47 for women. This means that with our assumptions pensioners in the future actually will be getting better of relative pensioners today.

Thus it seems as it is possible to obtain the aims of the pension accumulation reform but only together with other reform options. In this paper the solution has been to raise the pension age to 65 for both men and women and to rise the contribution rate to 24 % from 21 % when running the baseline scenario. These measures make the entire system financially stable and even increase the pension levels relative to final wages. This policy option also divides the increased burdens between the young (increased contribution rate) and the old (increased pension age) in a way that might be acceptable to all. The introduction of the pension accumulation scheme thus reduces the demographic pressure on the p-a-y scheme, which in principle is to the advantage of all Lithuanians in the long run. Further research will deal with sensitivity analysis as well as testing other policy options.

Acknowledgements

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Appendix:

Model Description and Formulae

"The Lithuanian Social Insurance Model"

Version 2007-08-23

Table of contents

1. General description of the model.

1.1 A cohort model

The model is a cohort model. Information in the model is organized into variables (arrays) which indicate events to the cohort during the year, the average status of the cohort during the year or at the end of the year etc. Usually these arrays use s = sex (gender), a = age and y = year as arguments, where a = age is in one-year age groups, $a = 0,...,100$ and y = BaseYear,...,EndYear. The difference between EndYear and BaseYear is the projection period and can be up to about 200 years. The argument "a" means age at the end of the year, i. e. $X(s, a, y)$ is the value of the variable X for the cohort $c = y - a$ in year y. All members of cohort $c = y - a$ have their a:th birthday during year y. Events during the year consequently occur in part at age a-1 (before the birthday) and in part at age a (after the birthday).

1.2 The general structure for the different levels of aggregation

$X(s, a, y)$

The general variable $X(s, a, y)$ by gender, age and year will be aggregated for all ages into the general aggregate level $X\_allages(s, y)$ by gender and year.

$$X\_allages(s, y) = \sum_a X(s, a, y)$$

The aggregate for all ages $\sum_a X\_allages(s, y)$ is then aggregated for both genders to the general yearly total $tot\_X(y)$

$$tot\_X(y) = \sum_a X\_allages(s, y)$$

2. Demography

2.1 Input data

To define a demographic scenario the following must be specified as input data to the model.

initPop	Initial base year population
prcntBornMales	The share of boys among the children born during a year. In percent.
FertilityRates(a, y)	Age specific fertility rates. The sum over ages is the Total Fertility Rate.
SurvivalRates(s, a, y)	Survival rates. The probability of survival from the beginning to the end of the year. By cohort.
ImmAgeDistr(s, a, y)	Age distribution of immigration, by gender and year. Adds to 100% for each gender and year.
EmAgeDistr(s, a, y)	Age distribution of emigration, by gender and year. Adds to 100% for each gender and year.
ImmigrationFlow_allages(s, y)	Total immigration by gender and year.
EmigrationFlow_allages(s, y)	Total emigration by gender and year.
EndYearPopulation(s, a, y)	Population by gender and age at the end of year y. Specified as input data for simulations with a pre-defined population.

2.2 The use of a pre-defined population

It is possible to make a run with the model which only imports a pre-defined population (size of population by gender and age) to the model for an optional projection period. This mechanism can be used to compare an external demographic projection with projections created within the model. There is a control parameter "preDefPopulation" which is set to TRUE or FALSE to decide if the pre-defined population is to be used. If it is used, the only variable which is calculated within the model is average population (AvgPopulation(s, a, y)). The reason is that, since survival rates and fertility rates are not specified, it is not possible to make a complete pension and budges simulation in this case.

2.3 Calculated variables and model equations

Lx(s, a, y)	Period survival curve by gender and year. Lx at birth = 10000 by gender, age and year.
Life_expectancy(s, a, y)	Life expectancy at each age a = 0,…,100 by gender, age and year.
Life_expect_examples(s, a, y)	Life expectancy at age a = 0, 55, 60, 65, 70 by gender, age and year.
LifeExpectMen0(y)	Life expectancy for men at age 0 by year.
LifeExpectWomen0(y)	Life expectancy for women at age 0 by year.

LifeExpectMen60(y)	Life expectancy for men at age 60 by year.
LifeExpectWomen60(y)	Life expectancy for women at age 60 by year.
EndYearPopulation(s, a, y)	Population at the end of the year by gender, age and year.
EndYearPopulation_allages(s, y)	Population at the end of the year by gender and year.
tot_EndYearPopulation(y)	Population at the end of the year by year.
AvgPopulation(s, a, y)	Average population by gender, age and year.
AvgPopulation_allages(s, y)	Average population by gender and year.
tot_AvgPopulation(y)	Total average population by year.
Births(a, y)	Number of children born by age of mother and year.
tot_Births(y)	Total number of births by year.
Deaths(s, a, y)	Deaths by gender, age and year.
Deaths_allages(s, y)	Deaths by gender and year.
tot_Deaths(y)	Total deaths by year.
Immigration(s, a, y)	Immigration by gender, age and year.
Immigration_allages(s, y)	Immigration by gender and year.
tot_Immigration(y)	Total immigration by year.
Emigration(s, a, y)	Emigration by gender, age and year.
Emigration_allages(s, y)	Emigration by gender and year.
tot_Emigration(y)	Total Emigration by year.
NetMigration(s, a, y)	Net migration by gender, age and year.
Netmigration_allages(s, y)	Net migration by gender and year.
Tot_NetMigration(y)	Total Net migration by year.
Fertility_Rates(a, y)	Age specific fertility rates by year.
Survival_Rates(s, a, y)	Survival rates by gender, age and year.
DemographicDependencyRatio(y)	Total demographic dependency ratio.
DemographicDependencyRatioOA(y)	Demographic dependency ratio of the aged.
DemographicDependencyRatioChild(y)	Demographic dependency ratio of the young.
DependencyRatio(y)	Dependency ratio of the aged by year.
tot_DependencyRatio(y)	Dependency ratio by year.
LifeExpectMen0(y)	Life expectancy for a newborn male.
LifeExpectWomen0(y)	Life expectancy for a newborn female.
LifeExpectMen65(y)	Life expectancy for a 65 year old male.
LifeExpectWomen(y)	Life expectancy for a 65 year old female.

Life expectancy is calculated using the formula $\sum_a Lx(s,a,y)/10000 - 0.5$. Note that the input data which is used (SurvivalRates(s, a, y)) is survival rates by cohort and not by age. The transformation to survival by age is made within the model.

Immigration and emigration by gender and age is calculated from the assumed total immigration and emigration by gender and year and from the age distributions.

Immigration(s, a, y) = ImmigrationFlow_allages(s, y) * ImmAgeDistr(s, a, y)

Emigration(s, a, y) = EmigrationFlow_allages(s, y) * EmAgeDistr(s, a, y)

NetMigration(s, a, y) = Immigration(s, a, y) - Emigration(s, a, y)

The number of deaths during year y is calculated from the size of the cohort in the beginning of the year and the survival probability of the cohort from the beginning to the end of the year.

Deaths(s, a, y) = EndYearPopulation(s, a - 1, y - 1)*(1 - ModeledSurvivalRates(s, a, y))

The population at the end of year y is calculated from the population at the end of the previous year and the survival probability and net migration during year y.

EndYearPopulation(s, a, y) = EndYearPopulation(s, a - 1, y - 1) * SurvivalRates(s, a, y) + Immigration(s, a, y) - Emigration(s, a, y)

Finally, the average population (midyear population) of cohort c = y – a during year y equals the average of the two consecutive end year populations.

avgPop(s,a,y) = (EndYearPop(s,a,y) + EndYearPop(s, a-1, y-1))/2

The demographic dependency ratio equals the sum of the young (below 20) and the aged (above 64) divided by total population of working age (in between).

DemographicDependencyRatio(y)= ($\sum_{a \geq 65}$ EndYearPopulation(s, a, y)+$\sum_{a \leq 19}$ EndYearPopulation(s, a, y))/($\sum_{20 \leq a \leq 64}$ EndYearPopulation(s, a, y))

The demographic dependency ratio of the aged equals the sum of the aged (above 64) divided by total population of working age (in between).

DemographicDependencyRatioOA(y)= $\sum_{a \geq 65}$ EndYearPopulation(s, a, y) /$\sum_{20 \leq a \leq 64}$ EndYearPopulation(s, a, y)

The demographic dependency ratio equals the sum of the young (below 20) divided by total population of working age (in between).

DemographicDependencyRatioChild(y)= $\sum_{a \leq 19}$ EndYearPopulation(s, a, y) /$\sum_{20 \leq a \leq 64}$ EndYearPopulation(s, a, y)

3. Labour market and contributors

When relating the model to data from the Labor Force Survey, average numbers for the whole year should be used.

Both the overall unemployment rate during each year during the projection period and age- and gender-specific unemployment rates are specified as assumptions. The latter might not agree with the former, depending on the size, age and gender distribution of the labor force. To guarantee that the overall (macro) unemployment rate is on the specified level, the age- and gender-specific unemployment rates are aligned (adjusted) proportionally by an adjustment factor which is calculated for each year.

3.1 Input data

To define a scenario the following must be specified as input data to the model.
In the model license owners are classified as self-employed.

shLabourForce(s,a,y)	Labour force participation rate (labour force as a share of population).
shprelUnemp(s,a,y)	Unemployed as a share of labour force.
UnemploymentRates(y)	Total average unemployment rate during year y.
shContribCW(s,a,y)	The share of employed who are contract worker contributors.
shContribCW(s, a, y)	The share of employed who are contract worker contributors.
shContribSE(s, a, y)	The share of employed who are self-employed contributors.
shContribSEFull(s, a, y)	The share of fully insured self-employed contributors.

3.2 Calculated variables and model equations

shLabourForce(s, a, y)	Share in labour force by gender, age and year.
shprelUnemp(s, ,a, y)	Share of preliminary unemployment by gender, age and year.
LabourForce(s, a, y)	Labour force by gender, age and year.
LabourForce_allages(s, y)	Labour force by gender and year.
tot_LabourForce(y)	Total labour force by year.
prelUnempl(s, a, y)	Preliminary unemployment by gender, age and year.
prelUnemp_allages(s, y)	Preliminary unemployment by gender and year.
tot_prelUnemp(y)	Total preliminary unemployment by year.

Employed(s, a, y)	Employed by gender, age and year.
Employed_allages(s, y)	Employed by gender and year.
tot_Employed(y)	Total employed by year.
ContribCW(s, a,y)	Contract worker contributors by gender, age and year.
ContribCW_allages(s, y)	Contract worker contributors by gender and year.
tot_ContribCW(y)	Total contract worker contributors by year.
ContribSE(s, a, y)	Self-employed contributors by gender, age and year.
ContribSE_allages(s, y)	Self-employed contributors by gender, and year.
tot_ConribSE(y)	Total self-employed contributors by year.
Contributors(s, a, y)	Contributors by gender, age and year.
Contributors_allages(s, y)	Contributors by gender and year.
tot_Contributors(y)	Total contributors by year.
ContribSEFull(s, a, y)	Self-employed contributors with full coverage by gender, age and year.
ContribSEFull_allages(s, y)	Self-employed contributors with full coverage by gender and year.
tot_ContribSEFull(y)	Self-employed contributors with full coverage by year.
ContribSEBasic(s, a, y)	Self-employed contributors with basic coverage by gender, age and year.
ContribSEBasic_allages(s, y)	Self-employed contributors with basic coverage by gender and year.
tot_ContribSEBasic(y)	Self-employed contributors with basic coverage by year.
LabourForceShPop(y)	Labour force as a share of average population by year
UnemployedShPop(y)	Unemployed as a share of average population by year
EmployedShPop(y)	Employed as a share of average population
ContribCWShPop(y)	Contract worker contributors as a share of average population
ContribSEShPop(y)	Self-employed contributors as a share of average population
ContributorsShPop(y)	Contributors as a share of average population
ContribSEFullShPop(y)	Self-employed contributors with full pension as a share of average population
ContribSEBasicShPop(y)	Self-employed contributors with basic pension as a share of average population
WorkingNonContribSh(y)	Working non contributors as a share of average population

The labour force equals the rate of labour force participation times the average population.

$$LabourForce(s,a,y)=shLabourForce(s,a,y)* avgPopulation(s,a,y)$$

The age and gender specific unemployment rates specified as input data -shprelUnemp(s,a,y) - are adjusted proportionally each year to produce the actual age and gender specific unemployment rates as used in the model - shUnemp(s,a,y) - which will be consistent with the overall unemployment rate during year y - shUnempAll(y) - which is also specified as input data. The adjustment factor - adjUnempl(y) - will depend on the age and gender distribution of the labour force during year y as well as on shprelUnemp(s,a,y).

$$shUnemp(s,a,y)= adjUnempl(y)*shprelUnemp(s,a,y)$$

Unemployment equal the share of unemployed times the population in the labour force.

$$Unemployed(s,a,y)=shUnemp(s,a,y)* LabourForce(s,a,y)$$

The employed equal the share of employed times the population in the labour force.

$$Employed(s,a,y)= (1-shUnemp(s,a,y))* LabourForce(s,a,y)$$

The number of contributors among contract workers (including military service) equal the share of contributors among contract workers times the number of employed.

$$ContribCW(s, a, y) = shContribCW(s, a, y) * Employed(s,a,y)$$

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The number of contributors among the self-employed equal the share of contributors among the self-employed times the number of employed.

$$ContribSE(s, a, y) = shContribSE(s, a, y) * Employed(s,a,y)$$

Total contributors equal the sum of self-employed contributors and the sum of working contributors.

$$Contributors(s,a,y)= ContribCW(s, a, y) + ContribSE(s, a, y)$$

The number of self-employed contributors who are covered for full pension equals the share of self-employed contributors who are covered for full pension of the total number of self-employed contributors.

$$ContribSEFull(s, a, y) = shContribSEFull(s,a,y) * ContribSE(s, a, y)$$

Self-employed contributors who are covered for basic pension only equals the share of self-employed contributors who are covered for basic pension only of the total number of self-employed contributors.

$$ContribSEBasic(s, a, y) = shContribSEBasic(s,a,y) * ContribSE(s, a, y)$$

The variable LabourForceShPop is calculated as the quota between the total labour force and the total average population at each given year.

$$\text{LabourForceShPop(y)} = \text{tot\_LabourForce(y)}/\text{tot\_avgPopulation(y)}$$

All other variables as shares of the population are calculated in the same manner.

4. Income and contributions

4.1 Input data

To define a scenario the following must be specified as input data to the model.

avgWageCW(baseYear)	Average *monthly* wage for all contract workers during the Base Year.
WageGrowthCW(y)	Rate of growth of the average contribution wage for contract workers.
prAllowances(s, a, y)	Allowances as a share of wage for contract workers.
shAvgWageCW(s,a, y)	Average monthly wage for contract workers by age and gender as a share of the average wage of men at age 40.
ContribRate(t, y)	Contribution rates by pension scheme tier and year.

4.2 Calculated variables and model equations

WagesumCW(s ,a , y)	Contract worker wage sum by gender, age and year.
WagesumCW_allages(s, y)	Contract worker wage sum by gender and year.
tot_WagesumCW(y)	Total contract worker wage sum by year.
InsuredIncome(s, a, y)	Insured income by gender, age and year.
InsuredIncome_allages(s, y)	Insured income by gender and year.
tot_InsuredIncome(y)	Total insured income by year.
avgWageCW(s, a, y)	Average wage for contract workers by gender, age and year.
avgWageCW_allages(s, y)	Average wage for contract workers by gender and year.
tot_avgWageCW(y)	Average wage for contract workers by year.

The relative wages for men and women and different ages are specified as input data - shAvgWageCW(s,a). shWageCW(a,s) is expressed as average contribution wage by age and gender during a reference year, for example the base year, and relative to a reference gender and age group, for example men of age 40. They are adjusted proportionally each year to produce the actual age and gender specific monthly wages as used in the model - WageCW(s, a, y) - which will be consistent with the overall average wage during year y - avgWageCW(y)- which in turn is determined by wage during the Base Year (avgWageCW(baseYear)) and the assumed growth rates of wages (WageGrowthCW(y)). The adjustment factor - adjWages(y) - will depend on the age and gender distribution of the labour force and of wages during year y.

The average wage of a contract worker equals the wage the previous year multiplied by the wage growth rate.

$$avgWageCW(y) = avgWageCW(y) * (1 + WageGrowthCW(y - 1))$$

The adjustment factor - adjWages(y) - is calculated in several steps.

The preliminary contract worker wage by gender and age equals the average wage for both sexes and all ages times the relative wage by gender and age (the gender and age distribution).

$$prelWageCW(s, a, y) = avgWageCW(y) * shAvgWageCW(s, a, y)$$

The preliminary contract worker wage sum equals the sum over age and gender of the preliminary contract worker average wage times the number of contributing contract workers at each age and gender.

$$prelWageSumCW(s, a, y) = prelWageCW(s, a, y) * Contributors(s, a, y)$$

Age and gender specific wages are then aggregated over ages as are the number of contributors.

$$prelWagesumCW\_allages(s, y) = \sum_a prelWageCW(s, a, y)$$

$$Contrib\_allages(s, y) = \sum_a Contributors\ (s, a, y)$$

Now the adjustment factor can be calculated as

$$adjWages(y) = avgWageCW(y) / \sum_s prelWagesumCW\_allages(s, y) / \sum_s Contributors\_allages(s, y))$$

Finally, the actual age and gender specific monthly wages as used in the model is

$$WageCW(s, a, y) = prelWageCW(s, a, y) * adjWages(y)$$

The contract worker wage sum equals the sum over age and gender of the preliminary contract worker average wage times the number of contributing contract workers at each age and gender.

$$WagesumCW(s, a, y) = WageCW(s, a, y) * Contributors(s, a, y)$$

$$WagesumCW\ \_alllages= \sum_a WagesumCW(s, a, y)$$

Insured income is calculated from wages and the assumed size of allowances as a share of the gender and age specific wage sum.

$$InsuredIncome(s, a, y) = WagesumCW(s, a, y) * (1 + prAllowances(s, a, y))$$

5. Old age pensions - Basic and Supplementary pension.

5.1 Input data

To define a scenario the following must be specified as input data to the model.

PensAgesByCohort(s, c)	Age of retirement (pensioning) in whole or half years of age.
avgVDP(y)	Established values for VDP.
avgVDPGrowth(y)	Assumed yearly growth rate.
K_before1995(s, c)	K before 1995 by gender and cohort.
S_before1995(s, c)	S before 1995 by gender and cohort.
K_Data_after1995(s, c, y)	K between 1996 and the year before the Base Year.
S_Data_after1995(s, c)	S between 1995 and the year before the Base Year.
BasicPension(baseYear)	Average Basic Pension during the Base Year.
BasicPensionGrowth(y)	The rate of growth (indexation) in real terms of Basic Pension.
FullSupplementaryPension(s, a, baseYear)	Average Supplementary Pension for pensioners during the Base Year before reduction because of participation in the Pension Accumulation Scheme.
shEarlyRetirment(s,c,t)	Share of early retirement by gender, cohort and time to statutory retirement age.
prPensionDeduction	Percentage of pension deduction for each year of early retirement.

Established values for avgVDP(y) are read as input data, and for the remaining years until EndYear avgVDP(y) is calculated from the assumed growth rates according to the formula:

$$avgVDP(y) = avgVDP(y - 1) * (1 + avgVDPGrowth(y))$$

5.2 Calculated variables and model equations

SharePensioners(s, a, y)	SharePensioners(s, a, y) multiplied by average population is the average number of pensioners during the year.
OldAgePensioners(s, a, y)	Old age pensioners by gender, age and year.
OldAgePensioners_allages(s, y)	Old age pensioners by gender and year.
tot_OldAgePensioners(y)	Total old age pensioners by year.
firstPensCohort(s)	The first cohort for which pensioning occurs between BaseYear and EndYear.
lastPensCohort(s)	The last cohort for which pensioning occurs between BaseYear and EndYear.
avgVDP(y)	Calculated values for insured income by year.

K_after1995(s, c)	K at retirement for the cohort. Used in the pension formula.
S_after1995(s, c)	S at retirement for the cohort. Used in the pension formula.
S_total(s, c)	All service years before and after 1995.
S_totalNewPensionersMen(y)	All service years before and after 1995 for the cohort of men who retires in year y.
S_totalNewPensionersWomen(y)	All service years before and after 1995 for the cohort of women who retires in year y.
BasicPension(y)	Average Basic Pension.
replacementRate_1(y)	Replacement rate from Basic Pension.
SystemDependencyRatio(y)	System dependency ratio by year.
FullSupplementaryPension(s, a ,y)	Full supplementary pension by gender, age and year.
FullSupplementaryPension_allages(s, y)	Full supplementary pension by gender, and year.
tot_FullSupplementaryPension(y)	Total full supplementary pension by year.
FullSupplementaryPensionExpenditures(s, a, y)	Full supplementary pension expenditures by gender, age and year.
FullSupplementaryPensionExpenditures_allages(s, y)	Full supplementary pension expenditures by gender and year.
tot_FullSupplementaryPensionExpenditures(y)	Total full supplementary pension expenditures by year.

The first and last cohort for which pensioning occurs between BaseYear and EndYear are calculated. These are the cohorts for which an old age pension will be calculated in the model according to the rules for pension benefit.

As a simplification in the model, all individuals in the cohort are assumed to retire at the same age. The retirement age might better be called the age of pensioning – the age when you start to collect a pension benefit, irrespective of whether you stop working. It must be specified in whole or half years of age (e. g. 60, 60.5, 61 or 61.5 etc).

For those who are pensioners already at the start of the Base Year, their future average pension and their pension costs are calculated on the basis of statistics about the average number of pensioners and the average pension benefit during the Base Year.

SharePensioners(s, a, y) is the average share of a year for which an individual in the cohort c = y- a is a pensioner. If the pension age of the cohort is a whole year of age SharePensioners(s, a, y) is 0 all years before pensioning, 0.5 during the year of pensioning and 1 all years after pensioning. If the pension age of the cohort is not a whole year of age, the cohort retires during two consecutive years. SharePensioners(s, a, y) is 0.125 during the first of these years and 0.875 during the second.

SharePensioners(s, a, y) multiplied by average population is the average number of pensioners during the year:

$$OldAgePensioners(s, a, y) = SharePensioners(s, a, y) * AvgPopulation(s, a, y)$$

$$\text{OldAgePensioners\_Allages}(s, y) = \sum_a \text{OldAgePensioners}(s, a, y)$$

5.2.1 The calculation of K and S for the Supplementary pension formula.

The annual average K before the Base Year are provided as input data. From the Base Year and during the whole projection period the annual K is calculated as:

$$\text{InsuredIncome}(s, y - c, y) / (\text{Contributors}(s, y - c, y) * 12 * \text{avgVDP}(y)).$$

K_after1995(s, c) is the value of K at retirement for the cohort c used in the formula for supplementary pension, and calculated from the annual K's according to the following assumptions.

We assume that the 25 best years used for calculation of K are the 25 last years before the year of pensioning. There are three cases:
   a) The number of years between 1995 and pensioning is less than 25.
   b) Some of the 25 years before pensioning occurs before the BaseYear.
   c) All the 25 years before pensioning occurs during the BaseYear or later.
 In case c only simulated data are used. In case a and b also estimated K-values provided as input data. Since there is no estimated K for 1995, in case b we use the last 24 years before pensioning if 1995 is included among the 25 and also in case a we use only 1996 and later.

S_after1995(s, c) is the value of S at retirement for the cohort c used in the formula for supplementary pension, and calculated from the share of contributors of the cohort, added for all years between the year when the cohort reaches age 15 to the year before pensioning..

$$\text{S\_after1995}(s, c) = \sum_y \text{Contributors}(s, y - c, y) / \text{AvgPopulation}(s, y - c, y).$$

For the cohorts who reach the age of 15 before the Base Year, the formula is:

$$\text{S\_after1995}(s, c) = \text{S\_Data\_after1995}(s, c) + \sum_y \text{CONTRIB}(s, y - c, y) / \text{AvgPopulation}(s, y - c, y).$$

For the cohorts who reach the age of 15 before the Base Year
S_total(s, c) = S_before1995(s, c) + S_after1995(s, c), and for later cohorts
S_total(s, c) = S_after1995(s, c).

S_totalNewPensionersMen(y) is S_total(s, c) for the cohort of men who retires in year y, and S_totalNewPensionersWomen(y) is S_total(s, c) for the cohort of women who retires in year y.

5.2.2 The calculation of Basic Pension.

The average Basic Pension during the Base Year and the assumed growth rate (indexation in real terms) of Basic Pension are provided as input data. Then the average Basic Pension for all projection years is calculated from the formula:

$$\text{BasicPension}(y) = \text{BasicPension}(y - 1) * (1 + \text{BasicPensionGrowth}(y)).$$

The replacement rate from Basic Pension is calculated as

replacementRate_1(y) = 12 * BasicPension(y) / avgWageCW(y).

5.2.2 The calculation of Supplementary Pension.

For the cohorts who reach the age of 15 before the Base Year, average supplementary pension at retirement is calculated as:

newSupplementaryPension(s, c) = 0.005 * avgVDP(pens_y) * (S_before1995(s, c) * K_before1995(s, c) + S_after1995(s, c) * K_after1995(s, c))

For later cohorts the formula is:

newSupplementaryPension(s, c) = 0.005 * avgVDP(pens_y) * S_after1995(s, c) * K_after1995(s, c)

avgVDP(pens_y) is the avgVDP during the year of retirement of the cohort.

The replacement rate from Supplementary Pension is calculated as

replacementRate_2(s, c) = 12 * newSupplementaryPension(s, c) / avgWageCW(pens_y)

The next step is to calculate Supplementary Pension for all years (y = c + a) after retirement. FullSupplementaryPension(s, a, y) is the average Supplementary Pension before reduction with respect to participation in the Pension Accumulation Scheme.

FullSupplementaryPension(s, a, c + a) = newSupplementaryPension(s, c) * (avgVDP(c + a) / avgVDP(pens_y)).

FullSupplementaryPension(s, a, baseYear) is the average Supplementary Pension for pensioners during the BaseYear before reduction because of participation in the Pension Accumulation Scheme and is provided as input data. Supplementary pension for existing pensioners during the BaseYear for all years (y = c + a) until EndYear or age 100 is then calculated as:

FullSupplementaryPension(s, a, c + a) = FullSupplementaryPension(s, baseYear - c, baseYear) * (avgVDP(c + a) / avgVDP(baseYear)).

Expenditures for Supplementary Pension is:

FullSupplementaryPensionExpenditures(s, a, y) = FullSupplementaryPension(s, a, y) * 12 * OldAgePensioners(s, a, y).

FullSupplementaryPensionExpenditures_allages(s, y) = $\sum_a$ FullSupplementaryPensionExpenditures(s, a, y).

FullSupplementaryPension_allages(s, y)
=FullSupplementaryPensionExpenditures_allages(s, y) / (OldAgePensioners_allages(s, y) * 12).

6. Old age pensions - The pension accumulation scheme.

6.1 Input data

To define a scenario the following must be specified as input data to the model.

tot_ContribRatel(y)	Total contribution rate.
ContribRateBasicPens(y)	Contribution rate basic pension.
ContribRateSupplPens(y)	Contribution rate supplementary pension.
ContribRatePA(y)	Contribution rate pension accumulation pension.
ParticipationPensAcc(s, c, y)	The share of the cohort which participates in the pension accumulation scheme.
prContribAdmin(baseYear)	Percentage rate administrative cost deducted from premium inflow.
RealRateOfReturn(y)	Real rate of return on pension accumulation funds.
RealGovBondRate(y)	Real government bond rate of return.
PAFundIn(s, c, baseYear)	Funds within the pension accumulation scheme at the end of the Base Year.
prFundAdmin(baseYear)	Percentage rate administrative cost deducted from fund value (asset management tax).
TransferToAnnuityCost	The percentage cost for transferring the accumulative funds into an annuity.
AnnuityLoading	The percentage rate deducted from the real rate of return in the annuity divisor, this rate equals the administrative cost for the annuity provider.

6.2 Calculated variables and model equations

RetiredPAPopulation(s, c)	The number of individuals in the cohort who are in the pension accumulation scheme at retirement.
ContribPA(s, a, y)	Contributions (premium inflow) to the pension accumulation scheme by gender, age and year.
ContribPA_allages (s, y)	Contributions to the pension accumulation scheme by gender and year.
tot_ContribPA(y)	Contributions to the pension accumulation scheme by year.
PAFund(s, c, y)	Pension accumulation funds by gender, cohort and year.
PAFund_allcohorts(s, y)	Pension accumulation funds by gender and year.
tot_PAFund(y)	Pension accumulation funds by year.
NetContribPA(s, a, y)	Contributions net of administrative cost deducted from premium inflow by gender, age and year.

NetContribPA_allages(s, y)	Contributions net of administrative cost deducted from premium inflow by gender and year.
tot_NetContribPA(y)	Contributions net of administrative cost deducted from premium inflow by year.
AdminContrib(s, a, y)	Administrative cost deducted from premium inflow by gender, age and year.
AdminContrib_allages(s, y)	Administrative cost deducted from premium inflow by gender and year.
tot_AdminContrib(y)	Administrative cost deducted from premium inflow by year.
prFundAdmin(y)	Percentage rate administrative cost deducted from fund value (asset management tax).
AnnualInterest(s, c, y)	Annual interest by gender, cohort and year.
AnnualInterest_allcohorts(s, y)	Annual interest by gender and year.
tot_AnnualInterest(y)	Total annual interest by year.
InheritLosses(s, c, y)	Inheritance losses by gender, cohort and year.
InheritLosses_allcohorts(s, y)	Inheritance losses by gender and year.
tot_InheritLosses(y)	Inheritance losses by year.
AnnuityDivisors(s, c)	Annuity divisors.
NetPAFund(s, c, y)	The net of the accumulated funds which is converted into an annuity.
LastYearFund(s, c)	The fund size in the year before entering old age pensioning.
AvgPAFundAtRetirement(s, c)	The average accumulated fund per person at the time of retirement.
AvgAnnuity(s, c)	The cohort annuity.
weightedSPASuppl(s, c)	Factor for reduction of supplementary pension for participants in the pension accumulation scheme.
replacementRatePA(s, c)	Replacement rate from the pension accumulation annuity.
SupplementaryPension(s, a, c + a)	Supplementary pension after reduction for participants in the pension accumulation scheme.
replacementRate_2(s, c)	Replacement rate from supplementary pension.
SupplementaryPensionExpenditures(s, a, y)	The supplementary pension expenditures.
SupplementaryPensionExpenditures_Allages(s, y)	The total supplementary pension expenditures for all ages by gender and year.
SupplementaryPension_allages(s, y)	Average supplementary pension for all ages.
avgPension(s, y)	Average Pensions by gender and year.
PensionCosts_allages(s, y)	Total pensions costs by gender and year.
Contrib_allages(s, y)	Contributions by gender and year.
rrMenSupplementary(y)	Replacement rate of supplementary pension for men by year.
rrMenPensionAccumulation(y)	Replacement rate of pension accumulation for men by year.

rrWomenSupplementary(y)	Replacement rate supplementary pension for women by year.
rrWomenPensionAccumulation(y)	Replacement rate of pension accumulation for women by year.
rrBasic(y)	Replacement rate for basic pension by year.
rrMenTotal(y)	Total replacement rate for men by year.
rrWomenTotal(y)	Total replacement rate for women by year.

RetiredPop(s, c) is calculated from totParticipationPensAcc(s, c, y) and AvgPopulation(s, a, y) at the age and year when the cohort retires. In the case that the cohort retires during two consecutive years, the average of AvgPopulation(s, a, y) during these years are used.

Contributions (premium inflow) to the pension accumulation scheme equal the contribution rate times the contract worker wage sum times the participation rate.

$$\text{ContribPA(s, a, y)} = \text{ContribRatePA(y)} * \text{WagesumCW(s, a, y)} * \text{ParticipationPA(s,a,y)}$$

The net contributions to the pension accumulation scheme equal the pension accumulation contributions after administrative costs.

$$\text{NetContribPA(s, a, y)} = \text{ContribRatePA(y)} * (1 - \text{prContribAdmin(baseYear)}) * \text{WagesumCW(s, a, y)}$$

Administrative cost deducted from premium inflow is:

$$\text{AdminContrib(s, a, y)} = \text{ContribRatePA(y)} * \text{prContribAdmin(baseYear)} * \text{WagesumCW(s, a, y)}$$

The percentage rate of administrative cost deducted from fund value (asset management tax) is assumed to be the same for all years:

$$\text{prFundAdmin(y)} = \text{prFundAdmin(baseYear)}$$

The annual interest during year y on funds in the beginning of year y and on net contributions during year y is:

$$\text{AnnualInterst(s, c, y)} = \text{PAFund(s, c, y - 1)} * (\text{RealRateOfReturn(y)}) + \text{NetContribPA(s, y - c, y)} * (\text{Sqr}(1 + \text{RealRateOfReturn(y)}) - 1)$$

Inheritance losses are distributed at the end of the year:

$$\text{InheritLosses(s, c, y)} = \text{PAFund(s, c, y - 1)} * (1 - \text{ModeledSurvivalRates(s, y - c, y)})$$

Inheritance losses are not allowed to be negative.

If InheritLosses(s, c, y) < 0 Then InheritLosses(s, c, y) = 0

Funds within the pension accumulation scheme at the end of year y is calculated as:

$$PAFund(s, c, y) = (PAFund(s, c, y - 1)$$
$$+ (AnnualInterst(s, c, y) - InheritLosses(s, c, y) + NetContribPA(s, y - c, y)) * (1 - prFundAdmin(y)))$$

Sum over cohorts to obtain aggregate values by gender and year:

$$AnnualInterest\_allcohorts(s, y) = \sum_c AnnualInterst(s, c, y)$$

$$InheritLosses\_allcohorts(s, y) = \sum_c InheritLosses(s, c, y)$$

$$PAFund\_allcohorts(s, y) = \sum_c PAFund(s, c, y)$$

Annuity divisors, AnnuityDivisors(s, c), are calculated according to the formula in appendix.

$$NetPAFund(s, c, y) = PAFund(s, c, y) * (1 - TransferToAnnuityCost)$$

LastYearFund(s, c) is NetPAFund(s, c, y) at the year of retirement of cohort c.

$$AvgPAFundAtRetirement(s, c) = LastYearFund(s, c) / RetiredPop(s, c)$$

$$AvgAnnuity(s, c) = AvgPAFundAtRetirement(s, c) / AnnuityDivisors(s, c)$$

Replacement rate from the pension accumulation annuity is:

$$replacementRatePA(s, c) = AvgAnnuity(s, c) / avgWageCW(pens\_y + c)$$

Supplementary pension within the PAYG scheme is reduced for participants in the pension accumulation scheme.

$$weightedSPASuppl(s, c) = S\_sum / S\_total(s, c)$$

Where S_sum = CONTRIB(s, y - c, y) / AvgPopulation(s, y - c, y) *
(ContribRatePA(y) / ContribRateSupplPens(y)) * totParticipationPensAcc(s, c, y)

$$SupplementaryPension(s, a, c + a) =$$
$$(1 - weightedSPASuppl(s, c)) * FullSupplementaryPension(s, a, c + a)$$

Replacement rate from supplementary pension is:

$$replacementRate\_2(s, c) = (1 - weightedSPASuppl(s, c)) * 12 * newSupplementaryPension(s, c) / avgWageCW(pens\_y)$$

$$SupplementaryPensionExpenditures(s, a, y) = SupplementaryPension(s, a, y) * 12 * OldAgePensioners(s, a, y)$$

SupplementaryPensionExpenditures_Allages(s, y) = $\sum_a$ SupplementaryPensionExpenditures(s, a, y)

SupplementaryPension_Allages(s, y) = SupplementaryPensionExpenditures_Allages(s, y) / (OldAgePensioners_Allages(s, y)*12)

Buffer fund …

avgPension(s, y) = 12 * (BasicPension(y) + SupplementaryPension_Allages(s, y))

totPensionCosts(s, y) = avgPension(s, y) * OldAgePensioners_Allages(s, y) + totContribPA(s, y)

totContrib(s, y) = totWagesumCW(s, y) * (ContribRateBasicPens(y) + ContribRateSupplPens(y))

…….

Replacement rates for new pensioners by year.

rrBasic(y) = replacementRate_1(y)

rrMenSupplementary(y) and rrWomenSupplementary(y) are replacementRate_2(s, c) for the cohort of men respective women who retire in year y.

rrMenPensionAccumulation(y) and rrWomenPensionAccumulation(y) are replacementRatePA(s, c) for the cohort of men respective women who retire in year y.

rrMenTotal(y) = rrBasic(y) + rrMenSupplementary(y) + rrMenPensionAccumulation(y)

and

rrWomenTotal(y) = rrBasic(y) + rrWomenSupplementary(y) + rrWomenPensionAccumulation(y)

Pension formula

totPensions(s,a,y)= totBasicPension(y)+ totSupplPension(s,a,y)+totAnnuity(s,a,y)

totPensions(s,a,y)=avgPensions(s,a,y)*avgOldagePensioneer(s,a,y)

totBasicPension(s,a,y)=avgBasicPension(y)*avgOldagePensioneer(s,a,y)

totSupplPension(s,a,y)=avgSupplPension(s,a,y)*avgOldagePensioneer(s,a,y)

weightedSPASuppl(s, c)= CONTRIB(s, y - c, y) / AvgPopulation(s, y - c, y) * (ContribRatePA(y) / ContribRateSupplPens(y)) * totParticipationPensAcc(s, c, y)/ ($\sum_y$ CONTRIB(s, y - c, y) / AvgPopulation(s, y - c, y) * (ContribRatePA(y) / ContribRateSupplPens(y)) * totParticipationPensAcc(s, c, y))

weightedSPASuppl(s, c) *is equal to the duration weighted participation in the pension participation scheme.*

PAFund(s,c,y)=(AnnualInterst(s,c,y)-InheritLosses+NetContirbPA+PAFund(s,c,y-1))*(1-prAdminFund(y))

Where

AnnualInterst(s,c,y) = PAFund(s,c,y-1)* RealRateOfReturn(y) + NetContribPA(s,y-c,y)* (sqrt(1+ RealRateOfReturn(y))-1)

The annual interest is the total interest income for the total pension accumulation fund. This variable is used when calculating the funds performance and fund accounts the 31ˢᵗ of December each year.

NetContribPA(s,a,y)= ContribPA (s,a,y)*(1- prAdminContrib(y))

The net contributions to the pension accumulation scheme equal the pension accumulation contributions after administration costs.

InheritLosses(s,c,y) = PensAccFund(s,c,y-1)*(1-ModeledSurvivalRates(s,y-c,y))

The inherit losses, the drain of the accumulated fund value, equals the deceased part of the total fund value

AdminContrib(y)=prAdminContrib(y)* ContribPA(s,a,y)

The administrative cost collected from the premium inflow in the pension accumulation scheme equals the average percentage rate of contribution fees times the total contributions to the scheme.

AdminFund(y)= prAdminFund(y)* PAFund(s,c+a)

The administrative cost for the fund management equal the fund value times the percentage rate of administrative fund costs.

NetPAFund(s, c, y) = PAFund(s, c, y) * (1 - TransferToAnnuityCost)

When a cohort retires the NetPAFund *is transferred into* LastYearFund(s,c) *which is transferred into an annuity, the cost of doing this is measured with* TransferToAnnuityCost

AvgPAFundAtRetirement(s, c) = LastYearFund(s, c) / RetiredPop(s, c)

AvgAnnuity(s, c) = AvgPAFundAtRetirement(s, c) / AnnuityDivisors(s, c)

Se appendix A about annuity divisors

$$\text{AvgPAFundAtRetirement}(s, c) = \text{LastYearFund}(s, c) / \text{RetiredPAPop}(s, c)$$

$$\text{AvgAnnuity}(s, c) = \text{AvgPAFundAtRetirement}(s, c) / \text{AnnuityDivisors}(s, c)$$

7. Budget

7.1 Calculated variables and model equations

BaseYearGDP(y)	Base Year Gross Domestic Product by year.
GDPWageShare(y)	Wage share of Gross Domestic Product by year.
Inflation(y)	Inflation rate by year

7.2 Revenues & Expenditures

GDP(y)	Gross Domestic Product by year.
CumulativeInflation(y)	Cumulative inflation rate from base year through all year until the endyear.
OldAgePensionRevenues(y)	Old age pension revenues by year.
OldAgePensionRevenuesShGDP(y)	Old age pension revenues as a share of GDP by year.
OldAgePensionRevenuesShWageSum(y)	Old age pension revenues as a share of the wage sum by year.
OldAgePensionExpenditures (y)	Old age pension expenditures by year.
OldAgePensionExpendituresShGDP(y)	Old age pension expenditures as a share of GDP by year.
OldAgePensionExpendituresShWageSum(y)	Old age pension expenditures as a share of the wage sum by year.
TransfersToPAFund(y)	Transfers to pension accumulation funds by year.
TransfersToPAFundShGDP(y)	Transfers to pension accumulation funds as a share of GDP by year.
TransfersToPAFundShWageSum(y)	Transfers to pension accumulation funds as a share of the wage sum by year.
Bufferfund(y)	Buffer fund by year (non existent in reality).
BufferfundShGDP(y)	Buffer fund as a share of GDP by year (non existent in reality).
BufferfundShWagesum(y)	Buffer fund as a share of the wage sum by year(non existent in reality).
SupplOldAgePensionExpenditures(y)	Supplementary pension expenditures by year.
SupplOldAgePensionExpendituresShGDP (y)	Supplementary pension expenditures as a share GDP by year.
SupplOldAgePensionExpendituresShWagesum(y)	Supplementary pension expenditures as a share of the wage sum by year.
DisabilityExpenditures(y)	Disability Expenditures by year.
DisabilityExpendituresShGDP(y)	Disability Expenditures as a share of GDP by year.

DisabilityExpendituresShWageSum(y)	Disability Expenditures as a share of the wage sum by year.
LossOfBreadWinnerExpenditures(y)	Loss of breadwinner expenditure by year.
LossOfBreadWinnerExpendituresShGDP(y)	Loss of breadwinner as a share of GDP by year.
LossOfBreadWinnerExpendituresShWageSum(y)	Loss of breadwinner as a share of the wage sum by year.
OrphansExpenditures(y)	Orphans expenditures by year.
OrphansExpendituresShGDP(y)	Orphans expenditures as a share of GDP by year.
OrphansExpendituresShWageSum(y)	Orphans expenditures as a share of wage sum by year.
WidowsWidowersExpenditures(y)	Widows and widowers expenditure by year.
WidowsWidowersExpendituresShGDP(y)	Widows and widowers expenditure as a share of GDP by year.
WidowsWidowersExpendituresShWageSum(y)	Widows and widowers expenditure as a share wages sum by year.
LossOfBreadWinnerExpenditures(y)	Loss of breadwinner expenditure by year.
LossOfBreadWinnerExpendituresShGDP(y)	Loss of breadwinner expenditure as a share of GDP by year.
LossOfBreadWinnerExpendituresShWageSum(y)	Loss of breadwinner expenditure as a share of the wage sum by year.
MaternityExpenditures(y)	Maternity Expenditures by year.
MaternityExpendituresShGDP(y)	Maternity Expenditures as share of GDP by year.
MaternityExpendituresShWageSum(y)	Maternity Expenditures as share of the wage sum by year.
ChildcareExpenditures(y)	Child care expenditures by year.
ChildcareExpendituresShGDP(y)	Child care expenditures as share of GDP by year.
ChildcareExpendituresShWageSum(y)	Child care expenditures as share of the wage sum by year.
SicknessExpenditures(y)	Sickness expenditures by year.
SicknessExpendituresShGDP(y)	Sickness expenditures as a share of GDP by year.
SicknessExpendituresShWageSum(y)	Sickness expenditures as a share of the wage sum by year.
UnemploymentExpenditures(y)	Unemployment expenditures by year.
UnemploymentExpendituresShGDP(y)	Unemployment expenditures as a share of GDP by year.
UnemploymentExpendituresShWageSum(y)	Unemployment expenditures as a share of the wage sum by year.
tot_Revenues(y)	Total revenues by year.
tot_RevenuesShGDP(y)	Total revenues as a share of GDP by year.
tot_RevenuesShWageSum(y)	Total revenues as a share of the wage sum by year.
tot_Expenditures(y)	Total expenditures by year.
tot_ExpendituresShGDP(y)	Total expenditures as a share of GDP by year.
tot_ExpendituresShWageSum(y)	Total expenditures as a share of the wage sum by year.

AnnualBudgetBalance(y)	Annual budget balance.
Bufferfund(y)	The accumulated annual balances, the buffer fund by year.

Contrib(s, y) = WagesumCW_allages(s, y) * (ContribRateBasicPens(y) + ContribRateSupplPens(y))

Total incomes to the pensions scheme equals the total covered wage sum times the total contribution rate.

PensionCosts(s, y) = avgPension(s, y) * OldAgePensioners_Allages(s, y) + ContribPA_allages(s, y)

where

avgPension_allages(s, y) = 12 * (BasicPension(y) + SupplementaryPension_allages(s, y))

Appendix A) Annuity divisors

The annuity is calculated as a temporary annuity from retirement age pa(s,c) to the age of 100. pa(s,c) is gender and cohort dependent.

The general formula for a discrete temporary annuity of value one monetary unit beginning from the age of *x* payable until *x+n*:

$$A(x) = \frac{N(x) - N(x+n)}{D(x)}$$

where

$$N(x) = \sum_{t=0}^{\infty} D(x+t)$$

an infinite annuity one monetary unit equals

$$A(x) = \frac{N(x)}{D(x)}$$

$$D(x) = l(x) \cdot v^x$$

$$v = \frac{1}{1+i}$$

i = Real rate of return – Loading

Appendix B) Budget and funds

AnnualBudgetBalance(y) = (tot_Revenues(y) – tot_Expenditures(y))

'1/2 year indexing moving the net to the end of year
EndYearBudgetBalance (y) = netFlow * Sqr(1 + RealGovBondRate(y))

Intrests = EndYearBudgetBalance (y)- AnnualBudgetBalance(y)

tmp = Bufferfund(y - 1)
Bufferfund(y) = (1 + RealGovBondRate(y)) * tmp
intrests = (Bufferfund(y) - tmp) + intrests

If the AnnualBudgetBalance(y) < 0 and the Bufferfund(y) + AnnualBudgetBalance (y) < 0 the the government have to borrow means.

If Bufferfund(y) <= 0 then government has to borrow the entire deficit

YearlyLoan(y) = - AnnualBudgetBalance (y)

If not then some of it is covered by the buffer fund Bufferfund(y) >= 0

YearlyLoan(y) = -(Bufferfund(y) + AnnualBudgetBalance (y))

If on the other hand the money is flowing in and the buffer fund is negative then the government can afford to pay some thing back.

ElseIf AnnualBudgetBalance(y) > 0 And Bufferfund(y) < 0

If -Bufferfund(y) > AnnualBudgetBalance (y) Then

All surplus used to pay back some of loans.

YearlyRepayment(y) = AnnualBudgetBalance (y)

If the surplus covers all the loans

YearlyRepayment(y) = -Bufferfund(y)

Bufferfund(y) = Bufferfund(y) + AnnualBudgetBalance(y)

There are two types of intrest in the model
```
    If intrests >= 0 Then
       IntrestsOnBufferfund(y) = intrests
       NegativeIntrestsOnBufferfund(y) = 0
    Else
       NegativeIntrestsOnBufferfund(y) = -intrests
       IntrestsOnBufferfund(y) = 0
```