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Maria Olofsson

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Postal address:
Mathematical Statistics
Dept. of Mathematics
Stockholm University
SE-106 91 Stockholm
Sweden

Internet:
http://www.math.su.se/matstat

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Abstract

There will be new accounting legislation (IAS) and changed regulatory requirements (Solvency II) for insurance undertakings within EU (the European Union). The Solvency II will be based on a more risk-based system than the present Solvency I. The regulator will most likely require that the liabilities (reserves) are valued at "fair value", i.e. some sort of market value. The fair value is, in terms of accounting, usually defined as a best estimate (BE) plus a margin usually called the market value margin (MVM). The MVM is a form of risk margin that is settled by the actors on the market. In terms of solvency, the BE is defined as the central estimate. In non-life insurance, this risk margin is the component of the value of insurance liabilities relating to the inherent uncertainty in the central estimate (the mean of the distribution of probable outcomes). We calculate this risk margin at the 75th percentile. Our guidance to this will be APRA's (Australian Prudential Regulation Authority's) newly released standards. The regulations in Australia require that the central estimate plus the risk margin of the insurance liabilities secure the 75th percentile of the underlying distribution. We will use the Australian approach to estimate

*Postal address: Dept of Mathematical Statistics, Stockholm University, SE-106 91 Stockholm, Sweden. E-mail: maria.olofsson@if.se, olofsson_maria@hotmail.com. Supervisor: Anders Martin-Löf.

the risk margins on some specific portfolios in the Swedish insurance company If P&C Insurance Ltd. The portfolios are Private Property (house owner, homeowner and holiday cottage), Motor TPL (Third Party Liability) and Liability. We will investigate both paid and incurred data (paid plus case reserves). We use the distribution-free method by Thomas Mack and a Bootstrap simulation for calculation of the risk margins. Today these estimates (the reserves) are usually based on a point estimate with reasonable and prudent assumptions and specific estimates of uncertainty are not made on a regular basis. At the end of the report, we will compare our results with the results of APRA for each portfolio. One problem that has followed us through the project is that incurred data includes negative incremental values. The bootstrap simulation is sensitive to negative values in the development triangle. The solution made here was to let the negative values in the development triangles equal zero.

PREFACE

This paper is mainly performed during the spring/summer 2005 at the Swedish insurance company If P&C Insurance Ltd, division Unit Corporate in Bergshamra (Stockholm). This task corresponds to 20 weeks of work and it constitutes my master thesis, comprised 20 points for a master's degree of a total 160 points in mathematical statistics, from the mathematical-computer science programme at the University of Stockholm.

I would like to take the opportunity to thank the insurance company If P&C Insurance Ltd for having offered me an office and through this offer have giving me a chance to get an insight in the working life.

I would like to express my gratitude to my two supervisors at If P&C Insurance Ltd and The Swedish Insurance Federation (Sveriges Försäkringsförbund), Ingrid Wrebo respectively Arne Sandström, for their high expectations both in quality and scope, for entrusting me with this responsibility and for being present when I needed help.

Finally, I would like to thank my supervisor at the University of Stockholm, Professor Anders Martin-Löf.

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1 Introduction

The correct estimation of the amount of money an insurance company should set aside to meet claims that arise in the future on written policies represents an important task for the insurance company, i.e. to get the correct picture of its liabilities. An insurance company needs to hold reserves because the timing of premiums receipt and claims payment does not coincide. There is a delay between the claim event and the claim settlement date and this means that the insurer must set up reserves in respect of those claims still to be settled.

The delay depends on how long time it takes from the day a loss occurs until the claim is settled and paid out from the insurance company to the policyholder. It can take a long time from the loss event to the reporting of the loss to the company, or it can take a long time from the day of reporting until the company knows the ultimate cost of the claim.

In the end of every reporting period (usually year and/or quarter) the insurance company shall present in the accounts how much of the money that is allocated for this incurred but not settled loss. This liability is often named as the loss reserve. The loss reserve is usually split-up into known and unknown claims reserve. This reserving is the essential part of estimating the claim cost. It is also necessary for business planning, budgeting and product pricing. In the long run the company's ability to price the products correctly will have the main influence on the company's solvency.

The problem of how to calculate the loss reserve is usually solved with statistical methods. Since the amount and timing of future claims are unknown, this creates an uncertainty over the amount of reserves that is necessary. The degree of uncertainty will depend on the class of business written (Line of Business, LoB), where we have short-tail and long-tail classes. A short-tail class is a business where the delay between the occurrence of a claim and the settlement is short, often less than a year. A long-tail business is a business where the delay between the occurrence of a claim and it being settled is long, more than couple of years.

Short-tail classes such as Motor (that, for example, cover fire and theft) insurance are considered less volatile, the uncertainty about the amount and timing of claims payments is typically resolved within a year. As a result, a company that is writing short-tail business is expected to hold smaller amounts of reserves than a company that is writing long-tail business whose uncertainty about the amount and timing of claims payments typically takes more than one year to solve, such as Motor TPL (third party liability, that covers personal injury) (ref [10]).

There are different statistical methods for solving this problem of how much money a company should set aside. The Chain Ladder method (CL) is probably the most popular one for estimating claims reserves. The main reason for this is its simplicity and the fact that it is distributions-free i.e. that it seems to work with almost no assumptions except for a consistent delay pattern in the payment of claims. This method is deterministic and you only get a point estimate.

In most cases data are set up in what we call a development triangle where the rows represent accident years and the columns development years. The elements d_{ij} in the development triangle represent what has been paid out during development year j for losses incurred during accident year i. The corresponding stochastic variable is denoted D_{ij}, i,j=1,2,...,m.

The payments during development year j are made j-1 years after the loss incurred. Usually we expect that all losses will be finally settled after m years. The diagonal (bottom left to top right) represents the claim payments in each payment year. We have that d_{ij} is known when $i + j \leq m+1$ and the task is to expand the triangle into a square contained with predictions of future values. The triangle with this future values is called "future triangle". See figure 1.1 and figure 1.2 for arrangement of the data.

	Development year					
Accident year	1	2	3	. . .	m-1	m
1	d_{11}	d_{12}	d_{13}	. . .	$d_{1,m-1}$	$d_{1,m}$
2	d_{21}	d_{22}	d_{23}	. . .	$d_{2,m-1}$	
.	. . .	. . .	. . .	. . .		
.	. . .	. . .	. . .			
m-1	$d_{m-1,1}$	$d_{m-1,2}$				
M	$d_{m,1}$					

Figure 1.1: Development triangle - Incremental data.

	Development year					
Accident year	1	2	3	. . .	m-1	m
1	d_{11}	d_{12}	d_{13}	. . .	$d_{1,m-1}$	$d_{1,m}$
2	d_{21}	d_{22}	d_{23}	. . .	$d_{2,m-1}$	$D_{2,m}$
.	. . .	. . .	. . .	. . .	. . .	. . .
.	. . .	. . .	. . .	. . .	. . .	. . .
m-1	$d_{m-1,1}$	$d_{m-1,2}$	$D_{m-1,3}$	. . .	$D_{m-1,m-1}$	$D_{m-1,m}$
M	$d_{m,1}$	$D_{m,2}$	$D_{m,3}$	. . .	$D_{m,m-1}$	$D_{m,m}$

Figure 1.2: Development triangle and future triangle - Incremental data.

The total of the loss reserve R, is the sum of all D_{ij} in the future triangle and they are estimated by $\hat{R} = \sum_{i+j>m+1} \hat{D}_{ij}$. You can say that the data are on the form incremental which means that what is really paid out during year j is really what is paid out during year j and not until year j. To get what is really paid out until year j you only take $C_{ij} = \sum_{k=1}^{j} D_{ik}$ which is said to be on a cumulative form (aggregated data) (ref [17]).

1.1 Purpose and description of the problem

Non-life insurance undertakings have different methods to decide on the loss reserve, one of the most common methods is the Chain Ladder method.

One aim of this paper is to define the terminology that is used to calculate the reserve. Usually the terminology is very vague with many different definitions for the same thing. The non-life insurance companies calculate a point estimate and say that this is their "best estimate (BE)", but what defines a best estimate? The regulator in Sweden SFSA (the Swedish Financial Supervisory Authority) says that these reserves should be prudent/sufficient. One problem that emerges is what defines prudent/sufficient? We have also that the BE should be within a range of "reasonable" estimates, what is "reasonable"?

Within EU (European Union) there will be new accounting legislation IAS (International Accounting Standards) and changed regulatory requirements (Solvency II). The Solvency II will be more risk based as compared to the present Solvency I system. This new solvency system will probably come into force after 2010.

In the new system, the regulator will most likely expect that the liabilities should be valued according to some "fair value" concept, i.e. some sort of market value. The fair value is most of the time defined as a BE (central estimate) plus a 75th risk margin or some margin called market value margin (MVM), which is a form of risk margin settled by the actors on the market.

One country that has developed some standards and ideas of how the 75th risk margin can be calculated is Australia. A good idea is to use the Australian approach to estimate the margins on some specific Swedish portfolios and see how that will affect the present reserve estimations. Today these estimates are usually based on point estimates with reasonable and prudent assumptions and specific estimates of uncertainty are not made on a regular basis.

2 The data

The data that are used in this paper are development triangles for three different LoB in the Swedish insurance company If P&C Insurance Ltd, Private Property (house owner, homeowner and holiday cottage), Motor TPL and Liability. All the amounts are recalculated in an invented currency, a-mark, for the protection of If P&C Insurance Ltd. The figures in the triangles are cumulative paid claims and cumulative incurred claims, both gross values. Paid claims are the booked payments and incurred claims are payments plus case reserves. The reserves are based on individual reserves from claim adjusters and/or statistical reserves for frequency claims. The development triangles are shown in Appendix A.1.

We have used the accident years from 1987 (i=0) to 2004 (i=17) which is enough for the purpose of this report but does not mean that we believe to have reached the ultimate claims amount for Motor TPL, which is long-tailed, after 17 years of development. When we took the cumulative triangles and made them into incremental triangles, some cells turned out to be negative and one of the methods we use, the bootstrap, is sensitive to negative values. The practical background is that sometimes we have negative payments i.e. payments that is coming into the company. As an example, this can happen when a company is paying for a Motor TPL loss and later on this loss went to another insurance company and the first company gets the money back. These negative values emerge most of the time in the last development years when the payments goes to zero. Most of the problems are in the incurred data and to overcome this problem we replace the negative values with zero whenever they show up. This is not a neat solution but it is a solution. A paper has looked into this problem, if there is something that would interest the reader more (ref [19].

3 Approaches

In this section, we present the approaches used in Australia and Sweden. In the last section, we will discuss accounting and risk margins.

3.1 The approach in Australia

APRA (Australian Prudential Regulation Authority) has released new standards for the determination of liability valuation and solvency for Australian non-life insurers. The regulations require that the central estimate plus the risk margin of the insurance liabilities secure the 75th percentile of the underlying distribution. The best estimate should be the central estimate plus the risk margin, neither overstates nor understates the expected outcome (ref [1]). The risk margin relates to the uncertainty in each of the central estimate values (the mean of the distribution of probable outcome). The actuaries are responsible for determining these risk margins. To ensure that insurers reported liabilities are broadly consistent and sufficiently rigorous across the industry the risk margin should

- be established on a basis that would be expected to secure the insurance liabilities of the insurer at a 75 % level of sufficiency.
- not be less than half of the coefficient of variation of the liability distribution (due to the highly skewed nature of the liability distributions of some classes of insurance). The coefficient of variation is defined as the ratio of the standard deviation and the mean (in this paper the central estimate).

These new principles and standards of practice provide only broad guidance to the actuary on what is "reasonable". This broad guidance is based on the principle that "reasonable" assumptions and models lead to "reasonable" estimates. It is hoped that these principles will help in the future so that the reserve do not get "to low" and the company get insolvent.

3.2 The approach in Sweden

In a typical reserve analysis, the actuary produces a range of reserve projections by accident year and line of business by application of several standard actuarial methods, like the deterministic Chain Ladder method. Although the supervisor in Sweden (SFSA) states that a range of reserves may be actuarially sound, it does not specify how this range may be determined. Based on current practice, the range of reasonable estimates is largely based on actuarial judgement.

The regulation in Sweden is not as thorough as in Australia. In Sweden, we only require that the insurance company calculates a "reasonable" point estimate of the total reserve i.e. a "best estimate" within a range of "reasonable" estimates. Fair and prudent!

The "best estimate" is defined as a point estimate that the actuary has decided is the best of all estimates. The board of directors makes the final decision of the reserve level in the company. The word "best" implies a particular point that is better than all others within the range of reasonable estimates. While different actuaries may produce different "best estimate" numbers, the range of best estimates among these actuaries should be narrower than the range of all "reasonable" estimates. For a particular actuary, there should be only one "best estimate" as of a given reserve date and it is up to the actuary to define what is best.

A reasonable estimate is defined as an estimate based on well tested method and assumptions. The board of directors believes that a reserve is "reasonable" if it is within the range of reasonable and adequate estimates of the actual outstanding loss.

The range of reasonable estimates is a range of estimates that would be produced by alternative sets of assumptions that the actuary judges to be sufficient, considering all the information reviewed by the actuary.

A reserve booked at the low end of the range of the possible outcomes would ordinarily not be within the range of reasonable estimates and so would not make a reasonable provision for all unpaid loss and loss expense obligations. In the end it is all up to the actuaries of the company and their judgement on what would be a reasonable estimate (ref [21]).

3.3 Accounting and risk margins

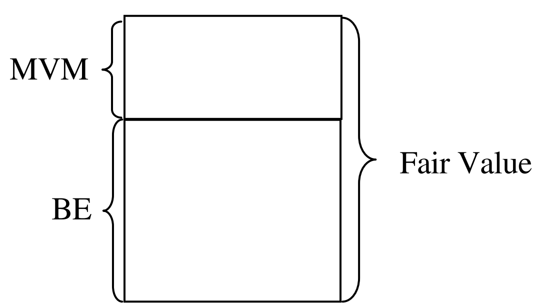
Accounting for insurance has been a top priority for the International Accounting Standards Board (IASB). Considerable work has been done in developing a new International Financial Reporting Standard (IFRS) that would value insurance contracts at fair value. In 2002, it became clear that the task could not be completed in time for the EU 2005 deadline. Consequently, the IASB decided to split the insurance project into two Phases (Phase I and Phase II) (ref [15]).

From a reserving perspective, the more interesting of these two Phases is Phase II, with the more challenging actuarial issues in fair value accounting of insurance contracts. Since this “fair value” for insurance liabilities is not yet defined, we only have a discussion on how to calculate it. Some of the discussion is on calculating risk margins and MVM, but how is still unclear. One thing that is mentioned in the discussion is whether we would work with discounted or undiscounted figures. Today we mainly value our liabilities without taking "time value of money" into account. This will probably change in the future. In this paper there is a short chapter on how to discount the figures (see chapter 6), but we will in the rest of this paper only work with undiscounted figures.

It is well known that the loss reserves are estimates of unknown future loss payments. Actual results will differ from estimated amounts, and the concept of risk margin reflects that fact. The greater the uncertainty, the larger is the risk margin. We can say that with more volatile portfolios, i.e. long-tailed portfolios like Motor TPL, have a larger risk margin most of the time since the uncertainty is greater. It is not obvious how that risk margin should be incorporated into statutory accounting, assuming that a risk margin is calculated and there is a discussion whether it should be incorporated or not (ref [18]).

The risk adjustments are referred to as "MVM" and should be set to be consistent with market-risk preferences. The market-based adjustment for risk and uncertainty effectively act as a market mechanism for pricing the uncertainty. However, one problem is that there is still no guidance on how this should be done (ref [15]). When we calculate the MVM in this paper we only add it to the expected reserve estimate (the central estimate) and the margin will reflect the risk and the uncertainty in the reserve since there are no agreed market risk margins to follow (ref [24]). We will say that the 75th percentile is our risk margin i.e. MVM.

In the future, there will be a problem to decide the MVM for a specific market. The discussion today is whether, for example, the SFSA should decide what the agreed market risk margins (MVM) would be for different portfolios, in discussions with the Swedish insurance industry. The SFSA will follow common guidelines that are set up. This will be the same for each country and their actors on the market. We will draw a picture (see below) just to see how this is done with the BE, MVM on a fair value basis when we have a market to follow. To have it on a fair value basis means also that consideration has been taken to the time value of money.



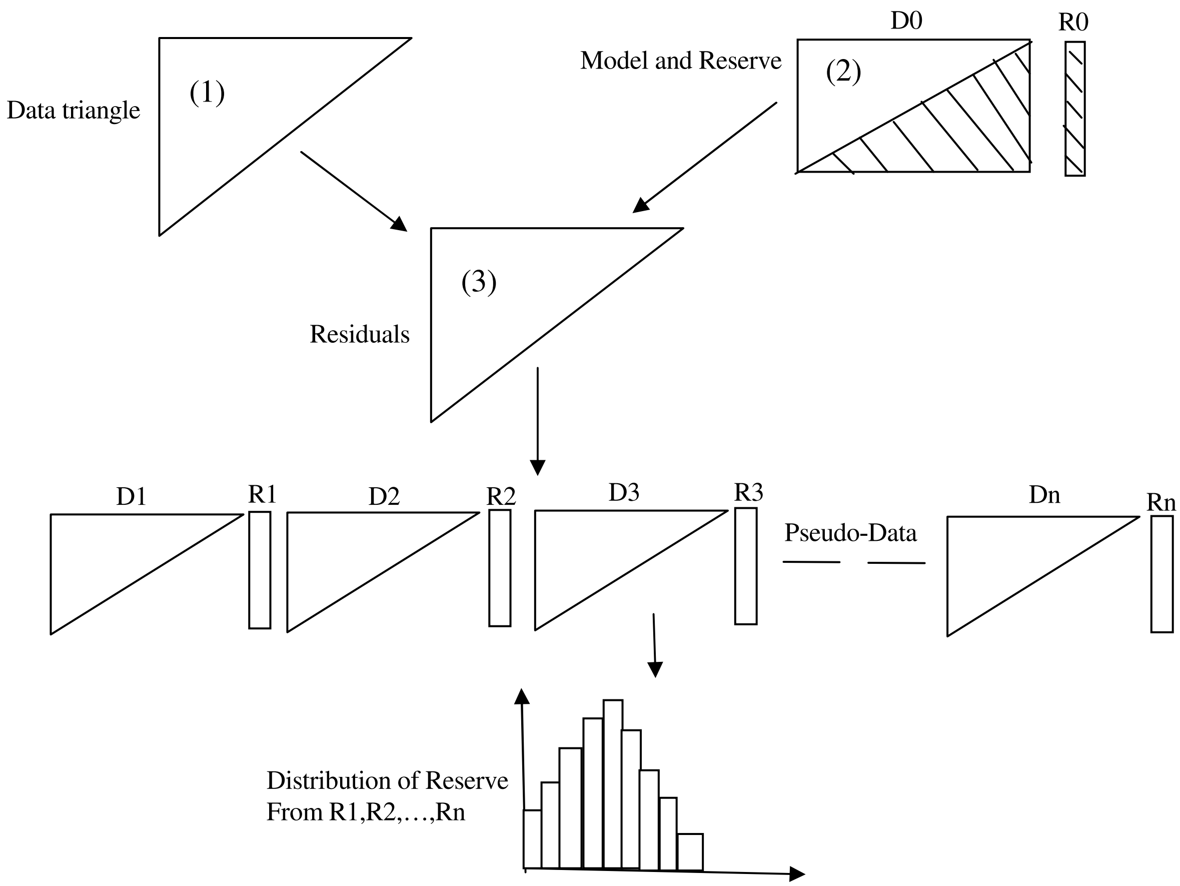
We have that the BE is the central estimate (discounted). The MVM is decided by the actors of the market, say SFSA co-operated with the Swedish insurance companies in Sweden. This is said to be on a fair value basis when we have a market to follow. Since we do not have any market to follow when we write this paper we do as follows. That is to have the central estimate (undiscounted) as a BE and then increase this estimate with the 75th percentile and say that this is on a fair value basis.

4 Description of the two methods

In this section, we will describe the two methods that we have used. There will only be a short description of each method since this is not the main purpose of the paper. If there is something that would interest the reader more, for example more thorough proofs, then we refer to the references at the end of this report.

4.1 Bootstrap simulation

The bootstrap has proved to be a very useful tool to assess the variability of the claim reserving predictions and to construct upper limits at an "adequate" confidence level. The method replaces analytic calculations of distribution and prediction error among parameters and estimates with an algorithm of simulation. This algorithm takes a given data sample and resample to new data sample through random selection with replacement**.** We will now illustrate this with a diagram below (ref [4]).



The basic data triangle ((1) in the diagram above) is taken and a reserving model is fitted to it. In the case of the chain ladder which we assume, this is that each accident year has its own "level" (of ultimate claims) and that there is then a development pattern that is constant across all accident years. This model not only projects future payments and hence allows one to make reserve estimates, it also produces a fitted model for the past data too ((2) in the diagram above).

The difference between (1), the actual data, and (2), the fitted data, gives us a set of residuals (shown as (3) in the diagram above). These residuals are a representation of how the real data and the model may differ. In other words, if the data are just a realization of some random process, another realization of the process could lead to another set of data that differs at any point from the model by any one of the set of residuals. The trick is then to produce many of these other possible realizations of the data by re-sampling the residuals and adding them to the fitted model to produce many sets of possible data triangles, known as pseudo-data (ref [12]). For each triangle produced, the reserving method is run, so that a series of reserve estimates is produced - pseudo-reserves. This is done many times say N, in this report is N=1000. We will now have a large collection of pseudo-reserves, which will have a certain distribution - that is they will have an expected size, and will vary around that expected size by certain amounts, which can be measured. It is the variation of these pseudo-reserves, which gives us a measure of the variability of the reserve estimates such as the prediction error. Since the bootstrap makes repeated use of the incremental data triangle, it is usually referred to as data re-sampling or data re-cycling (ref [23]).

In regression bootstrap, there are two possible techniques:

- Paired bootstrap - The resampling is done directly from the observations, which are assumed independent and identically distributed (iid).
- Residuals bootstrap - The resampling is applied to the residuals in the model and data are assumed independent but *not* identically distributed. However, the residuals are approximately iid.

In regression type problems, it is common to bootstrap the residuals. So in claim reserving only the latter method will be used, given the dependence between some observations and parameter estimates.

What we are interested in when we do a bootstrap is the incremental version of the fitted data. We are going to bootstrap based on the residuals between the incremental fitted payments and the incremental actual payments. We could bootstrap based on the difference between the cumulative actual data and the fitted data, but to justify inferring results from the bootstrapped reserves, we need to assume that all the residuals are independent. This is unlikely the case for cumulative data.

Before we start to explain the bootstrap in detail, we need some knowledge about Over-dispersed Poisson (OdP) and Generalized Linear Models (GLM) since the bootstrap simulation is built on this terminology. A summary is given in the appendices A.2.3 and A.2.4.

4.1.1 The bootstrap different steps

To implement a bootstrap analysis we need to choose a model, to define "adequate" residuals. An often suggested model is to assume that the incremental claims D_{ij} are distributed as independent OdP random variables with a logarithmic link function (η_{ij}), see England & Verall

(1999) and Appendix A.2.3. The OdP differs from the Poisson distribution in that the variance is not equal to the mean, but is proportional to the mean with a scale parameter ø. We have that

(4.1) $E[D_{ij}] = \mu_{ij}$ and $Var(D_{ij}) = \phi \nu(\mu_{ij}) = \phi \mu_{ij}^{\rho}$
where $\rho = 1$ and $\phi = 1$ in the OdP case.

(4.2) $\log(\mu_{ij}) = \eta_{ij}$

(4.3) $\eta_{ij} = d + \alpha_i + \beta_j, \qquad \alpha_1 = \beta_1 = 0$

i.e. equations (4.1-4.3) define a OdP (when the power $\rho = 1$) in which the response D_{ij} is modelled with a logarithmic link function (η_{ij}) and the variance is proportional to the mean. We use the logarithmic link function to re-parameterize the model so that the mean has a linear form instead of a multiplicative form (for a deviation of the re-parameterize see Appendix A.2.4). We can see that this structure is of a Chain Ladder type, in the sense that there is a parameter for each row i, and a parameter for each column j. Over-dispersion is introduced through the scale parameter ø, which is unknown and estimated as a part of the fitting procedure.

It should be noted that this model is robust for a small number of negative incremental claims, since the responses are the incremental claims themselves, see England & Verrall (1999). It is necessary to impose the restriction that the sum of incremental claims in every row and every column of the data triangle must be positive. Because of the logarithmic link function, fitted values are always positive. This usually makes the model unsuitable for use with incurred claims, which often include overestimates of case reserves in the early stages of development leading to a series of negative incremental incurred claims in the later stages of development.

Now we define the form of residual suitable for GLM:

unscaled Pearson residual non standardized (for standardized residuals see ref [19])

(4.4) $r_{ij}^{p} = \dfrac{d_{ij} - \hat{\mu}_{ij}}{\sqrt{\nu(\hat{\mu}_{ij})}} = \left\{\nu(\hat{\mu}_{ij}) = \text{variance function}\right\} = \dfrac{d_{ij} - \hat{\mu}_{ij}}{\sqrt{\hat{\mu}_{ij}^{\rho}}}$

where $\hat{\mu}_{ij}$ is the fitted incremental claims given by equation (4.1-4.3).

The scale parameter is estimated by

(4.5) $\hat{\phi}^{(p)} = \dfrac{\sum_{i+j \le m+1} r_{ij}^{(p)\,2}}{df}$

where df=n-p is the *degrees of freedom* in the model, n is the number of data points and p is the number of parameters estimated.

From (4.4) we get that d_{ij} is

(4.6) $d_{ij}^{*} = r_{ij}^{p} \sqrt{\hat{\mu}_{ij}} + \hat{\mu}_{ij}$

The bootstrap process starts with creating n residuals from our random sample d^* (development triangle). Then we resample with replacement from the residuals and we get a new pseudo triangle $\vec{d}^*$ by developing (4.6) to

(4.6') $\vec{d}_{ij}^* = r_{i'j'}\sqrt{\hat{\mu}_{ij}} + \hat{\mu}_{ij}$

where $r_{i'j'}$ mean that the residual is chosen randomly.

We tried to adjust a normal distribution with power $\rho = 0$ and $\phi = \sigma^2$ to the incurred claims triangles to overcome the smoothing of the data. The normal distribution does not have the same restrictions as the OdP. The prediction error (PE) got really high. So for the incurred triangles we had to smooth the data by letting the negative payments equal zero and then use the OdP. We use the OdP for both cumulative- and incurred data.

4.1.2 Prediction error (PE)

The idea with the PE is to capture on one hand the variation in the parameter (process) estimates and on the other hand the variation of the future. We have that the Prediction variance = Estimation variance + Process variance (ref [7]).

The mean square error of prediction is defined as

$$E\left[(y-\hat{y})^2\right] = E\left\{\left((y-E[y]) - (\hat{y}-E[y])\right)^2\right\} \approx E\left\{\left((y-E[y]) - (\hat{y}-E[\hat{y}])\right)^2\right\} =$$

$$= E\left[\left(y-E[y]\right)^2\right] - 2E\left[(y-E[y])(\hat{y}-E[\hat{y}])\right] + E\left[\left(\hat{y}-E[\hat{y}]\right)^2\right] \approx$$

$$= \left\{\begin{array}{l} \textit{Stochastic independence i.e. future observations are independent of past observations} \\ \Rightarrow \textit{the covariance vanishes} \end{array}\right\} =$$

4.7 $$\approx \underbrace{E\left[\left(y-E[y]\right)^2\right]}_{\textit{process variance}} + \underbrace{E\left[\left(\hat{y}-E[\hat{y}]\right)^2\right]}_{\textit{estimation variance}}$$

The prediction error is defined as

4.8 $PE = \sqrt{\textit{process variance} + \textit{estimation variance}}$

where the process variance reflects the noise inherent in the process. The estimation variance represents the uncertainty in the parameter estimates in the underlying model (ref [23]).

The process variance is defined as (see equation 4.1 above)

4.9 $Var(D_{ij}) = \phi\mu_{ij}^{\rho}$

The standard error of the reserve estimates are the square root of the estimation variance (ref [7]). To sum up

4.10 $\begin{cases} \text{Process variance} = \hat{\phi}^p \hat{\mu}_{ij} = \hat{\phi}^p R \\ \text{Estimation variance} = (SE_{bs}(R))^2 \end{cases}$

To obtain the bootstrap prediction errors of the reserves estimates, it is necessary to repeat the process a large number of times N (in this paper N=1000), each time creating a new bootstrap sample, and obtaining chain ladder reserve estimates. It is necessary to take account of the number of parameters used in fitting the model, to enable a proper comparison between the bootstrap PE's against other models PE's. The appropriate adjustment is to multiply the bootstrap estimation variance by n/(n-p). We have in the bootstrap sample that the PE error is the standard deviation of the bootstrap statistics (ref [6]). We get the following formula for the prediction error

(4.11) $PE^{bs}(R) = \sqrt{\underbrace{\hat{\phi}^{(p)} R}_{process\ variance} + \frac{n}{n-p} \underbrace{\left(SE^{bs}(R)\right)^2}_{estimation\ variance}}$

where R under the square root is an accident year total reserve and

(4.12) $SE^{bs}(R) = \sqrt{\frac{1}{N}\sum_{k=1}^{N}(\hat{R}^{bs} - \hat{R})^2}$

is the bootstrap standard error of the reserve estimate. $\hat{R}^{bs}$ is the bootstrap estimate, i.e. the standard errors are just the standard deviations of the N bootstrap reserves (ref [7]).

4.2 The distribution-free approach (the Thomas Mack approach)

The foundation of the distribution-free approach is the observation of three main assumptions, which are shown to underlie the traditional chain ladder technique. These are

(i) $\mathrm{E}[C_{i,k+1} \mid C_{i1},...,C_{ik}] = C_{ik} f_k \ ;\ 1 \le i \le I,\ 1 \le k \le I-1$

Because the chain ladder algorithm does not take into account any dependencies between accident years, we can additionally assume that the variables C_{ik} of different accident years, i.e.

(ii) $\{C_{i1},...,C_{iI}\}, \{C_{j1},...,C_{jI}\}\ ;\ i \ne j$, are independent

(iii) $\mathrm{Var}(C_{i,k+1} \mid C_{i1},...,C_{ik}) = C_{ik}\sigma_k^2\ ;\ 1 \le i \le I,\ 1 \le k \le I\text{-}1$ This is the variance assumption that is underlying the chain ladder method.

Where C_{ik} ($d_{ij} = c_{ij} - c_{i,j-1}$) denotes the cumulative total claims amount of accident year i up to development year k, either paid or incurred, f_k is the development factor from k to k+1 and σ_k are unknown parameters.

The aim is to estimate the ultimate claims amount C_{iI} and the outstanding claims reserve

(4.13) $R_i = C_{iI} - C_{i,I+1-i}$
for accident year i = 2,…,I.

The chain ladder method consists of estimating the f_k by

$$\textbf{(4.14)}\quad \hat{f}_k = \frac{\sum_{j=1}^{I-k} C_{j,k+1}}{\sum_{j=1}^{I-k} C_{jk}},\ 1 \le k \le I-1$$

which are simply the volume-weighted averages of the development factors in a particular column.

Thomas Mack shows in one of his papers (ref [13]) a corollary of assumption (iii) that the development factors are not correlated. That is, if we have a particularly high development factor in one period, there is no tendency for the subsequent factor to be particularly low (or high).

The chain ladder method also consists of estimating the ultimate claims amount C_{iI} by

$$\textbf{(4.15)}\quad \hat{C}_{iI} = C_{i,I+1-i} \cdot \hat{f}_{I+1-i} \cdot \ldots \cdot \hat{f}_{I-1}$$

Later on, we will need an estimator for σ_k^2. In the papers of Mack (1993) and (1994) (ref [13] and ref [14]), there is a derivation for σ_k^2 and it is shown that

$$\textbf{(4.16)}\quad \sigma_k^2 = \frac{1}{I-k-1}\sum_{i=1}^{I-k} C_{ik}\left(\frac{C_{i,k+1}}{C_{ik}} - \hat{f}_k\right)^2,\ 1 \le k \le I-2$$

is an unbiased estimator of σ_k^2, $1 \le k \le I-2$. An estimate is needed for σ_{I-1}. Mack suggests that if $\hat{f}_{I-1} = 1$ and if the claims development is believed to be finished after I-1 years, we can put $\hat{\sigma}_{I-1} = 0$. If not, he suggests we extrapolate the usually exponentially decreasing series $\hat{\sigma}_1, \ldots, \hat{\sigma}_{I-3}, \hat{\sigma}_{I-2}$ by one additional member, just by requiring that

$$\frac{\hat{\sigma}_{I-3}}{\hat{\sigma}_{I-2}} = \frac{\hat{\sigma}_{I-2}}{\hat{\sigma}_{I-1}}$$

This holds at least as long as $\hat{\sigma}_{I-3} > \hat{\sigma}_{I-2}$. This last possibility leads to

$$\textbf{(4.17)}\quad \sigma_{I-1}^2 = \mathrm{Min}\left(\frac{\hat{\sigma}_{I-2}^4}{\hat{\sigma}_{I-3}^2}, \mathrm{Min}\left(\hat{\sigma}_{I-3}^2, \hat{\sigma}_{I-2}^2\right)\right)$$

This problem does not exist if one is willing to assume that the data presented are fully mature, thus leading one to conclude no variance in the last factor or so.
The mean squared error of prediction $\mathrm{MSEP}(\hat{C}_{iI})$ of the estimator $\hat{C}_{iI}$ of C_{iI} is defined to be

$$\mathrm{MSEP}(\hat{\mathrm{C}}_{\mathrm{iI}}) = E[(\hat{\mathrm{C}}_{\mathrm{iI}} - \mathrm{C}_{\mathrm{iI}})^2 \mid \mathrm{D}]$$

where $\mathrm{D} = \{\mathrm{C}_{\mathrm{ik}} \mid i + k \le I + 1\}$ is the set of all data observed so far. We see that

$$\mathrm{MSEP}(\hat{\mathrm{R}}_{\mathrm{i}}) = \mathrm{E}[(\hat{\mathrm{R}}_{\mathrm{i}} - \mathrm{R}_{\mathrm{i}})^2 \mid D] = \mathrm{E}[(\hat{\mathrm{C}}_{\mathrm{iI}} - \mathrm{C}_{\mathrm{iI}})^2 \mid D] = MSEP(\hat{\mathrm{C}}_{\mathrm{iI}}).$$

Since we have the general rule of $\mathrm{E}[\mathrm{X} - \mathrm{a}]^2 = \mathrm{Var}(\mathrm{X}) + (\mathrm{E}[\mathrm{X}] - \mathrm{a})^2$, we get the following

$$\mathrm{MSEP}(\hat{\mathrm{C}}_{\mathrm{iI}}) = \mathrm{Var}(\mathrm{C}_{\mathrm{iI}} \mid \mathrm{D}) + (\mathrm{E}[\mathrm{C}_{\mathrm{iI}} \mid \mathrm{D}] - \hat{\mathrm{C}}_{\mathrm{iI}})^2$$

which shows that the mean squared error of prediction is the sum of the stochastic/statistical error (process variance) and the estimation variance. In other words Prediction variance = Process variance + Estimation variance the same as for the bootstrap (see chapter 4.1.2). We get that (see ref [13])

Process variance = $\hat{\mathrm{C}}_{\mathrm{iI}}^2 \sum_{k=I+1-i}^{I-1} \frac{\hat{\sigma}_{k+1}^2}{\hat{\mathrm{f}}_{\mathrm{k+1}}^2 \hat{\mathrm{C}}_{\mathrm{ik}}}$

Estimation variance = $\hat{\mathrm{C}}_{\mathrm{iI}}^2 \sum_{k=I+1-i}^{I-1} \frac{\hat{\sigma}_{k+1}^2}{\hat{\mathrm{f}}_{\mathrm{k+1}}^2 \sum_{q=1}^{I-k} \mathrm{C}_{\mathrm{qk}}}$

We have that

(4.18) $$\hat{\mathrm{MSEP}}(\hat{\mathrm{R}}_{\mathrm{i}}) = \hat{\mathrm{C}}_{\mathrm{iI}}^2 \sum_{k=I+1-i}^{I-1} \frac{\sigma_k^2}{\hat{\mathrm{f}}_{\mathrm{k}}^2} \left(\frac{1}{\hat{\mathrm{C}}_{\mathrm{ik}}} + \frac{1}{\sum_{j=1}^{I-k} \mathrm{C}_{\mathrm{jk}}} \right)$$

The square root $\mathrm{PE}(\hat{\mathrm{R}}_{\mathrm{i}})$ of an estimator of the mean squared error of prediction is defined to be the prediction error of $\hat{R}_i$. The prediction error is the standard deviation of the distribution of reserve estimates the same as for the Bootstrap.

Often the prediction error of the overall reserve estimate $\hat{R} = \hat{R}_2 + ... + \hat{R}_I$ is of interest. In this case we cannot simply add together the values of $(PE(\hat{R}_i))^2$, $2 \le i \le I$, because they are correlated via the common estimators $\hat{f}_k$ and $\hat{\sigma}_k$.

For the overall reserve MSEP we have the following formula

(4.19) $$MSEP(\hat{R}) = \sum_{i=2}^{I} \left\{ (s.e.(\hat{R}_i))^2 + \hat{C}_{iI} \left(\sum_{j=i+1}^{I} \hat{C}_{jI} \right) \sum_{k=I+1-i}^{I-1} \frac{2\hat{\sigma}_k^2 / \hat{f}_k^2}{\sum_{n=1}^{I-k} C_{nk}} \right\}$$

and the square root of this formula give us the prediction error for the overall reserve.
For more specific derivation of the formulas above, see ref [13].

5 Results applied on the data from If P&C Insurance Ltd

In this part, we show the overall result for gross paid/incurred losses for the three portfolios, Private Property, Motor TPL and Liability. It should be noted that in the incurred losses for bootstrap we had to "smooth" the data, which means that if we had negative increments in the development triangle we let them equal zero.

The natural goal for any reserving system is of course to estimate the size of the total reserve. The average reserves (i.e. the reserves for each accident year) are in practice just tools in order to make this estimation. In this section, we will only show the overall figures except for the example triangle in chapter 5.2. To see the average figures we refer to Appendix A.3. We start with a short note on how to calculate the prediction intervals.

5.1 Prediction interval

The interpretation of a prediction interval is "A $100(1-\alpha)$ per cent prediction interval is constructed according to a method such that $100(1-\alpha)$ per cent of all the prediction intervals contain the true value of the population parameter and α per cent of all intervals do not contain the true value of the population parameter" (ref [11]).

Up to now we only have estimates R_i and $PE(R_i)$ for the mean and the standard deviation of this distribution. If the volume of the outstanding claims is large enough we can, due to the central limit theorem, assume that this distribution function is a Normal distribution (see the Appendix for definition of the Normal distribution) with an expected value equal to the point estimate given by R_i and a standard deviation equal to the prediction error $PE(R_i)$.

Knowing that the volume of the outstanding claims is sufficiently large enough can be difficult. The more non-symmetric the distribution is the larger volume is needed to make the distribution approximately normally distributed. Therefore, the prediction level will be stated as just approximately.

A symmetric 75%-prediction interval for R_i is given by

(5.1) $\left(R_i - 0.674 * PE(R_i); R_i + 0,674 * PE(R_i)\right)$

Sometimes the distribution is rather skewed and then we use the log-normal distribution. Since the Normal distribution may not be a good approximation to the true distribution of R_i .

A log-normal 75%-prediction interval for R_i is given by

(5.2) $\left(R_i * \exp\left\{ -0.674 * \sqrt{\sigma_i^2} - \frac{\sigma_i^2}{2} \right\}; R_i * \exp\left\{ 0.674 * \sqrt{\sigma_i^2} - \frac{\sigma_i^2}{2} \right\} \right)$

To get to this result we approximate the unknown distribution of R_i by a log-normal distribution with parameters μ_i and σ_i^2 (see the Appendix for definition of the log-normal distribution) such that mean values as well as variances of both distributions are equal i.e. such that (ref [14])

(5.3) $\exp\left\{ \mu_i + \frac{\sigma_i^2}{2} \right\} = R_i$

(5.4) $\exp\left\{ 2\mu_i + \sigma_i^2 \right\} \exp\left\{ (\sigma_i^2) - 1 \right\} = \left(PE(R_i) \right)^2$

This leads to

(5.5) $\sigma_i^2 = \ln\left(1 + \frac{(PE(R_i))^2}{R_i^{\ 2}} \right)$

(5.6) $\mu_i = \ln\left(R_i \right) - \frac{\sigma_i^2}{2}$

To calculate the log-normal prediction interval we need first to calculate 75% prediction limit for the overall outstanding claims reserve R. After that we allocate this overall amount to accident year i=1987,…,2004 in such a way that we reach the same level prediction for every accident year. Each level of prediction corresponds to a certain percentile t of the standard normal distribution. We therefore only have to choose t in such a way that (ref [14])

(5.7) $\sum_{i=1}^{I} R_i * \exp\left(t\sigma_i - \frac{\sigma_i^2}{2} \right)$ = total upper prediction limit/total lower prediction limit

This can easily be done in Excel with the "target seeker". We will only show the overall 75th percentile in the tables below since they are of interest to get the risk margins for each portfolio.

5.2 Procedure

Let us insert the table for portfolio Private Property for Mack, just to exemplify the calculations that we use (Table 5.1). To get a more lucid table we only show the figures from accident year 1995 and forward. The calculations are the same for the other portfolios except that the formulas are different between Mack and Bootstrap. For the Mack approach, we use the formulas in chapter 4.2 and for the bootstrap approach, we use the formulas in chapter 4.1. To see the other portfolios average figures go to Appendix A.3.

Table 5.1: Portfolio Private Property (house owner, home owner and holiday cottage)

Paid data	currency:a-mark							
(Mack)	(1)	(2)	(3)	(4)	(5)	(6)		
Accident Year	*Paid Losses to Date*	*Loss Development Factor*	*Ultimate Losses*	*Total Reserves*	*Prediction Error of Reserves*	*Ratio of Prediction Error to Expected Reserves*		
1995	1 440 265	1,003	1 443 897	3 632	3 716	102%		
1996	1 632 105	1,003	1 637 564	5 459	4 067	74%		
1997	1 710 042	1,005	1 719 421	9 379	6 339	68%		
1998	1 633 819	1,007	1 645 451	11 631	6 516	56%		
1999	1 762 682	1,011	1 781 980	19 299	7 140	37%		
2000	1 735 297	1,016	1 763 514	28 217	7 961	28%		
2001	1 830 360	1,026	1 878 432	48 073	9 436	20%		
2002	1 874 795	1,045	1 959 824	85 029	13 131	15%		
2003	1 791 654	1,095	1 961 883	170 229	24 863	15%	**25th percentile:**	**1 064 238**
2004	1 030 655	1,730	1 782 527	751 872	96 798	13%	**75th percentile:**	**1 205 775**
Total	26 188 268		27 325 929	**1 137 661**	105 529	**9%**	**Risk margin:**	**6%**

(1) Is the last diagonal from the development triangle (appendix A.1, figure 1).
(2) Calculated with formula **(4.14)** from chapter **4.2**.
(3) = (1) * (2)
(4) = (3) - (1), also called the central estimate.
(5) In this model is the prediction error an estimate for the standard deviation of the outstanding claims reserve.
Calculated with formula **(4.18)** and for the overall reserve with formula **(4.19)** both from chapter **4.2**.
(6) = (5) / (4) = coefficient of variation = CV
We calculate the 75th percentile with a log-normal distribution.

We begin by estimating the central estimate of outstanding claims liabilities for each portfolio analysed using both methods. The estimates are based on the Chain Ladder valuation approach applied to gross paid claims and to gross incurred claims (see (4) in Table 5.1).

Then we estimate the prediction error, or uncertainty, in the central estimates of the outstanding claims liabilities, for each portfolio. We use two methods, "the distribution-free approach", developed by Thomas Mack (1993) (in the tables reduced to Mack) and the bootstrap method. Both methods are described earlier in the paper. For an example how this can look like see column (5) in Table 5.1.

Whilst we believe that these methods produce reliable results, it is possible that the use of other methods could produce significantly different results. To estimate the 75th percentile of the gross outstanding claims liability (i.e. as required by the APRA standard), we need to follow some sort of distribution. For the model developed by Thomas Mack, we need to choose a distribution by looking at the prediction errors. For the bootstrap, we do a best curve fitting using some statistical program (in this report we use @Risk, an ad-in for Excel). @Risk gives us a distribution that fits the data best for each portfolio, since we have 1000 simulated reserves. The most commonly used distributions in stochastic loss reserving are the normal and the log-normal distribution, see Appendix A.2.1 and A.2.2.

For the model of Mack, it is difficult to decide which distribution that fits the data best. It is individual from data to data. There is not any "rule of thumb", as mentioned before we need to look at the data, especially on the ratio $PE(R_i)/R_i$, and then decide which distribution to use. The log-normal distribution is the most common to use according to Mack (ref [14]). Often the ratio $PE(R_i)/R_i$ is very high and therefore is the log-normal distribution a much better fit to the data than the Normal distribution since the data are skewed. For the distribution-free method, we use the log-normal distribution.

The uncertainty in the reserve estimates can vary considerably depending on the maturity and the stability of a company's data. One reason why we calculate these margins is to be on the safe side with volatile portfolios with a high level of uncertainty. If there is a very volatile portfolio it is good to add a contingency margin to the reserves.

The ratio between the **overall** 75th percentile and the **overall** reserve estimate is called the risk margin (see the grey figure in Table 5.1 above). This figure is a way toward fair value or “fair value like” measurement. The discussion on what will be “fair value like” is not finalized yet. The new standards will probably be decided in connection with Solvency II and the IFRS process and this will probably take a couple of years (as mentioned in chapter 3.3).

It would perhaps be interesting to see the risk margin for all LoB in an overall figure. This is under discussion but there is no good conclusion on how this will be done yet. We cannot add all the overall reserves for every LoB together and than take a prediction error from that and get a risk margin for all LoB together. This process is the same as assuming 100% correlation between lines. Generally, there is some level of independencies between lines (i.e. less than 100% correlation). Further research is needed to develop additional formulas for calculating the covariances between LoB (ref [21]). We cannot get an overall figure for all LoB together today but we can get an overall figure for every LoB, see the tables below.

5.2.1 Comparison of the results between the two methods (Mack and Bootstrap)

We will now show a comparison of the results for the three portfolios in the following two tables, the figures are in a-mark. We start with total figures for gross paid claims (Table 5.2) with enclosed comments and the same for gross incurred claims (Table 5.3). Note that we use “smooth” data for the incurred loss in the bootstrap.

Table 5.2: Total figures for respective method and portfolio (paid data)

Mack

Portfolio	25th percentile	Reserve	75th percentile	75th percentile (%)
Private Property	1 064 238	1 137 661	1 205 775	6%
Motor TPL	8 895 634	9 269 913	9 626 868	4%
Liability	3 504 661	4 175 994	4 734 569	13%

Bootstrap (SMO)

Portfolio	25th percentile	Reserve	75th percentile	75th percentile (%)
Private Property	1 080 113	1 137 661	1 195 209	5%
Motor TPL	8 876 824	9 269 913	9 663 001	4%
Liability	3 584 941	4 175 994	4 678 623	12%

Prediction error (CV)

Portfolio	Mack	Bootstrap
Private Property	9%	7%
Motor TPL	6%	6%
Liability	23%	20%

Comparing the two methods for each portfolio, looking at the total reserve figures, we can say that they produce broadly identical results. This is not strange since both methods in the ground are built on the deterministic Chain Ladder technique.

In Mack we use a log-normal distribution to calculate the 75th percentile. In bootstrap we use @Risk to fit a distribution to the 1000 simulated reserves, see result below

Private Property	Normal
Motor TPL	Normal
Liability	Log-normal

For example we need according to portfolio Private Property in Mack 1.205.775 a-mark in order to have sufficient reserves at least 75% of the time. A loss reserve margin (risk margin) is an amount needed over and above the expected reserves to reflect the inherent "riskiness" of the reserves.

Notice that we get wider prediction interval for Mack than for Bootstrap except for Motor TPL. There is no general rule that log-normal distribution give wider prediction intervals. It depends on the percentile chosen and on the size of the ratio $PE(R_i)/R_i$. The log-normal approximation only prevents a negative lower confidence limit.

Note that the 75th percentile (in %) is reassuringly close between Mack and Bootstrap. We have that the risk margins overall for Mack and Bootstrap is not less than half of the CV for each portfolio, which are one of the standards in APRA (see chapter 3.1 in this paper).

Let us now present the results for the incurred data.

Table 5.3: Total figures for respective method and portfolio (incurred data)

Mack

Portfolio	25th percentile	Reserve	75th percentile	75th percentile (%)
Private Property	253 323	328 640	386 514	18%
Motor TPL	6 725 703	7 517 357	8 218 667	9%
Liability	944 902	1 265 406	1 504 609	19%

Bootstrap (SMO)

Portfolio	25th percentile	Reserve	75th percentile	75th percentile (%)
Private Property	355 263	405 761	449 540	11%
Motor TPL	7 269 912	7 876 452	8 482 991	8%
Liability	1 792 353	2 186 434	2 580 515	18%

Prediction error (CV)

Portfolio	Mack	Bootstrap (SMO)
Private Property	26%	18%
Motor TPL	15%	11%
Liability	29%	27%

We can say that the two methods for each portfolio produce broadly similar results, looking at the total reserve figures. The difference depends on that in the bootstrap case we use “smooth” data in the development triangles. We overestimate the figures in the bootstrap by putting the negative increments to zero in the development triangles. Since we have been tamper with the data, we need to be careful when we draw conclusion from this part.

In this case, we also do a best curve fitting in the bootstrap with the statistical program @Risk for the 1000 simulated reserves. The result is

Private Property	Log-normal
Motor TPL	Normal
Liability	Normal

The prediction intervals are wider with Mack (except for Liability) and we note that Mack gives us larger 75th percentiles (in %) than the bootstrap. The ratio between the 75th percentile and the reserve is lower in the bootstrap method. This depends on that we have overestimated the development triangles so that the reserves in the bootstrap get higher than the ones in Mack.

We have that the risk margins overall for Mack and Bootstrap is not less than half of the CV for each portfolio, which are one of the standards in APRA (see chapter 3.1 in this paper).

In the incurred case we can say that the distribution-free method (Mack) is a better fit to the data rather than the bootstrap. Since we have been tamper with the data in the bootstrap. It is difficult to say which of the two methods that is to prefer in the paid case. Both of the methods show fair results.

It is judgemental to choose a MVM for every LoB since we do not have any market to follow (see section 3.3). The data in this report does not differ much in the paid method (se table 5.2). We can take 4% to be MVM for Motor TPL for example. If the data would differ than a solution would be to take the figure that is in the middle for the two methods (the distribution-free and the Bootstrap) for the paid data. This gives us two “MVM’s” for every LoB (the distribution-free paid method, the bootstrap paid method) and the MVM we choose is the average of the two. We do not take the incurred data into the MVM calculations, since we have tampered with that data. If the incurred data would be correct then we get 4 “MVM’s” (the distribution-free paid method, the bootstrap paid method, the distribution-free incurred method and the bootstrap incurred method) and the MVM we will choose is the average of the four.

The MVM is a provision for risk and uncertainty in a specific portfolio (ref [15]). The market that we would follow would probably be set-up by the SFSA. This is still under discussion so this is only an assumption on how it will be.

We must bear in mind that the PE can only reflect the estimation error and the process (Stochastic/statistical) error, but not the model (specification) error (see chapter 4.1.2), i.e. the fact that the model chosen can be wrong or that the future development may not be in accordance with past experience.

In summary, both methods seem to show a good consensus of results compared to each other. Which model is best to use in order to obtain the best estimate for reserve uncertainty? Clearly, there is no single answer to this question. Different models will suit different problems or data sets. Under any circumstances, the data should be examined in detail in order to find an appropriate model, rather than using the same modelling approach in all circumstances. Reserving is a practical data analysis exercise and it is vital to try to understand and learn from the data rather than impose the same approach in all situations.

In general, we have that the stochastic methods described in this paper are better suited to paid data rather than incurred data. This is because case estimates are set individually and sometimes a little "conservatively", resulting in over-estimation when considered in aggregate, leading to negative incremental amounts in the later stages of development. One solution to this was to put all negative increments to zero (smooth data) (ref [19]). This "exercise" solution might make the results for the incurred method misleading. Therefore, be careful when you draw conclusions from the tables concerning incurred data in the bootstrap.

Since this approach on how to calculate fair value, MVM's and risk margins is still under construction. Take the results in this paper as a guideline for further investigation on how to calculate this different risk based margins. There will probably be more changes in this area until the IASB are ready to come forward with a finished set of rules and regulations.

5.3 Compare APRA's increase with the estimated increase from this paper

We will present a comparison between the different increases (risk margins) in this section, the ones that APRA is using (Table 5.4) with the ones calculated in this report (Table 5.5-5.8).

The figures that APRA presents in their report (ref [1] and ref [4]) are net (gross + reinsurance) figures. We believe that reinsurance has very little influence on our portfolios. So we can compare our gross results with APRA's net results. APRA recommends the following CV and risk margins for each portfolio.

Table 5.4: Summary of CV and Risk Margin for each portfolio using APRA's standards.

Class	CV (%)	APRA Risk Margin (%)
Private Property	around 13-14	around 9
Motor TPL	around 12-13	around 8
Liability	around 19	around 13

The risk margins presented in APRA's report (ref [1]) should be considered as guides only and representative of an "industry average" portfolio. This can lead to very different results when we compare the figures to each other. Since our portfolios is probably not any "industry average". The specific characteristics of an individual insurer's portfolio may result in significantly different liability distributions than presented.

Tables 5.5 and 5.6 show the risk margin and CV for each portfolio using the paid data and tables 5.7 and 5.8 show the risk margin and CV for each portfolio using the incurred data. We use the figures from table 5.2 (paid data) and 5.3 (incurred data).

Table 5.5: Summary of Risk Margin for each portfolio using the figures from this report.
(Paid data)

Class	MACK Risk Margin (%)	BOOTSTRAP Risk Margin (%)
Private Property	6	5
Motor TPL	4	4
Liability	13	12

Table 5.6: Summary of CV for each portfolio using the figures from this report.
(Paid data)

Class	MACK CV (%)	BOOTSTRAP CV (%)
Private Property	9	7
Motor TPL	6	6
Liability	23	20

Table 5.7: Summary of Risk Margin for each portfolio using the figures from this report.
(Incurred data)

Class	MACK Risk Margin (%)	BOOTSTRAP (smooth data) Risk Margin (%)
Private Property	18	11
Motor TPL	9	8
Liability	19	18

Table 5.8: Summary of CV for each portfolio using the figures from this report.
(Incurred data)

Class	MACK CV (%)	BOOTSTRAP (smooth data) CV (%)
Private Property	26	18
Motor TPL	15	11
Liability	29	27

We will in this section also have a table (Table 5.9) that shows what “MVM’s” we got with the calculations from this paper. See chapter 5.2.1 for the procedure for calculation of the MVM when there is no market to follow.

Table 5.9: Summary of choosen MVM for each portfolio using the figures from this report.

Class	MVM (%)
Private Property	5,5
Motor TPL	4
Liability	12,5

If we compare these “MVM’s” with the risk margins that APRA has compiled (se table 5.4) we get a lower result for each portfolio. These differences can depend on different reasons. One can be that APRA has an “industry average” portfolio (see section above in this chapter). Another reason can be that we have outliers in the development triangles or in the long-tailed portfolios Motor TPL and Liability we have not taken any concern for tail approximation (i.e. we do not have full history for the data. To get full history we have to do our own estimates by looking at the data and try to do a tail approximation).

6 Inflation-adjusted method

There is no requirement concerning discounted reserves, although that is very likely to be required in Solvency II. This is a heated discussion today and it is not decided how this is going to be.

At this point, having obtained estimates of future payments, estimates of reserves on a discounted basis can also be calculated. In practice, most companies choose not to discount outstanding payment liabilities, as this provides a useful, implicit margin against claims deteriorating significantly. Inflation is often taken into account implicitly (historical inflation reflects the future).

The basic chain ladder method does not explicitly allow for any calendar year effect such as inflation. The development factors calculated are based on payments from many different calendar periods. Past inflation is implicit in the factors derived, but it is not clear what assumption is actually made for future inflation, other than that it is a weighted average of past values.

In stating the model, it is essential to consider incremental, rather than cumulative data, so that the calendar year influence is applied to the correct year.

First, past inflation rates are required to adjust past claim payments to current monetary values, here we use Consumer Priceindex (KPI, KonsumentPrisIndex), which we get from Statistics Sweden (SCB, Statistiska CentralByrån). Then assumptions for future inflation rates are required, which are applied to the future incremental values, here we use the 2% inflation target from the Swedish central bank (Riksbanken).

One important rule to remember, when we choose the inflation rate and the interest rate, is that the inflation should be reflected in the cash flows, in a way that is consistent with the interest rates used for discounting.

If this small section is something that would interest the reader more, see (ref [16]).

7 Discussion and Conclusions

The data on which we have tested the models (distribution-free and Bootstrap) have been both paid losses and incurred losses. The models seem to work worse on the incurred data than on the paid data. The paid loss triangle is independent of claim adjusters or actuaries' opinions on reserves but it is sensitive to changes in payout patterns. The incurred loss triangle is independent of actuaries, but is sensitive to both changes in payout patterns and claim adjuster's case reserving practice. It happens that the claim adjuster is over-reserving or under-reserving. Over-reserving can happen when case reserves are set higher than actual future payments and the contrary with under-reserving. In an ideal world with perfect case reserving, this would not happen, because, when reported, case reserves would be set at exactly the future paid amounts. The inference is that actual incurred amounts have errors, because the case reserves cannot be set with perfect foresight. We should be careful when we draw conclusions from the incurred data since this data can be over-estimated/under-estimated.

It is interesting to note that the prediction errors of the reserves totals and the risk margins for the two approaches are reassuringly close. It is not always like this and care must be taken in making inferences from the results. In particular, the accuracy and interpretation of accident year prediction errors needs careful consideration. Clearly, it is not appropriate to consider approximate 75% prediction intervals when the prediction error is a large percentage of the reserve estimate. It is best to use the accident year prediction error as a crude means of assessing confidence in the reserve estimates.

The prediction intervals computed from the forecast distribution are conditional on the assumptions about the future remaining true. This is a very big problem since it is difficult to predict the future.

The background of choosing the distribution-free method and the bootstrap method is because both of these methods are very simple to work with and they are the two main alternatives on the market today. You only have to set up the methods in for example Matlab or Excel. The bootstrap procedure is in practice more expedient than the distribution-free approach because the bootstrap does not require the summation of a large collection of terms.

What we have to have in mind is that both the bootstrap and the distribution-free method are built on the assumptions from the deterministic Chain Ladder. One well-known weak point of the Chain Ladder method is that the estimators of the last two or three factors f_I, f_{I-1}, f_{I-2} rely on very few observations and the fact that the known claims amount C_{I1} of the last accident year forms a very uncertain basis for the projection to ultimate. A good idea would be to have another method tried that does not build on the Chain Ladder assumptions.

We have calculated risk margins and MVM's since increasing concern about the solvency issues makes it imperative that actuaries draw quantitative as well as qualitative conclusions about the sufficiency of loss reserves to cover future losses. This is a problem since it is difficult to decide for example what underlying distribution is the best fit for the data. We have only been trying the log-normal distribution in Mack, maybe another skewed distribution than the log-normal for example the Gamma would be a better fit. To try different distributions is time-consuming for a company. This whole approach with risk margins, MVM and discount will take more time than the approach that we are practising today in Swedish non-life insurance companies. However, if the direction of these standards will get really plain and the different definitions well explained then this might be a way to go. It is a very good idea to see if a company has "solvent" reserves.

This new approach will raise a number of challenging issues, but it will probably also make the accounts more understandable for the ones that are not actuaries since they get a figure on the uncertainty or the risk with the specific portfolio. When the new standards are finalized, the profession will need to develop practical approaches to doing the required reserve analyses. There is still a long way to go in this area.

The fair value of a liability consists of the estimated reserve (in the future discounted for the time value of money), and a risk adjustment. In this paper, we only present the reserve as undiscounted since the standards on how to discount and how to present it at fair value with market-based assumptions is unclear. It is done in the same way when these assumptions are clarified. Therefore, in the future one only has to change the estimated reserve to a discounted reserve with market-based assumptions that the MVM is build on. There obviously remains much yet to be done.

Let us now summarize the different terms that we have used in this paper. We first calculated our loss reserve which we call our central estimate. If we put on a 75th risk margin to this we get a best estimate. The MVM which was an average between the two methods (bootstrap/distribution-free) and the two data sets (paid/incurred) will reflect the margin for risk and uncertainty for each LoB. This is said to be on a fair value basis undiscounted. We might consider that any carried reserves above the excepted value to be "reasonable". The range from the expected value to 75% could be "reasonable and prudent/sufficient" and the range above 75% could be "reasonable and conservative".

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A Appendix

A.1 Data triangles

Where data triangles 1, 2,…,6 are on cumulative form (c_{ij}) i.e. what is really paid out until year j.

	Development year																	
Acc. Year	År0	År1	År2	År3	År4	År5	År6	År7	År8	År9	År10	År11	År12	År13	År14	År15	År16	År17
1987	592 793	881 200	924 999	949 580	963 023	969 830	974 477	975 373	977 224	978 122	978 139	978 496	979 332	979 355	979 355	979 355	979 604	979 604
1988	544 208	908 058	953 695	971 340	977 435	984 518	986 485	988 870	989 172	990 176	990 430	990 965	991 941	992 010	992 010	992 010	992 658	
1989	570 296	966 028	1 011 185	1 025 904	1 041 333	1 049 498	1 056 052	1 054 298	1 055 443	1 056 010	1 056 107	1 056 319	1 054 135	1 054 135	1 054 150	1 054 150		
1990	792 248	1 226 719	1 268 053	1 288 054	1 299 226	1 312 329	1 314 244	1 315 009	1 316 239	1 316 546	1 316 911	1 316 911	1 317 039	1 317 094	1 317 095			
1991	876 351	1 317 064	1 359 785	1 377 939	1 387 972	1 394 756	1 399 552	1 400 938	1 401 415	1 402 467	1 404 573	1 407 454	1 407 642	1 407 976				
1992	825 600	1 206 753	1 254 469	1 271 535	1 280 314	1 287 374	1 290 681	1 293 433	1 295 268	1 295 765	1 296 397	1 296 089	1 296 517					
1993	827 097	1 203 613	1 244 882	1 262 068	1 274 940	1 281 721	1 288 166	1 292 291	1 294 498	1 296 118	1 304 602	1 305 772						
1994	868 649	1 269 599	1 329 030	1 350 565	1 359 329	1 365 583	1 372 577	1 375 153	1 389 246	1 391 698	1 392 821							
1995	859 379	1 335 354	1 389 068	1 411 986	1 423 177	1 430 500	1 435 092	1 438 029	1 439 503	1 440 265								
1996	985 842	1 520 041	1 577 759	1 603 233	1 616 638	1 621 624	1 627 557	1 629 502	1 632 105									
1997	1 028 683	1 588 141	1 644 946	1 674 532	1 695 048	1 697 388	1 704 568	1 710 042										
1998	900 203	1 468 753	1 555 012	1 604 531	1 620 378	1 627 222	1 633 819											
1999	933 927	1 590 070	1 693 195	1 731 626	1 753 252	1 762 682												
2000	912 799	1 565 314	1 680 695	1 715 101	1 735 297													
2001	1 026 116	1 700 024	1 796 844	1 830 360														
2002	1 136 370	1 788 668	1 874 795															
2003	1 088 606	1 791 654																
2004	1 030 655																	

Figure 1: Development triangle (Paid claims). LoB Private Property. Currency a-mark.

	Development year																	
Acc. Year	År0	År1	År2	År3	År4	År5	År6	År7	År8	År9	År10	År11	År12	År13	År14	År15	År16	År17
1987	673 063	1 164 752	1 278 793	1 345 847	1 412 297	1 485 786	1 548 904	1 602 774	1 663 673	1 739 629	1 762 956	1 776 977	1 797 374	1 845 339	1 911 331	1 932 377	1 964 540	1 988 129
1988	755 919	1 351 781	1 491 679	1 586 522	1 661 176	1 715 522	1 791 443	1 823 572	1 917 596	1 982 438	2 027 480	2 091 013	2 144 981	2 168 023	2 247 651	2 271 349	2 294 543	
1989	790 574	1 484 064	1 628 685	1 753 698	1 821 415	1 888 796	1 954 126	2 048 619	2 119 339	2 209 771	2 253 231	2 319 371	2 389 097	2 418 630	2 458 333	2 498 755		
1990	930 291	1 556 529	1 724 017	1 819 354	1 921 473	1 972 228	2 069 104	2 210 495	2 300 188	2 354 205	2 394 869	2 471 034	2 573 759	2 605 852	2 632 775			
1991	906 157	1 507 255	1 672 255	1 787 307	1 871 079	1 929 454	1 990 463	2 069 980	2 132 745	2 234 061	2 309 806	2 403 897	2 447 440	2 511 339				
1992	727 591	1 170 747	1 295 602	1 373 802	1 451 386	1 494 445	1 581 465	1 660 484	1 702 081	1 808 043	1 851 930	1 901 076	1 964 995					
1993	611 610	994 733	1 104 742	1 207 988	1 291 203	1 353 598	1 407 596	1 492 742	1 557 076	1 700 146	1 740 624	1 766 283						
1994	617 752	1 000 405	1 127 207	1 221 654	1 302 676	1 364 567	1 481 496	1 542 311	1 605 539	1 716 257	1 783 747							
1995	588 493	1 015 620	1 148 195	1 258 436	1 349 224	1 440 759	1 562 639	1 643 954	1 716 262	1 813 423								
1996	570 433	983 910	1 134 063	1 237 805	1 324 095	1 421 843	1 502 402	1 573 292	1 669 333									
1997	611 166	1 026 096	1 189 702	1 287 393	1 382 844	1 475 139	1 553 575	1 687 717										
1998	638 079	1 105 404	1 242 794	1 342 450	1 432 413	1 500 968	1 651 144											
1999	662 501	1 123 256	1 288 096	1 420 148	1 522 918	1 673 786												
2000	657 909	1 189 991	1 355 593	1 472 612	1 591 819													
2001	722 724	1 312 752	1 490 903	1 641 956														
2002	750 637	1 305 774	1 498 335															
2003	790 443	1 344 130																
2004	845 054																	

Figure 2: Development triangle (Paid claims). LoB Motor TPL (third party liability). Currency a-mark.

	Development year																	
Acc. Year	År0	År1	År2	År3	År4	År5	År6	År7	År8	År9	År10	År11	År12	År13	År14	År15	År16	År17
1987	39 804	70 467	104 095	141 563	164 496	175 375	185 290	201 221	395 563	390 895	401 572	423 531	427 630	440 111	443 902	458 793	461 701	462 896
1988	29 446	93 530	135 965	176 343	193 863	211 489	247 722	263 721	268 106	287 634	306 500	321 597	328 677	330 066	331 850	332 270	332 910	
1989	23 424	87 108	149 687	185 244	213 792	230 853	230 797	234 341	280 734	398 171	401 871	404 235	408 363	412 403	422 490	424 455		
1990	10 874	108 059	190 807	224 771	264 986	288 045	293 388	321 233	330 000	384 879	390 571	391 081	390 655	390 655	395 644			
1991	46 928	159 449	232 888	288 276	311 290	340 422	367 943	438 623	441 238	449 555	440 921	441 101	441 689	441 689				
1992	51 121	170 770	204 986	249 779	274 848	319 948	332 218	335 090	360 112	367 604	367 367	368 160	368 592					
1993	68 241	174 615	206 403	231 733	246 676	276 434	278 758	281 764	285 850	305 223	305 752	305 752						
1994	94 452	262 343	376 951	421 889	487 593	636 087	647 835	755 239	755 706	759 854	761 766							
1995	86 957	288 911	402 681	445 837	478 258	502 843	520 306	545 810	548 156	556 677								
1996	88 663	297 178	441 935	517 098	566 636	593 781	640 700	679 808	698 095									
1997	84 501	327 197	457 943	526 594	592 330	652 213	752 351	831 262										
1998	121 395	345 805	460 845	576 483	652 326	709 234	740 327											
1999	115 064	405 741	569 424	759 205	863 323	949 827												
2000	114 568	361 974	580 481	666 930	729 803													
2001	133 359	366 362	546 169	648 922														
2002	117 053	350 333	525 063															
2003	110 588	294 948																
2004	114 245																	

Figure 3: Development triangle (Paid claims). LoB Liability. Currency a-mark.

Development year

Acc. Year	År0	År1	År2	År3	År4	År5	År6	År7	År8	År9	År10	År11	År12	År13	År14	År15	År16	År17
1987	823 875	955 223	961 631	976 361	979 210	979 768	982 074	982 008	981 690	980 999	980 808	980 580	979 966	979 835	979 840	979 848	979 838	979 953
1988	816 469	990 695	996 637	1 000 175	991 797	995 334	993 710	992 997	991 829	992 227	992 716	991 758	992 656	992 588	992 615	992 634	993 607	
1989	916 770	1 076 259	1 063 422	1 068 961	1 071 855	1 071 189	1 064 480	1 058 183	1 057 814	1 058 148	1 056 800	1 057 025	1 054 515	1 054 536	1 054 562	1 054 596		
1990	1 160 996	1 347 734	1 331 192	1 338 337	1 337 935	1 338 734	1 333 480	1 332 223	1 331 250	1 330 128	1 329 892	1 319 492	1 319 517	1 319 213	1 319 319			
1991	1 314 593	1 444 286	1 422 899	1 417 929	1 418 629	1 414 807	1 415 796	1 414 292	1 412 957	1 408 733	1 412 148	1 411 209	1 410 861	1 412 438				
1992	1 085 695	1 328 007	1 318 445	1 311 943	1 302 182	1 304 769	1 302 719	1 300 679	1 302 183	1 301 492	1 300 075	1 299 102	1 299 245					
1993	1 133 271	1 325 752	1 315 123	1 293 090	1 295 367	1 296 147	1 303 301	1 302 555	1 304 066	1 311 196	1 311 033	1 310 110						
1994	1 193 290	1 417 593	1 398 896	1 395 119	1 390 905	1 388 016	1 393 313	1 392 866	1 403 548	1 404 327	1 406 442							
1995	1 206 797	1 468 748	1 455 070	1 449 063	1 445 154	1 448 694	1 449 790	1 448 521	1 448 653	1 448 455								
1996	1 375 364	1 675 044	1 648 698	1 644 883	1 647 180	1 645 220	1 646 514	1 643 575	1 644 547									
1997	1 459 409	1 742 793	1 725 774	1 724 176	1 730 060	1 717 119	1 713 658	1 716 773										
1998	1 271 196	1 625 922	1 651 575	1 665 781	1 653 224	1 641 157	1 646 814											
1999	1 360 787	1 814 785	1 823 521	1 790 720	1 783 231	1 785 106												
2000	1 400 694	1 806 822	1 773 570	1 746 378	1 753 735													
2001	1 531 643	1 908 938	1 855 739	1 858 117														
2002	1 656 791	1 944 773	1 942 961															
2003	1 684 702	1 983 498																
2004	1 368 338																	

Figure 4: Development triangle (Incurred claims). LoB Private Property. Currency a-mark.

Development year

Acc. Year	År0	År1	År2	År3	År4	År5	År6	År7	År8	År9	År10	År11	År12	År13	År14	År15	År16	År17
1987	1 177 990	1 527 942	1 580 696	1 638 457	1 736 118	1 829 004	1 968 645	1 996 931	2 030 405	2 101 542	2 111 538	2 069 164	2 093 866	2 107 748	2 151 695	2 201 904	2 215 096	2 197 932
1988	1 463 536	1 716 374	1 787 908	1 863 885	1 927 263	2 027 909	2 123 422	2 184 984	2 267 794	2 278 020	2 356 091	2 371 982	2 433 653	2 496 765	2 518 958	2 540 109	2 555 455	
1989	1 570 898	1 941 641	1 982 212	2 065 220	2 237 614	2 314 512	2 413 082	2 462 463	2 510 021	2 556 522	2 591 638	2 685 496	2 712 115	2 792 316	2 780 975	2 782 694		
1990	1 796 104	2 023 756	2 099 442	2 284 719	2 420 085	2 517 376	2 601 501	2 637 910	2 706 630	2 772 450	2 782 983	2 832 472	2 881 698	2 907 333	3 054 286			
1991	1 676 800	1 937 634	2 173 553	2 298 176	2 385 682	2 438 634	2 473 806	2 641 552	2 695 620	2 759 274	2 785 247	2 902 599	2 927 469	3 224 853				
1992	1 268 385	1 561 066	1 687 911	1 755 247	1 829 113	1 857 790	1 970 022	2 076 655	2 115 426	2 266 246	2 302 187	2 329 401	2 288 566					
1993	1 198 181	1 416 074	1 518 785	1 577 007	1 669 992	1 806 820	1 854 983	1 926 188	1 948 099	2 072 100	2 120 643	2 123 749						
1994	1 187 360	1 466 176	1 559 126	1 645 825	1 756 036	1 817 945	1 949 481	2 036 277	2 086 155	2 174 967	2 183 927							
1995	1 198 312	1 506 768	1 615 534	1 656 663	1 762 870	1 897 309	2 062 118	2 149 108	2 378 430	2 407 106								
1996	1 273 312	1 551 548	1 671 462	1 705 412	1 843 772	1 917 260	2 018 953	2 059 660	2 140 169									
1997	1 315 113	1 612 824	1 670 786	1 760 969	1 878 267	1 976 874	2 076 076	2 113 673										
1998	1 311 382	1 613 887	1 639 411	1 699 009	1 808 047	1 869 269	2 061 598											
1999	1 363 435	1 737 124	1 784 020	1 952 930	2 045 330	2 173 403												
2000	1 351 360	1 741 042	1 761 589	1 887 545	1 996 416													
2001	1 557 018	1 961 984	2 039 645	2 083 197														
2002	1 489 683	1 825 342	1 878 743															
2003	1 488 205	1 827 275																
2004	1 570 953																	

Figure 5: Development triangle (Incurred claims). LoB Motor TPL (third party liability). Currency a-mark.

Development year

Acc. Year	År0	År1	År2	År3	År4	År5	År6	År7	År8	År9	År10	År11	År12	År13	År14	År15	År16	År17
1987	115 119	210 392	623 356	633 715	763 735	765 047	779 115	649 015	635 498	488 884	494 865	527 709	506 209	502 736	495 034	471 401	473 533	471 811
1988	98 744	176 308	316 872	318 465	304 781	330 801	333 598	294 951	303 099	309 092	336 125	338 743	346 052	343 516	336 001	336 183	335 138	
1989	105 266	209 101	278 224	289 635	302 269	341 274	406 555	437 022	441 516	530 788	534 183	536 744	422 488	425 546	448 397	447 602		
1990	92 136	185 256	238 018	263 035	273 460	277 534	304 425	356 955	363 150	396 126	400 036	397 976	394 703	394 705	400 216			
1991	154 730	323 339	352 816	461 909	357 872	435 251	463 369	491 155	495 365	490 319	445 763	445 616	446 208	446 642				
1992	280 369	354 808	357 395	360 207	372 786	418 538	380 860	380 081	382 378	369 755	368 229	370 974	371 406					
1993	231 269	289 501	340 337	374 188	376 572	335 291	330 255	341 431	340 657	341 511	341 740	325 607						
1994	588 938	799 752	735 396	738 842	743 536	784 813	790 255	782 364	778 123	778 842	778 728							
1995	536 639	712 155	666 725	672 739	666 655	648 912	612 280	609 157	611 212	613 063								
1996	299 345	528 461	609 989	667 425	705 909	778 470	803 614	823 166	819 970									
1997	362 057	595 628	631 261	644 559	657 846	709 688	819 858	884 382										
1998	417 294	649 182	660 852	757 355	819 471	820 401	825 058											
1999	445 969	790 974	952 070	1 039 395	1 120 885	1 100 936												
2000	505 803	807 989	810 934	834 633	838 180													
2001	526 476	766 142	800 173	801 401														
2002	532 232	951 217	1 011 036															
2003	516 456	717 950																
2004	462 027																	

Figure 6: Development triangle (Incurred claims). LoB Liability. Currency a-mark.

A.2 Distributions

A.2.1 Definition of the normal distribution

We say that the stochastic variable X is normally distributed with parameters μ and σ^2 if the density of X is given by

$$f_X(x;\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

We have for this distribution that $E(X) = \mu$ and $Var(X) = \sigma^2$.

A.2.2 Definition of the log-normal distribution

A log-normally distributed stochastic variable can be written as e^X, where X is normally distributed with parameters μ and σ^2. The density for a log-normally distributed variable is

$$f_X(x;\mu,\sigma^2) = \frac{1}{\sigma x\sqrt{2\pi}} e^{-(\ln x-\mu)^2/2\sigma^2}, \; x > 0.$$

We have for this distribution that $E(X) = e^{\mu+\sigma^2/2}$ and $Var(X) = e^{2\mu+\sigma^2}(e^{\sigma^2}-1)$.

A.2.3 Definition of the Over-dispersed Poisson (OdP)

The function of variance is $v(\mu) = \mu$, since we have an over-dispersed Poisson with p=1. We have the following function of frequency given the scale parameter ϕ for an over-dispersed Poisson variable

$$f_D(d) = \exp\left\{\frac{d\log(\mu)-\mu}{\phi} - \frac{d}{\phi}\log(\phi) - \log\left(\frac{d}{\phi}\right)\right\} = \exp\left\{\frac{d\log(\mu)-\mu}{\phi} + \log\left(\frac{\phi^{-\frac{d}{\phi}}}{\frac{d}{\phi}}\right)\right\}$$

where $d = \phi n$, $n \in \{0,1,2\ldots\}$, otherwise 0. We have for these distributions that $Var(D) = \phi\mu$ $\Rightarrow \sqrt{Var(D)} = \sqrt{\phi}\sqrt{\mu}$ i.e. the standard deviation is proportional to $\sqrt{\mu}$.

A.2.4 Assumptions for GLM

(A1) $\mu_{ij} \equiv E[D_{ij}] = \gamma_i \delta_j$, där $\sum_{\mathrm{j=1}}^{m} \delta_{\mathrm{j}} = 1$**.**

(A2) $D_{ij}, \mathrm{D}_{\mathrm{i}',j'}$, independent when $(i, j) \neq (i', j')$.

(A3) The distribution for D_{ij} belong to the EDM family and it has function variance as $v(\mu) = \mu^p$, p=1 in this paper since we are going to use over-dispersed Poisson.

Assumption (A1) mean that the paid claims are modelled multiplicative where we have that γ_i is parsed as a level parameter, or more specific the expected ultimate (where ultimate means up to the latest development year observed in the triangle) claims, for accident year i. Parameter δ_j is parsed as the expected proportion paid claims that rises during development year j. Assumption (A2) mean that the payments in different cells are assumed independent and assumption (A3) that they have a variance proportional to the expected value raised to p (in this case to 1).

We get the following if we take the logarithm of both sides of (A1)

(A1') $\log(\mu_{ij}) = \log(\gamma_i) + \log(\delta_j)$**,** where $\log(\sum_{j=1}^{m} \delta_j) = 0$**.**

Let us make a suitable parameter switch

$$c = \log(\gamma_1 \delta_1), \alpha_{\mathrm{i}} = \log(\gamma_i / \gamma_1), \beta_{\mathrm{j}} = \log(\delta_j / \delta_1) \Leftrightarrow \gamma_i = e^{c+\alpha_i} (\sum_{j=1}^{m} e^{\beta_j}), \delta_{\mathrm{j}} = \frac{e^{\beta_j}}{\sum_{j=1}^{m} e^{\beta_j}}$$

which give us

(A1'') $\log(\mu_{ij}) = c + \alpha_i + \beta_j$**,** there $\alpha_1 = \beta_1 = 0$.

Exponential dispersion-models, EDM, is the family of distributions whose function of frequency can be expressed as $f_D(d) = \exp\left\{ \frac{y\theta - b(\theta)}{\phi / w} + c(y; \phi, w) \right\}$, for the use of EDM in GLM see (ref [15]). Assumption (A1), (A2) and (A3) define a generalised linear model, GLM.

A.3 Tables for paid/incurred losses for each accident year

To get more understandable and readable tables we only show data from year 1995 to 2004. Table with figures for Private Property portfolio paid data see chapter 5.2, Table 5.1. We start with paid data for each of the two methods, Mack and Bootstrap.

Table 1: Portfolio Private Property (house owner, home owner and holiday cottage)

Paid data (Bootstrap) currency:a-mark

Accident Year	(1) Paid Losses to Date	(2) Loss Development Factor	(3) Ultimate Losses	(4) Total Reserves	(5) Prediction Error of Reserves	(6) Ratio of Prediction Error to Expected Reserves
1995	1 440 265	1,003	1 443 897	3 632	3 863	106%
1996	1 632 105	1,003	1 637 564	5 459	4 746	87%
1997	1 710 042	1,005	1 719 421	9 379	6 173	66%
1998	1 633 819	1,007	1 645 451	11 631	6 832	59%
1999	1 762 682	1,011	1 781 980	19 299	8 766	45%
2000	1 735 297	1,016	1 763 514	28 217	10 547	37%
2001	1 830 360	1,026	1 878 432	48 073	13 789	29%
2002	1 874 795	1,045	1 959 824	85 029	18 432	22%
2003	1 791 654	1,095	1 961 883	170 229	26 424	16%
2004	1 030 655	1,730	1 782 527	751 872	68 634	9%
Total	26 188 268		27 325 929	**1 137 661**	85 321	**7%**

25th percentile:	**1 080 113**
75th percentile:	**1 195 209**
Risk margin:	**5%**

(1) Is the last diagonal from the development triangle when it is recalculated on incremental form (appendix A.1, figure 1).
(2) Calculated with formula **(4.14)** from chapter **4.2**.
(3)= (1) * (2)
(4) = (3) - (1)
(5) In this model is the prediction error an estimate for the standard deviation of the outstanding claims reserve.
Calculated with formula **(4.11)** from chapter **4.1.2**.
(6) = (5) / (4) = coefficient of variation = CV
We get that the normal distribution is the best fit to the data (using @Risk). With this distribution we estimate the 75th percentile.

Table 2: Portfolio Motor TPL (third party liability)

Paid Method (Mack) currency:a-mark

Accident Year	(1) Paid Losses to Date	(2) Loss Development Factor	(3) Ultimate Losses	(4) Total Reserves	(5) Prediction Error of Reserves	(6) Ratio of Prediction Error to Expected Reserves
1995	1 813 423	1,169	2 119 040	305 617	14 535	19%
1996	1 669 333	1,227	2 049 104	379 771	17 861	19%
1997	1 687 717	1,277	2 155 578	467 862	19 212	16%
1998	1 651 144	1,340	2 213 232	562 088	21 913	16%
1999	1 673 786	1,414	2 367 435	693 650	25 921	15%
2000	1 591 819	1,484	2 362 420	770 601	28 645	15%
2001	1 641 956	1,574	2 584 745	942 790	31 104	13%
2002	1 498 335	1,698	2 543 944	1 045 610	32 285	12%
2003	1 344 130	1,907	2 563 368	1 219 238	34 377	11%
2004	845 054	3,278	2 770 324	1 925 269	45 759	10%
Total	32 857 265		42 127 178	**9 269 913**	135 830	**6%**

25th percentile:	**8 895 634**
75th percentile:	**9 626 868**
Risk margin:	**4%**

(1) Is the last diagonal from the development triangle (appendix A.1, figure 2).
(2) Calculated with formula **(4.14)** from chapter **4.2**.
(3) = (1) * (2)
(4) = (3) - (1), also called the central estimate.
(5) In this model is the prediction error an estimate for the standard deviation of the outstanding claims reserve.
Calculated with formula **(4.18)** and for the overall reserve with formula **(4.19)** both from chapter **4.2**.
(6) = (5) / (4) = coefficient of variation = CV
We calculate the 75th percentile with a log-normal distribution.

Table 3: Portfolio Motor TPL (third party liability)

Paid Method (Bootstrap)	currency:a-mark (1)	(2)	(3)	(4)	(5)	(6)
Accident Year	*Paid Losses to Date*	*Loss Development Factor*	*Ultimate Losses*	*Total Reserves*	*Prediction Error of Reserves*	*Ratio of Prediction Error to Expected Reserves*
1995	1 813 423	1,169	2 119 040	305 617	57 728	19%
1996	1 669 333	1,227	2 049 104	379 771	64 873	17%
1997	1 687 717	1,277	2 155 578	467 862	72 841	16%
1998	1 651 144	1,340	2 213 232	562 088	80 630	14%
1999	1 673 786	1,414	2 367 435	693 650	92 130	13%
2000	1 591 819	1,484	2 362 420	770 601	99 205	13%
2001	1 641 956	1,574	2 584 745	942 790	112 881	12%
2002	1 498 335	1,698	2 543 944	1 045 610	120 921	12%
2003	1 344 130	1,907	2 563 368	1 219 238	136 635	11%
2004	845 054	3,278	2 770 324	1 925 269	224 689	12%
Total	32 857 265		42 127 178	**9 269 913**	582 794	**6%**

25th percentile:	**8 876 824**
75th percentile:	**9 663 001**
Risk margin:	**4%**

(1) Is the last diagonal from the development triangle when it is recalculated on incremental form (appendix A.1, figure 2).
(2) Calculated with formula **(4.14)** from chapter **4.2**.
(3)= (1) * (2)
(4) = (3) - (1)
(5) In this model is the prediction error an estimate for the standard deviation of the outstanding claims reserve.
Calculated with formula **(4.11)** from chapter **4.1.2**.
(6) = (5) / (4) = coefficient of variation = CV
We get that the normal distribution is the best fit to the data (using @Risk). With this distribution we estimate the 75th percentile.

Table 4: Portfolio Liability

Paid Method (Mack)	currency:a-mark (1)	(2)	(3)	(4)	(5)	(6)
Accident Year	*Paid Losses to Date*	*Loss Development Factor*	*Ultimate Losses*	*Total Reserves*	*Prediction Error of Reserves*	*Ratio of Prediction Error to Expected Reserves*
1995	556 677	1,078	600 205	43 528	23 178	53%
1996	698 095	1,147	800 942	102 847	79 847	78%
1997	831 262	1,234	1 025 833	194 571	185 499	95%
1998	740 327	1,341	993 004	252 676	187 137	74%
1999	949 827	1,423	1 351 661	401 835	230 160	57%
2000	729 803	1,577	1 151 229	421 425	216 942	51%
2001	648 922	1,761	1 142 519	493 598	217 604	44%
2002	525 063	2,108	1 106 993	581 929	221 313	38%
2003	294 948	3,043	897 657	602 710	209 417	35%
2004	114 245	9,484	1 083 550	969 304	329 333	34%
Total	9 582 873		13 758 868	**4 175 994**	942 863	**23%**

25th percentile:	**3 504 661**
75th percentile:	**4 734 569**
Risk margin:	**13%**

(1) Is the last diagonal from the development triangle (appendix A.1, figure 3).
(2) Calculated with formula **(4.14)** from chapter **4.2**.
(3) = (1) * (2)
(4) = (3) - (1), also called the central estimate.
(5) In this model is the prediction error an estimate for the standard deviation of the outstanding claims reserve.
Calculated with formula **(4.18)** and for the overall reserve with formula **(4.19)** both from chapter 4.2.
(6) = (5) / (4) = coefficient of variation = CV
We calculate the 75th percentile with a log-normal distribution.

Table 5: Portfolio Liability

Paid Method (Bootstrap) Accident Year	currency:a-mark (1) Paid Losses to Date	(2) Loss Development Factor	(3) Ultimate Losses	(4) Total Reserves	(5) Prediction Error of Reserves	(6) Ratio of Prediction Error to Expected Reserves		
1995	556 677	1,078	600 205	43 528	37 112	85%		
1996	698 095	1,147	800 942	102 847	58 390	57%		
1997	831 262	1,234	1 025 833	194 571	84 459	43%		
1998	740 327	1,341	993 004	252 676	98 979	39%		
1999	949 827	1,423	1 351 661	401 835	132 840	33%		
2000	729 803	1,577	1 151 229	421 425	134 564	32%		
2001	648 922	1,761	1 142 519	493 598	154 217	31%		
2002	525 063	2,108	1 106 993	581 929	184 512	32%		
2003	294 948	3,043	897 657	602 710	214 637	36%	**25th percentile:**	**3 584 941**
2004	114 245	9,484	1 083 550	969 304	517 510	53%	**75th percentile:**	**4 678 623**
Total	9 582 873		13 758 868	**4 175 994**	832 352	**20%**	**Risk margin:**	**12%**

(1) Is the last diagonal from the development triangle when it is recalculated on incremental form (appendix A.1, figure 3).
(2) Calculated with formula **(4.14)** from chapter **4.2**.
(3)= (1) * (2)
(4) = (3) - (1)
(5) In this model is the prediction error an estimate for the standard deviation of the outstanding claims reserve.
Calculated with formula **(4.11)** from chapter **4.1.2**.
(6) = (5) / (4) = coefficient of variation = CV
We get that the log-normal distribution is the best fit to the data (using @Risk). With this distribution we estimate the 75th percentile.

We will now present the tables for incurred data.

Table 6: Portfolio Private Property (house owner, home owner and holiday cottage)

Incurred method Mack Accident Year	currency:a-mark (1) Incurred Losses to Date	(2) Loss Development Factor	(3) Ultimate Losses	(4) Total Reserves	(5) Prediction Error of Reserves	(6) Ratio of Prediction Error to Expected Reserves		
1995	1 473 515	1,002	1 476 039	2 524	1 981	78%		
1996	1 679 606	1,002	1 683 753	4 147	3 521	85%		
1997	1 751 791	1,004	1 758 099	6 307	5 292	84%		
1998	1 671 438	1,004	1 677 808	6 370	5 240	82%		
1999	1 825 396	1,005	1 834 993	9 597	6 319	66%		
2000	1 814 178	1,006	1 825 077	10 899	6 548	60%		
2001	1 911 316	1,007	1 925 118	13 802	7 262	53%		
2002	1 944 773	1,009	1 963 054	18 281	10 201	56%		
2003	1 983 498	1,011	2 006 063	22 565	13 035	58%	**25th percentile:**	**253 323**
2004	1 368 338	1,222	1 672 722	304 384	80 007	26%	**75th percentile:**	**386 514**
Total	26 924 053		27 252 694	**328 640**	85 842	**26%**	**Risk margin:**	**18%**

(1) Is the last diagonal from the development triangle when it is recalculated on incremental form (appendix A.1, figure 4).
(2) Calculated with formula **(4.14)** from chapter **4.2**.
(3) = (1) * (2)
(4) = (3) - (1), also called the central estimate.
(5) In this model is the prediction error an estimate for the standard deviation of the outstanding claims reserve.
Calculated with formula **(4.18)** and for the overall reserve with formula **(4.19)** both from chapter 4.2.
(6) = (5) / (4) = coefficient of variation = CV
We calculate the 75th percentile with a log-normal distribution.

Table 7: Portfolio Private Property (house owner, home owner and holiday cottage)

Incurred method ***Bootstrap*** ***(smooth data**)***	currency:a-mark							
	(1)	(2)	(3)	(4)	(5)	(6)		
Accident Year	*Incurred Losses to Date*	*Loss Development Factor*	*Ultimate Losses*	*Total Reserves*	*Prediction Error of Reserves*	*Ratio of Prediction Error to Expected Reserves*		
1995	1 473 515	1,002	1 476 039	2 524	4 646	184%		
1996	1 679 606	1,002	1 683 753	4 147	5 905	142%		
1997	1 751 791	1,004	1 758 099	6 307	7 158	113%		
1998	1 671 438	1,004	1 677 808	6 370	7 147	112%		
1999	1 825 396	1,005	1 834 993	9 597	8 719	91%		
2000	1 814 178	1,006	1 825 077	10 899	9 235	85%		
2001	1 911 316	1,007	1 925 118	13 802	10 362	75%		
2002	1 944 773	1,009	1 963 054	18 281	11 837	65%		
2003	1 983 498	1,011	2 006 063	22 565	13 148	58%	**25th percentile:**	**355 263**
2004	1 368 338	1,222	1 672 722	304 384	50 819	17%	**75th percentile:**	**449 540**
Total	27 417 985		27 823 746	**405 761**	71 338	**18%**	**Risk margin:**	**11%**

***smooth data meens that all data in the development triangle that is negative is put to 0.*

(1) Is the last diagonal from the development triangle when it is recalculated on incremental form (appendix A.1, figure 3).

(2) Calculated with formula **(4.14)** from chapter **4.2**.

(3)= (1) * (2)

(4) = (3) - (1)

(5) In this model is the prediction error an estimate for the standard deviation of the outstanding claims reserve.
Calculated with formula **(4.11)** from chapter **4.1.2**.

(6) = (5) / (4) = coefficient of variation = CV

We get that the log-normal distribution is the best fit to the data (using @Risk). With this distribution we estimate the 75th percentile.

Table 8: Portfolio Motor TPL (third party liability)

Incurred method ***Mack***	currency:a-mark							
	(1)	(2)	(3)	(4)	(5)	(6)		
Accident Year	*Incurred Losses to Date*	*Loss Development Factor*	*Ultimate Losses*	*Total Reserves*	*Prediction Error of Reserves*	*Ratio of Prediction Error to Expected Reserves*		
1995	2 407 106	1,107	2 664 888	257 782	159 576	62%		
1996	2 140 169	1,142	2 443 586	303 416	162 041	53%		
1997	2 113 673	1,178	2 490 290	376 617	178 300	47%		
1998	2 061 598	1,217	2 508 824	447 226	185 201	41%		
1999	2 173 403	1,282	2 786 876	613 472	208 301	34%		
2000	1 996 416	1,340	2 675 675	679 259	209 616	31%		
2001	2 083 197	1,418	2 954 963	871 766	225 499	26%		
2002	1 878 743	1,488	2 795 548	916 805	226 882	25%		
2003	1 827 275	1,560	2 850 161	1 022 887	243 629	24%	**25th percentile:**	**6 725 703**
2004	1 570 953	1,908	2 996 696	1 425 743	274 861	19%	**75th percentile:**	**8 218 667**
Total	40 663 997		48 181 354	**7 517 357**	1 123 349	**15%**	**Risk margin:**	**9%**

(1) Is the last diagonal from the development triangle (appendix A.1, figure 5).

(2) Calculated with formula **(4.14)** from chapter **4.2**.

(3) = (1) * (2)

(4) = (3) - (1), also called the central estimate.

(5) In this model is the prediction error an estimate for the standard deviation of the outstanding claims reserve.
Calculated with formula **(4.18)** and for the overall reserve with formula **(4.19)** both from chapter 4.2.

(6) = (5) / (4) = coefficient of variation = CV

We calculate the 75th percentile with a log-normal distribution.

Table 9: Portfolio Motor TPL (third party liability)

Incurred method	currency:a-mark							
Bootstrap	(1)	(2)	(3)	(4)	(5)	(6)		
(smooth data**)						*Ratio of*		
		Loss			*Prediction*	*Prediction Error*		
	Incurred Losses	*Development*	*Ultimate*	*Total*	*Error of*	*to Expected*		
Accident Year	*to Date*	*Factor*	*Losses*	*Reserves*	*Reserves*	*Reserves*		
1995	2 407 106	1,116	2 685 699	278 592	103 906	37%		
1996	2 140 169	1,151	2 462 668	322 499	110 660	34%		
1997	2 113 673	1,187	2 509 738	396 065	124 654	31%		
1998	2 061 598	1,226	2 528 416	466 818	135 477	29%		
1999	2 173 403	1,292	2 808 639	635 236	162 360	26%		
2000	1 996 416	1,351	2 696 570	700 154	175 289	25%		
2001	2 083 197	1,430	2 978 039	894 842	201 320	22%		
2002	1 878 743	1,500	2 817 379	938 636	210 908	22%		
2003	1 827 275	1,572	2 872 419	1 045 144	233 828	22%	**25th percentile:**	**7 269 912**
2004	1 570 953	1,922	3 020 098	1 449 145	310 006	21%	**75th percentile:**	**8 482 991**
Total	40 663 997		48 540 449	**7 876 452**	899 257	**11%**	**Risk margin:**	**8%**

***smooth data meens that all data in the development triangle that is negative is put to 0.*

(1) Is the last diagonal from the development triangle when it is recalculated on incremental form (appendix A.1, figure 5).
(2) Calculated with formula **(4.14)** from chapter **4.2**.

(3)= (1) * (2)

(4) = (3) - (1)

(5) In this model is the prediction error an estimate for the standard deviation of the outstanding claims reserve.
Calculated with formula **(4.11)** from chapter **4.1.2**.

(6) = (5) / (4) = coefficient of variation = CV

We get that the normal distribution is the best fit to the data (using @Risk). With this distribution we estimate the 75th percentile.

Table 10: Portfolio Liability

Incurred method	currency:a-mark							
Mack	(1)	(2)	(3)	(4)	(5)	(6)		
						Ratio of		
		Loss			*Prediction*	*Prediction Error*		
	Incurred Losses	*Development*	*Ultimate*	*Total*	*Error of*	*to Expected*		
Accident Year	*to Date*	*Factor*	*Losses*	*Reserves*	*Reserves*	*Reserves*		
1995	722 076	1,000	722 076	10 419	77 246	741%		
1996	823 166	1,000	823 166	13 936	124 555	894%		
1997	884 382	1,001	885 352	16 018	130 849	817%		
1998	825 058	1,004	828 550	18 499	142 138	768%		
1999	1 120 885	1,031	1 155 160	59 104	184 374	312%		
2000	838 180	1,038	869 747	78 260	169 816	217%		
2001	801 401	1,031	826 057	101 784	181 908	179%		
2002	1 011 036	1,058	1 069 358	194 139	225 528	116%		
2003	717 950	1,124	806 960	243 963	325 205	133%	**25th percentile:**	**944 902**
2004	462 027	1,561	721 269	504 332	349 119	69%	**75th percentile:**	**1 504 609**
Total	11 651 153		12 916 559	**1 265 406**	360 879	**29%**	**Risk margin:**	**19%**

(1) Is the last diagonal from the development triangle when it is recalculated on incremental form (appendix A.1, figure 6).
(2) Calculated with formula **(4.14)** from chapter **4.2**.
(3) = (1) * (2)
(4) = (3) - (1), also called the central estimate.
(5) In this model is the prediction error an estimate for the standard deviation of the outstanding claims reserve.
Calculated with formula **(4.18)** and for the overall reserve with formula **(4.19)** both from chapter 4.2.
(6) = (5) / (4) = coefficient of variation = CV
We calculate the 75th percentile with a log-normal distribution.

Table 11: Portfolio Liability

*Incurred method Bootstrap (smooth data**)* Accident Year	currency:a-mark (1) Incurred Losses to Date	(2) Loss Development Factor	(3) Ultimate Losses	(4) Total Reserves	(5) Prediction Error of Reserves	(6) Ratio of Prediction Error to Expected Reserves		
1995	722 076	1,040	751 288	29 212	43 752	150%		
1996	823 166	1,068	879 378	56 212	60 540	108%		
1997	884 382	1,073	949 302	64 920	66 580	103%		
1998	825 058	1,108	914 152	89 093	75 142	84%		
1999	1 120 885	1,153	1 292 500	171 614	110 356	64%		
2000	838 180	1,207	1 011 720	173 540	109 244	63%		
2001	801 401	1,262	1 011 353	209 952	122 350	58%		
2002	1 011 036	1,334	1 348 558	337 522	167 526	50%		
2003	717 950	1,517	1 088 945	370 995	176 540	48%	**25th percentile:**	**1 792 353**
2004	462 027	2,368	1 093 979	631 952	292 539	46%	**75th percentile:**	**2 580 515**
Total	12 622 954		14 809 387	**2 186 434**	584 265	**27%**	**Risk margin:**	**18%**

***smooth data meens that all data in the development triangle that is negative is put to 0.*

(1) Is the last diagonal from the development triangle when it is recalculated on incremental form (appendix A.1, figure 6).

(2) Calculated with formula **(4.14)** from chapter **4.2**.

(3)= (1) * (2)

(4) = (3) - (1)

(5) In this model is the prediction error an estimate for the standard deviation of the outstanding claims reserve.
Calculated with formula **(4.11)** from chapter **4.1.2**.

(6) = (5) / (4) = coefficient of variation = CV

We get that the normal distribution is the best fit to the data (using @Risk). With this distribution we estimate the 75th percentile.

Viewing the prediction errors as a percentage of reserve estimates we can see that the earlier accident years give us large prediction errors. It should be noted that the reserve estimate is very low, and a large prediction error is not unexpected. In reality for earlier accident years, we have for short-tailed business that the reserves are known and probably fairly determined for the claims still to be settled. The uncertainty for Private Property portfolio should be negligible. For long-tailed business such as Motor TPL and sometimes Liability, we have an uncertainty that is **not** negligible. Often for long-tailed business we do not have full history for the data. We do our own estimates for example tail approximation to get full history. In this paper, we have not done any approximation in the tail, this can make the data insecure for portfolios like Liability (see table 10, column 6 above).