Mathematical Statistics
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Analysing the Accident Frequency of the Commercial Airlines Total Loss Catastrophes

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Abstract

Nowadays, actuaries and analysts frequently use predictive modeling in several capacities, in everything from pricing, reserving, reinsurance to identifying future customers. Basically, predictive modeling is a process by which one uses statistical analysis of data to make predictions about future events. Use of this technique will allow considering all possible factors simultaneously, permit for the nature of random processes and will provide necessary diagnostics. It will remove a potential "double-counting" of the variables and can explain interaction effects.

In this paper, I will present a model for the accident frequency of the commercial aviation catastrophes exceeding the cost of 10 million USD using the technique of predictive modeling. The paper will include a discussion of the various aspects of designing such a model, including the type of rating factors, accuracy of the data, homogeneity of the classes and level of available information needed.

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Summary

This paper is a Diploma thesis in mathematical statistics and was written for Inter Hannover Scandinavian Branch and the Stockholm University. Proposal for this paper came from Lars Klingberg, senior actuary at Hannover Re. The main goal for this project is to find a model for accident frequency for aviation catastrophes that exceed the cost of 10 million USD based on the past claims data. It is reasonable to think that the accident frequency of aviation catastrophes depends on different factors and therefore the importance of identifing those factors. This paper will examine the significance of the aircraft age, geographic area, the type of the aircraft and some others variables on an accident frequency. The second objective of this paper is to give a comparison of the new pricing model with the existing rating system MARTHA that has been in use for almost 30 years. Today, this rating system is used to estimate the risk premium of the insured airline. The structure of this paper will be as follows: first I am going to look at how predictive modeling can be used in calculating the frequency of accidents for airline hull insurance, using separate models for Partial and Total Losses. Next, I will take a look at combining pricing models for the two claim groups. Finally, I will describe a possible pricing algorithm that applies for both groups and make a comparison to the rating system MARTHA. This paper is intended to be for practical use and will present recommendations for future studies and research.

Acknowledgement

I would like to express my gratitude towards Lars Klingberg, senior actuary at the Hannover Re for his involvement and many contributions to this project. His systematic pedagogical and editorial verifications contributed with many improvements to this paper's content and greatly improved the final product. I am also thankful to Esbjörn Ohlsson,Thomas Höglund and Louise af Klintberg my mentors at the Stockholm University, who had willingly contributed with their knowledge and time to this paper. Finally I would like to thank my colleagues at Inter Hannover for their valuable help with materials, ideas, news and statistics that helped to make this project possible and my husband for his patience and encouragement during this project.

Purpose

In this paper, I will present a model for the accident frequency of the commercial aviation catastrophes exceeding the cost of 10 million USD using the technique of predictive modeling. The paper will include a discussion of the various aspects of designing such a model, including the type of rating factors, accuracy of the data, homogeneity of the classes and level of available information needed.

Simply put, this paper will examine the significance of the aircraft age, geographic area, the type of the aircraft and some others variables on an accident frequency. The second objective of this paper is to give a comparison of the new model with the existing rating system MARTHA that has been in use for almost 30 years. This paper is intended to be for practical use and will present recommendations for future studies and research.

Chapter 1

Introduction

1.1 Background information

Airlines nowadays, buy a wide variety of insurance covers, but the main aviation risks are for hull and aviation liability. The hull policy covers the air and ground risks from accidental damage and the liability policy covers the risk from legal action by customers and third parties in respect of injury or physical damage.

To spread the catastrophe risk costs, several insurance companies share the risk, by each taking a few percent of the contract. Depending on the size of aircraft, geographical area of operation and the relative legal requirement, limits can range anywhere from 250 million USD to 2 billion USD, calculated as a dollar amount per 1000 revenue passenger kilometers (i.e. seats filled with "earning" passengers). The hull rate is calculated as the percentage of the insured fleet value and can be up to 200 million USD for aircraft hull. The insurers provide these liability limits to the airline for each aircraft, each take-off and hence each occurrence, there is no limit to the number of occurrences covered in a given policy period.

By the nature of the business, the airline can be involved in different kinds of accidents. Those reported accidents can be broken into three groups of claims. The first group consists of every-day claims such as lost luggage, generally grouped as Minor claims below a certain limit. The second group consists of Major Partial claims arising out of minor damage to the hull of an aircraft, while the third group contains aircraft crashes resulting in the Total Loss of an aircraft, fatalities, property damage and third party damages. The Total Loss can be very expensive for the airline. For a jumbo jet (B747) the hull value may amount to some 150 million USD. In the event of a fatal accident, the potential liability for the passenger awards could reach 1.5 billion USD , this without taking any third party losses on the ground into consideration. While there is a degree of consistency in the number of accidents every year, the consequences and costs of these accidents can vary dramatically. But in spite of the overwhelming value of these exposures, airline can go about their business knowing that the aviation insurance industry has relieved them from those enormous exposures and cover the world's collective fleets of aircraft valued at more than 570 billion USD.

The accident frequency model that I will present in this paper depends on the information about an airline's fleet, the aircraft's generation (or age of the aircraft), destination profiles, the manufacturer of the aircraft and a few other rating factors.

The underlying unit for an insurance premium is an exposure base, but this varies depending on the characteristics of the insurance coverage. Airlines, as a custom, report the number of departures, kilometers flown and hours flown over a one-year period for the insures. Any of these parameters could serve as an exposure base. The main decision is which of these factors is more accurate to use, but also which one is considered standard in airlines business. I will use the policy period as unit of exposure. However, there is no significant difference in results when one uses departures as unit of exposure.

1.2 Rating Factors

In the insurance business, both the costs of the policy and the need for that policy are highly dependent on the characteristics of the individual risk. In addition, the exposure to accidents changes with a significant number of factors. Many factors that are related to the accident frequency and accident cost cannot be objectively defined and rated. Some of these cannot be used because they are either unspecified or too personal. The variables that are left may be used in risk classification.

The categorization of the rating factors represented below are used both in calculating the frequency of accidents for aviation insurance such as hull insurance, and in the rating system MARTHA. The main rating factors are the size of the airline fleet, the aircraft's generation, destination profiles, manufacturer of the aircraft, social area, aircraft class and changes over the time. The time factor was included in the model because there is evidence that the probability of an airline catastrophe has changed over the years.

1.2.1 Aircraft class and Aircraft Generation

The first Boeing 747 (jumbo jet) was built in 1969 and was the first wide body aircraft. This aircraft laid the ground for commercial airline industry as we know it today. Western built aircrafts can be divided in three technical generations (GEN), depending on the engines, type of cockpit, age, etc. The first generation was designed and built in 1950 – 1960. Only few of them are in the commercial traffic today. The second generation was designed in 1960 – 1970, and they are used today for both domestic and international traffic. Aircrafts built after 1970 are included in the third generation.

GEN:

1. Jet aircraft, First generation - B707, DC8, Caravelle ...
2. Jet aircraft, Second generation - A300, B727, B737, B747, DC10 ...
3. Jet aircraft, Third generation - A310, A320, A330, B747-400 ...

Major improvements in aircraft engines have given quieter flight, greater comfort and long-distance travel with few stops. Aircrafts and their systems have become more reliable. Even though, work continues with safety improvements, technology and education of the pilots. Certain enhancements are incorporated with each introduction of the new aircraft type.

> "Each generation of modern airplanes is more reliable and safer than its predecessors. Extremely clever pieces of kit in the cockpit have become compulsory throughout much of Europe and North America. Improvements in technology over the past 10 years have pilots a clearer picture of their proximity to the ground when landing in poor visibility and told pilots to ascend or descend to avoid a collision with another aircraft."[24]

In spite of the technical improvements, such as reduced noise and fuel consumption, greater engine and system reliability, all of the latest aircrafts have the same speed as the first generation jet aircraft. However, there are noticeable differences in safety performance of the different aircraft generations.

Besides the generation classification, aircraft's can as well be grouped into three classes (ACL) depending on the purpose of use and range of flight.

ACL:

1. LR – long range flight

2. SR — short range flight
3. MR – medium range flight

1.2.2 Manufacturer

An accident involving an airline, presents significant financial exposure to the company, as well as, the manufacturer of the aircraft. An aircraft accident may involve liability exposure on the part of an aircraft manufacturer. Manufacturers typically use disclaimers of liability, limited warranties and requirements of indemnification from purchasers to limit their liability [4].

Nowadays, it continues to be Boeing and Airbus/EADS that are dominating the market manufacturing sector and in particular, generating the lion-share of premium and aircraft units for the market (se fig. 1.1). At present, Boeing is the market leader generating in excess of 189 million USD worth of premium. This situation generates harder competition and faster technical development. Manufacturer, as the rating factor has complicating classification of the sub-classes and the underlying data becomes a problem during rating. This factor is one of many possible factors that are related to the accident frequency and accident cost but problematic in objective definion and rating. According to the volume of production, I divided worldwide manufacturers (MFR) into six different classes.

MFR:

1. BOE — Boeing
2. ABI — Airbus Industry
3. MDO — McDonnell Douglas
4. BAS — British Airspace Systems
5. FOK — Fokker
6. Others – small manufactures.

1.2.3 Social area and Geographic Area

The global aircraft fleet is spread over five continents, many countries and thousands of different locations. Geography is just as important as aircraft generation, when considering the appropriate policy price. There are extensive amount of data that illustrates significant variations in the level of cover

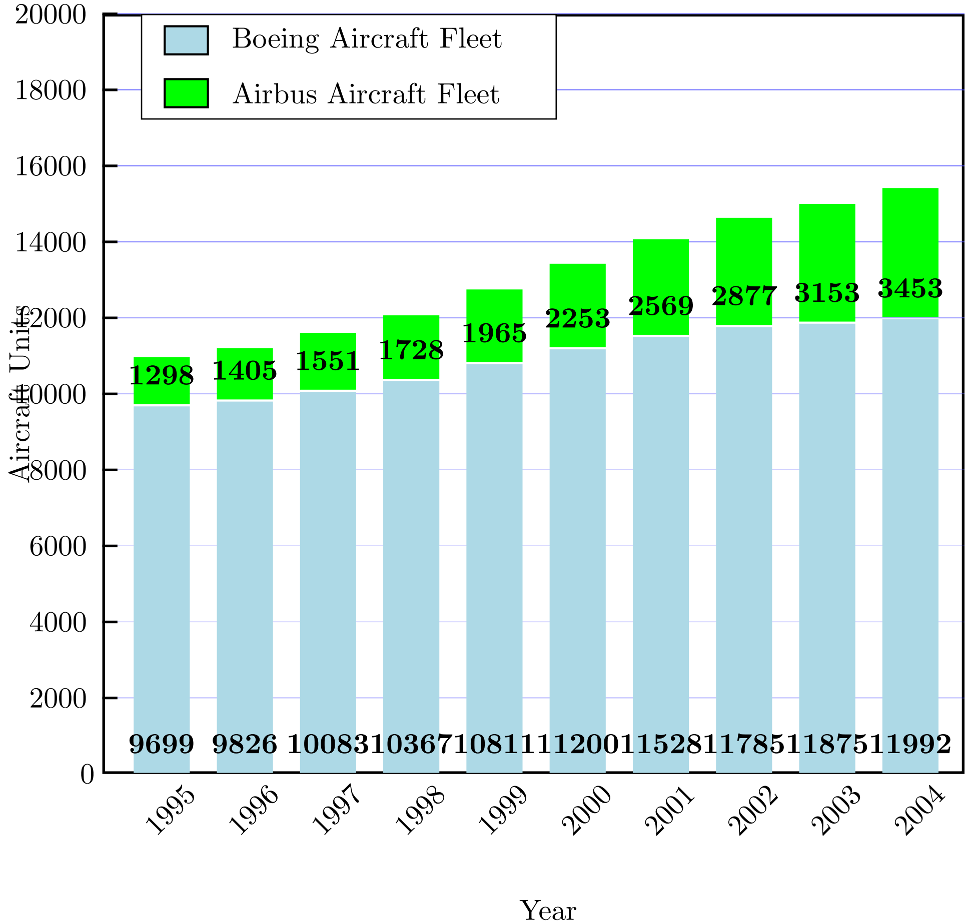

Figure 1.1: Total Active Fleet (Airbus vs. Boeing Fleets 1995 - 2004)

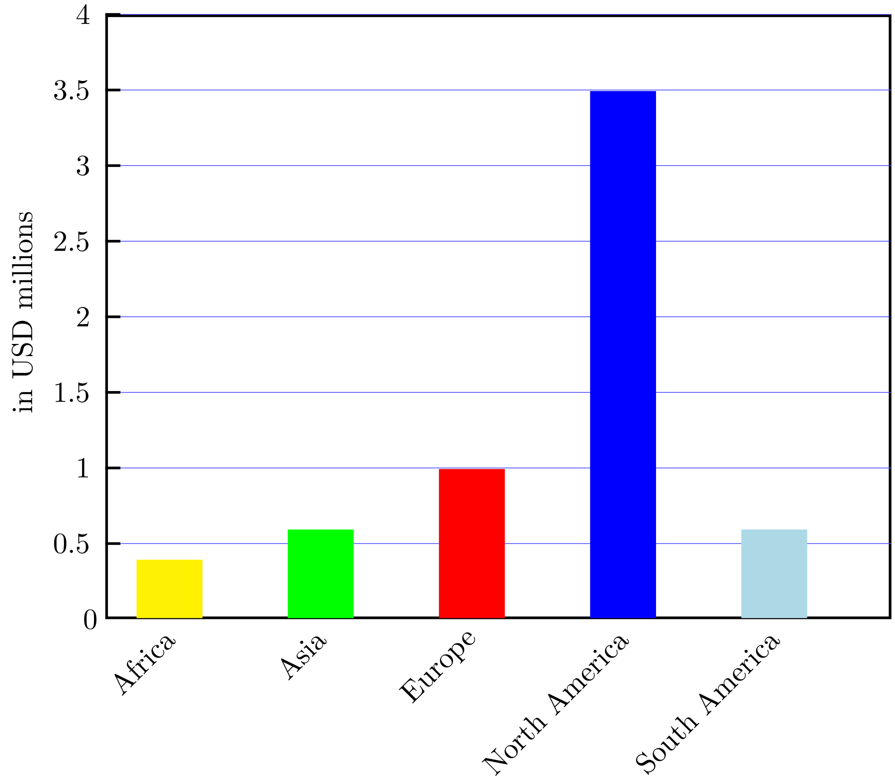

Figure 1.2: Passenger awards by region

purchased country by country. And it is no surprise that the company in the USA tend to buy greater levels of insurance than other countries, largely due to the extraordinary levels of litigation experiences when compared to any other country in the world (se fig.1.2)[26].

Many researchers have used normalized accident and incident data to analyze the safety of the U.S. commercial aviation industry [25]. Conclusions of those studies are that the risk of death for air travelers is exceptionally small, that this risk fell dramatically between the 1970s and the 1980s, and has remained at these lower levels since then. These studies have found that companies based in the U.S. and other developed countries consistently have lower accident rates than companies based in less developed countries (se fig.1.3)[25].

The chart 1.3 looks at Total Losses, distinguishing between western countries (North America, USA, Europe and Australasia), intermediate and developing countries:

A. US, Europe, Japan, Australia

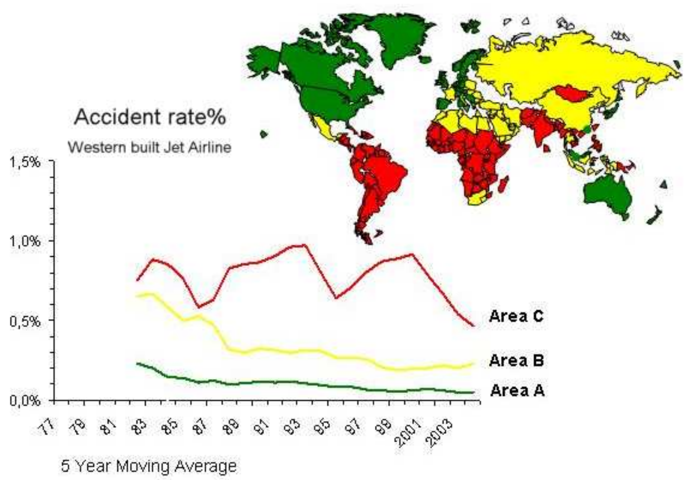

Figure 1.3: Accident rate for Social Area

B. Intermediate

C. Developing Countries

It shows that the average loss ratio for the industrialized countries is less than for the others. There has been a progressive improvement in the developed countries rate which has been fallen for a decade now. European and American airlines have gone more than three years without a crash attributed to aircraft or pilot failure. The last was in November 2001 when an American Airlines jet plunged into Rockaway Beach soon after taking off from New York, killing 260 people. For the developing countries, the same period shows a smaller but still improvement of the safety.

There is as well a classification of the operation areas of the airline into eight regions, called Geographic Areas (GAC).

GAC:

1. USA and Canada
2. Europe
3. South and Central America

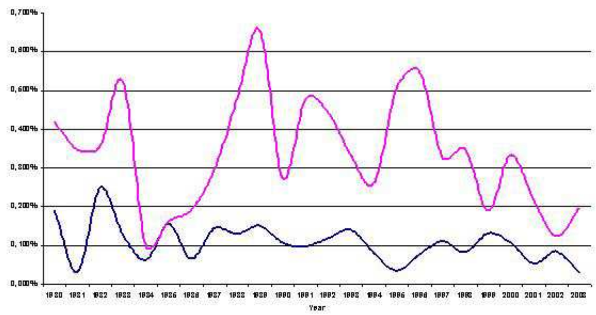

Figure 1.4: Accident rate for Large and Small Operators

4. Africa
5. Middle East
6. Far East and Australia
7. Eastern Europe
8. former Soviet Union

1.2.4 Fleet value

It is said that many small and medium sized companies have exposures that could produce significant claims. Large claims don't just happen to the large companies; a claim for a small to medium sized company can proportionately be more threatening. Most often it is the combination of pilot competence, maintenance quality, financial stability and management attitude.

The main and traditional tool for handling the risks is to relocate them with use of insurances [23]. That brings a tendency to overlook other solutions besides the insurance. In order to prevent major losses it is important to have a significant risk management thinking throughout the organization. It ought to be in every company's interest to be aware of the present risks and to have a clear line of action in case of an unexpected event. All staff at all levels have to be trained to deal with the consequences of a disaster. With this in mind, it is useful to define "safety performance", as performance which includes both negative outcomes (like accidents and incidents) and positive outcomes (like safe uneventful flights). The main principle of the risk management would be to ensure that most/all "safety performance" outcomes are positive [25].

Carrying out the risk management generally obstructed be the size of the company. There is strong correlation between the level of awareness of risk management and the implementation of it. Management guidelines affect the performance of risk management. The companies with a risk management policy often get an improved insurance coverage and have reached further in the security and safety (se fig. 1.4).

I separate small and large airlines by fleet value (FLV).

FLV:

1. Fleet value less than 1 Billion USD.
2. Fleet value more than 1 Billion USD.

1.2.5 Time

There has been a noticeable decrease in the number of fatal airline accidents in the past ten years (se figure 1.5). The rolling three-year average illustrates the average number of 50 fatal accidents in 1994 has been reduced to a current average of 35. And on more interesting note, the year 2003 showed a record low, contrasting to the 72 fatal accidents and 2,539 fatalities in 1972.

Technology, risk management, safety management systems, engineering reliability and maintainability were the foundations to the strategies that contributed to reducing the accident rate from 1,9 per million departures in 1994 to 0,68 in 2003.

The graph below (1.6) illustrates that worldwide airline traffic faces an average of one thousand passenger fatalities. Although the number of fatalities is rather stable over the years, the cost of any occurrence greatly varies from accident to accident. Accidents and incident rates have declined noticeably over time. This is even through the overall expansion of the industry. Air transportation seems to be getting safer. Did the advanced technology finally came to grasp with the risk of flight?

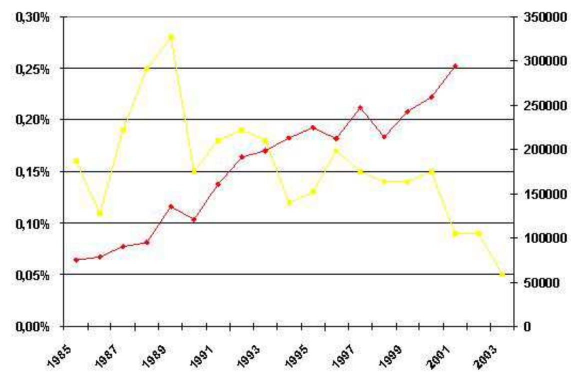

Figure 1.5: Departures vs. Accident rate

With those assumptions the absolute number of the Total Losses for Western built aircraft will, optimistically, remain stable, pessimistically - the time will demonstrate.

I have defined TIME by separating the claims into four classes.

TIME:

1. 1971 — 1978
2. 1979 — 1986
3. 1987 — 1994
4. 1995 — 2003

1.3 Method

For finding the factors that influence the occurence of the Total Loss (TL) and of the occurence of the Major Partial Loss (MPL), I use a multiple regression method known as the Generalized Linear Model (GLM). The benefit of using GLM over other methods is that the model is formulated with a

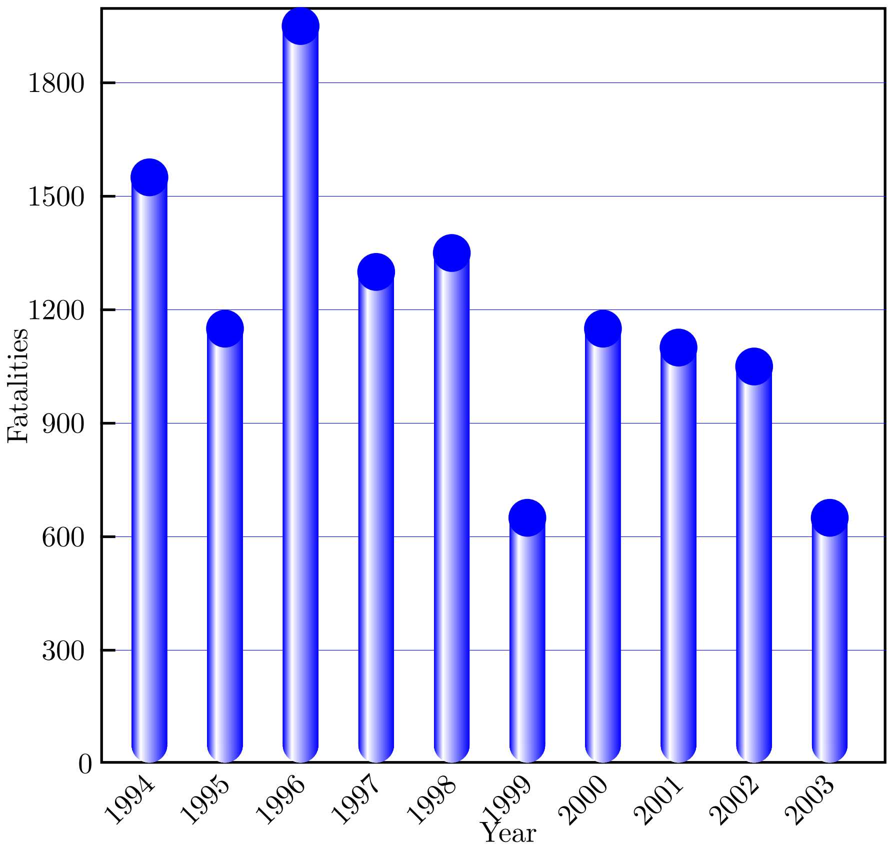

Figure 1.6: Severity of accidents

statistical support. This allows considering all possible factors simultaneously, to group factors, exclude non-significant variables, test all possible combinations in search of interactions and allows standard statistical tests, such as χ^2-tests and F-tests, to be used for comparing models.

This thesis is based on a multifactor approach. That means that for every combination of rating factors, the raw claim frequency is used to calculate expected accident frequency. Later expected accident frequency and average cost for each type of claim are combined to produce theoretical premium rates for each cell. But this paper is mainly concerned with expected accident frequency.

In order to have a reliable theoretical support the main focus was on literature concerning ratemaking, generalized linear models, modeling techniques and aviation. These topics establish the base of this thesis and with a strong connection to the theory, the model for the accident frequency of the commercial airlines catastrophes was developed.

The statistical software package SAS is used for this project.

1.4 Source data

The data base used for the calculations is the Inter Hannover data base Maria Market III and it contains current information about airline companies between 1971 and 2003. To estimate accident frequency the data was divided into two data files. One of the files contained all information concerning the accidents and the second file covered exposure information. This paper mainly focuses on hull insurance and two groups of claims such as Major Partial claims (between 1-10 million USD) and Total Loss claims that exceed 10 million USD. The objective was to calculate the frequency of accident for hull insurance for the two claims groups separately and then combine the two pricing models into one model.

Chapter 2

The statistical model

We would like to aim for a model that will reflect the experience gained in the last thirty years. Find all recognisable and relevant factors that influence the expected accident frequency. Especially, to calculate a response variable (in this case accident frequency) using a series of the explanatory variables (or rating factors).

Nowadays, large data storage capabilities allow for a greater number of rating and underwriting variables to be tracked and analyzed. For that purpose, many factors can be found to be predictive of frequency or loss severity. An example of possible aviation rating factors is shown in figure 2.1.

Unfortunately there are still limitations associated with multivariate approache:

- low volumes of data across dimensions, and
- variables with large number of rating levels

To decrease the effect of these limitations, we apply dimension reduction techniques to our multivariate solution. This will reduce the dimensionality of a data set by reducing a number of underlying factors. Given a table of data, the columns represent the facts of the data, and rows represent the observations. Because the number of rating levels grows exponentially with the inclusion of each level, the dimension reduction technique focuses on the reducing the number of columns (associations among variables) and the number of rows (associations among observations).

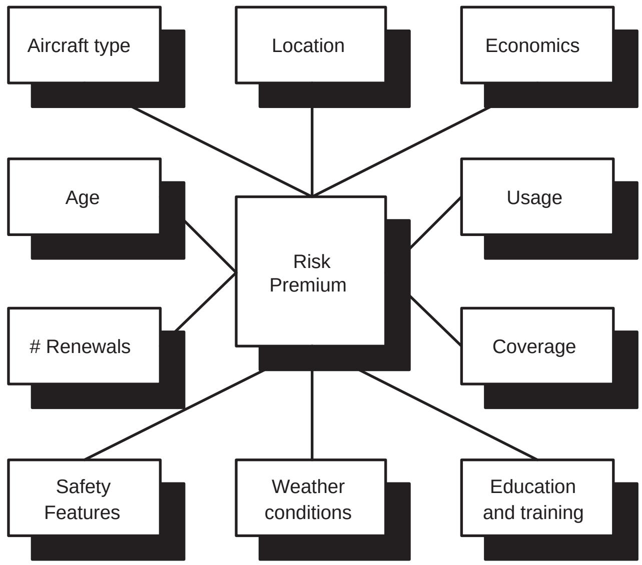

Figure 2.1: Risk Premium

The goal is to identify a number of linear combinations of the original factors that will account for a sufficient amount of information exhibited in the original set.

Let us assume that $i = (j, k, l, m, n, o, p)$ is the combination of the rating factors: aircraft class (ACL), generation (GEN), manufacturer (MFR), social area (SOC), fleet value (FLV), geographic area (GAC) and variable time (TIME). Next, assume that X_i is the number of accidents (as distinct from the number of damaged aircrafts) for $i = (j, k, l, m, n, o, p)$ and w_i is the exposure period of the risk. Under the assumption of independency, homogeneity and that accidents happens independent of each other, X_i has Poisson distribution.

$$X_i \sim \text{Po}(\mu_i w_i)$$

with discrete probability distribution [7]

$$f_{X_i}(x_i; \mu_i) = e^{-\mu_i w_i} \frac{(\mu_i w_i)^{x_i}}{x_i!} \tag{2.1}$$

$$x_i = 0, 1, 2, ...$$

The accident frequency Y_i for the combination of the observations $i = (j, k, l, m, n, o, p)$, is the number of accidents X_i divided by the exposure period w_i, i.e. $Y_i = \frac{X_i}{w_i}$.

$$E(X_i) = \mu_i w_i \Longrightarrow E(Y_i) = \mu_i \tag{2.2}$$

$$Var(X_i) = w_i\mu_i \Longrightarrow Var(Y_i) = \mu_i/w_i \tag{2.3}$$

[7]

with discrete probability distribution

$$f_{Y_i}(y_i;\mu_i) = P(Y_i = y_i) = P(X_i = w_iy_i) = e^{-\mu_i w_i}\frac{(\mu_i w_i)^{w_iy_i}}{w_iy_i!} \tag{2.4}$$

$$y_i = 0, \frac{1}{w_i}, \frac{2}{w_i}, ...$$

Within non-life actuarial practice this *"relative"* **Poisson distribution** commonly used for the accident frequency and claims assumptions. [7]

The benefits with this distribution is that parameter estimates stay unchanged if group by unique combination of rating factor and is unvarying to measures of time. Furthermore, Poisson distribution makes it easier to update parameters by adding experience to prior experience and exposure to prior exposure.

Going back to the expectation of accident frequency $E(Y_i) = \mu_i$, instead of modeling μ_i we can model the monotonous function $g(\cdot)$ of μ_i where $i = (j, k, l, m, n, o, p)$. Let

$$g(\mu_i) = \log(\mu_i)$$

This log-link form is common for calculations of frequency models and guarantees that the fitted frequencies are positive (Brockman and Wright [1]).

That is

$$\log(\mu_i) = \alpha + \beta_j + \gamma_k + \delta_l + \epsilon_m + \zeta_n + \eta_o + \theta_p$$

Taking exponential on both sides gives

$$\mu_i = e^{(\alpha+\beta_j+\gamma_k+\delta_l+\epsilon_m+\zeta_n+\eta_o+\theta_p)}$$

or

$$= A \times B_j \times C_k \times D_l \times E_m \times F_n \times G_o \times H_p$$

$$\text{where } B_j = e^{\beta_j} \text{ etc}$$

and $i = (j, k, l, m, n, o, p)$.

If there is interaction between factors, we can include in the model terms of type κ_{jm}. The term κ_{jm} in this case, represent interaction between the factors aircraft class (ACL) and social area (SOC), i.e. the effect of one variable depends on the other variable. Then the model becomes

$$\mu_i = e^{\alpha+\beta_j+\gamma_k+\delta_l+\epsilon_m+\zeta_n+\eta_o+\theta_p+\kappa_{jm}}$$

or

$$\log(\mu_i) = \alpha + \beta_j + \gamma_k + \delta_l + \epsilon_m + \zeta_n + \eta_o + \theta_p + \kappa_{jm}$$

the model with interaction.

Subsections 3.2 and 3.3 begin with the examination of parameters and hypothesis testing. The fit of the model to the data can be considered through the *deviance*:

Let $l(\hat{\mu})$ be the log likelihood of the current model at the Maximum Likelihood estimate, and let $l(y)$ be the log likelihood of the full (saturated) model. The deviance D is defined as :

$$D = 2(l(y) - l(\hat{\mu}))$$

If the model is true, the deviance will asymptotically tend towards the χ^2-distribution.

A second and more important use of the deviance is in evaluating competing models. The test starts with two hypotheses, one called the null hypothesis H_0 and another called an alternative hypothesis H_1. Assume that a model gives a deviance D_1 on df_1 degrees of freedom, and that a simpler model produces deviance D_2 on df_2 degrees of freedom. For comparison of the two models it is necessary to calculate the difference in deviance, $(D_2 - D_1)$, and relate this to the χ^2 distribution with $(df_2 - df_1)$ degrees of freedom. This would give a large sample test of the significance of the parameters that are included i an alternative model H_1 but not in the null model H_0. This, of course, requires that the parameters included in model H_0 is a subset of the parameters of model H_1, i.e., that the models are hierarchic [7] .

If the p-value for the observed data is significant, the null hypothesis H_0 is rejected and the alternative hypothesis H_1 is set as the result of the observed data.

The Proc GenMod in SAS offers ("Type 3 test" [7]) p-values both for the likelihood test with χ^2-distrubution and for the F-test with F-distribution.

A common weakness of this approach is that the choice of the significance level is subjective. In fact by changing the significance level, any wanted result can be obtained.

Chapter 3

Estimating parameters and hypothesis testing

3.1 Rating model

Since one of the purposes of this paper is to design a model that may be used as a tool for further analyses and evaluations, it is vital for the result to be clear and understandable.

There are a few basic steps that must be followed in order to obtain a useful model:

- include all factors in the initial analysis, (both factors with continuous scale and categorical scale can be used in the model)
- test interactions before simplifying the model, otherwise there is a risk of removing a significant parameter,
- handle related factors (e.g. SOC and GAC),
- compare two alternative models, with one model as a subset of the other,
- exclude non - significant variables from the model. This is easiest and most straight-forward way to simplify a model.
- while the factor might be significant, it may be possible to band certain levels within the factor to create a more parsimonious model. The standard error of the parameter estimate identifies the potential grouping,

- and at the end of the day, for every conclusion about the model there has to be a reasonable explanation for the parameter's predictive value.

There is also some additional consideration in the analysis: should the observations be weighted? But considering that the variability of a particular record will be proportional to the exposure, this will bring a natural weight in terms of exposures. For example, the severity is more credible if weighted by the number of claims they are based on. Frequencies are more credible if weighted by exposures.

This paper focuses on hull insurance with claims divided into two groups: Major Partial claims (between 1-10 million USD) and Total Loss claims that exceed 10 million USD. It provides separate subsections for each of these claims, with different rating models.

It was intended initially to use both geographic area and social area for building of the pricing model, but there is certain association between these two rating factors. The rating system MARTHA uses geographic area to classify social area for a particular airline. This classification is based on the Total Loss Factor (the probability for a Total Loss for one aircraft under one year). The Total Loss Factor is calculated for each individual airline, based on the number of accidents and number of aircraft in operation for the last seven years. Then, with the underwriter's experience and intuition, airlines are categorized in three social area classes. This way both rating factors are linked to the pricing method.

The final rating model now includes the rating factors: aircraft class (ACL), generation (GEN), manufacture (MFR), social area (SOC), fleet value (FLV) and time factor (TIME).

$$\log(\mu_{jklmnp}) = \alpha + \beta_j + \gamma_k + \delta_l + \epsilon_m + \zeta_n + \theta_p$$

The levels for these factors are represented in table (3.1) below:

Level	ACL	GEN	MFR	SOC	FLV	TIME
1	LR	1	BOE	USA, Europe Australia	$<$ 1 Bill. USD	1971–1978
2	SR	2	ABI	Intermediate Countries	$>$ 1 Bill. USD	1979–1986
3	MR	3	MDO	Developing Countries		1987–1994
4			BAS			1995–2003
5			FOK			
6			Others			

Table 3.1: Rating factors

3.2 Calibration of the model for Major Partial Loss (MPL)

The total number of rating cells in the model is $3 \times 3 \times 6 \times 3 \times 2 \times 4 = 1296$ for the accident frequency analysis. Each cell consists of nine items: the level of each of the six rating factors, the exposure w_i (i.e. number of policy years) and the number of accidents X_i, where $i = (j, k, l, m, n, p)$ every combination of rating factors. The table (3.2) shows a fraction of the design matrix for a particular combination of rating parameters for Major Partial losses:

	ACL	GEN	MFR	SOC	FLV	TIME	number of accidents X_i	exposure w_i
Level	1	1	1	1	1	1	$\emptyset^1$	1257,06
	1	1	1	1	1	2	1	194,79
	1	1	1	1	1	3	$\emptyset^1$	30,30
	1	1	1	2	1	3	$\emptyset^1$	119,21
	1	1	1	3	1	2	1	453,97
	1	1	1	2	1	2	4	404,63
	1	1	1	2	1	3	$\emptyset^1$	119,21

[1] Accidentally empty cell

Table 3.2: Number of reported damage accidents and aggregate exposure by aircraft type, generation, manufacture, social area, fleet value and time changes

The base level for each factor is chosen according to the volume of exposure for each level and tables below show the number of accidents, exposure and claims frequency for each factor.

ACL

level	number of accidents X_i	exposure w_i (number of policy years)	claims freq (promille)
1	67	53789,44	1,246
2	63	127084,83	0,496
3	48	42772,76	1,122
Total	178	223647,03	0,796

GEN

level	number of accidents X_i	exposure w_i (number of policy years)	claims freq (promille)
1	12	19700,53	0,609
2	91	125756,32	0,723
3	75	78190,18	0,959
Total	178	223647,03	0,796

MFR

level	number of accidents X_i	exposure w_i (number of policy years)	claims freq (promille)
1	71	127281,47	0,55
2	37	18386,19	2,012
3	43	56434,10	0,762
4	13	7082,05	1,836
5	7	6363,80	1,091
6	7	8099,43	0,864
Total	178	223647,03	0,796

SOC

level	number of accidents X_i	exposure w_i (number of policy years)	claims freq (promille)
1	78	167343,76	0,46
2	59	37918,22	1,556
3	41	18385,05	2,23
Total	178	223647,03	0,796

Or with the other words, the base level for each factor is:

FLV

level	number of accidents X_i	exposure w_i (number of policy years)	claims freq (promille)
1	85	72591,80	1,17
2	93	151055,22	0,616
Total	178	223647,03	0,796

TIME

level	number of accidents X_i	exposure w_i (number of policy years)	claims freq (promille)
1	1	26520,07	0,04
2	38	45328,04	0,84
3	66	66956,75	0,99
4	73	84842,17	0,86
Total	178	223647,03	0,796

Table 3.3: Number of accidents, exposure and claims frequency for each factor

ACL– level 2	GEN– level 2	MFR– level 1
SOC– level 1	FLV– level 2	TIME– level 4

This leads to the model:

$$\log(\text{expected number of accidents}) = \begin{array}{rcl} \alpha & + & \text{log(exposure)} \\ & + & \text{effect due to aircraft type} \\ & + & \text{effect due to generation} \\ & + & \text{effect due to manufacture} \\ & + & \text{effect due to social area} \\ & + & \text{effect due to fleet value} \\ & + & \text{effect due to time changes} \end{array}$$

and with base levels in bold:

$$\log(\text{expected number of accidents}) = \begin{array}{rcll} \alpha & + & \text{log(exposure)} & \\ & + & \beta_1 & \text{when ACL is LR} \\ & + & \boldsymbol{\beta_2} & \text{when ACL is SR} \\ & + & \beta_3 & \text{when ACL is MR} \\ & + & \gamma_1 & \text{when GEN} = 1 \\ & + & \boldsymbol{\gamma_2} & \text{when GEN} = 2 \\ & + & \gamma_3 & \text{when GEN} = 3 \\ & + & ... & \end{array}$$

or simply

$$\log(\mu_{jklmnp}) = \alpha + \beta_j + \gamma_k + \delta_l + \epsilon_m + \zeta_n + \theta_p$$

where α is the constant parameter, β_j is aircraft class (ACL), γ_k is the effect of the variable generation (GEN), δ_l is variable manufacture (MFR), ϵ_m is the effect of the social area (SOC), ζ_n is fleet value (FLV) and θ_p is the effect of the variable time (TIME) on the data.

The tables in next subsections, show the results of testing the fit of a multiple Poisson model to the observed accident frequency $Y_i = \frac{X_i}{w_i}$,where $i = (j, k, l, m, n, p)$, using SAS code given in the appendix.

3.2.1 Factors that have influence on the probability of an MPL event

I start with the testing of interactions before simplifying the model. It begins with two hypotheses. Since, I am working with the hierarchic models, it makes sense to compare two alternative models, with one as a subset of the other. Assume that a model gives a deviance D_1 on df_1 degrees of freedom, and that a simpler model produces deviance D_2 on df_2 degrees of freedom. For comparison of the two models it is necessary to calculate the difference in deviance, $(D_2 - D_1)$, and relate this to the χ^2 distribution with $(df_2 - df_1)$ degrees of freedom. [7] .

The sample of the analysis is represented in the table (3.4). For more detailed information about the interaction test results se table (3.5)

Null Hypoth. H_0	Alter. Hypoth. H_1	$\Pr \geq \chi^2$ versus null hypoth.
All variables (no interaction)	All incl. $SOC * FLV$	**0,0189**
All variables (no interaction)	All incl. $GEN * SOC$	0,0940
All variables (no interaction)	All incl. $GEN * FLV$	0,3698
All variables (no interaction)	All incl. $ACL * SOC$	**0,0030**
All variables (no interaction)	All incl. $ACL * FLV$	0,7001

Table 3.4: Interactions, Major Partial Losses

Note The most significant interaction terms for the accident frequency on this basis are $SOC * FLV$ and $ACL * SOC$. Because interaction terms make a fitted model more difficult to understand, it is preferable to include interaction factors in the model only if they improve the data so much that the chance of this would be less than 1/20 (5%) if the interaction factor had no impact. Table (3.4) show that the probability value for $ACL*SOC$ is 0,0030 which is significant at the 5% significance level. The probability value for the interaction $SOC*FLV$ is 0,0189 which is also significant at the 5% significance level.

Table 3.5: LR Statistics For Type 3 Analysis

Null Hypoth. H_0	Alt. Hypoth. H_1	F Value	Pr > F	Chi- Square	Pr $\geq \chi^2$	Significance
All Variables	All incl. SOC*FLV	3,97	0,0197	7,94	**0,0189**	***
All Var.	All incl. GEN*SOC	1,98	0,0964	7,93	0,0940	no
All Var.	All incl. GEN*FLV	0,99	0,3707	1,99	0,3698	no
All Var.	All incl. ACL*SOC	4,00	0,0034	16,01	**0,0030**	***
All Var.	All incl. ACL*FLV	0,36	0,7003	0,71	0,7001	no
All Var.	All incl. GEN*MFR	0,58	0,4466	0,58	0,4461	no
All Var.	All incl. FLV*TIME	2,44	0,0642	7,32	0,0625	no
All Var.	All incl. ACL*MFR	1,33	0,25	1,33	0,2493	no
All Var.	All incl. ACL*GEN	1,43	0,2240	5,71	0,2217	no
All Var.	All incl. ACL*TIME	1,18	0,3141	7,10	0,3115	no
All Var.	All incl. GEN*TIME	0,45	0,6403	0,89	0,6399	no
All Var.	All incl. MFR*SOC	1,22	0,2758	12,21	0.2714	no
All Var.	All incl. MFR*FLV	2,83	0,0934	2,83	0,0926	no
All Var.	All incl. MFR*TIME	0,83	0,6393	12,51	0,6399	no
All Var.	All incl. SOC*TIME	1,32	0,2517	1,32	0,2510	no
All Var. and ACL*SOC	All Var. and ACL*SOC and SOC*FLV	6,14	0,0024	12,29	**0,0021**	***

Following the examination of interactions, it is now time to examine the significance of each rating factor for the model and to group classes within each factor to more compact clusters.

At first, one might think, that a good model is the one that fits data very well i.e. that places fitted values very close to existing data. By including enough parameters in the model it is possible to make it fit the data close and by having as many parameters as observations, model, without a doubt, will fit perfectly. With this, however, there is no reduction in complexity. This *simplicity*, is a essential feature of the rating model, with the ambition not to include unnecessary extra parameters.

The process starts with the model which includes all rating factors. Part of the output is as follows:

```
                    The GENMOD Procedure
                     Model Information
Data Set                                 WORK.RELATION2
Distribution                             Poisson
Link Function                            log
Dependent Variable                       SKADEFREKVENS
Scale Weight variable                    DURATION
```

The output in the table (3.6) below, contains an Analysis of Parameter estimates. This gives estimates of model parameters, their standard errors, and a Wald test of each parameter in form of $\geq \chi^2$test.

LR Statistics For Type 3 Analysis

Source	Num DF	F Value	Pr > F	Chi-Square	Pr > Chi-Squ
ACL	2	93.47	<.0001	186.94	<.0001
GEN	2	23.26	<.0001	46.53	<.0001
MFR	5	5.63	<.0001	28.13	<.0001
SOC	2	26.74	<.0001	53.49	<.0001
FLV	1	26.67	<.0001	26.67	<.0001
TIME	3	30.60	<.0001	91.81	<.0001

Note The first parameter Intercept is the cell with each factor at the base level. The estimated accident frequency in this cell is exp(0,0291) = 1,0295. The remaining parameters represent how accident frequency in other cells relates to this cell. For example, the accident frequency is higher in a cell with ACL *LR* than in base cell ACL *SR* by factor

Parameter	Estimate	Standard Error	Pr $\geq \chi^2$
Intercept	0,0291	0,1338	
ACL *LR*	1,6009	0,1171	**<,0001**
ACL *SR*	BAS	.	.
ACL *MR*	0,8030	0,1470	**<,0001**
GEN *1*	-1,3253	0,2281	**<,0001**
GEN *2*	BAS	.	.
GEN *3*	-0,2064	0,1212	0,0887
MFR *Boeing*	BAS	.	.
MFR *Airbus*	0,2977	0,1510	**0,0487**
MFR *McDonell Douglas*	-0,2370	0,1204	**0,0489**
MFR *BAS*	0,5591	0,2434	**0,0216**
MFR *Fokker*	0,1244	0,3045	0,6828
MFR *Others*	-0,9926	0,3008	**0,0010**
SOC *USA,Europe*	BAS	.	.
SOC *Intermediate*	0,6916	0,1094	**<,0001**
SOC *Develop. Countries*	0,8142	0,1362	**<,0001**
FLV < *1 Bill$*	0,5602	0,1067	**<,0001**
FLV > *1 Bill$*	BAS	.	.
TIME *1971 - 1978*	-4,1791	1,0456	**<,0001**
TIME *1979 - 1986*	-0,2377	0,1379	0,0847
TIME *1987 - 1994*	0,0034	0,1084	0,9747
TIME *1995 - 2003*	BAS	.	.

Table 3.6: Parameter estimates for the base factors, Major Partial Losses

exp (1,6009) = 4,957. Similarly, accident frequency is lower for GEN *1* than the base level GEN *2* by a factor exp (-1,3253) = 0,266.

The useful rule-of-thumb in model building is to keep in the model all terms that are significant at 5% level. In this case, the process would start with the full model. It will then move to the more predictive rating model with the simplified rating structure.

Note For the factor GEN *3* the estimate -0,2064 at this level is very unreliable and insignificant. It is better to join this factor into new cluster with base level GEN *2*. The factor GEN *1*, on the other hand, is significant at 5% level, which can be interpreted as generation *1* aircrafts are the safest. But knowing the past statistics, nature of the flights and usage of the first generation aircrafts it is reasonable to question the quality of the old data.

Parameter	*exp*(estimate)	*Lower 95%*	*Upper 95%*
ACL *LR*	4,957	3,941	6,236
ACL *SR*	1,00	-	-
ACL *MR*	2,232	1,673	2,978
GEN *1*	0,266	0,170	0,416
GEN *2*	1,00	-	-
GEN *3*	0,814	0,641	1,032
MFR *Boeing*	1,00	-	-
MFR *Airbus*	1,347	1,002	1,810
MFR *McD Doug*	0,789	0,623	0,999
MFR *BAS*	1,749	1,086	2,818
MFR *Fokker*	1,132	0,624	2,057
MFR *Others*	0,371	0,206	0,668
SOC *USA,Europe*	1,00	-	-
SOC *Intermediate*	1,997	1,611	2,475
SOC *Dev.*	2,257	1,728	2,948
FLV < *1 Bill$*	1,751	1,421	2,158
FLV > *1 Bill$*	1,00	-	-
TIME *1971 - 1978*	0,015	0,002	0,119
TIME *1979 - 1986*	0,788	0,602	1,033
TIME *1987 - 1994*	1,003	0,811	1,241
TIME *1995 - 2003*	1,00	-	-

Table 3.7: Confidence intervall for the base factors, Major Partial Losses

Note The factor MFR *Fokker* with the small amount of data sample will give more accurate test when combined with the MFR *Others* which is significant at level of 5%.

Note The low accident frequency of the factor TIME *1971 - 1978* and insignificance of the remaining levels for this factor, can appear some what strange at this point. In this cell we have only one major partial accident. This depends on the fact that there is not much information and data that we have from 1970's. The regular and systematic gathering of the data started in the early 80's. It is reasonable to assume that the safety, technology and education of the pilots, continuously improves over the time. Certain improvements are incorporated with each introduction of the new aircraft types and TIME factor at this stage can be in conflict with the aircraft generation factor. After further analysis with altered clustering, rating factor TIME was deleted from the model because of the insignificance for the accident frequency.

This suggests a model that includes all initial factors but with different clustering of the levels. Parameter estimates are given in table (3.8).

Factor	Estimate	Prob-value
Intercept	-0,0599	
ACL *LR*	1,5435	<,0001
ACL *SR*	BAS	
ACL *MR*	0,7011	<,0001
GEN *1*	-1,3203	<,0001
GEN *2*	BAS	
MFR *Boeing*	BAS	
MFR *Airbus*	0,2828	0,0570
MFR *McD Doug*	-0,2228	0,0647
MFR *BAS*	0,5141	0,0346
MFR *Others*	-0,5506	0,0107
SOC *USA,Europe*	BAS	
SOC *Intermediate*	0,7113	<,0001
SOC *Dev.*	0,8730	<,0001
FLV < *1 Bill$*	0,5469	<,0001
FLV > *1 Bill$*	BAS	

Table 3.8: Rating parameters and their significance for the model

Note (*table 3.8)* In this model, factor MFR *McD Doug* (p-value 0,0647) is not significant for the test at a 5%-significance level and therefore moved to the factor MFR *Boeing* because of the purchase agreement between two companies. For the factor MFR *Airbus*, which is slightly insignificant it is reasonable to test this factor separately without any changes. However after further analysis factor MFR was deleted from the model because of the insignificance (for result of the test se table 3.9).

Note (*table 3.9)* The exponential model without rating factor MFR.

Now, it remains to investigate whether the interactions between the terms $SOC * FLV$ and $ACL * SOC$ in the model would improve the fit. To check this, interaction terms were added to the model (se LR Statistics For Type 3 Analysis).

```
LR Statistics For Type 3 Analysis
```

Factor	Estimate	Prob-value
Intercept	-0,1050	
ACL *LR*	1,4492	<,0001
ACL *SR*	BAS	
ACL *MR*	0,7553	<,0001
GEN *1*	-1,3054	<,0001
GEN *2*	BAS	
SOC *USA,Europe*	BAS	
SOC *Intermediate*	0,8007	<,0001
SOC *Dev.*	0,95421	<,0001
FLV < *1 Bill$*	0,4884	<,0001
FLV > *1 Bill$*	BAS	

Table 3.9: Final rating parameters and their significance for the model without MFR factor

Source	Num DF	F Value	Pr > F	Chi-Square	Pr > Chi-Squ
ACL	2	171.00	<.0001	342.00	<.0001
GEN	1	142.12	<.0001	142.12	<.0001
SOC	2	143.10	<.0001	286.20	<.0001
FLV	1	34.39	<.0001	34.39	<.0001
SOC*FLV	2	19.59	<.0001	39.18	<.0001
ACL*SOC	4	17.56	<.0001	70.23	<.0001

This suggested a model that includes only aircraft class, aircraft generation, social area and fleet value together with two-ways interactions $SOC * FLV$ and $ACL * SOC$.

Since the interaction between aircraft class and social area and the interaction between social area and fleet value are included in the model, the main effects of the aircraft class, social area and fleet value should also be included.

The presence of an interaction, $ACL * SOC$, means that combination of the range of the flight and geography of the flight have a certain influence on the probability of the accident. This without a doubt can be explained by the fact that most of the companies in developing countries are operating on domestic and medium range flights only.

The presence of an interaction, $SOC * FLV$, shows the importance of the social area and fleet value for the frequency of Major Partial Losses. Right combination of large company and industrialised country can radically improve the safety of flying. This without a doubt brings better manufacturing service, maintenance and improved routines.

The resulting model has the deviance/df $= 309$ compared to the saturated model deviance/df $= 1081$.

The resulting estimated model for the probability of the Major Partial Losses is:

$$\log\left(\text{expected number of accidents}\right) = \begin{array}{rl} \alpha & + \ \log(\text{exposure}) \\ & + \ \text{effect due to aircraft type} \\ & + \ \text{effect due to generation} \\ & + \ \text{effect due to social area} \\ & + \ \text{effect due to fleet value} \\ & + \ \text{effect due to ACL*SOC} \\ & + \ \text{effect due to SOC*FLV} \end{array}$$

and with more complete level information:

$$\log\left(\text{expected number of accidents}\right) = \begin{array}{rll} \alpha & + \ \log(\text{exposure}) & \\ & + \ \beta_1 & \text{when ACL is LR} \\ & + \ \beta_2 & \text{when ACL is SR} \\ & + \ \beta_3 & \text{when ACL is MR} \\ & + \ \gamma_1 & \text{when GEN} = 1 \\ & + \ \gamma_{2,3} & \text{when GEN} = 2\text{, GEN} = 3 \\ & + \ \epsilon_1 & \text{when SOC (USA,Europe)} \\ & + \ \epsilon_2 & \text{when SOC (Intermediate)} \\ & + \ \epsilon_3 & \text{when SOC (Dev.)} \\ & + \ \zeta_1 & \text{when FLV (}< 1 \text{ Bill USD)} \\ & + \ \zeta_2 & \text{when FLV (}> 1 \text{ Bill USD} \\ & + \ \kappa_{jm} & \text{with interaction ACL*SOC} \\ & + \ \rho_{mn} & \text{with interaction SOC*FLV} \end{array}$$

3.3 Calibration of the model for Total Loss (TL)

The objective of this evaluation is to show the factors that have a significant infuence on the Total Loss accidents and represent them in the rating model.

The base levels for each factor are chosen accordingly to the volume of exposure for each level with $i = (j, k, l, m, n, p)$. Tables below, show the number of accidents, exposure and claims frequency for each factor.

ACL

level	number of accidents X_i	exposure w_i (number of policy years)	claims freq (promille)
1	66	53770,21	1,23
2	70	126953,42	0,55
3	29	42652,58	0,68
Total	165	223376,21	0,74

GEN

level	number of accidents X_i	exposure w_i (number of policy years)	claims freq (promille)
1	30	19700,53	1,52
2	90	125686,64	0,72
3	45	77989,04	0,58
Total	165	223376,21	0,74

MFR

level	number of accidents X_i	exposure w_i (number of policy years)	claims freq (promille)
1	65	127117,56	0,51
2	21	18304,05	1,15
3	50	56427,43	0,89
4	12	7067,92	1,69
5	11	6363,80	1,73
6	6	8095,44	0,74
Total	165	223376,21	0,74

The base levels are the same as in the previous subsection:

ACL– level 2	GEN– level 2	MFR– level 1
SOC– level 1	FLV– level 2	TIME– level 4

SOC

level	number of accidents X_i	exposure w_i (number of policy years)	claims freq (promille)
1	63	167343,76	0,37
2	48	37895,86	1,27
3	54	18136,58	2,98
Total	165	223376,21	0,74

FLV

level	number of accidents X_i	exposure w_i (number of policy years)	claims freq (promille)
1	78	72421,01	1,08
2	87	150955,20	0,58
Total	165	223376,21	0,74

TIME

level	number of accidents X_i	exposure w_i (number of policy years)	claims freq (promille)
1	20	26520,07	0,74
2	34	45328,04	0,75
3	54	66945,88	0,81
4	57	84582,22	0,67
Total	165	223376,21	0,74

Table 3.10: Number of accidents, exposure and claims frequency for each factor

and rating model is

$$\log(\mu_{jklmnp}) = \alpha + \beta_j + \gamma_k + \delta_l + \epsilon_m + \zeta_n + \theta_p$$

with α as the constant parameter, β_j is the effect of the aircraft class (ACL), γ_k is the variable generation (GEN), δ_l is variable manufacture (MFR), ϵ_m is the effect of the social area (SOC), ζ_n is fleet value (FLV) and θ_p is the effect of the variable time (TIME).

3.3.1 Factors that have influence on the probability of an TL event

The process starts with the saturated model which include all rating factors and proceeds with the investigation whether any interactions between the terms in the model would improve the fit. Since there are six variables, the model was tested with all possible pair interactions.

The results of the testing the interactions are represented in the tables (3.11) and (3.12).

Null Hypoth. H_0	Alter. Hypoth. H_1	Pr $\geq \chi^2$ versus null hypoth.
All variables (no interaction)	All incl. SOC*FLV	0,4316
All variables (no interaction)	All incl. GEN*SOC	0,3620
All variables (no interaction)	All incl. GEN*FLV	0,2058
All variables (no interaction)	All incl. ACL*SOC	0,5862
All variables (no interaction)	All incl. ACL*FLV	0,7991
All variables (no interaction)	All incl. ACL*TIME	0,7488
All variables (no interaction)	All incl. GEN*TIME	0,2433

Table 3.11: Interactions for the base factors, Total Loss

Table 3.12: LR Statistics For Type 3 Analysis

Null Hypoth. H_0	Alt. Hypoth. H_1	F Value	Pr > F	Chi- Square	Pr $\geq \chi^2$	Significance
All Variables	All incl. SOC*FLV	0,95	0,4647	3,81	0,4316	no
All Var.	All incl. GEN*SOC	1,08	0,3637	4,34	0,3620	no
All Var.	All incl. GEN*FLV	1,58	0,2071	3,16	0,2058	no
All Var.	All incl. ACL*SOC	0,53	0,5867	1,07	0,5862	no
All Var.	All incl. ACL*FLV	0,22	0,7992	0,45	0,7991	no
All Var.	All incl. GEN*MFR	1,71	0,1049	11,98	0,1013	no
All Var.	All incl. FLV*TIME	0,97	0,4069	2,91	0,4058	no
All Var.	All incl. ACL*MFR	1,01	0,4263	8,10	0,4241	no
All Var.	All incl. ACL*GEN	0,84	0,5030	3,34	0,5020	no
All Var.	All incl. ACL*TIME	0,58	0,7485	3,46	0,7488	no
All Var.	All incl. GEN*TIME	1,36	0,2440	1,36	0,2433	no
All Var.	All incl. MFR*SOC	1,77	0,0645	17,70	0,0603	no
All Var.	All incl. MFR*FLV	0,10	0,7511	0,10	0,7509	no
All Var.	All incl. SOC*TIME	2,28	0,1320	2,28	0,1312	no

Note None of the interactions are significant for the Total Loss.

A standard analysis of the data material would be to test whether there is dependence between the original rating factors and accident frequency through a χ^2 test. The observed numbers in each cell are assumed to be generated from the Poisson distribution with the log link. Thus, the parameter estimates for this model are as follows in table(3.13) and table (3.14).

Parameter	Estimate	Standard Error	$\Pr \geq \chi^2$
Intercept	-0,0599	0,1383	
ACL *LR*	-0,0154	0,1365	0,9102
ACL *SR*	BAS	.	.
ACL *MR*	-0,1946	0,2006	0,3320
GEN *1*	0,37	0,1692	**0,0292**
GEN *2*	BAS	.	.
GEN *3*	-0,8070	0,1564	**<,0001**
MFR *Boeing*	BAS	.	.
MFR *Airbus*	0,1282	0,2252	0,5691
MFR *McDonell Douglas*	0,4249	0,1171	**0,0003**
MFR *BAS*	0,5505	0,2366	**0,0195**
MFR *Fokker*	0,3914	0,2094	0,0615
MFR *Others*	-0,7887	0,3951	**0,0459**
SOC *USA,Europe*	BAS	.	.
SOC *Intermediate*	1,1575	0,1295	**<,0001**
SOC *Develop. Countries*	1.9565	0,1290	**<,0001**
FLV < *1 Bill$*	0,1940	0,1152	0,0921
FLV > *1 Bill$*	BAS	.	.
TIME *1971 - 1978*	-0,460	0,1905	**0,0157**
TIME *1979 - 1986*	-0,1361	0,1415	0,3363
TIME *1987 - 1994*	0,0729	0,1238	0,5562
TIME *1995 - 2003*	BAS	.	.

Table 3.13: Parameter estimates for the base factors, Total Loss

The Type 3 test for original model produces following results:

```
                LR Statistics For Type 3 Analysis

Source  Num DF  F Value    Pr > F   Chi-Square  Pr > Chi-Squ
ACL          2     0.48    0.6212         0.95        0.6208
GEN          2    16.62    <.0001        33.24        <.0001
```

MFR	5	5.10	0.0002	25.50	0.0001
SOC	2	112.92	<.0001	225.85	<.0001
FLV	1	2.84	0.0927	2.84	0.0919
TIME	3	3.27	0.0213	9.81	0.0203

Parameter	*exp*(estimate)	*Lower 95%*	*Upper 95%*
ACL *LR*	0,985	0,754	1,287
ACL *SR*	1,00	-	-
ACL *MR*	0,823	0,556	1,220
GEN *1*	1,446	1,040	2,015
GEN *2*	1,00	-	-
GEN *3*	0,446	0,328	0,606
MFR *Boeing*	1,00	-	-
MFR *Airbus*	1,137	0,731	1,768
MFR *McD. Doug.*	1,529	1,216	1,924
MFR *BAS*	1,734	1,091	2,757
MFR *Fokker*	1,479	0,981	2,229
MFR *Others*	0,454	0,209	0,986
SOC *USA,Europe*	1,00	-	-
SOC *Intermediate*	3,182	2,468	4,101
SOC *Dev. Count.*	7,074	5,494	9,110
FLV $<$ *1 Bill$*	1,214	0,969	1,522
FLV $>$ *1 Bill$*	1,00	-	-
TIME *1971 - 1978*	0,631	0,435	0,917
TIME *1979 - 1986*	0,873	0,661	1,152
TIME *1987 - 1994*	1,076	0,844	1,371
TIME *1995 - 2003*	1,00	-	-

Table 3.14: Confidence intervall for the base factors, Total Loss

Note(*table 3.13*) There is no indication of the association between aircraft class and probability of accident frequency.

Note There is no indications for the correlation between company size and frequency of the Total Loss accident.

Note For the factor GEN *1* the estimated accident frequency is very unreliable because of the small amount of accidents in this class.

Note The MFR *Fokker* factor does not effect the frequency of the Total Loss accidents. The small amount of data allows us to combine *Fokker* and *Others* with each other.

Table 3.15: LR Statistics For Type 3 Analysis without variables ACL and FLV

Source	Num DF	F Value	Pr > F	Chi-Square	Pr > Chi-Squ
GEN	2	23.83	<.0001	47.65	<.0001
MFR	5	4.60	0.0007	23.00	0.0003
SOC	2	125.51	<.0001	251.02	<.0001
TIME	3	2.33	0.0769	7.00	0.0720

Note The factor MFR *Airbus*,which is insignificant, is reasonable to test first separately without any changes. The clustering with other levels would bring difficulties in understanding of the resulting model. Later on this level is treated as insignificant.

Note British Airspace (MFR *BAS*) significant for both Major Partial and Total Loss claims. But after some changes in clustering, this level became insignificant and tested in the same way with level *Others* and base level *Boeing*.

Note The low accident frequency of the factor TIME *1971 - 1978* and insignificance of the remaining factors can appear some what strange as it was with Major Partial Losses. It is reasonable to assume that the safety and technology continuously improves over the time. Certain improvements are incorporated with each introduction of the new aircraft types and TIME factor are correlated with the aircraft generation factor. Just to test the factor TIME *1971 - 1978* one more time, I am joining factors TIME *1979 - 1986*, TIME *1987 - 1994* and base level TIME *1995 - 2003* into a new cluster. With this clustering, rating factor TIME has not effect on accident frequency i.e.the factor is insignificant (se test result 3.15).

The new main effect model without variables ACL, FLV and TIME are given in table (3.16). There are less parameters than in the original model and factor MFR *McD. Doug.* is tested for the significance.

The factor MFR class *McD. Doug.* is the McDonnell Douglas manufactured aircrafts. The cost for the repairs of the damaged airplane in most of those cases exceeds the agreed value of the airplane. As the result of this, most of the McDonnell Douglas claims were classified as the Total Loss claims. This would explain the small amount of major partial claims and significant frequency of Total Losses. It is significant factor but because of the business changes in the structure of the company it is no longer possible to classify McDonnell Douglas in future as the separate factor.

Factor	Estimate	Prob-value
Intercept	-0,0596	
GEN *1*	0,2274	0,2136
GEN *2*	BAS	
GEN *3*	-0,8228	<,0001
MFR *Boeing*	BAS	
MFR *McD. Doug.*	0,3925	0,0089
SOC *USA,Europe*	BAS	
SOC *Intermediate*	1,2281	<,0001
SOC *Dev.*	2,1317	<,0001

Table 3.16: Rating parameters and their significance for the model

Factor	Estimate	Prob-value
GEN *1, 2*	BAS	
GEN *3*	-0,8837	<,0001
SOC *USA,Europe*	BAS	
SOC *Intermediate*	1,1621	<,0001
SOC *Dev.*	2,0849	<,0001

Table 3.17: Final rating parameters and their significance for the model

The test of the model with just two rating factors, such as generation and social area demonstrated the significance of mentioned variables for the claim frequency model (se table 3.17). The interaction term GEN*SOC is insignificant of the frequency of Total Losses (se table 3.12).

The Type 3 test for rating model produce the following results:

```
                LR Statistics For Type 3 Analysis

Source  Num DF  F Value    Pr > F   Chi-Square  Pr > Chi-Squ
GEN        1    794.57     0.0013    794.57          <.0001
SOC        2   2207.19     0.0005   4414.39          <.0001
```

The model fits well, with the deviance/df = 65 compared to the saturated models deviance/df = 899.

The resulting model for the Total Loss accidents includes :

$$\log\left(\text{expected number of accidents}\right) = \begin{array}{rcl} \alpha & + & \log(\text{exposure}) \\ & + & \text{effect due to generation} \\ & + & \text{effect due to social area} \end{array}$$

and with more complete level information:

$$
\log\left(\text{expected number of accidents}\right) = \begin{array}{ccll}
\alpha & + & \log(\text{exposure}) & \\
 & + & \gamma_{1,2} & \text{when GEN} = 1,\ \text{GEN} = 2 \\
 & + & \gamma_3 & \text{when GEN} = 3 \\
 & + & \epsilon_1 & \text{when SOC (USA,Europe)} \\
 & + & \epsilon_2 & \text{when SOC (Intermediate)} \\
 & + & \epsilon_3 & \text{when SOC (Dev.)}
\end{array}
$$

This model relates with the general knowledge of the underwriters that Total Loss claims depends mainly on generation of the airplane and social area of the operation.

Chapter 4

Conclusion

When using the statistical data to build a model, the process must end with a "winner". But it is important to remember that whatever model is selected, it is only a rough guess of reality.

"All models are wrong, but some models are useful."[18]

With that in mind, the resulting models from the previous chapters, for the Major Partial Loss is:

$$\log(\text{expected number of accidents}) = \begin{array}{lll} \alpha + & \log(\text{exposure}) & \\ + & \beta_j & \text{effect due to ACL} \\ + & \gamma_k & \text{effect due to GEN} \\ + & \epsilon_m & \text{effect due to SOC} \\ + & \zeta_n & \text{effect due to FLV} \\ + & \kappa_{jm} & \text{effect due to ACL*SOC} \\ + & \rho_{mn} & \text{effect due to SOC*FLV} \end{array}$$

and for the Total Loss:

$$\log(\text{expected number of accidents}) = \begin{array}{lll} \alpha + & \log(\text{exposure}) & \\ + & \gamma_k & \text{effect due to GEN} \\ + & \epsilon_m & \text{effect due to SOC} \end{array}$$

The frequency of the Total Losses has drastically decreased over the recent years. This can relates to the two factors: improved engine and system in new generation aircraft and enhanced cockpit technology, which provides better situational awareness to flight crews.

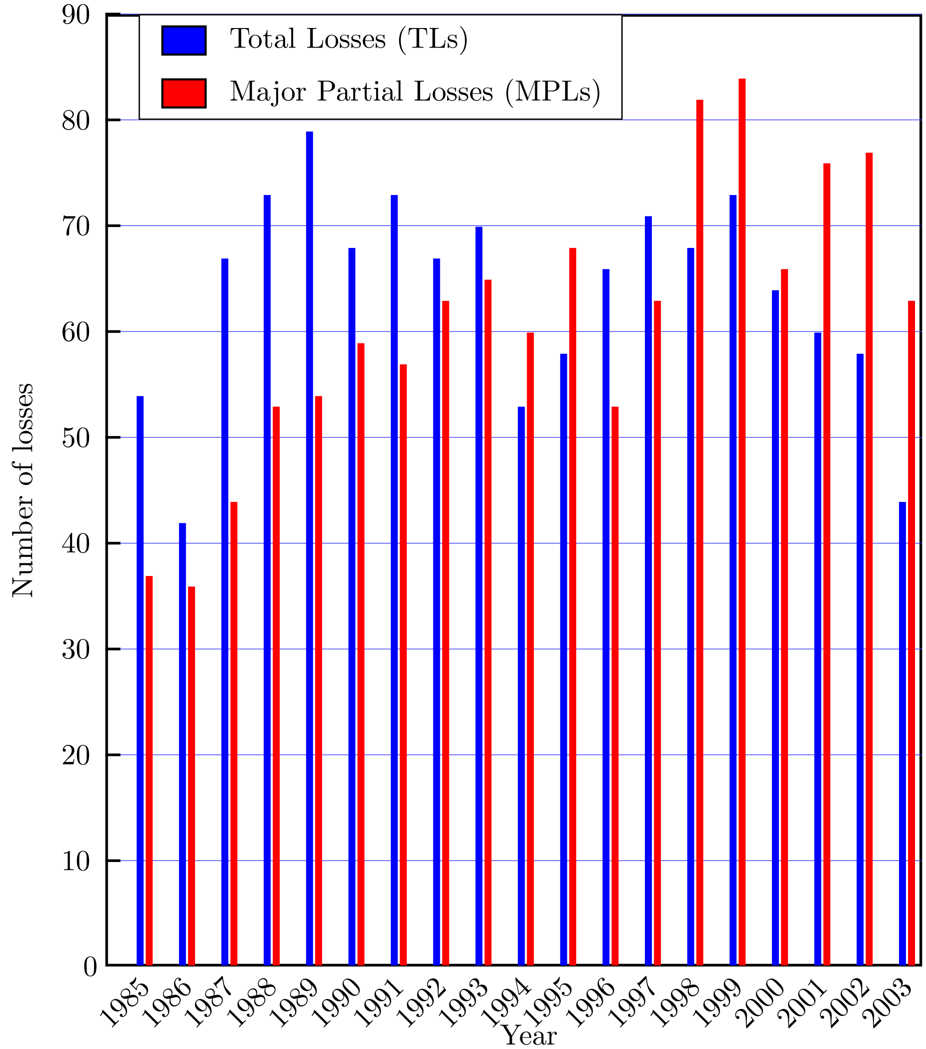

Figure 4.1: Total vs. Major Partial Losses

Nevertheless, while the statistics for Total Losses show the improvement, the constant increase in the number of Major Partial Losses over the past is reason to concern (se figure 4.1). Since 1998, Major Partial Losses have begun to outnumber Total Losses. Note, that the classification of the Major Partial Loss - a loss less than 10 million USD or a minimum 1 million USD - has not been changed to reflect historical inflation. But it seems rational to explain the rise in Major Partial Losses by the increasing complexity of aircraft structures, the use of composite materials and the introduction of more sensitive and fragile equipment on board [26].

In addition, natural catastrophes and other ”events” in addition contribute to the Major Partial Losses. Hailstorm is a typical example. One such storm

Figure 4.2: Hail can cause major damage to an aircraft

in April 1999, damaged 45 aircraft, resulting in a loss of 61 million USD (se figure 4.2).

Despite all the improvements in aviation business modeling of the risk, nevertheless, remains an art. Data will point at several possible models and it is easy to fall in love with one model, to the exclusion of alternatives.

The rating model with only rating variables generation and social area, in some way simplifies the understanding of Major Partial Losses but does not evaluate the probability of the claims. The fact that the model includes interactions complicates the interpretation of the accident frequency but can not be overlooked. We might, even want to try other variables in the model (i.e., pilots flight hours), but at some point over fitting and quality of the underlying data becomes a problem. There is reason to suspect, that the same model for both claims groups will not be 'right' for both of them. Some alternatives will fit data better for total claims than for major partial claims and vice versa. There is always the dilemma between finding the simplest model and which fits the data closest.

The rating system MARTHA uses three factors such as generation of the airplane, social area and fleet value of the airline for calculating the risk premium for Total Loss accident. Because of the complexity of the model for the Major Partial Losses this rating system instead uses specified percentage ratio between Total Losses and Major Partial Losses. There is no reason to make overall changes in this rating system with the principle - a simpler model is preferred. Even the fact that factor fleet value is insignificant for Total Losses, does not validate the exclusion of this factor from the model. It is significant for the Major Partial Losses and provides information for the underwriter's judgment.

The disadvantage with the generalized linear model (GLM) is that it assumes completely multiplicative or completely additive structure. However many rating models have mixed structures. The suggestion for the further studies would be to test mixed structure for accident frequency. That is, to use attained multiplicative results even for estimation of additive factors i.e. to achieve the hybrid model.

In addition to recognize and model customers' premium expectations i.e. to verify the underlying signals from the customer surveys and reflect it in the model.

Furthermore consider about credibility i.e. how much weight to assign to given information. This approach is very judgmental but this is the unique way of handling a unique challenges.

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