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Pricing Equity within the Framework of a Structural Model - Empirical Study of Swedish Investment Companies

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Abstract

The main aim with this thesis is pricing equity of investment companies within, as the name suggests, the framework of a structural model by Reneby (1998). This includes explaining the model and the techniques involved, such as structural models, modelling of each element in the capital structure, different approaches of parameter estimation, sums of correlated lognormals and moment matching. Emphasis is put on investment companies, including an empirical study where model parameters have been estimated by using the observed underlying assets. Results imply that the relation between theoretical and observed equity value appear to be a mean reverting process.

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1 Introduction

This thesis treats the pricing of equity within the framework of the structural model of Reneby (1998). The structural model will be presented and applied in an empirical study of investment companies. Investment companies are particularly of interest as their underlying assets are often traded. Before presenting the Reneby model, structural models are introduced.

Emphasis is put on the latter half, embracing investment companies, data analysis, and approaches for estimating parameters. Within the analysis, focus is set on comparing theoretical and observed equity prices for investment companies. The theoretical equity price is given by applying the proposed pricing formula for equity onto the observed underlying portfolio. Model parameters are estimated empirically by using the observed underlying assets. The distribution of the underlying portfolio is assumed to be lognormal, whereupon the model parameters are given by moment matching.

This approach is primarily applicable on investment companies, and therefore a section on estimating parameters describes two general approaches, one by Ronn & Verma (1986) and one by Duan (1994). One result is that the relations between theoretical and observed prices appear to have mean reverting characters, which motivates the usage of structural models within equity trading. This idea of pricing equity of investment companies show to be applicable as an analysis tool for investment companies.

2 Structural Models

When pricing credit derivatives, equity is given as a by-product. The two general approaches are structural models and reduced form models. Within reduced form models, the credit risk is modelled exogenously, whereas the credit risk is incorporated within a structural model. Reduced form models are often called intensity-based models, as the main emphasis is put on the modelling of the random time of default. Typically the random default time is defined as the jump time of some one-jump process. See Bielecki & Rutkowski (2002) for details.

The structural approach to modelling default risk attempts to describe the underlying characteristics of an issuer via a stochastic process representing the total value of the assets of a firm or a company. When the value of these assets falls below a certain threshold, the firm is considered to be in default.

Historically, this is the oldest approach to the quantitative modelling of credit risk for valuation purposes, originating with the work of Black & Scholes (1973) and Merton (1974). As the fundamental process being described is the value of the firm, these models are alternatively called *firm value models*. As the name implies, this approach is more suited to the study of corporate issuers, where an actual firm value can be identified, for example using balance sheet data. For sovereign issuers, the concept of a total asset value is much less clear cut, though attempts have been made to adapt this approach to sovereign credit risk using national stock indices as proxies for firm values, c.f. Lehrbass (2000).

2.1 The Merton Model

Within the Merton model, it is assumed that the firm's capital structure consists of:

- debt with a notional amount N, in the form of zero coupon bonds with maturity T and total value today, time t, equal to $B^d(t,T)$
- equity with total value today, time t, equal to $S(t)$.

At each time before the bonds mature, $t \leq T$, the total market value of the firm's assets is denoted by $V(t)$. The value $V(t)$ is referred to as firm value or asset value interchangeably. Further, $V(t)$ is assumed to evolve according to a geometric Brownian motion.

Definition 1 (Geometric Brownian Motion) *Let $V(t)$ be a geometric Brownian motion with drift term μ and diffusion term σ. Then the dynamics of $V(t)$ under the objective probability measure is given by the following stochastic differential equation,*

$$dV(t) = \mu V(t)dt + \sigma V(t)dW(t) \tag{1}$$

where $W(t)$ denotes a $Wienerprocess$. This can equally be written as

$$V(t) = V(0)\exp\left(\left(\mu - \frac{1}{2}\sigma^2\right) + \sigma W(t)\right). \tag{2}$$

Firms have limited liability. Therefore, by the fundamental balance sheet equation, the firm's total assets must equal the sum of its equity and liabilities. This means that the stock price and the bond price are linked to the firm value via the equation

$$V(t) = S(t) + B^d(t,T). \tag{3}$$

The fundamental assumption of the Merton model is that default of the bond can only take place at its maturity, since this is the only date on which a repayment is due. The payoff at maturity is therefore

$$B^d(T,T) = \min(V(T), N). \tag{4}$$

If the firm value is greater than the redemption value of the debt, then the firm is solvent and the debt is worth par. If the firm value is less than the redemption value, the firm is insolvent and bondholders have first claim on its assets. This means that the shareholders are residual claimants with a payoff at maturity of

$$S(T) = \max(V(T) - N, 0). \tag{5}$$

2.2 Valuation

In effect, the shareholders are long in a European call option on the firm value. Subject to the assumption that V evolves according to a geometric Brownian motion, see Definition 1, this can be priced just as in the Black & Scholes model.

Using (3), the value of the corporate bond can be implied out. If $P(t)$ denotes the price of a put option on the firm value with a strike of N, and

$B(t,T)$ is the price of a non defaultable zero coupon bond with notional N and maturity T, basic put-call parity implies that

$$B^d(t,T) = V(t) - S(t) = B(t,T) - P(t). \tag{6}$$

The bondholders have sold the shareholders an option to put the firm back to them for the bond's notional value at maturity T. It is this additional risk which makes them demand a yield spread over the default free zero coupon bond.

The market price $B^d(0,T)$ of the risky debt is calculated from the Black & Scholes option pricing formula. Now introduce the quotient

$$d = \frac{N\exp(-rT)}{V(0)}, \tag{7}$$

where r is the risk free interest rate. This is the debt-to-assets ratio when the nominal value of the debt is discounted at the market's risk free interest rate. It is one way of measuring the leverage of the firm. Clearly, a higher value of d leads to a greater degree of risk for the firm. Also, define

$$h_1 = \frac{-1}{\sigma_F\sqrt{T}}\left(\frac{1}{2}\sigma_F^2 T + \log d\right) \quad \text{and} \quad h_2 = -\left(h_1 + \frac{2\log d}{\sigma_F\sqrt{T}}\right), \tag{8}$$

where σ_F is the volatility of the firm value. Then, the market value of risky debt in the Merton model is given by

$$B^d(0,T) = N\exp(-rT)\left(\Phi(h_1) + \frac{1}{d}\Phi(h_2)\right), \tag{9}$$

where Φ denotes the cumulative distribution function of the standard normal distribution. The definition of the T-maturity credit spread s implies that

$$s = -\frac{1}{T}\log\left(\frac{B^d(0,T)}{N}\right) - r. \tag{10}$$

2.3 Calibration

The decisive pricing inputs of the Merton model are the volatility σ_F of the firm value and the degree d of the firm's leverage. Though the book value of assets and the notional value of outstanding debt can be deduced from a firm's balance sheet, this information is updated on a relatively infrequent basis when compared to financial market data. For pricing purposes the total market value of all the firm's assets is needed. This cannot be observed directly, but must be estimated. Consequently, there is no time series readily available for it, and its volatility must also be estimated.

For a publicly traded company, the model can be used to imply out the firm value and its volatility from the notional of the outstanding debt and stock market data. Recall that the stock is a call option on the firm value. As such its price is given by the equivalent of the Black & Scholes formula. In the notation of (8) this is

$$S = V\Phi(h_2) - K\exp(-rT)\Phi(h_1). \tag{11}$$

Also the stock's delta with respect to the firm value is given by $\Delta = \Phi(h_2)$. A simple calculation then shows that the volatility σ_S of the stock price is given by

$$\sigma_S = \frac{\sigma_F V \Delta}{S}. \tag{12}$$

Taking the outstanding notional N, as well as the stock price S and its volatility σ_S as given, (11) and (12) can be solved simultaneously for the firm value V and its volatility σ_F. This has to be done numerically. These parameters can then be used as inputs for the valuation of debt.

2.4 Extensions of the Merton Model

The Merton model is the benchmark for all structural models of default risk. However, some of its assumptions pose severe limitations. The capital structure of the firm is very simplistic as it is assumed to have issued only zero coupon bonds with a single maturity. Geske (1977) and Geske & Johnsson (1984) analyse coupon bonds and different capital structures in the Merton framework. The analytical valuation of bonds is still possible using methods for compound options.

Also, the evolution of the risk-free term structure is deterministic. Several authors have extended the model by combining the mechanism for default with various popular interest rate models. Among these are Shimko, Tejima & van Deventer (1993), who use the Vasicek (1977) specification for the (default) risk free short rate. Using the techniques well known from Gaussian interest rate models, credit spreads can be computed. The results are compatible with those in the deterministic case, and credit spreads are generally an increasing function of the short rate volatility and its correlation with the firm value.

2.5 The Passage Time Mechanism

It is clearly unrealistic to assume that the default of an issuer only becomes apparent at the maturity of the bond, as there are usually indenture provisions and safety covenants protecting the bondholders during the life of the bond. As an alternative, the time of default can be modelled as the first time the firm value crosses a certain boundary. This specifies the time τ of default as a random variable given by

$$\tau = \min\left(\tau \geq 0 | V(t) = N(t)\right), \tag{13}$$

where N(t) denotes some time-dependent, and possibly stochastic boundary. This passage time mechanism for generating defaults was first introduced by Black & Cox (1976), and has been extended to stochastic interest rates by, among others, Longstaff & Schwartz (1995) and Briys & de Varenne (1997).

The main mathematical difficulty in a passage time model is the computation of the distribution of default times, which is needed for risk neutral pricing. If the dynamics of the firm asset value are given by a (continuous) diffusion process, this is a reasonably tractable problem. However, if the paths of the firm asset value are continuous, this has an important practical consequence for the behaviour of credit spreads. If the firm value is strictly above the default barrier, then a diffusion process cannot reach it in the next instant - default cannot occur suddenly. Therefore, in a diffusion model, short term credit spreads must tend towards zero; this is at odds with empirical evidence. One remedy is to allow jumps in the firm value, c.f. Schönbucher (1996) or Zhou (1997). The analytic computation of the passage time distribution, however, becomes much more complicated, and often recourse to simulation is the only option.

Allowing jumps in the firm value also introduces additional volatility parameters. The total volatility of the firm value process is determined by that of the diffusion component, as well as by the frequency and size of jumps. Qualitatively, it can be said that early defaults are caused by jumps in the firm value, whereas defaults occurring later are due primarily to the diffusion component. The additional variables give more freedom in calibrating to a term structure of credit spreads, but also post the problem of parameter identification.

2.6 Practical Applications of Firm Value Models

The fact that firm value models focus on fundamentals makes them useful to analysts adopting a bottom-up approach. For example, corporate financiers may find them useful in the design of the optimal capital structure of a firm. Investors, on the other hand, can use them to assess the impact of proposed changes of the capital structure on credit quality. One caveat to this is that it is extremely difficult to apply the firm value model in special situations such as takeovers or leveraged buyouts, where debt might become riskier while equity valuations increase.

The calibration of a firm value model is very data intensive. Moreover, this data is not readily available. It is a non trivial task to estimate asset values and volatilities from balance sheet data. If one follows the firm value concept to its logical conclusion, then it is necessary to take into account all of the various claims on the assets of a firm - a highly unfeasible task. Furthermore, fitting a term structure of bond prices would require a term structure of asset value volatilities and asset values, which is simply not observable.

In terms of analytical tractability, one has to note that firm value models quickly become cumbersome and slow to compute when we move away from the single zero coupon bond debt structure. If a coupon paying bond instead is introduced into the debt structure, then its pricing is dependent on whether the firm value is sufficient to repay the coupon interest on the coupon payment dates. Mathematically the form of the equations become equivalent to pricing a compound option. Similarly, if the issuer has two zero coupon bonds outstanding, the price of the longer maturity bond is conditional on whether the company is solvent when the shorter maturity bond matures. This also makes the pricing formulae very complicated. The pricing of credit derivatives with more exotic payoffs is beyond the limits of this model.

Finally, if a diffusion process is used for the firm value, default is predictable in the sense that one can see it coming as the asset price falls. This means that default is never a surprise. In the real world, it sometimes is. For example, the default of emerging market sovereign bonds is not just caused by an inability to pay, which can be modelled within a firm value approach, but also by an unwillingness to pay, which cannot.

3 Reneby's Model

In this section the structural model of Reneby (1998) will be presented, and the corresponding closed-form formula for pricing equity derived.

Arbitrage opportunities are ruled out and investors are price takers. Furthermore, for at least some large investors, there are no restrictions on short selling stocks or risk free bonds. These can be traded costlessly and continuously in time. There are no assumptions about the tradability of corporate bonds.

The state process $U(t)$ used in this model is firm value, where the firm value is assumed to be observed only at certain times. At companies financial reports for instance, which often is four times a year. Hence, firm value can be seen as a non traded asset, at least not continuously traded. The firm value process is assumed to be lognormal, and thus a geometric Brownian motion, see Definition 1.

It is acknowledged that assets generate revenue that is not reinvested. This "free cash flow", or liquid assets, can be used to service debt or be paid out as dividends to shareholders. Assume that a constant fraction β of the return from assets is not reinvested.

Assumption 1 (Economic Setting) *The state variable of firm value is assumed to be traded at discrete times. Debt is modelled by a coupon bond with infinite maturity, and is assumed to increase with time. Further the risk free interest rate is assumed to be constant, and the standard economic assumptions by Black & Scholes (1973) and Merton (1974) are made.*

3.1 Firm Value

As the state process of firm value $U(t)$ is considered not to be continuously traded, this would correspond to an incomplete market setup. The process drift under the risk neutral probability measure $\mathbf{Q}^B$ would thus be given by $\mu - \lambda\sigma$, where λ denotes the market price of risk.

Definition 2 (Firm Value - Discrete Process) *Define $U(t)$ as the discrete firm value process, where $U(t)$ is assumed to have the following dynamics under the objective probability measure $\mathbf{P}$,*

$$dU(t) = \mu U(t)dt + \sigma U(t)dW(t).$$

As $U(t)$ is assumed to be traded only at certain times, this is an incomplete market setting. Hence the dynamics under the risk neutral probability measure $\mathbf{Q}^B$ is given by

$$dU(t) = (\mu - \lambda\sigma)U(t)dt + \sigma U(t)dW^B(t).$$

W denotes a Wiener process under the objective probability measure $\mathbf{P}$, whereas W^B denotes a Wiener process under the risk neutral probability measure $\mathbf{Q}^B$.

As the state process in the model is a discrete firm value process, we would like to somehow transform this into a continuous process. Assuming that there exists a process $V(t)$ such that $V(t) = F(t, U(t))$, then the arbitrage free price of this process would be given by

$$V(t) = e^{-r(T-t)}\mathbf{E}^B\left[U(T)\right].$$

Note that T is a point in time when U is traded. The $\mathbf{Q}^B$-dynamics of $U(t)$, $dU(t) = (\mu - \lambda\sigma)U(t)dt + \sigma U(t)dW^B(t)$, can equally be written as

$$U(T) = U(t)\exp\left(\left(\mu - \lambda\sigma - \frac{1}{2}\sigma^2\right)(T-t) + \sigma\left(W^B(T) - W^B(t)\right)\right).$$

By taking the logarithm of $U(T)$, this shows to be a linear expression of a normal variable W^B.

Consider two random variables, Y and Z. A basic result from probability theory shows that if $Y \in N(\mu, \sigma^2)$ and $Z = aY + b$, where a and b are constants, this implies that $Z \in N(a\mu + b,\ a^2\sigma^2)$. By the moment generating function of a normally distributed random variable, $\mathbf{E}^B\left[U(T)\right]$ can be expressed as

$$\mathbf{E}^B\left[U(T)\right] = U(t)e^{(\mu-\lambda\sigma)(T-t)}.$$

Definition 3 (Firm Value - Continuous Process) *Let $U(t)$ be defined as above. Assume that $V(t)$ can be described as a continuous function of $U(t)$. Then the risk neutral value of $V(t)$ is given by*

$$V(t) = e^{-r(T-t)}\mathbf{E}^B\left[U(T)\right].$$

We define $V(t)$ as the continuous firm value process $V(t)$, where

$$V(t) = U(t)e^{(\mu-\lambda\sigma-r)(T-t)}. \tag{14}$$

By simply using Itô calculus, the dynamics of $V(t)$ under the objective probability measure $\mathbf{P}$ is calculated.

Let $V(t) = F(t, U(t))$, where F is the continuous and twice differentiable function given by (14). Then, by Itô,

$$dF = \frac{\partial F}{\partial t}dt + \frac{\partial F}{\partial U}dU + \frac{1}{2}\frac{\partial^2 F}{\partial U^2}(dU)^2,$$

where the partial derivatives are

$$\begin{aligned}
\tfrac{\partial F}{\partial t} &= -\left(\mu - \lambda\sigma - r\right) U(t)e^{(\mu-\lambda\sigma-r)(T-t)}, \\
\tfrac{\partial F}{\partial U} &= e^{(\mu-\lambda\sigma-r)(T-t)}, \\
\tfrac{\partial^2 F}{\partial U^2} &= 0.
\end{aligned}$$

Using the $\mathbf{P}$-dynamics of U, the dynamics of $V(t)$ is given by

$$dV(t) = (r + \lambda\sigma)V(t)dt + \sigma V(t)dW^B(t).$$

Note that the following rules of calculation have been used,

$$(dt)^2 = 0, \quad dtdW(t) = 0, \quad (dW(t))^2 = dt, \tag{15}$$

see Björk (1998). A fraction β of the asset return is assumed not to be reinvested. This generated cash flow is henceforth excluded from the drift of the firm value process [1]. Thus, the $\mathbf{Q}^B$-dynamics of $V(t)$ are given by

$$dV(t) = (r + \lambda\sigma - \beta)V(t)dt + \sigma V(t)dW^B(t).$$

3.2 Debt

Debt is composed of bank loans, bonds, accounts payable, salaries due, accrued taxes etc. Money due to suppliers, employees and the government are substitutes for other forms of debt. Part of the price of a supplied good and part of a salary paid can be viewed as corresponding to compensation for the debt, that it in substance consitutes. The cost of debt consequently includes not only regular interest payments to lenders and coupons to bondholders, but also fractions of most other payments made by a company. Debt service therefore can be argued to take place more or less every day. To price equity strictly one would have to take account for all these individual payments.

[1] Reneby chooses to exclude this free cash flow from the firm value drift, and reintroduce it as a separate term in the assumed value composition of equity.

To obtain a simple, closed form pricing formula for equity, debt service is assumed to take place continuously. Considering the frequency of actual payments, this is a fair hypothesis, at least if no individual payment is very large. Moreover, since most companies do not have a maturity, fixed or otherwise, the firm is assumed, on conditional on no default, to continue its operations forever.

Total debt is allowed to increase its debt over time. As time passes and the value of its assets increases, it is reasonable to expect debt obligations to increase as well. Otherwise, the debt to equity ratio would tend towards zero as time goes by. As financial distress one way or the other is related to total debt, this implies that the risk of default disappears with time.

Definition 4 (Nominal Debt) *Define $N(t)$ as a continuous process for nominal debt, where $N(t)$ is assumed to grow with the rate α. Hence $N(t)$ has the following dynamics*

$$dN(t) = \alpha N(t)dt.$$

Definition 5 (Coupon Payments) *Define $C(t)$ as a process for coupon payments, where $C(t)$ is assumed to grow with the rate α, the same rate as for nominal debt. Hence, $C(t)$ has the following dynamics*

$$dC(t) = \alpha C(t)dt.$$

Coupon payments are tax deductible, and we therefore need the corporate tax rate, which we denote by ζ.

Within the pricing of equity, Reneby includes the market price of future loans. Denote market price of future loans by $d(t, V(t))$. The market price of a loan is expressed out of a debtor's point of view. Hence, the value of the loan consists in its coupons and the expected repayment of the nominal amount, as there is a risk that the borrower goes into bankruptcy. However this term is not strictly defined, and will therefore be treated in the preceding discussion section.

3.3 Default - Financial Distress

The firm is assumed to enter into financial distress, and start some form of reorganisation or possibly file for bankruptcy, if the value of its assets falls below the reorganisation (default) barrier. Within the calculations of equity, we use the same barrier as in Reneby.

Definition 6 (Default Barrier) *Define $L(t)$ as the default barrier.*

The setup for structural models, is that the firm is expected to go into bankruptcy when firm value hits the default barrier. In this situation, the firm is assumed to realise all assets and thereafter pay costs of default. When this has been done, an estimated fraction of the remaining value will be paid out to debt holders and equity holders, due to assumed violations of the absolute priority rule.

Assumption 2 (Payouts in Default) *Reneby argues that the absolute priority rule at default is violated, and henceforth the fraction of the assets paid out to debt holders and equity holders are being estimated with constant fractions of the firm value in default. The fraction paid out to debt holders, the debt recovery rate, is denoted by δ and the fraction to equity holders by ε.*

There are several ways of viewing default. Reneby discusses the following three approaches of default, of which he uses the first one in his model.

First approach is to choose the barrier so that it equals the total amount of nominal debt, or a fraction thereof. In many countries, corporate law states that financial distress occurs when the value of the firms assets reaches some lower level, usually related to the total nominal value of outstanding debt. Apart from this judicial view, there are several economic justifications. One is to view the barrier as the level of asset value that is necessary for the firm to retain sufficient credibility to continue its operations or where, due to some covenant, it voluntarily files for bankruptcy. Another is to think of the barrier as the asset value at which it is no longer possible to honour the payments be it by selling assets or issuing new securities. The default barrier would for this approach be

$$L(t) = N(t).$$

Second approach is based on the supposition that equity holders are too small and scattered to contribute funds to satisfy creditors and avoid a reorganisation situation. Thus, assuming that internally generated funds are the only means to service debt, the firm is solvent as long as internally generated funds exceed the current coupon, i.e. as long as $\beta V(t) \geq C(t)$. Hence, the default barrier would for this approach be

$$L(t) = \frac{C(t)}{\beta}.$$

Third approach is the level of asset value at which equity holders are no longer willing to contribute funds to stave off financial distress. This choice of barrier is the lowest possible since a lower equity value is not concordant with limited liability. Using this alternative, the level of barrier is endogenously determined within the model. The default barrier would for this approach be

$$L(t)|_{\beta>0,r\neq\alpha} = \frac{\zeta\frac{r}{r-\alpha}\theta(r-\alpha)-\theta(r)}{\varepsilon(1+\theta(r-\alpha))-\theta^V+\delta\frac{N(0)}{L(0)}(\theta(r)-\theta(r-\alpha))}N(t),$$

$$\text{where} \quad \theta(\rho) = \frac{\sqrt{\left(\frac{r-\beta-\alpha-0.5\sigma}{\sigma}\right)^2+2\rho}+\frac{r-\beta-\alpha-0.5\sigma}{\sigma}}{\sigma}, \quad \rho = r-\alpha,\ r$$

$$\text{and} \quad \theta^V = \frac{\sqrt{\left(\frac{r-\beta-\alpha+0.5\sigma}{\sigma}\right)^2+2\beta}+\frac{r-\beta-\alpha+0.5\sigma}{\sigma}}{\sigma}.$$

See Appendix in Reneby (1998) for details.

Assumption 3 (Default Barrier) *We assume the default barrier*

$$L(t) = N(t).$$

Definition 7 (Default Process) *Define X as the default process, where*

$$X(t) = \frac{1}{\sigma}\log\frac{V(t)}{L(t)}.$$

The event of default is defined as $X(\tau) = 0$, where τ denotes the time of default.

The dynamics of the default process is derived by using Itô calculus.

Let $X(t) = F(t, V(t), L(t))$, where F is the continuous and twice differentiable function $X(t) = \sigma^{-1}\log V(t)/L(t)$. Then, by Itô,

$$dF = \frac{\partial F}{\partial t}dt + \frac{\partial F}{\partial V}dV + \frac{\partial F}{\partial L}dL + \frac{1}{2}\left(\frac{\partial^2 F}{\partial V^2}(dV)^2 + \frac{\partial^2 F}{\partial L^2}(dL)^2 + 2\frac{\partial^2 F}{\partial V\partial L}dV\,dL\right),$$

where the derivatives are $\frac{\partial F}{\partial t} = 0$, $\frac{\partial F}{\partial V} = \frac{1}{\sigma V}$, $\frac{\partial F}{\partial L} = -\frac{1}{\sigma L}$, $\frac{\partial^2 F}{\partial V^2} = -\frac{1}{\sigma V^2}$, $\frac{\partial^2 F}{\partial L^2} = \frac{1}{\sigma L^2}$, and $\frac{\partial^2 F}{\partial V\partial L} = 0$.

As the barrier $L(t)$ only is time dependent, and henceforth without any driving diffusion process, the dynamics of $L(t)$ under the objective probability measure $\mathbf{P}$ are given by,

$$dL(t) = \nu L(t)dt,$$

with some drift ν. The dynamics of $V(t)$ are given by $dV(t) = (r + \lambda\sigma - \beta)V(t)dt + \sigma V(t)dW(t)$.

By applying the rules of calculation stated in equation (15) onto the dynamics of $V(t)$ and $L(t)$, this implies that $(dL)^2$, $dVdL$, $(dL)^2 = \nu^2 L^2 (dt)^2$, and $dVdL$ all equal 0. Hence, by using either dynamics of V,

$$\begin{aligned} dF &= \tfrac{\partial F}{\partial t}dt + \tfrac{\partial F}{\partial V}dV + \tfrac{\partial F}{\partial L}dL + \tfrac{1}{2}\tfrac{\partial^2 F}{\partial V^2}(dV)^2 \\ &= \tfrac{1}{\sigma V}\left((r + \lambda\sigma - \beta)V(t)dt + \sigma V(t)dW(t)\right) \\ &\quad -\tfrac{1}{\sigma L}(\nu L(t)dt) - \tfrac{1}{2\sigma V^2}\left(\sigma^2 (V(t))^2 dt\right). \end{aligned}$$

The dynamics of X under the objective probability measure $\mathbf{P}$ are hence given by,

$$dX(t) = \mu_X dt + dW(t),$$

where the drift is given by

$$\mu_X = \frac{r + \lambda\sigma - \beta - \nu - \frac{1}{2}\sigma^2}{\sigma}.$$

Using the barrier $L(t) = N(t)$, the drift term ν equals α.

Definition 8 (Default Density Function) *The density function of the default process X under the risk neutral probability measure $\mathbf{Q}^m$ is defined as*

$$f^m(s; X(t)) = \frac{X(t)}{\sqrt{2\pi(s-t)^3}} \exp\left(-\frac{1}{2}\left(\frac{X(t) - \mu_X^m(s-t)}{\sqrt{s-t}}\right)^2\right).$$

This is the density of a first passage time, see Ross (2000) for details on hitting times.

Remark 1 (Ericsson (1997)) *Consider the default process X defined earlier, with drift μ_X^m. If $\left(\mu_X^m\right)^2 + 2\rho \geq 0$, then the following holds for the density function of X,*

$$\int_t^\infty e^{-\rho(s-t)} f^m(s; X(t))ds = \exp\left(-X(t)\left(\sqrt{\left(\mu_X^m\right)^2 + 2\rho} + \mu_X^m\right)\right).$$

See Ericsson (1997) pp. 139 - 140 for details. Note that for $\rho = 0$, this implies

$$\int_t^\infty f^m(s; X(t))ds = e^{-2X(t)\mu_X^m}.$$

3.3.1 Importance of an increasing barrier

If a model does not account for an actual increase in total debt, and the therewith associated growth of the reorganisation barrier, the distribution of bankruptcy probabilities over time will be at odds with reality. Specifically, the default intensity will be heavily skewed to the early part of the company's life. This means that the probability of default, conditional on no previous default, very quickly approaches zero. In other words, if a firm does not go bankruptcy in the immediate future, it probably never will.

This feature of a constant barrier, infinite horizon model severly biases the subsequent bond price estimates. Consider a long term corporate coupon bond floated by a company with a solid financial situation. In reality, even if the risk of default within the next five years is negligible, there is usually a risk that the company will encounter financial difficulties before the maturity of the bond. This uncertainty is naturally reflected in real prices, but is not taken into account by a constant barrier model. Note that this critique is only directly applicable when the barrier is the only trigger of default. If, for example, repayment of the principal also is critical, the skew of the default intensity will be mitigated.

The reason for the declining bankruptcy density is that the expected value of the asset value exponentially increases, and that therefore the expected ratio of asset value to the barrier (which determines the bankruptcy intensity) rapidly becomes very large. The remedy is, of course, to allow the barrier to grow as well.

3.4 Equity

Having discussed the setting for the firm value process, modelling of debt and default process, we shall now derive a closed form formula for pricing equity. Thus, with a slight amendment of Reneby's approach. Reneby is performing the calculating of the equity pricing function for a arbitrary interval, $[t, T]$. However, we make the distinction of choosing to view the interval $[0, T]$, as we are only interested in the current value of equity by today ($t = 0$). Henceforth, all calculations will be adjusted to comprise this interval.

Assumption 4 (Equity Value Composition) *Assume that the value of equity is composed by a call option on firm value with nominal debt as face value and with infinite maturity, generated cash flow, coupon payments and taxes, additional borrowing and costs of reorganisation, and violations to the absolute priority rule.*

Definition 9 (Equity Pricing Function) *Let E denote the firms equity value. Define $\Pi(t, V(t))$ as the continuous pricing function of equity.*

With the assumed composition of equity value above, Reneby applies risk neutral valuation onto each element. Hence discounted expectations are calculated under the martingale measure $\mathbf{Q}^B$. Conditioning of the effect by default is included.

Definition 10 *Let Π_i denote the i:th element of which the pricing function is composed. Then the elements are defined as,*

$$\begin{aligned}
\Pi_1 &= \mathbf{E}^B\left[e^{-rT}(V(T) - N(T))^+ I(\tau > T)\right], \\
\Pi_2 &= \mathbf{E}^B\left[\int_0^T e^{-rs}\beta V(s) I(\tau > s) ds\right], \\
\Pi_3 &= -\mathbf{E}^B\left[\int_0^T e^{-rs}(1-\zeta) C(s) I(\tau > s) ds\right], \\
\Pi_4 &= \mathbf{E}^B\left[\int_0^T e^{-rs} d(s, V(s)) I(\tau > s) ds\right], \\
\Pi_5 &= \mathbf{E}^B\left[e^{-r\tau}\varepsilon L(\tau) I(\tau \leq T)\right].
\end{aligned}$$

Within Reneby's model, the value of equity is calculated as a limit value as T tends to infinity. The probabilistic expression of the equity function is then given by the following proposition.

Proposition 1 (Equity Pricing Function - Probabilistic Expression) *Given the assumptions for the composition of equity value above, the equity pricing function can be expressed as*

$$\Pi(0, V(0)) = \lim_{T\to\infty} \sum_i \Pi_i(0, V(0); T).$$

Definition 11 (Asset Claim) *Define Ω as a claim on the firms assets, where*

$$\Omega = \lim_{T\to\infty} (\Pi_1 + \Pi_2).$$

To derive an expression for Ω, Π_1 and Π_2 are treated separately.

3.4.1 Calculating $\mathbf{Lim}_{T\to\infty}\mathbf{\Pi_1}$

Note that $\lim_{T\to\infty} \Pi_1$ is a European call option with infinite maturity. By separating the part containing firm value and the part containing nominal debt, this can be expressed as

$$\Pi_1 = e^{-rT}\mathbf{E}^B\left[V(T) I(\tau > T)\right] - e^{-rT}\mathbf{E}^B\left[N(T) I(\tau > T)\right].$$

By using the dynamics of N, we see that $N(T) = N(0)e^{\alpha T}$. Inserting this into the expression above containing nominal debt, this can equally be written as

$$N(0)e^{(r-\alpha)T}\mathbf{E}^B\left[I(\tau > T)\right].$$

For an indicator function I of an event A, we know that $\mathbf{E}\left[I(A)\right] = P(A)$. Hence this implies that $\mathbf{E}^B\left[I(\tau > T)\right] = \mathbf{Q}^B(\tau > T)$. We are interested in calculating the limit as maturity tends to infinity, and thus calculating

$$\lim_{T\to\infty}\frac{\mathbf{Q}^B(\tau > T)}{e^{(r-\alpha)T}}$$

for all combinations of r and α.

With a probability defined as a real number between 0 and 1, the limit for $r > \alpha$ is given by

$$\lim_{T\to\infty}\frac{\mathbf{Q}^B(\tau > T)}{e^{(r-\alpha)T}} \;\leq\; \frac{1}{e^{(r-\alpha)T}} \longrightarrow 0 \text{ as } T \to \infty.$$

For $r = \alpha$, the limit is equal to $\lim_{T\to\infty}\mathbf{Q}^B\left(\tau > T\right)$ or equally

$$\lim_{T\to\infty}\int_T^\infty \frac{X(0)}{\sqrt{2\pi s^3}}\exp\left(-\frac{\left(X(0)-\mu_X^B s\right)^2}{2s}\right)ds.$$

As $s > 0$ for all s, the exponential term will always be less than 1. Hence $\exp\left(-\frac{1}{2}\cdot q^2\right) \leq 1$. Considering this, we get

$$\lim_{T\to\infty}\int_T^\infty \frac{X(0)}{\sqrt{2\pi s^3}}\exp\left(-\frac{\left(X(0)-\mu_X^B s\right)^2}{2s}\right)ds \;\leq\; \lim_{T\to\infty}\int_T^\infty \frac{X(0)}{\sqrt{s^3}}ds =$$

$$= \frac{2X(0)}{\sqrt{T}} \longrightarrow 0 \text{ as } T \to \infty.$$

By applying L'Hôpital's rule onto the third limit value problem, when $r < \alpha$, the limit $\lim_{T\to\infty}e^{-(r-\alpha)T}\mathbf{Q}^B(\tau > T)$ shows to equal zero as well. The limit value is equally given by the quote between the first derivatives, and thus

$$\lim_{T\to\infty}\frac{\mathbf{Q}^B(\tau>T)}{e^{(r-\alpha)T}} = \lim_{T\to\infty}\frac{\frac{X(0)}{\sqrt{2\pi T^3}}\exp\left(-\frac{\left(X(0)-\mu_X^B T\right)^2}{2T}\right)}{(r-\alpha)\exp\left((r-\alpha)T\right)} =$$

$$= \frac{X(0)}{\sqrt{2\pi}(r-\alpha)}\lim_{T\to\infty}\frac{\exp\left(-\frac{\left(X(0)-\mu_X^B T\right)^2}{2T}\right)}{T^{3/2}\exp\left((r-\alpha)T\right)} =$$

$$= \frac{X(0)}{\sqrt{2\pi}(r-\alpha)}\lim_{T\to\infty}\frac{\exp\left(-\frac{(X(0))^2}{2T}+X(0)\mu_X^B-\frac{(\mu_X^B)^2T}{2}\right)}{T^{3/2}\exp\left((r-\alpha)T\right)}.$$

To ease calculations, we introduce $A = \alpha - r$, where $A > 0$. For the call option to be positive, β must equal zero. Hence we have that $\beta = 0$. Recalling $\mu_X^B = (r - \alpha - \beta - 0.5\sigma^2)/\sigma = -(A + 0.5\sigma^2)/\sigma$, we get

$$\frac{-X(0)}{A\sqrt{2\pi}} \lim_{T\to\infty} \frac{\exp\left(-\frac{(X(0))^2}{2T} + X(0)\mu_X^B - \frac{(\mu_X^B)^2 T}{2}\right)}{T^{3/2}\exp(-AT)} =$$

$$= \frac{-X(0)\exp\left(X(0)\mu_X^B\right)}{A\sqrt{2\pi}} \cdot \lim_{T\to\infty} \frac{\exp\left(AT - \frac{(X(0))^2}{2T} - \frac{(\mu_X^B)^2 T}{2}\right)}{T^{3/2}} =$$

$$= \frac{-X(0)}{A\sqrt{2\pi}} \exp\left(\frac{-X(0)\left(A + 0.5\sigma^2\right)}{\sigma}\right) \cdot \lim_{T\to\infty} \frac{\exp\left(-\frac{(X(0))^2}{2T}\right)\exp\left(AT - \frac{\left(A+0.5\sigma^2\right)^2 T}{2\sigma^2}\right)}{T^{3/2}}.$$

Note that the constant term before the limit value is negative. The first exponential expression within the limit value, $e^{-(X(0))^2/2T} \longrightarrow 0$ as $T \to \infty$. As the second exponential expression is more dominant than the other remaining term, $T^{-3/2}$, we continue by looking at the argument of the exponential expression. Hence, we see that

$$\left(A - \frac{\left(A+0.5\sigma^2\right)^2}{2\sigma^2}\right)T = -\frac{1}{2\sigma^2}\left(A^2 - A\sigma^2 + \frac{\sigma^4}{4}\right)T = -\frac{1}{2\sigma^2}\left(A - \frac{\sigma^2}{2}\right)^2 T,$$

which clearly tends to $-\infty$ as T tends to ∞. Thus, we have that the second exponential expression tends to 0 as $T \to 0$ which implies that $\lim_{T\to\infty} e^{-(r-\alpha)T}\mathbf{Q}^B(\tau > T) = 0$.

Hence, for all α and r we have that

$$\lim_{T\to\infty} \frac{\mathbf{Q}^B(\tau > T)}{e^{(r-\alpha)T}} = 0.$$

The part of the call option containing nominal debt vanishes as maturity tends to infinity. This implies that

$$\lim_{T\to\infty} \Pi_1 = \lim_{T\to\infty} e^{-rT}\mathbf{E}^B\left[V(T)I(\tau > T)\right].$$

Lemma 1 (Numeraires and Change of Probability Measures) *For a general probability measure* $\mathbf{Q}^j$*, it holds that*

$$\mathbf{E}^j[mY] = \mathbf{E}^j[m]\,\mathbf{E}^m[Y]$$

where $\mathbf{E}^m$ *is the expected value under probability measure* $\mathbf{Q}^m$ *defined through*

$$\begin{cases} d\mathbf{Q}^m = \mathbf{R}^{j\to m} d\mathbf{Q}^j, \\ \mathbf{R}^{j\to m} = \frac{m}{\mathbf{E}^j[m]}. \end{cases}$$

By applying the lemma above, we get

$$\mathbf{E}^B\left[V(T)I(\tau > T)\right] = \mathbf{E}^B\left[V(T)\right]\mathbf{E}^V\left[I(\tau > T)\right]$$

which can be further expressed as $\mathbf{E}^B\left[V(T)\right]\mathbf{Q}^V(\tau > T)$.

Remark 2 (Remark by Reneby - Default Dynamics) $\mathbf{Q}^V$ *is the measure under which price processes normalised with*

$$e^{-rT}\mathbf{E}^B\left[V(T)\right] = e^{-\beta T}V(0)$$

are martingales. The Girsanov kernel used to go from the pricing measure is $h^{B\to V} = -\sigma$.

By using the remark above, the limit value of the call option can be expressed as

$$\lim_{T\to\infty} e^{-rT}\mathbf{E}^B\left[V(T)I(\tau > T)\right] = \lim_{T\to\infty} V(0)e^{-\beta T}\mathbf{Q}^V(\tau > T).$$

Note that the limit value for a $\beta > 0$ corresponds to the limit problem within the part of nominal debt earlier. Letting $\beta = 0$ and by rewriting the probability, we get the expression $V(0)\left(1 - \lim_{T\to\infty}\mathbf{Q}^V(\tau \le T)\right)$. However, by applying the definition of default density function along with the remark by Ericsson, we see that $\lim_{T\to\infty}\mathbf{Q}^V(\tau \le T) = \exp\left(-2X(0)\mu_X^V\right)$. With X defined as $X(0) = \sigma^{-1}\log V(0)/L(0)$, this can be express as

$$\exp\left(-2X(0)\mu_X^V\right) = \left(\frac{V(0)}{L(0)}\right)^{-2\mu_X^V/\sigma}.$$

Hence we have the following limit value of the call option

$$\lim_{T\to\infty}\mathbf{E}^B\left[e^{-rT}(V(T) - N(T))I(\tau > T)\right] = V(0)\left(1 - \left(\frac{V(0)}{L(0)}\right)^{-2\mu_X^V/\sigma}\right),$$

which by considering the limit value

$$\lim_{T\to\infty}\frac{\mathbf{Q}^V(\tau > T)}{e^{\beta T}} = 0, \ \ \beta > 0,$$

implies that

$$\lim_{T\to\infty}\Pi_1 = \begin{cases} V(0)\left(1 - \left(\frac{V(0)}{L(0)}\right)^{-2\left(\frac{r-\alpha+0.5\sigma^2}{\sigma^2}\right)}\right) & \text{when } \alpha < r + 0.5\sigma^2 \text{ and } \beta = 0 \\ 0 & \text{when } \alpha \ge r + 0.5\sigma^2 \text{ or } \beta > 0. \end{cases}$$

3.4.2 Calculating $\mathbf{Lim}_{T\to\infty}\mathbf{\Pi_2}$

Now consider the part Π_2, which can be seen as a cash claim. As β is a constant, and the exponential function deterministic, we extract this out of the expectation value, and get

$$\mathbf{E}^B\left[\int_0^T e^{-rs}\beta V(s)I(\tau > s)ds\right] = \beta\int_0^T e^{-rs}\mathbf{E}^B\left[V(s)I(\tau > s)\right]ds.$$

By Remark 2, we can express $e^{-rs}\mathbf{E}^B\left[V(s)\right] = e^{-\beta s}V(0)$. Hence we have the expression

$$\Pi_2 = \beta V(0)\int_0^T e^{-\beta s}\mathbf{Q}^V(\tau > s)ds.$$

By using partial integration, we get

$$\begin{aligned}
&\beta\textstyle\int_0^T e^{-\beta s}\mathbf{Q}^V(\tau > s)ds = \\
&= \beta\left(\left[\tfrac{-e^{-\beta s}}{\beta}\left(1-\mathbf{Q}^V(\tau \le s)\right)\right]_{s=0}^T - \textstyle\int_0^T \tfrac{e^{-\beta s}}{\beta}f^V(s;X(0))ds\right) \qquad (16)\\
&= \left(1 - e^{-\beta T}\left(1-\mathbf{Q}^V(\tau \le T)\right)\right) - \textstyle\int_0^T e^{-\beta s}f^V(s;X(0))ds.
\end{aligned}$$

Rewriting the probability and taking the limit value, we recognise the limit value $\lim_{T\to\infty} e^{-\beta T}\mathbf{Q}^V(\tau > T)$. By

$$\frac{\mathbf{Q}^V(\tau > T)}{e^{\beta T}} \le \frac{1}{e^{\beta T}} \longrightarrow 0 \text{ as } T \to \infty,$$

Remark 1 and that $\Pi_2 = 0$ if $\beta = 0$, we have that

$$\lim_{T\to\infty}\Pi_2 = \begin{cases} V(0)\left(1 - \left(\frac{V(0)}{L(0)}\right)^{-\left(\sqrt{\left(\mu_X^V\right)^2+2\beta}+\mu_X^V\right)/\sigma}\right) & \text{when } \beta > 0 \\ 0 & \text{when } \beta = 0 \end{cases}$$

We therefore have the following lemma.

Lemma 2 (Value of Asset Claim)

$$\Omega = \begin{cases} V(0)\left(1 - \left(\frac{V(0)}{L(0)}\right)^{-\left(\sqrt{\left(\frac{r-\beta-\alpha+0.5\sigma^2}{\sigma}\right)^2+2\beta}+\frac{r-\beta-\alpha+0.5\sigma^2}{\sigma}\right)/\sigma}\right) & \textit{when } \beta > 0 \textit{ or } (\beta = 0 \textit{ and } \alpha < r + 0.5\sigma^2) \\ 0 & \textit{when } \beta = 0 \textit{ and } \alpha \ge r + 0.5\sigma^2. \end{cases}$$

Definition 12 (Default Claims) *Define G as the value of a claim paying off unity in financial distress,*

$$G(\tau, V(\tau)) = 1.$$

Define G^α as the value of a claim paying off $e^{\alpha\tau}$ in financial distress,

$$G^\alpha(\tau, V(\tau)) = e^{\alpha\tau}.$$

Now consider the expectation values under the martingale measure $\mathbf{Q}^B$ for these claims. The expectation value $\mathbf{E}^B\left[e^{-\rho\tau}\right]$ (ρ arbitrary) is solved by simply using the definition of the expectation value of a continuous stochastic process Y with density function $f_Y(t)$, $\mathbf{E} = \int y f_Y(y)dy$. Thus

$$\mathbf{E}^B\left[e^{-\rho\tau}\right] = \lim_{T\to\infty}\int_0^T e^{-\rho s} f^B(s, X(0))ds = \left(\frac{V(0)}{L(0)}\right)^{-\left(\sqrt{\left(\mu_X^B\right)^2+2\rho}+\mu_X^B\right)/\sigma} \tag{17}$$

by Remark 1 and by the definition of X. Note that the restriction $\left(\mu_X^B\right)^2 + 2\rho \geq 0$ for the remark by Ericsson to hold. To conclude,

Lemma 3 (Value of Default Claim G) *The value of the dollar-in-default claim G, is*

$$G(0, V(0)) = \left(\frac{V(0)}{L(0)}\right)^{-\left(\sqrt{\left(\frac{r-\beta-\alpha-0.5\sigma^2}{\sigma}\right)^2+2r}+\frac{r-\beta-\alpha-0.5\sigma^2}{\sigma}\right)/\sigma}.$$

Lemma 4 (Value of Default Claim G^α) *The value of the claim G^α, is*

$$G^\alpha(0, V(0)) = \left(\frac{V(0)}{L(0)}\right)^{-\left(\sqrt{\left(\frac{r-\beta-\alpha-0.5\sigma^2}{\sigma}\right)^2+2(r-\alpha)}+\frac{r-\beta-\alpha-0.5\sigma^2}{\sigma}\right)/\sigma}.$$

3.4.3 Calculating $\mathbf{Lim}_{T\to\infty}\left(\mathbf{\Pi_3} + \mathbf{\Pi_4}\right)$

The third part, Π_3, of the equity pricing function is the effect of coupon payments C, adjusted by the tax reduction calculated on a corporate tax rate ζ. We divide this effect of total coupon payments into one term of coupon payments of current loans, and one term of coupon payments to future loans. Hence we can write $C(s) = C(0) + (C(s) - C(0))$. By using the dynamics of

C, we can write $C(s) = C(0)e^{\alpha s}$. The effect of coupon payments, then can be written as

$$\begin{aligned}\Pi_3 &= -\mathbf{E}^B\left[\int_0^T e^{-rs}(1-\zeta)C(s)I(\tau > s)ds\right] \\ &= -(1-\zeta)\mathbf{E}^B\left[\int_0^T e^{-rs}C(0)I(\tau > s)ds\right] \\ &\quad -(1-\zeta)\mathbf{E}^B\left[\int_0^T e^{-rs}C(0)\left(e^{\alpha s}-1\right)I(\tau > s)ds\right].\end{aligned}$$

The fourth part, Π_4, of the equity price function is the effect of future loans. However this part is needed to be discussed further, we assume that the relation above holds, and continue the discussion in the forthcoming discussion section.

Reneby simply assumes that the value of a loan constitutes of its coupons and estimated default payout to the debtor. By assuming this and using the dynamics of C, Definition 5, and L, where $L(\tau) = L(0)e^{\alpha\tau}$, we get the following relation:

$$\begin{aligned}\Pi_4 &= \mathbf{E}^B\left[\int_0^T e^{-rs}d(s, V(s))I(\tau > s)ds\right] \\ &= \mathbf{E}^B\left[\int_0^T e^{-rs}(C(s)-C(0))I(\tau > s)ds\right] + \delta\mathbf{E}^B\left[e^{-r\tau}(L(\tau)-L(0))\right] \\ &= \mathbf{E}^B\left[\int_0^T e^{-rs}C(0)\left(e^{\alpha s}-1\right)I(\tau > s)ds\right] + \delta\mathbf{E}^B\left[e^{-r\tau}L(0)\left(e^{\alpha\tau}-1\right)\right].\end{aligned}$$

By adding these two parts, Π_3 and Π_4, we get

$$\begin{aligned}\Pi_3 + \Pi_4 &= \zeta\mathbf{E}^B\left[\int_0^T e^{-rs}C(0)\left(e^{\alpha s}-1\right)I(\tau > s)ds\right] \\ &\quad -(1-\zeta)\mathbf{E}^B\left[\int_0^T e^{-rs}C(0)I(\tau > s)ds\right] \\ &\quad +\delta\mathbf{E}^B\left[e^{-r\tau}L(0)\left(e^{\alpha\tau}-1\right)\right].\end{aligned}$$

Extracting $C(0)$ and $L(0)$ out of the expectation values, and slightly rewriting the expression implies

$$\begin{aligned}\Pi_3 + \Pi_4 &= -C(0)\mathbf{E}^B\left[\int_0^T e^{-rs}I(\tau > s)ds\right] \\ &\quad +\zeta C(0)\mathbf{E}^B\left[\int_0^T e^{-(r-\alpha)s}I(\tau > s)ds\right] \\ &\quad +\delta L(0)\mathbf{E}^B\left[e^{-r\tau}\left(e^{\alpha\tau}-1\right)\right].\end{aligned} \tag{18}$$

By rewriting these three terms, we recognise the following two structures, $\mathbf{E}^B\left[e^{-r\tau}\right]$ and $\mathbf{E}^B\left[e^{-(r-\alpha)\tau}\right]$. Referring to equation (17), Lemma 3 and 4, we see that $\mathbf{E}^B\left[e^{-r\tau}\right] = G$ and $\mathbf{E}^B\left[e^{-(r-\alpha)\tau}\right] = G^\alpha$.

Now, going back to $\Pi_3 + \Pi_4$, this will simplify the calculations considerably. Start by consider the first term in (18). Hence, by the same reasoning as in equation (16), and partial integration we get the following expression:

$$
\begin{aligned}
&\mathbf{E}^B\left[\int_0^T e^{-rs} I(\tau > s) ds\right] = \\
&= \left[\frac{-e^{-rs}}{r}\left(1 - \mathbf{Q}^B(\tau \leq s)\right)\right]_{s=0}^T - \int_0^T \frac{e^{-rs}}{r} f^B(s; X(0)) ds \\
&= \frac{1}{r}\left(\left(1 - e^{-rT}\left(1 - \mathbf{Q}^B(\tau \leq T)\right)\right) - \int_0^T e^{-rs} f^B(s; X(0)) ds\right)
\end{aligned}
$$

Calculating the limit value, once again, the limit value problem $\lim_{T\to\infty} e^{-rT}\mathbf{Q}^B(\tau > T)$ appears. Assuming that $r > 0$, we get that the limit equals zero. By applying the remark by Ericsson we get that

$$
\lim_{T\to\infty} \int_0^T e^{-rs} f^B(s; X(0)) ds = \exp\left(-X(0)\left(\sqrt{\left(\mu_X^B\right)^2 + 2r} + \mu_X^B\right)\right),
$$

which by using the definition of the default process can be written as

$$
\lim_{T\to\infty} \int_0^T e^{-rs} f^B(s; X(0)) ds = \left(\frac{V(0)}{L(0)}\right)^{-\left(\sqrt{\left(\mu_X^B\right)^2 + 2r} + \mu_X^B\right)/\sigma}.
$$

Hence, we end up with the following expression

$$
\begin{aligned}
&\lim_{T\to\infty}\left(-C(0)\mathbf{E}^B\left[\int_0^T e^{-rs} I(\tau > s) ds\right]\right) = \\
&= \quad -\frac{C(0)}{r}\left(1 - \left(\frac{V(0)}{L(0)}\right)^{-\left(\sqrt{\left(\mu_X^B\right)^2 + 2r} + \mu_X^B\right)/\sigma}\right) = -\frac{C(0)}{r}(1 - G),
\end{aligned}
$$

using the previously definition of G. Now continue with the second term in equation (18), which is solved in a similar manner to the previous term.

Start by rewriting the expression to $\zeta C(0) \int_0^T e^{-(r-\alpha)s} \mathbf{E}^B\left[I(\tau > s)\right] ds$, where the expectation of the indicator function once again is given by $\mathbf{E}^B\left[I(\tau > s)\right] = \mathbf{Q}^B(\tau > s)$. Hence,

$$
\zeta C(0)\mathbf{E}^B\left[\int_0^T e^{-(r-\alpha)s} I(\tau > s) ds\right] = \zeta C(0) \int_0^T e^{-(r-\alpha)s} \mathbf{Q}^B(\tau > s) ds.
$$

As we are interested in the limit value of this expression we get the following result, by the same reasoning as above, including the remark by Ericsson, partial integration etc,

$$\lim_{T\to\infty} \zeta C(0)\mathbf{E}^B\left[\int_0^T e^{-rs}e^{\alpha s}I(\tau>s)ds\right] = \zeta\frac{C(0)}{r-\alpha}\left(1-\left(\frac{V(0)}{L(0)}\right)^{-\left(\sqrt{\left(\mu_X^B\right)^2+2(r-\alpha)}+\mu_X^B\right)/\sigma}\right).$$

This second term should however be treated for $r \neq \alpha$ and $r = \alpha$. The first approach is however already solved, and by using the definition of G^α, this can be expressed as

$$\lim_{T\to\infty} \zeta C(0)\mathbf{E}^B\left[\int_0^T e^{-rs}e^{\alpha s}I(\tau>s)ds\right] = \zeta\frac{C(0)}{r-\alpha}\left(1-G^\alpha\right).$$

For $r = \alpha$, Reneby has used

$$\lim_{\alpha\to r}\frac{1}{r-\alpha}(1-G^\alpha)$$

to get the result $\left(\beta+0.5\sigma^2\right)^{-1}\log\left(V(0)/L(0)\right)$.

Definition 13 *Define H(0, V(0)) as*

$$H(0,V(0)) = \begin{cases} \frac{1}{r-\alpha}(1-G^\alpha) & \text{when } r \neq \alpha \\ \log\left(\frac{V(0)}{L(0)}\right)^{\frac{1}{\beta+0.5\sigma^2}} & \text{when } r = \alpha. \end{cases}$$

H(0, V(0)) will be suppressed to H.

The third term in equation (18) , is solved by using the two previously derived structures, $\mathbf{E}^B\left[e^{-r\tau}\right]$ and $\mathbf{E}^B\left[e^{-(r-\alpha)\tau}\right]$. By slightly rewriting the expression, it can be written as $\delta L(0)\left(\mathbf{E}^B\left[e^{-(r-\alpha)\tau}\right]-\mathbf{E}^B\left[e^{-r\tau}\right]\right)$.Note that the limit value as T tends to infinity is the same as the expression above, as this does not depend on T. Using the definitions for G and G^α, we can express the third term as

$$\delta L(0)\mathbf{E}^B\left[e^{-r\tau}\left(e^{\alpha\tau}-1\right)\right] = \delta L(0)\left(G^\alpha - G\right).$$

Having calculated these three terms of $\Pi_3+\Pi_4$, and also the limit value as T tends to infinity, we have expressed the following

$$\lim_{T\to\infty}(\Pi_3+\Pi_4) = -\frac{C(0)}{r}(1-G)+\zeta C(0)H+\delta L(0)\left(G^\alpha-G\right).$$

3.4.4 Calculating $\mathbf{Lim}_{T\to\infty}\mathbf{\Pi_5}$

The last term Π_5 of the pricing function of equity corresponds to the payout at default to equity holders, due to abbreviations of the absolute priority rule. Using the assumed dynamics for $L(t)$, where ν denotes some drift, we can express $L(\tau) = L(0)e^{\nu\tau}$. Note that in the case of $L(t) = N(t)$, we simply set $\nu = \alpha$. As ε is assumed to be constant, Π_5 can therefore be expressed as

$$\begin{aligned}\mathbf{E}^B\left[e^{-r\tau}\varepsilon L(\tau)I(\tau \le T)\right] &= \varepsilon\mathbf{E}^B\left[e^{-r\tau}L(0)e^{\alpha\tau}I(\tau \le T)\right] \\ &= \varepsilon L(0)\mathbf{E}^B\left[e^{-(r-\alpha)\tau}I(\tau \le T)\right],\end{aligned}$$

where the limit as $T \to \infty$ for the last expectation value is recognised as G^α. Hence,

$$\lim_{T\to\infty}\Pi_5 = \varepsilon L(0)G^\alpha$$

Proposition 2 (Equity Price - Closed Form Pricing Formula) *Given Proposition (1) and Lemma (2), (3) and (4), the deterministic function of equity value is given by*

$$\Pi(0, V(0)) = \Omega - \frac{C(0)}{r}(1 - G) + \zeta C(0)H + \delta L(0)\left(G^\alpha - G\right) + \varepsilon L(0)G^\alpha,$$

where $\Omega(0, V(0))$ is suppressed to Ω. G, G^α and H accordingly.

3.5 Model Discussion

Reneby assumes a lognormal process U for modelling firm value, with a certain drift and diffusion term. Generated cash flow β is assumed to be included within the drift term, then excluded from the drift to be reintroduced in a separate definition of equity value. Further nominal debt N is modelled with a coupon bond with infinite maturity, where nominal debt and consequently also coupons C are assumed to grow at a certain rate α. Having stated these, equity is defined as a call option on firm value, as in Merton's model (1974), and some additional terms. Beside this call option, where the β is excluded, this generated cash flow is reintroduced as a separate term which has a positive affection on equity value. Then equity is argued to be negatively affected by the net effect of coupon payments after tax deduction. The two last terms argued to affect equity value are market price of future borrowings d, and an expected payoff to equity holders at default. As Reneby defines dividends to equity holders as the free generated cash flow minus coupon payments ex tax plus market price of future borrowings, the equity value is henceforth argued to be affected by a call option on firm value, dividends accrued to equity holders, and an expected payoff to share holders in default.

The net effect of letting coupon payments ex tax and market value of future borrowings affect equity, can be seen as the realised difference between market and nominal value of debt. This difference can be argued to gain equity holders, as it can be used as collateral for additional borrowings. However, this motivation differs from Reneby's initial motivation, thus with the same result.

4 Investment Companies

One aim with this thesis is to investigate the fitting of firm value for a structural model. Listed investment companies are used as object for the study, as their underlying assets in most cases also are listed. With observed market prices for the underlying assets, firm value is observed. Thus, by using investment companies, the modelled firm value can be compared with its observed value. Furthermore, with observed firm value, the parameters of the assumed firm value process can be empirically estimated.

4.1 Underlying Assets

As we observe listed investment companies, their assets are almost completely in shares. The main part of the holdings is shares in listed companies, yet a part is often in non listed companies. These companies have normally a minor part of their assets in risk free assets. We assume that the remaining assets, beside shares, are in a risk free asset. Hence the assets almost completely constitutes of holdings in listed and non listed companies. Not having the observed market values of the non listed companies, except at discrete times, the returns of the non listed companies shares are approximated with the OMX index. The OMX index is chosen as it represents the most traded companies on Stockholm Stock Exchange and other listed companies henceforth are more or less correlated with these. Note that all underlying assets are correlated.

Share prices are assumed to be lognormally distributed, which also is assumed for the OMX index. The dividing into listed shares, non listed shares and risk free holdings, is used when calculating the firm value. Hence, all underlying assets are assumed to be lognormally distributed in the parameter estimation. Let S denote the underlying assets.

Definition 14 (Underlying Assets) *Let S denote underlying correlated assets, and assume that every underlying asset S_i evolves according to a geometric Brownian motion, see Definition 1 with drift term μ_{S_i} and diffusion term σ_{S_i}. Let $R_{S_i}(t) = S_i(t)/S_i(0)$ denote the underlying asset return. By dividing both sides with $S_i(0)$, taking the logarithm, and then the expectation value, we get the following distribution of the underlying asset log-return,*

$$\log R_{S_i}(t) \in N\left(\left(\mu_{S_i} - \frac{1}{2}\sigma_{S_i}^2\right)t,\ \ \sigma_{S_i}^2 t\right).$$

4.2 Portfolio of Underlying Assets

Firm value of an investment company can thus be regarded as the value of a portfolio of the underlying assets. With the underlying assets defined as above, the portfolio can be regarded as a finite sum of correlated lognormal random variables. However, the sum of correlated lognormal random variables is not lognormal itself, and therefore several estimations of the sums distribution have been proposed. Yet, a common approach is still by assuming that the sum is approximately lognormal. In a study by Milevsky & Posner (1998), it shows that an infinite sum of correlated lognormally distributed random variables is reciprocal gamma distributed, but under certain restrictions for the drift and just the fact that it is for an infinite sum. However, it shows that the reciprocal gamma distribution is at least as good approximation of the finite sum, as the lognormal distribution is, irrespective of drift conditions. To ease calculations, the portfolio distribution will be approximated to be lognormal.

Definition 15 (Portfolio of Underlying Assets) *Let S denote underlying assets and a_i denote the absolute holding of the i:th asset. Then define $P(t)$ as the portfolio of underlying assets, where*

$$P(t) = \sum_i a_i S_i(t).$$

Approximate P(t) by a geometric Brownian motion with drift μ_P and diffusion term σ_P. By denoting the portfolio return with $R_P(t) = P(t)/P(0)$, the return on underlying assets $R_{S_i}(t) = S_i(t)/S_i(0)$, and the relative holding at time t by $w_i(t) = a_i S_i(t)/P(t)$, this can alternatively be expressed as

$$R_P(t) = \sum_i w_i(0) R_{S_i}(t).$$

With the approximation of $P(t)$ above, and similar reasoning as for the underlying assets, the distribution for the portfolios log-returns is given by

$$\log R_P(t) \in N\left(\left(\mu_P - \frac{1}{2}\sigma_P^2\right)t,\ \ \sigma_P^2 t\right).$$

4.3 Estimating Portfolio Parameters

To estimate portfolio drift and diffusion parameters, expression for the portfolio return is used along with the assumed distributions of the portfolio and sum of correlated lognormals. The expressions for portfolio parameters are then derived, by matching the two first moments of the sum and portfolio respectively. When estimating portfolio parameters, variances and correlations for the underlying assets are needed. The variances are estimated in the same way as for the variance of observed equity in the section of optimal parameter setting. Both variances and correlations are calculated by using stock returns. A useful result from probability theory is given by the following lemma.

Lemma 5 (Moment For a Lognormal Random Variable) *For two random variables X and Y, it holds that if X is lognormally distributed, then X is equally distributed with e^Y, where Y has a normal distribution. Thus,*

$$X \in LN\left(\mu, \sigma^2\right) \;\Rightarrow\; X \stackrel{d}{=} e^Y;\;\; Y \in N\left(\mu, \sigma^2\right),$$

for some mean μ and variance σ^2. Further, this implies equality between the moments of X and e^Y, and hence

$$\mathbf{E}\left[X^k\right] = \mathbf{E}\left[e^{kY}\right],$$

where the right hand expression in the equality is recognised as the moment generating function $\psi_Y(k)$ of Y. Hence we get the following expression for the k:th moment of X,

$$\mathbf{E}\left[X^k\right] = \exp\left(k\mu + \frac{1}{2}\sigma^2 k^2\right).$$

See Gut (1995) for proof.

Applying the Lemma 5 onto the portfolio return $R_P(t)$, we see that the first two moments are given by

$$\mathbf{E}\left[(R_P(t))^k\right] = \exp\left(k \cdot \left(\mu_P - \frac{1}{2}\sigma_P^2\right) t + \frac{1}{2}\left(\sigma_P^2 t\right) k^2\right);\;\; k = 1, 2,$$

which after some shorter rewriting can be expressed as

$$\begin{aligned} \mathbf{E}\left[R_P(t)\right] &= \exp\left(\mu_P t\right), \\ \mathbf{E}\left[(R_P(t))^2\right] &= \exp\left(\left(2\mu_P + \sigma_P^2\right) t\right). \end{aligned}$$

Now consider the corresponding sum, $\sum_i w_i(0) R_{S_i}(t)$, for which we want to calculate the following

$$\mathbf{E}\left[\left(\sum_i w_i(0) R_{S_i}(t)\right)^k\right];\;\; k = 1, 2.$$

Note that $w_i(0)$ are constants. Starting with the first expression, when $k = 1$, this can be written as $\sum_i w_i(0)\mathbf{E}\left[R_{S_i}(t)\right]$. By the previously defined distribution of $R_{S_i}(t)$, and applying the Lemma (5) onto the expectation value, we see that this can be expressed as

$$\mathbf{E}\left[\sum_i w_i(0)R_{S_i}(t)\right] = \sum_i w_i(0)\exp\left(\mu_{S_i}t\right).$$

Proceeding with the second expression, when $k = 2$, we use that $\left(\sum_i b_i\right)^2 = \sum_i \sum_j b_i b_j$. Hence we can rewrite the squared sum, by letting

$$\left(\sum_i w_i(0)R_{S_i}(t)\right)^2 = \sum_i \sum_j w_i(0)w_j(0)R_{S_i}(t)R_{S_j}(t).$$

The expectation value can thus be written as

$$\sum_i \sum_j w_i(0)w_j(0)\mathbf{E}\left[R_{S_i}(t)R_{S_j}(t)\right],$$

which leaves us with the expectation value $\mathbf{E}\left[R_{S_i}(t)R_{S_j}(t)\right]$. By using the inverse relation between the exponential and the logarithm functions along with elementary logarithm rules, the expectation value can be expressed as

$$\mathbf{E}\left[\exp\left\{\log R_{S_i}(t) + \log R_{S_j}(t)\right\}\right].$$

From probability theory we have that the sum of two normally distributed random variables $Z = Y_i + Y_j$, where $Y_i \in N\left(\mu_i, \sigma_i^2\right)$, is normally distributed with mean $\mu_Z = \mu_i + \mu_j$ and variance $\sigma_Z^2 = \sigma_i^2 + \sigma_j^2 + 2\rho_{ij}\sigma_i\sigma_j$. Applying this onto the sum $Z = \log R_{S_i}(t) + \log R_{S_j}(t)$, we get that Z is normally distributed with mean μ_Z and variance σ_Z given by

$$\begin{aligned}\mu_Z &= \left(\mu_{S_i} + \mu_{S_j} - \tfrac{1}{2}\left(\sigma_{S_i}^2 + \sigma_{S_j}^2\right)\right)t,\\ \sigma_Z^2 &= \left(\sigma_{S_i}^2 + \sigma_{S_j}^2 + 2\rho_{ij}\sigma_{S_i}\sigma_{S_j}\right)t.\end{aligned}$$

The correlation between $S_i(t)$ and $S_j(t)$ is denoted by ρ_{ij}. We continue by using the Lemma (5) onto the expectation value, and see that this can be expressed as

$$\mathbf{E}\left[\exp\left(\log R_{S_i}(t) + \log R_{S_j}(t)\right)\right] = \exp\left(\left(\mu_{S_i} + \mu_{S_j} + \rho_{ij}\sigma_{S_i}\sigma_{S_j}\right)t\right).$$

Hence we have the following expression for the second moment of the sum,

$$\mathbf{E}\left[\left(\sum_i w_i(0)R_{S_i}(t)\right)^2\right] = \sum_i \sum_j w_i(0)w_j(0)\exp\left(\left(\mu_{S_i} + \mu_{S_j} + \rho_{ij}\sigma_{S_i}\sigma_{S_j}\right)t\right).$$

Having calculated the first two moments of the portfolio and the corresponding sum of correlated lognormals, we want to derive the portfolio parameters out of the matched moment expressions. Thus, we use the following derived moment matching relations for deriving the portfolio parameter expressions,

$$\begin{aligned}\exp\left(\mu_P t\right) &= \textstyle\sum_i w_i(0)\exp\left(\mu_{S_i} t\right), \\ \exp\left(\left(2\mu_P + \sigma_P^2\right)t\right) &= \textstyle\sum_i \sum_j w_i(0) w_j(0) \exp\left(\left(\mu_{S_i} + \mu_{S_j} + \rho_{ij}\sigma_{S_i}\sigma_{S_j}\right)t\right).\end{aligned}$$

Start with the relation of first moments. By taking the logarithm and then divide both sides with t, we get the following expression for the portfolio drift term,

$$\mu_P = \frac{1}{t}\log\sum_i w_i(0)\exp\left(\mu_{S_i} t\right).$$

Continuing with the relation of second moments, we take the logarithm and use the derived expression for μ_P. Thus, we get the following expression for σ_P^2,

$$\sigma_P^2 = \frac{1}{t}\log\frac{\sum_i\sum_j w_i(0)w_j(0)\exp\left(\left(\mu_{S_i} + \mu_{S_j} + \rho_{ij}\sigma_{S_i}\sigma_{S_j}\right)t\right)}{\left(\sum_i w_i(0)\exp\left(\mu_{S_i} t\right)\right)^2}.$$

5 Data

Swedish listed investment companies are used for the study, as the main part of their underlying assets also are listed companies. The investment companies are:

1. Bure
2. Custos
3. Industrivärden
4. Investor
5. Invik
6. Kinnevik
7. Latour
8. Lundbergs
9. Ratos
10. Svolder
11. Öresund

Underlying assets constitute beside listed companies, of holdings in non-listed companies and other miscellaneous assets. The returns of the non-listed underlying assets are estimated by the OMX index. Also listed underlying assets which are traded in foreign markets are approximately estimated by OMX. We chose to approximate with OMX as these shares are in several cases companies parallel listed in Norway and Sweden. Further, we make the assumption that the remaining assets constitute risk free holdings, and thus model these with a bank account paying risk free interest. The risk free interest rate is approximated by the 3-month STIBOR, which is the inter offer rate between Stockholm banks. The Swedish corporate tax rate, used in the equity pricing function, is $\zeta = 28\%$. The pricing function of equity also includes estimated fractions paid out to equity holders ε and debt holders δ in default. We estimate the default payout to equity holders to be $\varepsilon = 10\%$ and the fractions to debt holders, the debt recovery rate, to $\delta = 40\%$.

5.1 Market Data

Stock prices for each of the investment companies and its underlying listed companies have been used. For many of the investment companies, there are several share types, where some are hardly traded. At all times the most traded share type has been used to represent the whole share capital. This assumption gives better approximation for the share volatility, as the less traded share type in most cases is quoted at the same price as the more traded type. Trades in these shares are often made through verbal agreements as the amount traded constitute significant parts of the company's total share capital. In market data, there shows to be missing quotes at several dates. These missing data are approximated by the mean of the adjacent market values. Daily market data of stock prices and the government bond, which is used for estimating the risk free interest rate, have been provided by

Stockholmsbörsen (www.stockholmsborsen.se). We use monthly data of the inflation rate, which is provided by Statistiska Centralbyrån (www.scb.se).

5.2 Companies Financial Information

Companies financial information is provided in monthly analyses from Swedbank Markets, and quarterly financial reports by each company. Within the monthly analyses from Swedbank Markets, market values of each underlying asset is given. Furthermore, the analyses include calculation of net asset value showing net effects of nominal debt or market value of assets and debt. As these appear in netted posts, we use companies financial reports for estimating the monthly data. However the analyses gives information about net asset value, amount of issued shares and dividends. Most important is the valuing of each underlying asset, for traded as well as non traded assets. For each of these, relative portfolio weights and market values are given. Within the pricing function of equity, we need an estimation of the free generated cash flow β, estimated coupon payments C, nominal debt N, estimation of the growth rate α of nominal debt and coupons equally, booked asset value and market price of asset U.

First we want an estimation of the free generated cash flow parameter β. By using Renebys definition of dividends [2], slightly amended, we define β as the coupon payments plus dividends, all divided by asset value. Assuming this, we use data from the Swedbank Market analyses of predicted dividends. We also assume that the generated cash flow must cover administration costs, and therefore let $\beta \geq 2$ %.

Second, we want to estimate the growth rate α of nominal debt and coupons. A modest assumption would be that the growth rate of nominal debt should be at least in line with, if not exceeding, the rate of inflation. Hence, we use the inflation rate as an estimate of α.

Third, we want estimated coupon payments C, nominal debt N, total asset value U. These are not explicitly given through the analyses from Swedbank Markets as some appear in netted posts. However, the asset value U is assumed to be the sum of traded and non traded underlying assets and, where the "net cash holdings/debt" is positive, cash holdings. When these "net

[2] Reneby defines dividends as the generated cash flow minus coupon payments plus market value of future borrowings. We make a slight amendment of the definition, by excluding the market value of future borrowings. Thus, we get $\beta =$ (coupon payments + dividends)/U.

cash holdings/debt" is negative, this is assumed to be total debt N and that no cash holdings exist.

5.3 Theoretical Equity Prices

Within the analysis, observed and theoretical equity prices are used. The observed equity prices are simply the observed prices of the investment companies share prices, whilst the theoretical equity prices are obtained by using observed share prices of the underlying assets and applying the equity pricing function.

5.3.1 Parameter Estimation

In calculating theoretical equity prices, parameter estimations of the underlying assets are needed. These have been estimated for a long period, where all historical data of each company is being used, and rolling 1-year and 3-month periods. The rolling 1-month and the long period estimation, show to give quite similar estimate values. Note that the proposed model by Reneby is defined for an infinite time interval. For many companies the investment horizon is no longer than 5 years, perhaps 1 or 2 years. However the rolling 1-year period estimations would suit our purpose better than the rolling 3-month, where the monthly variance in estimatie values is considerably high. See Appendix for parameter estimates.

5.3.2 Rebalancing Underlying Portfolio

By using monthly weights and financial data from the analyses, the development of theoretical equity prices can incorporate the effect of rebalancing the portfolio of underlying assets. Hence the development of theoretical equity prices is composed of monthly developments with a monthly rebalancing of the portfolio of underlying assets. A remark would be that several of the investment companies have a very long term investment horizon. Variations in weights in the underlying assets for these companies, are likely due to short term trading. However, some of the companies are more active in rebalancing their portfolios, which effects a monthly adjustment in portfolio weights sufficiently covers.

6 Analysis

Within this section we shall analyse relations between theoretical and observed equity prices and returns. Note that Reneby has assumed risk free market, and hence set $\lambda = 0$. With $\lambda \neq 0$, the relation between discount to net asset value and market price of risk would be interesting to analyse. Note that the chosen barrier is $L = N$.

6.1 Theoretical and Observed Equity Prices

Starting off with an illustrative plot over the theoretical and observed equity price for Industrivärden (Figure 1), we see that the theoretical equity price is higher than the observed price over the whole interval. This is the case for most of the investment companies, with a few exceptions. Notably is the difference between theoretical and observed prices. A possibility would be that the two time series have better matching by another barrier, like $L(t) = C(t)/\beta$ for instance. When comparing the plots of theoretical and observed equity values for the two barriers, the second proposed barrier, $L(t) = C(t)/\beta$, showed to give at least as good fitting to data, as the first barrier. See Appendix for complete plots with respective barrier.

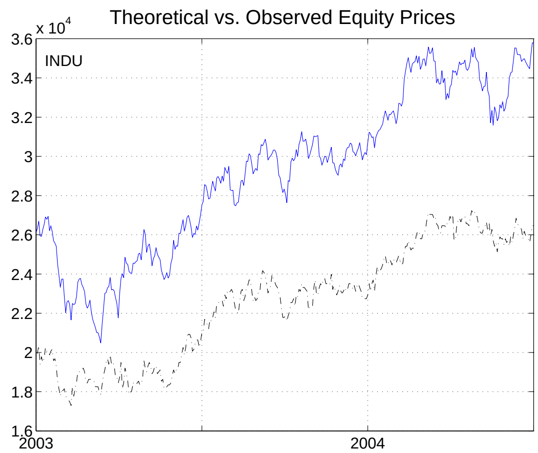

Figure 1

Figure 1 shows the theoretical and observed equity prices for Industrivärden, where the observed trajectory is dashed. 1-year rolling periods have been used for parameter estimations, and the barrier used is $L = N$.

Within the analysis, parameters have been estimated by using the observed underlying portfolios. However, this approach is particularly applicable on investment companies. Estimating parameters by more theoretical approaches, might increase the matching by the theoretical and observed prices. In the next section, two alternative approaches of estimating the parameters will therefore be presented, one by Ronn & Verma (1986) and one by Duan (1994).

We are lead into closer investigation of the absolute and relative relations between theoretical and observed equity prices. By plotting the absolute and relative relations with the respective sample mean, we see a conspicuous mean reverting character. See Figure 2 and 3.

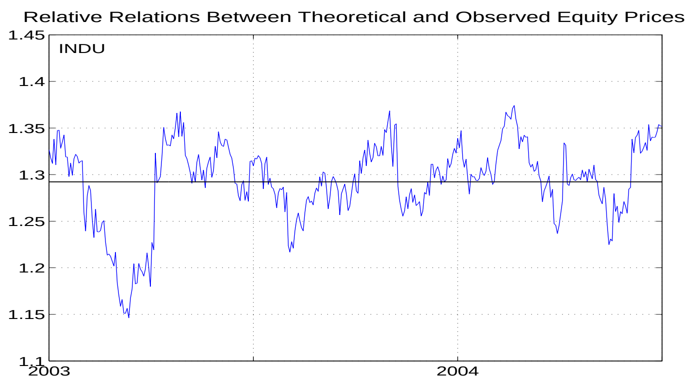

Figure 2

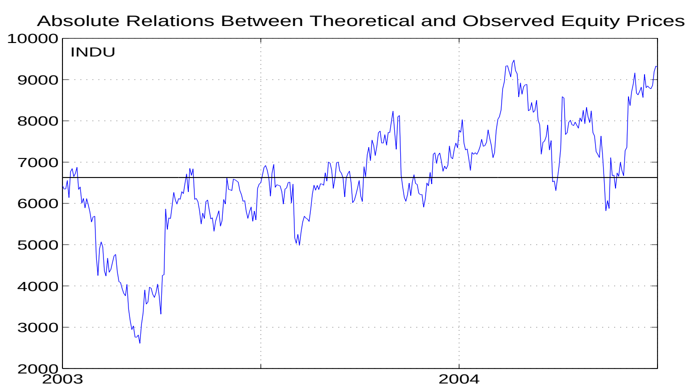

Figure 3

Figure 2 and 3 shows the relative and absolute relations between theoretical and observed equity prices for Industrivärden.

The mean reverting character of the relative relations is most significant for Industrivärden, Investor and Lundbergs. For the absolute relations, Kinnevik and Svolder have the most significant mean reverting characters, which both where not as clear when looking into the relative relations. Industrivärden, Investor and Lundbergs have also clear mean reverting characters for the absolute relations, thus with a small positive drift, as it seems.

Notable is that some companies have high degree of mean reversion within the investigated time interval. However, for some this behaviour is not as obvious and a longer time interval might be needed. Thus the intensity of the reversion varies from company to company.

Looking at applications, the mean reverting character of the relative relations between theoretical and observed equity value can be used for trading purposes. For instance, take a short position in the underlying portfolio and long in the investment company in an "over-the-mean" point. Then net these positions in an "under-the-mean" point. Thus, by contrary positioning in the underlying portfolio and the investment company, these gains can be realisable.

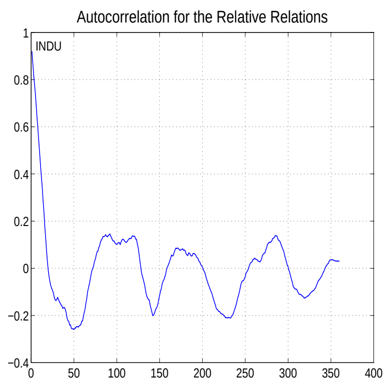

Figure 4

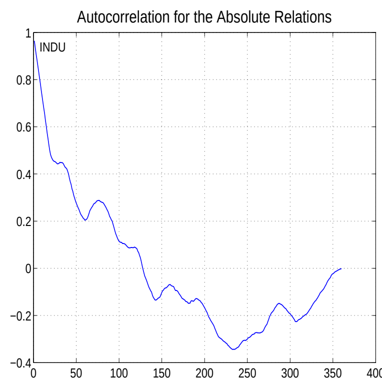

Figure 5

Figure 4 shows the autocorrelation of the relative relations of Industrivärden, whereas Figure 5 shows the autocorrelation of the absolute relations.

For the absolute and relative relations, the autocorrelation is calculated. For many of the investment companies, there is a positive autocorrelation for 100 days. The slower the decrease, the farther away in time does the correlation exist. Also here, special attention should be paid to Industrivärden, Investor and Lundbergs. However, for several companies the autocorrelation quickly approaches zero, and then fluctuates close to zero. The autocorrelation for the relative respectively absolute relationship between theoretical and observed equity prices is illustrated for Industrivärden in Figure 4 and Figure 5.

6.2 Theoretical and Observed Equity Returns

The theoretical and observed equity returns have been analysed, both daily, weekly and monthly. By plotting the returns, we found a linear dependence between the theoretical and observed returns. This dependence exist for the daily, weekly as well as for the monthly returns. However the range differs with time, where the returns are greater with longer time period. We illustrate this with Kinnevik in Figure 6.

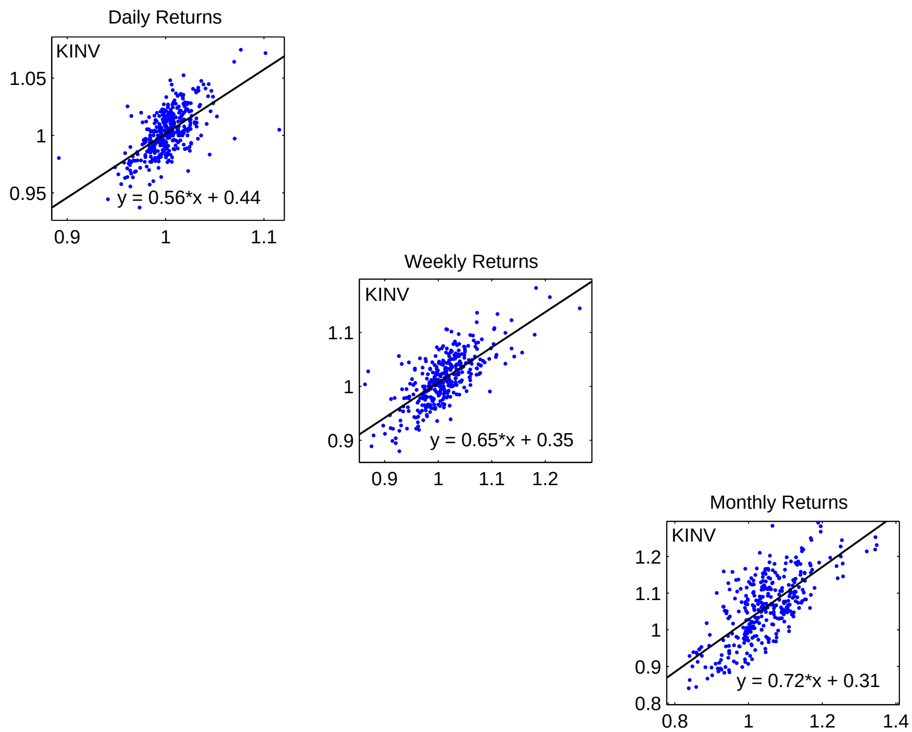

Figure 6

Figure 6 shows the daily, weekly and monthly returns for Kinnevik with theoretical returns represented by the horizontal axis and observed returns by the vertical axis.

7 Estimating Parameters

As firm value is the driving process in a structural model, its dynamics is of interest. Apart from firm value itself, its drift and diffusion parameters are estimated. There are several ways of estimating these, of which two will be presented.

The first approach, proposed by Ronn & Verma (1986), is by estimating firm value and its volatility parameter, by solving an equation system of the equity pricing function and the expression for the equity diffusion parameter. Within the equation system, observed market value of equity and an empirical estimation of its diffusion parameter are being used.

The second approach, is by maximum likelihood estimating the firm value parameters. Also here observed market value of equity is being used. This proposed setup by Duan (1994), incorporates expressing the likelihood function for equity by using its relation to firm value. Before describing the first approach, let us start with the prerequisites of equity.

7.1 Equity

Within this section the distribution of equity logreturns will be defined, empirical parameter estimations for equity derived, and the general parameter expressions of its pricing function derived. These will all show to be useful in the first approach of estimating firm value parameters.

7.1.1 Distribution of Logreturns

As market price of equity is given by the market value of company's stock, and stock prices are assumed to be lognormally distributed with mean μ_E and volatility σ_E, the equity return can be expressed as,

$$R_E(t) = \exp\left(\left(\mu_E - \frac{1}{2}\sigma_E^2\right)t + \sigma_E W(t)\right).$$

Now take the logarithm of both sides. By elementary probability theory the logreturns can be expressed as $\log R_E(t) = \left(\mu_E - \frac{1}{2}\sigma_E^2\right)t + \sigma_E W(t)$, which along with the fact that $W(t) \in N(0,t)$ yields the following distribution for the logreturns of equity,

$$\log R_E(t) \in N\left(\left(\mu_E - \frac{1}{2}\sigma_E^2\right)t,\ \ \sigma_E^2 t\right).$$

7.1.2 Empirical Parameter Estimations

Thus we have a normal distribution for the logreturns of equity. One way of estimating the mean and variance, is by estimating these with the sample mean and sample variance of daily observed logreturns. Note that the daily logreturns have the mean $\left(\mu_E - \frac{1}{2}\sigma_E^2\right)\Delta t$ and variance $\sigma_E^2 \Delta t$. The i-indexed equity value mark the i:th observed equity value, t denotes the time in years. Letting n denote the last observation, we get the following relation, $n = t/\Delta t$ [3] The sample mean of the daily logreturns is given by

$$m = \frac{1}{n}\sum_{i=1}^{n-1} \log \frac{E_{i+1}}{E_i}.$$

By rewriting the sum as $\sum_{i=1}^{n-1}(\log E_{i+1} - \log E_i)$, we see that the sample mean can be expressed as

$$m = \frac{1}{n}\log \frac{E_n}{E_1}.$$

With sample mean denoted by m and sample variance denoted by s^2, the sample variance of the daily logreturns is given by

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n-1}\left(\log \frac{E_{i+1}}{E_i} - m\right)^2,$$

which by using $\mathbf{E}[X-\mu]^2 = \mathbf{E}[X^2] - \mu^2$, can equally be expressed as

$$\frac{1}{n-1}\sum_{i=1}^{n-1}\left(\log \frac{E_{i+1}}{E_i}\right)^2 - \frac{n}{n-1}m^2.$$

By using the sample mean and sample variance as estimates for the drift and diffusion parameters of the daily returns of equity, we thus get the following estimates,

$$\begin{aligned} \left(\mu_E - \tfrac{1}{2}\sigma_E^2\right)\Delta t &= \tfrac{1}{n}\log \tfrac{E_n}{E_1}, \\ \sigma_E^2 \Delta t &= \tfrac{1}{n-1}\textstyle\sum_{i=1}^{n-1}\left(\log \tfrac{E_{i+1}}{E_i}\right)^2 - \tfrac{n}{n-1}\left(\tfrac{1}{n}\log \tfrac{E_n}{E_1}\right)^2. \end{aligned}$$

By slightly rewriting these, and using that $n = t/\Delta t$, we get

$$\begin{aligned} \mu_E &= \tfrac{1}{t}\log \tfrac{E_n}{E_1} + \tfrac{1}{2}\sigma_E^2, \\ \sigma_E^2 &= \tfrac{n}{t(n-1)}\textstyle\sum_{i=1}^{n-1}\left(\log \tfrac{E_{i+1}}{E_i}\right)^2 - \tfrac{1}{t(n-1)}\left(\log \tfrac{E_n}{E_1}\right)^2. \end{aligned}$$

[3] Usually Δt is approximated by 1/250, as the number of trading days in a year is approximately 250.

7.1.3 Parameters of Pricing Function

We now derive general parameter expressions for the pricing function of equity, by using Itô calculus. Note that the assumed dynamics of V is given by

$$dV(t) = \mu V(t)dt + \sigma V(t)dW(t).$$

Assume that the pricing function of equity is of the form $\Pi(t) = F(t, V(t))$, where F is some continuous and twice differentiable function. Then, by Itô and Equation (15), we get

$$\begin{aligned} dF &= \tfrac{\partial F}{\partial t}dt + \tfrac{\partial F}{\partial V}dV(t) + \tfrac{1}{2}\tfrac{\partial^2 F}{\partial V^2}(dV(t))^2 = \\ &= \left(\tfrac{\partial F}{\partial t} + \mu V(t)\tfrac{\partial F}{\partial V} + \tfrac{1}{2}\sigma^2(V(t))^2\tfrac{\partial^2 F}{\partial V^2}\right)dt + \sigma V(t)\tfrac{\partial F}{\partial V}dW(t). \end{aligned}$$

Thus, the dynamics of Π under the objective probability measure $\mathbf{P}$ is given by

$$d\Pi(t) = \mu_\Pi \Pi(t)dt + \sigma_\Pi \Pi(t)dW(t),$$

where the drift and diffusion terms are given by

$$\begin{aligned} \mu_\Pi &= \frac{\frac{\partial F}{\partial t} + \mu V(t)\frac{\partial F}{\partial V} + \frac{1}{2}\sigma^2(V(t))^2\frac{\partial^2 F}{\partial V^2}}{F}, \\ \sigma_\Pi &= \frac{\sigma V(t)\frac{\partial F}{\partial V}}{F}. \end{aligned}$$

The last relation, the definition of equity's diffusion term, will be used in the first approach of estimating firm value parameters.

7.2 First Approach of Estimating Firm Value Parameters

One way of estimating firm value and its parameters is by solving an equation system of the pricing function for equity Π and the expression for its diffusion parameter σ_Π. The diffusion parameter expression is derived by applying Itô calculus onto the pricing function of equity, and is expressed as

$$\sigma_\Pi = \frac{\sigma V(t)\frac{\partial \Pi}{\partial V}}{\Pi(t)}.$$

By using observed market prices of equity E and its empirical estimated volatility $\hat{\sigma}_E^2$, only firm value $V(t)$ and the firm value diffusion parameter σ remain unknown. Thus, firm value and its diffusion parameter are given as answers to the equation system

$$\begin{cases} E^*(t) &= \Pi(t, V(t)) \\ \hat{\sigma}_E &= \frac{\sigma V(t)\frac{\partial \Pi}{\partial V}}{E^*(t)}, \end{cases}$$

which is the approach used by Ronn & Verma (1986). Note that the market price of risk λ, which is included in the firm value drift, is not given by the calculations. To estimate this, we use equity market data. The market price of risk is defined as follows.

Definition 16 (Market Price of Risk) *Assume that the market is free of arbitrage. Then there exists a universal process λ such that*

$$\lambda = \frac{\mu_F - r}{\sigma_F}$$

holds for all contracts F, whether it is the underlying or derivative.

The firm value drift term under the objective probability measure $\mathbf{P}$ is given by $\mu = r + \lambda\sigma$. Thus, having estimated market price of risk λ and its volatility σ along with a given risk free interest rate r, the drift term is estimated.

7.3 Firm Value

Before proceeding with the second approach of estimating the parameters of firm value, the prerequisites will be presented. Within the second approach, the density function of firm value logreturns and the related likelihood function are being used. Alternatively the density function of the logreturns can be expressed as a conditional density function of firm value, where firm value is conditioned on the previous firm value.

Definition 17 (Firm Value Logreturns) *Assume that the firm value process $V(t)$ follows a geometric Brownian motion, with drift $(r + \lambda\sigma)$ and diffusion term σ. Define $\log R_V(t)$ as the logreturns of firm value, where $R_V(t) = V(t)/V(t-1)$, gives us the following distribution of the logreturns,*

$$\log R_V(t) \in N\left(\left(r + \lambda\sigma - \frac{1}{2}\sigma^2\right)\Delta t,\ \ \sigma^2\Delta t\right).$$

Note that we approximately set $\Delta t = 1/250$.

Let g denote the density function of the logreturns. With firm value logreturns defined as above, the corresponding density function g is defined as,

$$g = \frac{1}{\sqrt{2\pi\sigma^2\Delta t}} \exp\left(-\frac{\left(\log R_V(t) - \left(r + \lambda\sigma - \frac{1}{2}\sigma^2\right)\Delta t\right)^2}{2\sigma^2\Delta t}\right).$$

Denote the likelihood function with L_{R_V}. By using that the likelihood function is proportional to its related density function, we define the likelihood function L_{R_V} as,

$$L_{R_V} = \frac{1}{\sigma^n} \exp\left(-\frac{1}{2\sigma^2 \Delta t} \sum_{i=1}^{n} \left(\log_{R_V(t)} - \left(r + \lambda\sigma - \frac{1}{2}\sigma^2\right) \Delta t\right)^2\right).$$

Hence, by reintroducing the firm value returns expressed as fractions, we see that the density functions for the log firm value, conditioned on previous observation. Thus, the conditional density function for the log firm value is expressed as,

$$g\left(\log v(t) \middle| \log v(t-1); \lambda, \sigma\right) =$$
$$\frac{1}{\sqrt{2\pi\sigma^2 \Delta t}} \exp\left(-\frac{\left(\log v(t) - \left(\log v(t-1) + \left(r + \lambda\sigma - \frac{1}{2}\sigma^2\right)\Delta t\right)\right)^2}{2\sigma^2 \Delta t}\right).$$

By symmetry, the likelihood function for the firm value logreturns can be viewed as the likelihood function for the conditional log firm value, where log firm value is conditioned on the previous log firm value. Hence, the likelihood function can be expressed as,

$$L_{\log V(t)}(\lambda, \sigma) =$$
$$= \tfrac{1}{\sigma^n} \exp\left(-\tfrac{1}{2\sigma^2 \Delta t} \textstyle\sum_{i=1}^{n} \left(\log v(t) - \left(\log v(t-1) + \left(r + \lambda\sigma - \tfrac{1}{2}\sigma^2\right) \Delta t\right)^2\right)\right).$$

7.4 Second Approach of Estimating Firm Value Parameters

The second approach of estimating firm value and its parameters is by maximum likelihood estimation of the relevant parameters of the firm value process, $V(t)$. This is accomplished by using a time series of market prices of equity, $E^* = \{E_i^*, i = 1, \ldots, n\}$ [4].

As the available data is the result of a transformation involving the unknown parameters of the postulated model, the critical task for the estimation is the

[4] Note that the convention of denoting observed random variables with small letters is deviated from. As the observed values of the random variable E would have been denoted e, these might be mixed up with the conventional irrational number e. Therefore the observed values of E is denoted by E^*.

derivation of the loglikelihood function for the observed data. Ordinarily, deriving the likelihood function for the transformed data is a straightforward task when the transformation does not involve the unknown parameters of the original random variables. Since the transformation involve unknown parameters and the inverse transformations may not have analytical solutions, the likelihood functions can be difficult to derive. The proposed approach by Duan (1994) uses standard theory on differentiable transformations to define the likelihood function of the transformed data. This approach provides a framework in which numerical assessment of maximum likelihood estimates can be carried out.

We thus require the likelihood function of the observed price variable. Note that subscript i is used to index observations, in contrast to referring to a point t in time in years. Typically E_n^* is the current stock price and E_1^* is the stock price $n-1$ days ago. Define f as the conditional density for E_i^*, where $f(E_i^*|E_{i-1}^*;\lambda,\sigma)$. The corresponding loglikelihood function will thus be expressed as

$$L_E(E^*;\lambda,\sigma) = \sum_{i=2}^{n} \log f(E_i^*|E_{i-1}^*;\lambda,\sigma).$$

We note that V is a vector of the unobserved random variable denoting the firm value process. It's density function $g(v_i;\lambda,\sigma)$, as previously defined, is assumed to be continuously twice differentiable in both v, λ and σ. Also note that E^* is a vector of the observed random variable E, which is denoting equity value. By considering the pricing function of equity as a data transformation, the vector of observed equity prices E^* can be argued to be resulting from a data transformation of the unobserved vector V. This transformation from $\mathbf{R}^n$ to $\mathbf{R}^n$ is thus a function of the unknown parameters λ and σ. Denote this transformation Π. By inverting the pricing function and using observed equity values, the observed firm values can be implied out. Thus,

$$E_i^* = \Pi(v_i,t_i;\lambda,\sigma) \Longleftrightarrow v_i = \Pi^{-1}(E_i^*,t_i;\lambda,\sigma).$$

By using the transformation theorem from probability theory (see Gut (1995)), the following theorem is yielded.

Theorem 1 *The conditional density f of every E_i^*, $i=1,\ldots,n$, is given by*

$$f_E(E_i^*|E_{i-1}^*;\lambda,\sigma) = g_V(\Pi^{-1}(E_i^*;\lambda,\sigma);\lambda,\sigma)\cdot\left(\frac{\partial v_i}{\partial E_i^*}\right),$$

where Π^{-1} *is the unique inverse of* Π*, and where*

$$J = \left|\frac{d(v)}{d(E^*)}\right| = \left|\left\{\frac{\partial v_i}{\partial E_i^*}\right\}_{i=1}^{n}\right|,$$

that is, J *is the Jacobian.*

As the transformation is performed on an element-by-element basis, the Jacobian is a diagonal matrix. Let L_V denote the loglikelihood function for V. By using that the determinant of a diagonal matrix equals the product of its diagonal elements, the loglikelihood functions are expressed as,

$$L_E(E^*; \lambda, \sigma) = L_V(\Pi^{-1}(E_i^*; \lambda, \sigma); \lambda, \sigma) - \sum_{i=1}^{n} \log\left|\frac{\partial E_i^*}{\partial v_i}\right|.$$

8 Conclusions

The results of comparing relative as absolute relations between theoretical and observed equity values show to have mean reverting characters. Another result is the positive autocorrelations for these time series.

Also clear linear dependence between the corresponding returns is observed. With the clear linear dependence between returns on underlying portfolio and the investment company, an increase (decrease) in one of the two almost certain is followed by an increase (decrease) in the other.

For parameter estimations, the rolling 1-year periods yields sufficient volatility estimates. The effects of estimated financial data, the frequency of rebalancing the underlying portfolio, and finding the optimal process to describe the mean reverting processes (amongst others, the Ornstein-Uhlenbeck process approximation and its mean reversion), could all be discussed further.

Last, the barrier $L = C/\beta$ show to have the at least as good fitting as $L = N$. The barrier $L = C/\beta$ fitted the observed prices even better for Invik and Kinnevik.

9 Acknowledgements

I would like to thank my supervisor Ola Hammarlid at Department of Mathematics at Stockholm University for guidance and fruitful discussions, Christian Brunlid at Swedbank Markets for providing me with analyses of investment companies, Jin-Chuan Duan for providing me with a copy of his article from 1994, and Joel Reneby for answering questions regarding his model. Also I would like to thank my friends Andreas Nordvall Lagerås and Jens Ahlström for taking their time to read and for giving me feedback.

10 References

1. T. R. Bielecki and M. Rutkowski. "Credit Risk: Modelling, Valuation and Hedging". Springer-Verlag, 2002.

2. T. Björk. "Arbitrage Theory in Continuous Time". Oxford University Press, 1998.

3. J. C. Duan. "Maximum Likelihood Estimation Using Price Data of the Derivative Contract". *Mathematical Finance*, 2, 1994, pp 155 - 167.

4. J. Ericsson and J. Reneby. "A Framework for Valuing Corporate Securities". Stockholm School of Economics, 1995.

5. J. Ericsson and J. Reneby. "Implementing Firm Value Based Pricing Models". Stockholm School of Economics, 1997.

6. A. Gut. "An Intermediate Course in Probability". Springer-Verlag, 1995.

7. H. E. Leland. "Corporate Debt Value, Bond Covenants, and Optimal Capital Structure". *The Journal of Finance*, 4, 1984, pp 1213 - 1252.

8. M. A. Milevsky and S. E. Posner. "Asian Options, the Sum of Lognormals, and the Reciprocal Gamma Distribution". *Journal of Financial and Quantitative Analysis*, 3, Sep 1998, pp 409 - 422.

9. M. A. Milevsky and S. E. Posner. "A Closed-Form Approximation for Valuing Basket Options". *The Journal of Derivatives*, 1998, pp 54 - 61.

10. D. O'Kane and L. Schlögl. "Modelling Credit: Theory and Practice". Lehman Brothers International, Feb 2001.

11. J. Reneby. "Pricing Corporate Debt". Stockholm School of Economics, 1998.

12. E. I. Ronn and A. K. Verma. "Pricing Risk-Adjusted Deposit Insurance: An Option-Based Model". *The Journal of Finance*, 4, 1986, pp 871 - 895.

13. S. M. Ross. "Introduction to Probability Models - Seventh Edition". Academic Press, 2000.

14. P. J. Schönbucher. "Credit Risk Modelling and Credit Derivatives". Rheinischen Friedrich-Wilhelms-Universität Bonn, 2000.

A Appendix

A.1 Autocorrelation Function

The autocorrelation function $A(k)$, for a process $X(t)$ with mean $\bar{X}$, is defined as

$$A(k) = \frac{\sum_{t=k+1}^{T} \left(X(t-k) - \bar{X}\right) \left(X(t) - \bar{X}\right)}{\sum_{t=1}^{T} \left(X(t) - \bar{X}\right)^2}$$

By plotting the autocorrelation for all k, that is for all observations of X, we see positive autocorrelation approximately 100 days for most of the investment companies.

A.2 Plots

All plots refers to the period from 2003-01-02 to 2004-06-28.

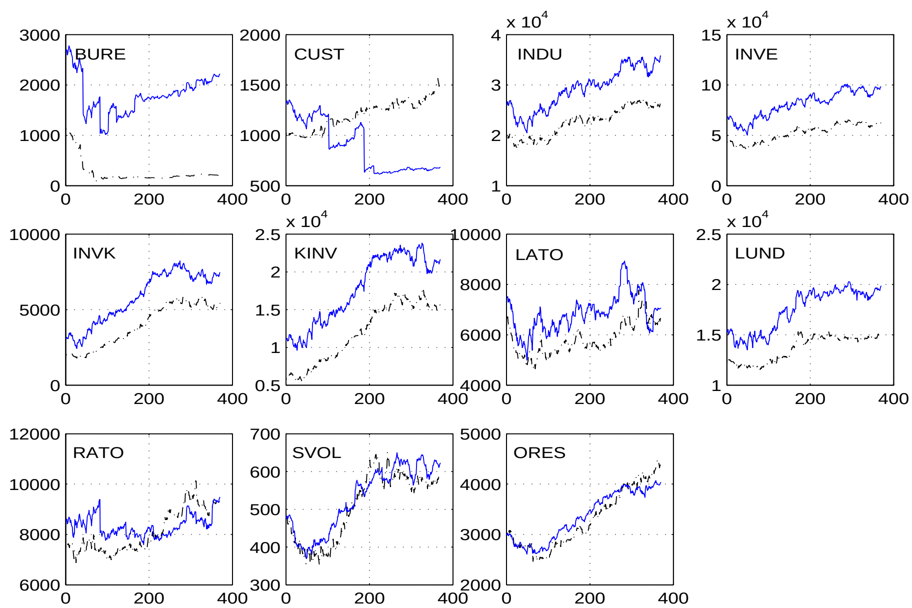

Figure A1

Figure A1: Theoretical and observed prices, with barrier $L = N$. The observed price trajectory is dashed, the theoretical continuous.

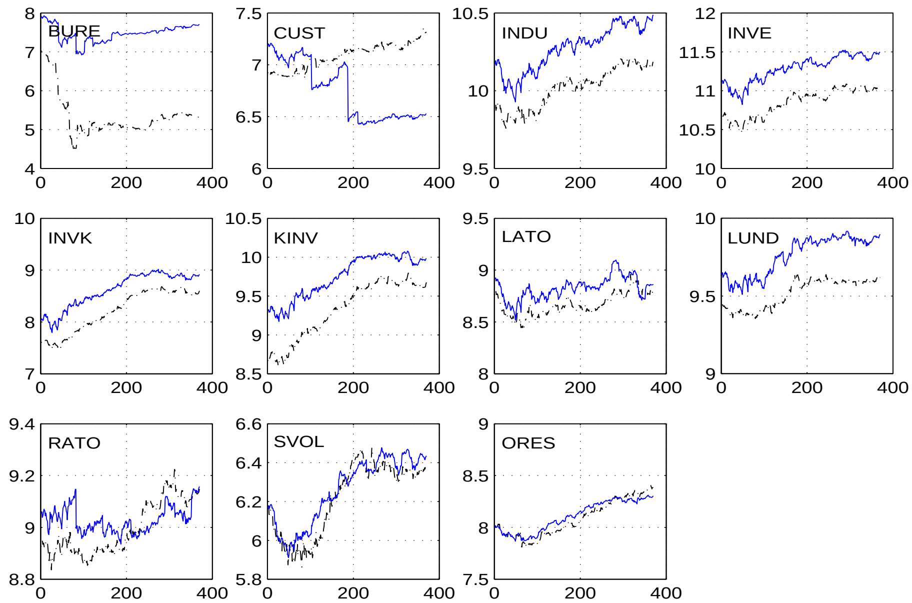

Figure A2

Figure A2: Theoretical and observed logprices, with barrier $L = N$. The observed logprice trajectory is dashed, the theoretical continuous.

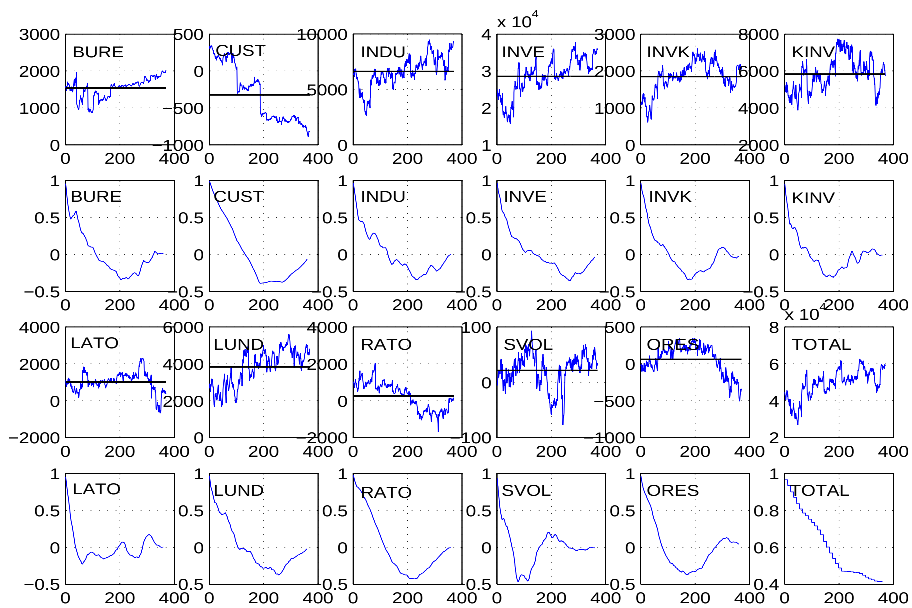

Figure A3

Figure A3: Absolute relations with corresponding autocorrelations. Barrier $L = N$.

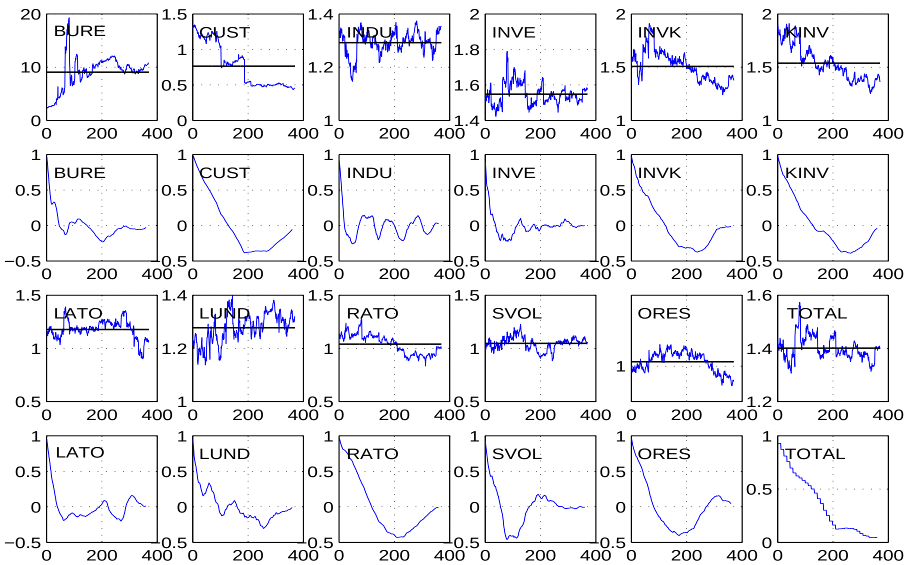

Figure A4

Figure A4: Relative relations with corresponding autocorrelations. Barrier $L = N$.

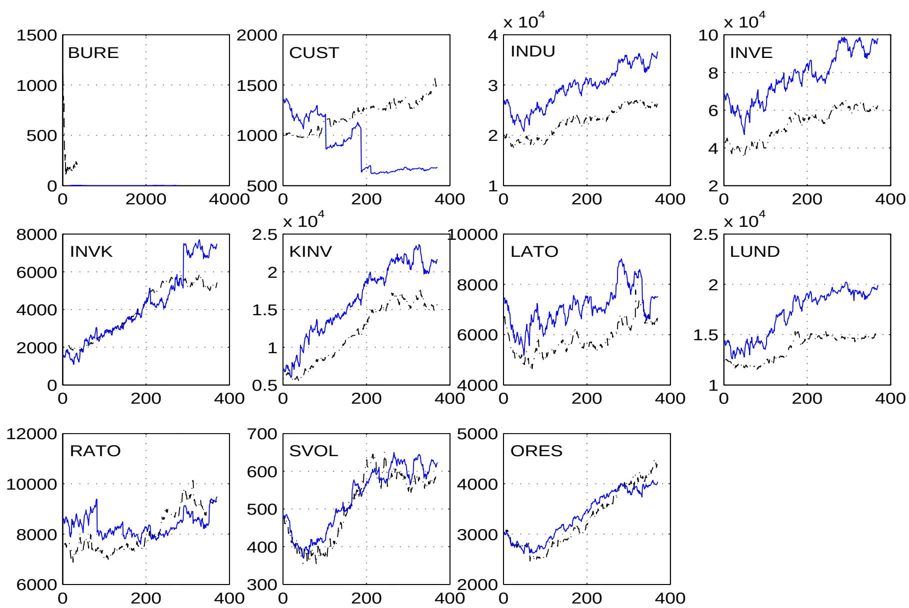

Figure A5

Figure A5: Theoretical and observed prices, with barrier $L = C/\beta$. The observed price trajectory is dashed, the theoretical continuous.

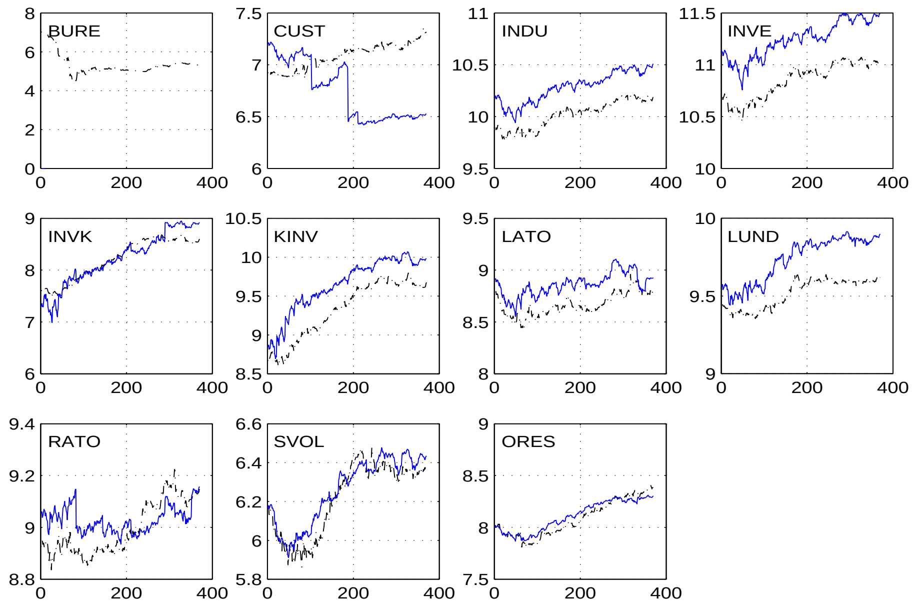

Figure A6

Figure A6: Theoretical and observed logprices, with barrier $L = C/\beta$. The observed price trajectory is dashed, the theoretical continuous.

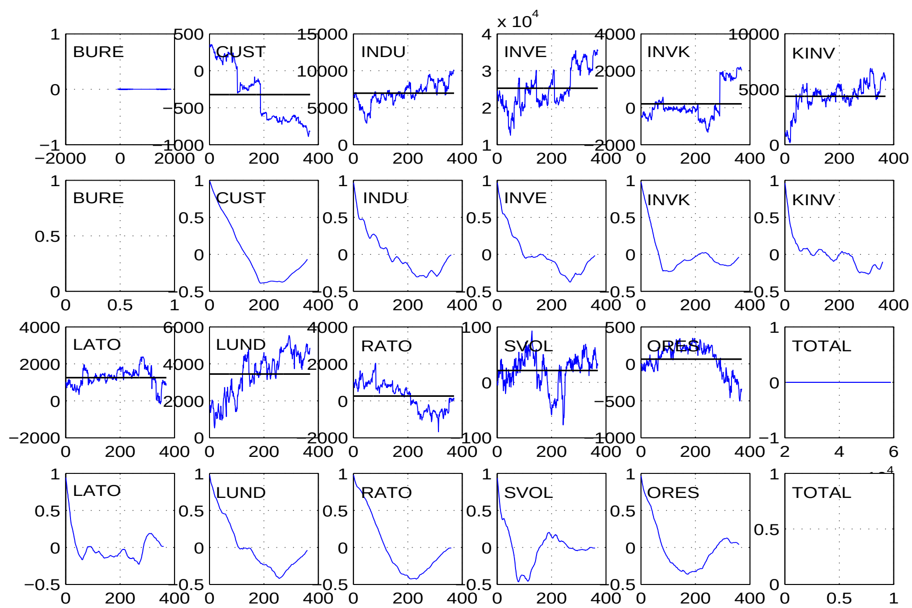

Figure A7

Figure A7: Absolute relations with corresponding autocorrelations. Barrier $L = C/\beta$.

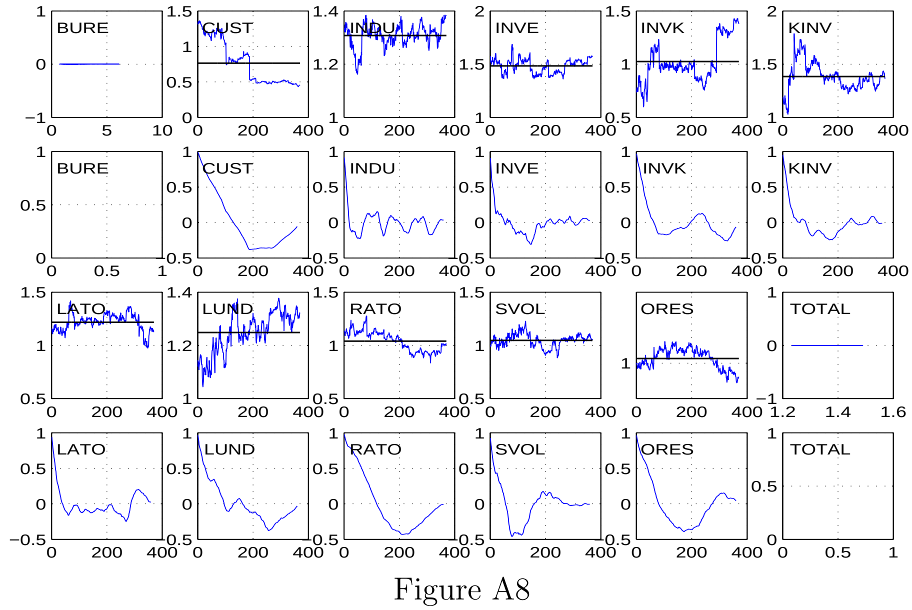

Figure A8: Relative relations with corresponding autocorrelations. Barrier $L = C/\beta$.

A.3 Data Tables

BURE	0301	0302	0303	0304	0305	0306	0307	0308	0309
U	3608	3401	2433	2439	2041	2610	2182	1697	1787
N	1024	1019	1074	1074	1114	1270	1020	313	72
C	40	39	39	39	38	39	30	9	2
Utd	104	78	0	0	0	0	0	0	0
β	0.0399	0.0344	0.0160	0.0160	0.0186	0.0149	0.0137	0.0053	0.0012
$DirAvk$	0.0920	0.0910	0	0	0	0	0	0	0
α	0.0280	0.0340	0.0310	0.0230	0.0190	0.0180	0.0180	0.0170	0.0170
r	0.0388	0.0383	0.0364	0.0363	0.0344	0.0304	0.0290	0.0292	0.0292
E^*	1135	854	323	260	93	90	387	342	1526
$\hat{\mu}_E$	0.0181								
$\hat{\sigma}_E$	0.4086								
$\hat{\lambda}_E$	-0.0506								
P^*									
$\hat{\mu}_P$	-0.0873	-0.1000	-0.1982	-0.0945	-0.0650	-0.0539	-0.0361	0.0023	0.0115
$\hat{\sigma}_P$	0.3600	0.3578	0.3641	0.3622	0.3795	0.3890	0.3928	0.4366	0.4359
$\hat{\lambda}_P$	-0.3504	-0.3865	-0.3698	-0.3612	-0.2618	-0.2166	-0.1657	-0.0617	-0.0405
$\hat{\mu}_{P_{1y}}$	-0.4119	-0.4123	-0.4401	-0.4968	-0.2663	-0.1713	-0.0322	0.2015	0.2568
$\hat{\sigma}_{P_{1y}}$	0.3655	0.3663	0.3807	0.3817	0.3868	0.3854	0.3787	0.3578	0.3543
$\hat{\lambda}_{P_{1y}}$	-1.2334	-1.2302	-1.2516	-1.3965	-0.7772	-0.5234	-0.1617	0.4814	0.6423
$\hat{\mu}_{P_{3m}}$	0.5292	-0.1628	-0.7757	-0.4843	0.2417	0.4792	0.9015	1.1453	0.8072
$\hat{\sigma}_{P_{3m}}$	0.4054	0.2941	0.2802	0.3024	0.2717	0.3010	0.2792	0.3233	0.2483
$\hat{\lambda}_{P_{3m}}$	1.2095	-0.6838	-2.8985	-1.7213	0.7629	1.4910	3.1247	3.4518	3.1339

BURE	0310	0311	0312	0401	0402	0403	0404	0405	0406
U	1674	2188	2219	2219	2221	2539	2323	2436	2458
N	0	546	546	546	552	645	338	419	423
C	0	16	16	15	15	16	7	9	9
Utd	0	0	0	0	0	0	0	0	0
β	0	0.0073	0.0072	0.0068	0.0068	0.0063	0.0030	0.0037	0.0037
$DirAvk$	0	0	0	0	0	0	0	0	0
α	0.0140	0.0140	0.0140	0.0080	-0.0030	-0.0010	0.0030	0.0070	0.0050
r	0.0292	0.0292	0.0290	0.0280	0.0263	0.0246	0.0219	0.0220	0.0220
E^*	1384	1384	1319	1410	1668	1707	1914	1966	1862
$\hat{\mu}_E$									
$\hat{\sigma}_E$									
$\hat{\lambda}_E$									
P^*									
$\hat{\mu}_P$	-0.0232	-0.0070	-0.0041	0.0020	0.0134	0.0228	0.0180	0.0138	0.0078
$\hat{\sigma}_P$	0.2656	0.2364	0.2432	0.2421	0.2429	0.2599	0.2519	0.2413	0.2401
$\hat{\lambda}_P$	-0.1972	-0.1532	-0.1361	-0.1073	-0.0532	-0.0068	-0.0155	-0.0340	-0.0591
$\hat{\mu}_{P_{1y}}$	0.2322	0.1717	0.0790	0.1954	0.3064	0.4217	0.3879	0.2821	0.2540
$\hat{\sigma}_{P_{1y}}$	0.2447	0.1962	0.1921	0.1935	0.2009	0.2177	0.1993	0.1791	0.1743
$\hat{\lambda}_{P_{1y}}$	0.8295	0.7262	0.2602	0.8651	1.3943	1.8244	1.8366	1.4526	1.3312
$\hat{\mu}_{P_{3m}}$	0.3366	0.3495	0.2942	0.4036	0.3921	0.5662	0.3001	0.0597	-0.1685
$\hat{\sigma}_{P_{3m}}$	0.1661	0.1675	0.1795	0.1499	0.1619	0.1961	0.1907	0.1375	0.1498
$\hat{\lambda}_{P_{3m}}$	1.8507	1.9123	1.4778	2.5053	2.2596	2.7612	1.4588	0.2744	-1.2720

CUST	0301	0302	0303	0304	0305	0306	0307	0308	0309
U	1365	1198	1136	1184	1208	868	889	938	1067
N	52	0	0	0	0	0	0	0	0
C	2	0	0	0	0	0	0	0	0
Utd	89	92	44	44	44	33	33	33	33
β	0.0667	0.0768	0.0387	0.0372	0.0364	0.0380	0.0371	0.0352	0.0309
$DirAvk$	0.0870	0.0910	0.0450	0.0430	0.0410	0.0380	0.0380	0.0380	0.0350
α	0.0280	0.0340	0.0310	0.0230	0.0190	0.0180	0.0180	0.0170	0.0170
r	0.0388	0.0383	0.0364	0.0363	0.0344	0.0304	0.0290	0.0292	0.0292
E^*	1019	1015	989	1041	1078	883	876	890	949
$\hat{\mu}_E$	0.0620								
$\hat{\sigma}_E$	0.4069								
$\hat{\lambda}_E$	0.0570								
P^*									
$\hat{\mu}_P$	-0.1901	-0.1404	-0.1409	-0.0559	-0.0056	-0.0377	0.0323	0.0345	0.0469
$\hat{\sigma}_P$	0.2446	0.2017	0.1933	0.1446	0.1381	0.2043	0.1759	0.1635	0.1690
$\hat{\lambda}_P$	-0.9358	-0.8860	-0.9172	-0.6378	-0.2894	-0.3333	0.0187	0.0324	0.1049
$\hat{\mu}_{P_{1y}}$	-0.2432	-0.1638	-0.2246	-0.1344	-0.0083	0.0085	0.1061	0.2094	0.2699
$\hat{\sigma}_{P_{1y}}$	0.2953	0.2657	0.2701	0.1975	0.1944	0.2813	0.2317	0.2385	0.2606
$\hat{\lambda}_{P_{1y}}$	-0.9551	-0.7607	-0.9666	-0.8641	-0.2198	-0.0778	0.3328	0.7553	0.9237
$\hat{\mu}_{P_{3m}}$	0.7548	0.1216	-0.4414	-0.1042	0.3581	0.8595	0.8408	0.2624	0.5644
$\hat{\sigma}_{P_{3m}}$	0.4249	0.1763	0.1645	0.1756	0.2362	0.3271	0.2241	0.1688	0.2168
$\hat{\lambda}_{P_{3m}}$	1.6849	0.4727	-2.9052	-0.8001	1.3705	2.5349	3.6218	1.3811	2.4690

CUST	0310	0311	0312	0401	0402	0403	0404	0405	0406
U	634	623	633	622	652	675	662	670	667
N	0	0	0	0	0	0	0	0	0
C	0	0	0	0	0	0	0	0	0
Utd	25	25	25	25	25	34	34	34	34
β	0.0394	0.0401	0.0395	0.0402	0.0383	0.0504	0.0514	0.0507	0.0510
$DirAvk$	0.0440	0.0430	0.0440	0.0420	0.0420	0.0530	0.0580	0.0530	0.0510
α	0.0140	0.0140	0.0140	0.0080	-0.0030	-0.0010	0.0030	0.0070	0.0050
r	0.0292	0.0292	0.0290	0.0280	0.0263	0.0246	0.0219	0.0220	0.0220
E^*	567	590	579	602	598	636	585	638	661
$\hat{\mu}_E$									
$\hat{\sigma}_E$									
$\hat{\lambda}_E$									
P^*									
$\hat{\mu}_P$	-0.0522	-0.0230	-0.0132	-0.0307	-0.0311	-0.0320	-0.0334	-0.0015	0.0014
$\hat{\sigma}_P$	0.1800	0.1397	0.1351	0.1337	0.1626	0.1660	0.1644	0.1247	0.1141
$\hat{\lambda}_P$	-0.4525	-0.3735	-0.3128	-0.4387	-0.3528	-0.3409	-0.3367	-0.1885	-0.1806
$\hat{\mu}_{P_{1y}}$	0.3102	0.2526	0.1690	0.1847	0.2580	0.3039	0.3033	0.1974	0.1797
$\hat{\sigma}_{P_{1y}}$	0.2507	0.1976	0.1866	0.1568	0.1681	0.1625	0.1530	0.1275	0.1189
$\hat{\lambda}_{P_{1y}}$	1.1211	1.1304	0.7506	0.9988	1.3785	1.7188	1.8392	1.3756	1.3257
$\hat{\mu}_{P_{3m}}$	0.3493	0.4061	0.2281	0.3182	0.2239	0.2876	0.1709	0.0714	-0.0297
$\hat{\sigma}_{P_{3m}}$	0.1700	0.1495	0.1409	0.1245	0.1022	0.0947	0.1026	0.0961	0.0894
$\hat{\lambda}_{P_{3m}}$	1.8837	2.5210	1.4128	2.3308	1.9325	2.7753	1.4525	0.5144	-0.5781

INDU	0301	0302	0303	0304	0305	0306	0307	0308	0309
U	27942	24841	24010	25989	27806	28544	29467	31146	32682
N	2100	2100	2100	3384	3050	3163	3163	3163	2850
C	81	80	76	123	105	96	92	92	83
Utd	1159	1159	966	966	966	966	966	966	966
β	0.0444	0.0499	0.0434	0.0419	0.0385	0.0372	0.0359	0.0340	0.0321
$DirAvk$	0.0570	0.0660	0.0520	0.0520	0.0500	0.0490	0.0450	0.0410	0.0400
α	0.0280	0.0340	0.0310	0.0230	0.0190	0.0180	0.0180	0.0170	0.0170
r	0.0388	0.0383	0.0364	0.0363	0.0344	0.0304	0.0290	0.0292	0.0292
E^*	19682	17289	17885	18108	18717	19170	20568	22781	23570
$\hat{\mu}_E$	0.1197								
$\hat{\sigma}_E$	0.3290								
$\hat{\lambda}_E$	0.2460								
P^*									
$\hat{\mu}_P$	-0.1036	-0.1259	-0.1033	-0.1023	-0.0685	-0.0701	-0.0608	-0.0443	-0.0291
$\hat{\sigma}_P$	0.2217	0.2259	0.2231	0.2258	0.2434	0.2481	0.2475	0.2746	0.2721
$\hat{\lambda}_P$	-0.6423	-0.7269	-0.6259	-0.6138	-0.4228	-0.4051	-0.3629	-0.2677	-0.2144
$\hat{\mu}_{P_{1y}}$	-0.2328	-0.2992	-0.3023	-0.2999	-0.1847	-0.1320	-0.0250	0.2845	0.3150
$\hat{\sigma}_{P_{1y}}$	0.3224	0.3567	0.3452	0.3513	0.4030	0.3875	0.3809	0.4754	0.4524
$\hat{\lambda}_{P_{1y}}$	-0.8425	-0.9462	-0.9810	-0.9570	-0.5437	-0.4192	-0.1418	0.5369	0.6317
$\hat{\mu}_{P_{3m}}$	1.1859	-0.0266	-0.5414	-0.3230	0.5263	0.6607	0.8011	0.7269	0.8907
$\hat{\sigma}_{P_{3m}}$	0.8495	0.3169	0.2794	0.2805	0.3176	0.4061	0.3334	0.3645	0.3700
$\hat{\lambda}_{P_{3m}}$	1.3502	-0.2047	-2.0680	-1.2812	1.5487	1.5522	2.3157	1.9140	2.3287

INDU	0310	0311	0312	0401	0402	0403	0404	0405	0406
U	31293	32467	32729	32219	33959	37195	37420	37362	37737
N	2850	2860	2860	1584	1610	3162	3914	4449	4458
C	83	84	83	44	42	78	86	98	98
Utd	966	966	966	966	966	1159	1159	1159	1159
β	0.0335	0.0323	0.0321	0.0313	0.0297	0.0333	0.0333	0.0336	0.0333
$DirAvk$	0.0420	0.0400	0.0410	0.0410	0.0380	0.0420	0.0430	0.0430	0.0440
α	0.0140	0.0140	0.0140	0.0080	-0.0030	-0.0010	0.0030	0.0070	0.0050
r	0.0292	0.0292	0.0290	0.0280	0.0263	0.0246	0.0219	0.0220	0.0220
E^*	22084	23221	22894	22990	24603	26534	26274	25992	25472
$\hat{\mu}_E$									
$\hat{\sigma}_E$									
$\hat{\lambda}_E$									
P^*									
$\hat{\mu}_P$	-0.0458	-0.0170	-0.0144	0.0111	0.0047	0.0238	0.0245	0.0271	0.0246
$\hat{\sigma}_P$	0.2611	0.2614	0.2559	0.2400	0.2828	0.2825	0.2795	0.2764	0.2709
$\hat{\lambda}_P$	-0.2873	-0.1766	-0.1697	-0.0705	-0.0763	-0.0027	0.0092	0.0184	0.0095
$\hat{\mu}_{P_{1y}}$	0.5075	0.3341	0.1748	0.3054	0.5304	0.6958	0.6868	0.4751	0.4451
$\hat{\sigma}_{P_{1y}}$	0.4762	0.3458	0.3102	0.3119	0.3629	0.3861	0.3520	0.3084	0.2973
$\hat{\lambda}_{P_{1y}}$	1.0043	0.8817	0.4700	0.8893	1.3889	1.7383	1.8892	1.4693	1.4231
$\hat{\mu}_{P_{3m}}$	0.4747	0.3021	0.0816	0.4642	0.4491	1.1748	0.9049	0.4015	0.0156
$\hat{\sigma}_{P_{3m}}$	0.3066	0.2213	0.1734	0.1356	0.2452	0.3856	0.4004	0.2771	0.1960
$\hat{\lambda}_{P_{3m}}$	1.4532	1.2330	0.3034	3.2174	1.7242	2.9829	2.2052	1.3691	-0.0328

INVE	0301	0302	0303	0304	0305	0306	0307	0308	0309
U	85908	74497	71598	77552	85846	88533	89280	93706	100920
N	19700	16089	16089	15739	18811	18643	18083	18493	18493
C	764	616	586	571	647	567	524	540	540
Utd	2493	2608	1534	1534	1534	1534	1534	1534	1534
β	0.0379	0.0433	0.0296	0.0271	0.0254	0.0237	0.0231	0.0221	0.0206
$DirAvk$	0.0580	0.0710	0.0440	0.0400	0.0370	0.0360	0.0340	0.0320	0.0270
α	0.0280	0.0340	0.0310	0.0230	0.0190	0.0180	0.0180	0.0170	0.0170
r	0.0388	0.0383	0.0364	0.0363	0.0344	0.0304	0.0290	0.0292	0.0292
E^*	42806	36901	35014	38742	41811	42806	45647	48176	55848
$\hat{\mu}_E$	0.1161								
$\hat{\sigma}_E$	0.3359								
$\hat{\lambda}_E$	0.2302								
P^*									
$\hat{\mu}_P$	-0.0381	-0.0450	-0.0421	-0.0373	-0.0002	-0.0040	0.0090	0.0236	0.0306
$\hat{\sigma}_P$	0.2432	0.2552	0.2499	0.2602	0.2734	0.2783	0.2729	0.2948	0.3023
$\hat{\lambda}_P$	-0.3161	-0.3265	-0.3143	-0.2829	-0.1266	-0.1236	-0.0734	-0.0188	0.0045
$\hat{\mu}_{P_{1y}}$	-0.3460	-0.3939	-0.4331	-0.4251	-0.2154	-0.1548	-0.0076	0.3026	0.3997
$\hat{\sigma}_{P_{1y}}$	0.4352	0.4653	0.4468	0.4627	0.4939	0.4714	0.4420	0.4725	0.4914
$\hat{\lambda}_{P_{1y}}$	-0.8844	-0.9289	-1.0507	-0.9972	-0.5058	-0.3928	-0.0828	0.5786	0.7538
$\hat{\mu}_{P_{3m}}$	1.1012	1.1331	-0.7379	-0.2795	0.5772	0.7815	0.9029	0.7690	1.1599
$\hat{\sigma}_{P_{3m}}$	1.3499	0.8803	0.3191	0.3182	0.3844	0.4425	0.3634	0.3361	0.4421
$\hat{\lambda}_{P_{3m}}$	0.7870	1.2437	-2.4266	-0.9925	1.4121	1.6973	2.4048	2.2011	2.5576

INVE	0310	0311	0312	0401	0402	0403	0404	0405	0406
U	98354	103685	103011	104711	111035	108343	107071	105917	106462
N	18493	19753	21692	21653	20891	13430	14084	14704	14967
C	540	577	629	606	549	330	308	323	329
Utd	1534	1534	1534	1534	1918	1918	1918	1918	1918
β	0.0211	0.0204	0.0210	0.0204	0.0222	0.0207	0.0208	0.0212	0.0211
$DirAvk$	0.0290	0.0280	0.0290	0.0280	0.0320	0.0310	0.0320	0.0330	0.0320
α	0.0140	0.0140	0.0140	0.0080	-0.0030	-0.0010	0.0030	0.0070	0.0050
r	0.0292	0.0292	0.0290	0.0280	0.0263	0.0246	0.0219	0.0220	0.0220
E^*	53546	54697	52468	54158	59372	61674	60067	58149	58605
$\hat{\mu}_E$									
$\hat{\sigma}_E$									
$\hat{\lambda}_E$									
P^*									
$\hat{\mu}_P$	0.0228	0.0492	0.0412	0.0448	0.0577	0.0688	0.0647	0.0656	0.0604
$\hat{\sigma}_P$	0.2912	0.2915	0.2957	0.2960	0.3153	0.3429	0.3382	0.3253	0.3184
$\hat{\lambda}_P$	-0.0220	0.0685	0.0412	0.0566	0.0996	0.1290	0.1266	0.1342	0.1205
$\hat{\mu}_{P_{1y}}$	0.5759	0.5029	0.2244	0.3828	0.5662	0.8089	0.7643	0.5169	0.4910
$\hat{\sigma}_{P_{1y}}$	0.5536	0.4412	0.3727	0.3675	0.3852	0.4055	0.3718	0.3220	0.3085
$\hat{\lambda}_{P_{1y}}$	0.9875	1.0736	0.5241	0.9653	1.4017	1.9344	1.9965	1.5369	1.5206
$\hat{\mu}_{P_{3m}}$	0.6790	0.4747	0.1993	0.4418	0.3911	1.1495	0.8830	0.3368	-0.0934
$\hat{\sigma}_{P_{3m}}$	0.3685	0.2312	0.1804	0.1381	0.2232	0.3832	0.3921	0.2727	0.2190
$\hat{\lambda}_{P_{3m}}$	1.7634	1.9268	0.9440	2.9972	1.6343	2.9354	2.1962	1.1543	-0.5269

INVK	0301	0302	0303	0304	0305	0306	0307	0308	0309
U	5338	4723	4843	5395	5651	6289	6359	6506	7088
N	2466	2150	1774	1852	1735	1724	1724	1740	1653
C	96	82	65	67	60	52	50	51	48
Utd	23	23	31	31	31	31	31	31	31
β	0.0223	0.0222	0.0198	0.0182	0.0161	0.0132	0.0127	0.0126	0.0111
$DirAvk$	0.0120	0.0130	0.0170	0.0150	0.0130	0.0110	0.0110	0.0100	0.0090
α	0.0280	0.0340	0.0310	0.0230	0.0190	0.0180	0.0180	0.0170	0.0170
r	0.0388	0.0383	0.0364	0.0363	0.0344	0.0304	0.0290	0.0292	0.0292
E^*	2010	1824	1847	2173	2483	2941	3081	3213	3639
$\hat{\mu}_E$	0.0737								
$\hat{\sigma}_E$	0.5063								
$\hat{\lambda}_E$	0.0690								
P^*									
$\hat{\mu}_P$	-0.3900	-0.3522	-0.2837	-0.2202	-0.1260	-0.0734	-0.0446	-0.0178	0.0321
$\hat{\sigma}_P$	0.5441	0.5334	0.5337	0.5219	0.5086	0.4918	0.4814	0.4680	0.4599
$\hat{\lambda}_P$	-0.7880	-0.7321	-0.5999	-0.4915	-0.3153	-0.2111	-0.1529	-0.1004	0.0062
$\hat{\mu}_{P_{1y}}$	-0.3990	-0.2858	-0.1140	-0.1553	0.2608	0.4750	0.5115	0.6289	0.7240
$\hat{\sigma}_{P_{1y}}$	0.5441	0.5471	0.5560	0.5338	0.4827	0.4512	0.4579	0.4535	0.4352
$\hat{\lambda}_{P_{1y}}$	-0.7880	-0.5923	-0.2705	-0.3589	0.4690	0.9855	1.0538	1.3224	1.5966
$\hat{\mu}_{P_{3m}}$	1.8862	0.3675	-0.0558	0.3838	0.8970	1.2670	1.5195	0.8608	0.7769
$\hat{\sigma}_{P_{3m}}$	0.5753	0.3916	0.4005	0.3801	0.3862	0.3830	0.3776	0.3486	0.2957
$\hat{\lambda}_{P_{3m}}$	3.2111	0.8406	-0.2303	0.9142	2.2337	3.2291	3.9470	2.3855	2.5284

INVK	0310	0311	0312	0401	0402	0403	0404	0405	0406
U	7489	8795	8864	8779	9390	8996	8636	8814	8578
N	1653	1659	1659	1659	1662	1773	1784	1791	1759
C	48	48	48	46	44	44	39	39	39
Utd	31	31	31	31	39	155	155	155	155
β	0.0105	0.0090	0.0089	0.0088	0.0088	0.0221	0.0224	0.0220	0.0226
$DirAvk$	0.0090	0.0060	0.0060	0.0060	0.0070	0.0280	0.0290	0.0290	0.0300
α	0.0140	0.0140	0.0140	0.0080	-0.0030	-0.0010	0.0030	0.0070	0.0050
r	0.0292	0.0292	0.0290	0.0280	0.0263	0.0246	0.0219	0.0220	0.0220
E^*	3880	5044	5122	5432	5657	5642	5385	5432	5160
$\hat{\mu}_E$									
$\hat{\sigma}_E$									
$\hat{\lambda}_E$									
P^*									
$\hat{\mu}_P$	0.0557	0.1499	0.1366	0.1414	0.1715	0.1618	0.1278	0.1129	0.0817
$\hat{\sigma}_P$	0.4517	0.4485	0.4368	0.4273	0.4257	0.4271	0.4175	0.3951	0.3901
$\hat{\lambda}_P$	0.0587	0.2691	0.2463	0.2654	0.3410	0.3212	0.2537	0.2301	0.1530
$\hat{\mu}_{P_{1y}}$	0.9608	0.7480	0.5740	0.7054	0.7991	0.8007	0.6860	0.5046	0.3425
$\hat{\sigma}_{P_{1y}}$	0.4070	0.3674	0.3475	0.3367	0.3342	0.3222	0.3041	0.2709	0.2544
$\hat{\lambda}_{P_{1y}}$	2.2891	1.9564	1.5685	2.0117	2.3122	2.4085	2.1837	1.7815	1.2600
$\hat{\mu}_{P_{3m}}$	0.8439	1.2416	0.8610	0.8207	0.3464	0.2194	-0.1284	-0.2373	-0.3949
$\hat{\sigma}_{P_{3m}}$	0.2909	0.2735	0.2661	0.2592	0.1951	0.1833	0.1859	0.1834	0.2289
$\hat{\lambda}_{P_{3m}}$	2.8001	4.4324	3.1266	3.0583	1.6408	1.0628	-0.8083	-1.4137	-1.8216

KINV	0301	0302	0303	0304	0305	0306	0307	0308	0309
U	16800	16627	17352	18683	18923	20593	20743	22213	23960
N	6312	6500	6830	6995	6694	6889	6889	7536	7425
C	245	249	249	254	230	209	200	220	217
Utd	126	189	315	315	315	315	315	315	315
β	0.0221	0.0263	0.0325	0.0305	0.0288	0.0254	0.0248	0.0241	0.0222
$DirAvk$	0.0220	0.0360	0.0540	0.0450	0.0400	0.0360	0.0350	0.0320	0.0280
α	0.0280	0.0340	0.0310	0.0230	0.0190	0.0180	0.0180	0.0170	0.0170
r	0.0388	0.0383	0.0364	0.0363	0.0344	0.0304	0.0290	0.0292	0.0292
E^*	5790	5192	5853	6923	7836	8686	9000	9913	11172
$\hat{\mu}_E$	0.1113								
$\hat{\sigma}_E$	0.4391								
$\hat{\lambda}_E$	0.1650								
P^*									
$\hat{\mu}_P$	-0.2325	-0.2259	-0.1985	-0.1802	-0.1311	-0.1131	-0.0962	-0.0991	-0.0777
$\hat{\sigma}_P$	0.4420	0.4292	0.4321	0.4341	0.4274	0.4257	0.4152	0.4278	0.4268
$\hat{\lambda}_P$	-0.6139	-0.6155	-0.5437	-0.4987	-0.3873	-0.3372	-0.3014	-0.2999	-0.2505
$\hat{\mu}_{P_{1y}}$	-0.4083	-0.3220	-0.2152	-0.2560	0.0928	0.2834	0.3097	0.4006	0.4542
$\hat{\sigma}_{P_{1y}}$	0.4524	0.4401	0.4515	0.4156	0.3609	0.3476	0.3581	0.3317	0.3522
$\hat{\lambda}_{P_{1y}}$	-0.9884	-0.8186	-0.5573	-0.7034	0.1618	0.7278	0.7839	1.1196	1.2065
$\hat{\mu}_{P_{3m}}$	1.5239	0.2325	-0.2160	0.1161	0.5426	0.7590	0.8983	1.4438	0.8293
$\hat{\sigma}_{P_{3m}}$	0.5423	0.3110	0.3106	0.3174	0.3368	0.3105	0.2800	0.6176	0.4033
$\hat{\lambda}_{P_{3m}}$	2.7387	0.6247	-0.8127	0.2515	1.5087	2.3469	3.1049	2.2904	1.9838

KINV	0310	0311	0312	0401	0402	0403	0404	0405	0406
U	25246	28213	28329	25824	26838	26241	26192	24954	24391
N	7425	7053	7053	4726	4646	4339	4302	4308	4519
C	217	206	205	132	122	107	94	95	99
Utd	315	315	315	378	378	441	441	441	441
β	0.0211	0.0185	0.0184	0.0197	0.0186	0.0209	0.0204	0.0215	0.0221
$DirAvk$	0.0270	0.0220	0.0220	0.0250	0.0230	0.0270	0.0280	0.0280	0.0300
α	0.0140	0.0140	0.0140	0.0080	−0.0030	−0.0010	0.0030	0.0070	0.0050
r	0.0292	0.0292	0.0290	0.0280	0.0263	0.0246	0.0219	0.0220	0.0220
E^*	11770	14413	14539	15231	16427	16112	15483	15609	14728
$\hat{\mu}_E$									
$\hat{\sigma}_E$									
$\hat{\lambda}_E$									
P^*									
$\hat{\mu}_P$	-0.0636	-0.0153	-0.0129	0.0144	0.0295	0.0122	0.0005	-0.0034	-0.0176
$\hat{\sigma}_P$	0.4247	0.4172	0.4152	0.4360	0.4268	0.4142	0.4004	0.3964	0.3828
$\hat{\lambda}_P$	-0.2184	-0.1067	-0.1010	-0.0311	0.0074	-0.0299	-0.0558	-0.0641	-0.1035
$\hat{\mu}_{P_{1y}}$	0.6902	0.5345	0.4013	0.6150	0.7787	0.7622	0.6989	0.5823	0.3782
$\hat{\sigma}_{P_{1y}}$	0.3794	0.3376	0.3445	0.3891	0.4120	0.3841	0.3541	0.3417	0.2834
$\hat{\lambda}_{P_{1y}}$	1.7420	1.4968	1.0809	1.5087	1.8260	1.9201	1.9119	1.6398	1.2567
$\hat{\mu}_{P_{3m}}$	0.8969	0.8913	0.7583	1.3918	0.7191	0.3340	0.0047	-0.1443	-0.3241
$\hat{\sigma}_{P_{3m}}$	0.4182	0.2425	0.2745	0.6089	0.4235	0.1686	0.1226	0.1663	0.2325
$\hat{\lambda}_{P_{3m}}$	2.0747	3.5629	2.6567	2.2396	1.6358	1.8347	-0.1404	-0.9997	-1.4886

LATO	0301	0302	0303	0304	0305	0306	0307	0308	0309
U	8878	7733	7226	7365	7794	7771	7655	8292	8778
N	1554	1554	1576	1546	1452	1917	1904	1907	1910
C	60	60	57	56	50	58	55	56	56
Utd	267	267	244	244	244	242	243	243	242
β	0.0368	0.0423	0.0417	0.0407	0.0377	0.0386	0.0389	0.0361	0.0339
$DirAvk$	0.0460	0.0560	0.0550	0.0550	0.0460	0.0530	0.0470	0.0460	0.0420
α	0.0280	0.0340	0.0310	0.0230	0.0190	0.0180	0.0180	0.0170	0.0170
r	0.0388	0.0383	0.0364	0.0363	0.0344	0.0304	0.0290	0.0292	0.0292
E^*	6374	5126	4797	4474	5515	5069	5127	5500	5904
$\hat{\mu}_E$	0.1830								
$\hat{\sigma}_E$	0.3095								
$\hat{\lambda}_E$	0.4658								
P^*									
$\hat{\mu}_P$	0.0817	0.0635	0.0464	0.0451	0.0755	0.0592	0.0729	0.0854	0.0874
$\hat{\sigma}_P$	0.3039	0.2925	0.2874	0.2903	0.2929	0.2845	0.2767	0.2793	0.2728
$\hat{\lambda}_P$	0.1413	0.0863	0.0346	0.0302	0.1404	0.1012	0.1586	0.2014	0.2135
$\hat{\mu}_{P_{1y}}$	-0.3135	-0.3612	-0.4354	-0.4799	-0.3165	-0.3035	-0.2032	-0.0237	0.0282
$\hat{\sigma}_{P_{1y}}$	0.3631	0.3655	0.3805	0.3942	0.3970	0.3644	0.3223	0.2914	0.2726
$\hat{\lambda}_{P_{1y}}$	-0.9702	-1.0930	-1.2399	-1.3094	-0.8839	-0.9164	-0.7205	-0.1815	-0.0036
$\hat{\mu}_{P_{3m}}$	0.2629	-0.2859	-0.8980	-0.6694	-0.2634	0.2479	0.5245	0.4117	0.7043
$\hat{\sigma}_{P_{3m}}$	0.3859	0.2881	0.2838	0.2698	0.3262	0.2996	0.2223	0.2954	0.2714
$\hat{\lambda}_{P_{3m}}$	0.5807	-0.1254	-3.2924	-2.6156	0.7020	0.7260	2.2294	1.2948	2.4874

LATO	0310	0311	0312	0401	0402	0403	0404	0405	0406
U	8329	8318	8263	8509	9122	10040	9763	8441	8283
N	1910	1661	1690	1690	1573	2430	2408	1823	1817
C	56	49	49	47	41	60	53	40	40
Utd	242	242	241	241	241	285	285	285	285
β	0.0358	0.0350	0.0351	0.0338	0.0309	0.0344	0.0346	0.0385	0.0392
$DirAvk$	0.0470	0.0470	0.0460	0.0470	0.0410	0.0450	0.0450	0.0450	0.0460
α	0.0140	0.0140	0.0140	0.0080	-0.0030	-0.0010	0.0030	0.0070	0.0050
r	0.0292	0.0292	0.0290	0.0280	0.0263	0.0246	0.0219	0.0220	0.0220
E^*	5287	5331	5324	5302	6135	6310	6266	6398	6179
$\hat{\mu}_E$									
$\hat{\sigma}_E$									
$\hat{\lambda}_E$									
P^*									
$\hat{\mu}_P$	0.0760	0.0923	0.0930	0.0983	0.1034	0.1250	0.1162	0.1227	0.1066
$\hat{\sigma}_P$	0.2643	0.2637	0.2649	0.2642	0.2779	0.2913	0.2864	0.3094	0.3107
$\hat{\lambda}_P$	0.1771	0.2393	0.2415	0.2661	0.2774	0.3446	0.3293	0.3256	0.2724
$\hat{\mu}_{P_{1y}}$	0.1458	0.1208	0.0144	0.1318	0.2287	0.3845	0.3855	0.2579	0.2271
$\hat{\sigma}_{P_{1y}}$	0.2605	0.2437	0.2390	0.2432	0.2737	0.3110	0.2977	0.2940	0.2743
$\hat{\lambda}_{P_{1y}}$	0.4475	0.3759	-0.0612	0.4270	0.7396	1.1575	1.2214	0.8026	0.7477
$\hat{\mu}_{P_{3m}}$	0.4643	0.2507	0.1232	0.5242	0.3680	0.6427	0.3791	0.3105	-0.3094
$\hat{\sigma}_{P_{3m}}$	0.2389	0.1951	0.2098	0.1879	0.2342	0.2686	0.2676	0.2331	0.1622
$\hat{\lambda}_{P_{3m}}$	1.8209	1.1353	0.4487	2.6404	1.4588	2.3009	1.3346	1.2380	-2.0436

LUND	0301	0302	0303	0304	0305	0306	0307	0308	0309
U	18573	17728	17795	17778	17523	17904	18448	18787	20760
N	3782	3663	3663	3779	3303	3254	2501	2605	2477
C	147	140	133	137	114	99	73	76	72
Utd	372	372	388	388	388	388	388	388	388
β	0.0279	0.0289	0.0293	0.0295	0.0286	0.0272	0.0250	0.0247	0.0222
$DirAvk$	0.0300	0.0330	0.0330	0.0340	0.0350	0.0330	0.0320	0.0310	0.0270
α	0.0280	0.0340	0.0310	0.0230	0.0190	0.0180	0.0180	0.0170	0.0170
r	0.0388	0.0383	0.0364	0.0363	0.0344	0.0304	0.0290	0.0292	0.0292
E^*	12353	11422	11794	11515	11236	11887	12291	12415	14370
$\hat{\mu}_E$	0.0787								
$\hat{\sigma}_E$	0.3181								
$\hat{\lambda}_E$	0.1253								
P^*									
$\hat{\mu}_P$	0.0819	0.0733	0.0765	0.0741	0.0787	0.0759	0.0823	0.0893	0.0952
$\hat{\sigma}_P$	0.1968	0.1938	0.1947	0.1987	0.2004	0.2014	0.2021	0.1997	0.2027
$\hat{\lambda}_P$	0.2187	0.1804	0.2059	0.1902	0.2213	0.2261	0.2639	0.3009	0.3257
$\hat{\mu}_{P_{1y}}$	-0.1350	-0.2242	-0.2630	-0.3055	-0.2122	-0.1824	-0.0779	0.1196	0.1735
$\hat{\sigma}_{P_{1y}}$	0.2716	0.2907	0.3034	0.3089	0.3196	0.3049	0.2777	0.2453	0.2505
$\hat{\lambda}_{P_{1y}}$	-0.6401	-0.9028	-0.9869	-1.1064	-0.7714	-0.6980	-0.3851	0.3684	0.5760
$\hat{\mu}_{P_{3m}}$	0.4204	-0.0847	-0.5102	-0.3679	0.2327	0.1388	0.4286	0.4279	0.6472
$\hat{\sigma}_{P_{3m}}$	0.3167	0.2112	0.2452	0.2505	0.2068	0.2089	0.1960	0.1879	0.1889
$\hat{\lambda}_{P_{3m}}$	1.2047	-0.5824	-2.2289	-1.6133	0.9588	0.5188	2.0391	2.1222	3.2715

LUND	0310	0311	0312	0401	0402	0403	0404	0405	0406
U	20181	20662	20973	21137	21278	22511	21205	21176	21218
N	2477	2477	2438	2438	2691	3408	2773	2779	2737
C	72	72	71	68	71	84	61	61	60
Utd	388	388	388	388	435	434	434	434	434
β	0.0228	0.0223	0.0219	0.0216	0.0238	0.0230	0.0233	0.0234	0.0233
$DirAvk$	0.0280	0.0270	0.0270	0.0270	0.0310	0.0310	0.0320	0.0310	0.0310
α	0.0140	0.0140	0.0140	0.0080	-0.0030	-0.0010	0.0030	0.0070	0.0050
r	0.0292	0.0292	0.0290	0.0280	0.0263	0.0246	0.0219	0.0220	0.0220
E^*	13719	14215	14153	14370	13967	14167	13764	14105	14136
$\hat{\mu}_E$									
$\hat{\sigma}_E$									
$\hat{\lambda}_E$									
P^*									
$\hat{\mu}_P$	0.0912	0.0981	0.0982	0.0992	0.1025	0.1040	0.0990	0.0986	0.0963
$\hat{\sigma}_P$	0.1989	0.1979	0.1932	0.1918	0.1895	0.1903	0.1882	0.1890	0.1878
$\hat{\lambda}_P$	0.3120	0.3483	0.3581	0.3712	0.4021	0.4174	0.4097	0.4051	0.3957
$\hat{\mu}_{P_{1y}}$	0.2369	0.2028	0.0997	0.1994	0.2850	0.3084	0.2885	0.2332	0.2347
$\hat{\sigma}_{P_{1y}}$	0.2444	0.2273	0.2148	0.2136	0.2117	0.2027	0.2069	0.1749	0.1601
$\hat{\lambda}_{P_{1y}}$	0.8499	0.7637	0.3289	0.8025	1.2222	1.4000	1.2885	1.2078	1.3281
$\hat{\mu}_{P_{3m}}$	0.4192	0.3796	0.1741	0.3845	0.2245	0.2740	0.0265	-0.0877	-0.2044
$\hat{\sigma}_{P_{3m}}$	0.1948	0.1813	0.1542	0.1413	0.1034	0.0840	0.1313	0.1074	0.1536
$\hat{\lambda}_{P_{3m}}$	2.0019	1.9326	0.9413	2.5236	1.9171	2.9680	0.0348	-1.0219	-1.4740

RATO	0301	0302	0303	0304	0305	0306	0307	0308	0309
U	8544	8593	8586	8536	8069	7919	7952	7922	7877
N	0	0	0	0	0	0	0	0	0
C	0	0	0	0	0	0	0	0	0
Utd	492	492	531	531	531	531	531	530	530
β	0.0576	0.0573	0.0618	0.0622	0.0658	0.0671	0.0668	0.0669	0.0673
$DirAvk$	0.0660	0.0720	0.0730	0.0710	0.0710	0.0760	0.0740	0.0730	0.0720
α	0.0280	0.0340	0.0310	0.0230	0.0190	0.0180	0.0180	0.0170	0.0170
r	0.0388	0.0383	0.0364	0.0363	0.0344	0.0304	0.0290	0.0292	0.0292
E^*	7513	7081	7277	7631	7356	7001	7350	7346	7346
$\hat{\mu}_E$	0.1028								
$\hat{\sigma}_E$	0.3739								
$\hat{\lambda}_E$	0.1712								
P^*									
$\hat{\mu}_P$	-0.0613	-0.0604	-0.0555	-0.0550	-0.0387	-0.0394	-0.0385	-0.0285	-0.0181
$\hat{\sigma}_P$	0.2439	0.2141	0.2066	0.2126	0.2266	0.2220	0.2287	0.2283	0.2211
$\hat{\lambda}_P$	-0.4106	-0.4608	-0.4449	-0.4293	-0.3224	-0.3145	-0.2950	-0.2527	-0.2141
$\hat{\mu}_{P_{1y}}$	-0.3554	-0.2885	-0.3071	-0.3346	-0.1886	-0.1400	-0.0562	0.0931	0.1191
$\hat{\sigma}_{P_{1y}}$	0.2714	0.2331	0.2304	0.2341	0.2681	0.2658	0.2546	0.2354	0.2212
$\hat{\lambda}_{P_{1y}}$	-1.4526	-1.4016	-1.4905	-1.5841	-0.8315	-0.6410	-0.3344	0.2714	0.4063
$\hat{\mu}_{P_{3m}}$	0.3886	-0.0784	-0.4579	-0.3169	0.2752	0.2987	0.4765	0.3080	0.3929
$\hat{\sigma}_{P_{3m}}$	0.3179	0.1974	0.1566	0.1923	0.2395	0.2188	0.1819	0.1490	0.1368
$\hat{\lambda}_{P_{3m}}$	1.1004	-0.5914	-3.1555	-1.8370	1.0057	1.2263	2.4599	1.8713	2.6579

RATO	0310	0311	0312	0401	0402	0403	0404	0405	0406
U	7869	7878	7991	8005	8239	9009	8421	8566	9156
N	0	0	0	0	0	0	0	0	0
C	0	0	0	0	0	0	0	0	0
Utd	530	530	530	530	530	570	570	570	570
β	0.0674	0.0673	0.0663	0.0662	0.0643	0.0632	0.0677	0.0665	0.0623
$DirAvk$	0.0730	0.0680	0.0660	0.0610	0.0600	0.0600	0.0610	0.0640	0.0620
α	0.0140	0.0140	0.0140	0.0080	-0.0030	-0.0010	0.0030	0.0070	0.0050
r	0.0292	0.0292	0.0290	0.0280	0.0263	0.0246	0.0219	0.0220	0.0220
E^*	7345	7895	8170	8759	8680	9505	9348	8837	9152
$\hat{\mu}_E$									
$\hat{\sigma}_E$									
$\hat{\lambda}_E$									
P^*									
$\hat{\mu}_P$	-0.0212	-0.0074	-0.0025	0.0111	0.0162	0.0110	0.0089	0.0118	0.0057
$\hat{\sigma}_P$	0.2193	0.2173	0.1914	0.1877	0.1991	0.1999	0.2207	0.2193	0.2122
$\hat{\lambda}_P$	-0.2299	-0.1685	-0.1646	-0.0901	-0.0507	-0.0681	-0.0588	-0.0464	-0.0769
$\hat{\mu}_{P_{1y}}$	0.2043	0.1794	0.0779	0.1917	0.2804	0.3010	0.3357	0.2486	0.2219
$\hat{\sigma}_{P_{1y}}$	0.2081	0.1830	0.1485	0.1699	0.1743	0.1485	0.1515	0.1416	0.1419
$\hat{\lambda}_{P_{1y}}$	0.8416	0.8207	0.3291	0.9638	1.4581	1.8613	2.0716	1.6006	1.4088
$\hat{\mu}_{P_{3m}}$	0.2712	0.3040	0.1368	0.3813	0.2306	0.3424	0.2100	0.1156	-0.0462
$\hat{\sigma}_{P_{3m}}$	0.1331	0.1414	0.1168	0.1373	0.1026	0.0938	0.1286	0.1442	0.1336
$\hat{\lambda}_{P_{3m}}$	1.8191	1.9434	0.9227	2.5729	1.9911	3.3892	1.4630	0.6491	-0.5105

SVOL	0301	0302	0303	0304	0305	0306	0307	0308	0309
U	476	415	398	393	412	436	467	495	552
N	0	0	0	0	0	0	0	0	0
C	0	0	0	0	0	0	0	0	0
Utd	29	29	19	19	19	19	19	19	19
β	0.0609	0.0699	0.0477	0.0483	0.0461	0.0436	0.0407	0.0384	0.0344
$DirAvk$	0.0610	0.0740	0.0530	0.0530	0.0510	0.0510	0.0480	0.0430	0.0380
α	0.0280	0.0340	0.0310	0.0230	0.0190	0.0180	0.0180	0.0170	0.0170
r	0.0388	0.0383	0.0364	0.0363	0.0344	0.0304	0.0290	0.0292	0.0292
E^*	474	390	365	365	374	378	397	448	512
$\hat{\mu}_E$	0.0605								
$\hat{\sigma}_E$	0.3500								
$\hat{\lambda}_E$	0.0619								
P^*									
$\hat{\mu}_P$	-0.0986	-0.1199	-0.1209	-0.1166	-0.0874	-0.0656	-0.0335	0.0316	0.0620
$\hat{\sigma}_P$	0.4216	0.4160	0.4094	0.4060	0.4119	0.4081	0.4114	0.4176	0.4211
$\hat{\lambda}_P$	-0.3260	-0.3804	-0.3841	-0.3765	-0.2956	-0.2353	-0.1519	0.0058	0.0779
$\hat{\mu}_{P_{1y}}$	-0.2532	-0.2820	-0.2905	-0.3470	-0.2126	-0.1614	-0.0398	0.1358	0.2295
$\hat{\sigma}_{P_{1y}}$	0.4437	0.4398	0.4314	0.4243	0.4295	0.3969	0.3714	0.3328	0.3444
$\hat{\lambda}_{P_{1y}}$	-0.6581	-0.7282	-0.7576	-0.9035	-0.5751	-0.4832	-0.1852	0.3203	0.5816
$\hat{\mu}_{P_{3m}}$	1.1328	0.2380	-0.5902	-0.2576	0.1353	0.3791	1.2475	1.4569	1.6238
$\hat{\sigma}_{P_{3m}}$	0.6275	0.3984	0.2963	0.2196	0.2179	0.2855	0.4159	0.4629	0.4558
$\hat{\lambda}_{P_{3m}}$	1.7435	0.5012	-2.1145	-1.3384	0.4629	1.2215	2.9300	3.0845	3.4981

SVOL	0310	0311	0312	0401	0402	0403	0404	0405	0406
U	533	590	579	575	632	629	623	616	616
N	0	0	0	0	0	0	0	0	0
C	0	0	0	0	0	0	0	0	0
Utd	29	29	29	29	32	32	32	32	32
β	0.0544	0.0492	0.0501	0.0504	0.0506	0.0509	0.0514	0.0519	0.0519
$DirAvk$	0.0520	0.0460	0.0530	0.0530	0.0540	0.0560	0.0570	0.0570	0.0580
α	0.0140	0.0140	0.0140	0.0080	-0.0030	-0.0010	0.0030	0.0070	0.0050
r	0.0292	0.0292	0.0290	0.0280	0.0263	0.0246	0.0219	0.0220	0.0220
E^*	557	627	539	544	589	568	557	558	552
$\hat{\mu}_E$									
$\hat{\sigma}_E$									
$\hat{\lambda}_E$									
P^*									
$\hat{\mu}_P$	0.0731	0.1104	0.1346	0.1257	0.1840	0.1710	0.1185	0.1174	0.1137
$\hat{\sigma}_P$	0.3912	0.3737	0.3949	0.3729	0.3902	0.3714	0.3273	0.3206	0.3158
$\hat{\lambda}_P$	0.1123	0.2173	0.2675	0.2619	0.4041	0.3943	0.2951	0.2975	0.2904
$\hat{\mu}_{P_{1y}}$	0.5010	0.4499	0.2930	0.4258	0.6275	0.6427	0.5863	0.5381	0.5117
$\hat{\sigma}_{P_{1y}}$	0.3687	0.3343	0.3427	0.3394	0.3897	0.3779	0.3334	0.3004	0.2839
$\hat{\lambda}_{P_{1y}}$	1.2797	1.2585	0.7702	1.1719	1.5427	1.6357	1.6928	1.7180	1.7246
$\hat{\mu}_{P_{3m}}$	0.8463	0.8696	0.7606	0.8828	0.8296	0.5566	0.3625	0.1054	-0.1014
$\hat{\sigma}_{P_{3m}}$	0.3806	0.3002	0.2778	0.2666	0.2911	0.2388	0.1830	0.1770	0.1789
$\hat{\lambda}_{P_{3m}}$	2.1465	2.7997	2.6333	3.2067	2.7598	2.2275	1.8606	0.4709	-0.6898

ORES	0301	0302	0303	0304	0305	0306	0307	0308	0309
U	3012	2841	2753	2625	2822	2848	2986	3094	3272
N	59	30	16	7	127	0	0	0	0
C	2	1	1	0	4	0	0	0	0
Utd	141	189	165	165	165	165	165	165	165
β	0.0475	0.0669	0.0603	0.0629	0.0599	0.0579	0.0553	0.0533	0.0504
$DirAvk$	0.0470	0.0690	0.0600	0.0650	0.0660	0.0630	0.0590	0.0570	0.0530
α	0.0280	0.0340	0.0310	0.0230	0.0190	0.0180	0.0180	0.0170	0.0170
r	0.0388	0.0383	0.0364	0.0363	0.0344	0.0304	0.0290	0.0292	0.0292
E^*	3003	2735	2752	2554	2506	2634	2794	2870	3105
$\hat{\mu}_E$	0.1167								
$\hat{\sigma}_E$	0.2676								
$\hat{\lambda}_E$	0.2910								
P^*									
$\hat{\mu}_P$	-0.0661	-0.0861	-0.0875	-0.0826	-0.0627	-0.0410	-0.0178	-0.0035	0.0392
$\hat{\sigma}_P$	0.1512	0.1511	0.1503	0.1498	0.1517	0.1401	0.1380	0.1478	0.1359
$\hat{\lambda}_P$	-0.6935	-0.8232	-0.8240	-0.7938	-0.6400	-0.5096	-0.3389	-0.2212	0.0733
$\hat{\mu}_{P_{1y}}$	-0.1033	-0.1507	-0.1671	-0.1892	-0.1170	-0.0743	-0.0100	0.1006	0.1776
$\hat{\sigma}_{P_{1y}}$	0.1876	0.2046	0.2093	0.2198	0.2276	0.2086	0.2035	0.2051	0.2166
$\hat{\lambda}_{P_{1y}}$	-0.7575	-0.9235	-0.9725	-1.0261	-0.6652	-0.5021	-0.1915	0.3481	0.6854
$\hat{\mu}_{P_{3m}}$	0.6641	0.0281	-0.4158	-0.2656	0.1008	0.2014	0.3695	0.4657	0.5703
$\hat{\sigma}_{P_{3m}}$	0.4196	0.1731	0.1764	0.1930	0.1675	0.1581	0.1774	0.1707	0.1744
$\hat{\lambda}_{P_{3m}}$	1.4905	-0.0588	-2.5632	-1.5643	0.3965	1.0814	1.9198	2.5568	3.1025

ORES	0310	0311	0312	0401	0402	0403	0404	0405	0406
U	3308	3610	3704	3746	3841	3926	3885	4011	3969
N	0	0	0	0	0	0	0	122	0
C	0	0	0	0	0	0	0	3	0
Utd	165	165	165	165	198	207	207	212	212
β	0.0499	0.0457	0.0445	0.0440	0.0515	0.0527	0.0533	0.0536	0.0534
$DirAvk$	0.0550	0.0480	0.0460	0.0460	0.0530	0.0520	0.0520	0.0520	0.0500
α	0.0140	0.0140	0.0140	0.0080	-0.0030	-0.0010	0.0030	0.0070	0.0050
r	0.0292	0.0292	0.0290	0.0280	0.0263	0.0246	0.0219	0.0220	0.0220
E^*	2992	3434	3594	3575	3754	3970	3961	4107	4234
$\hat{\mu}_E$									
$\hat{\sigma}_E$									
$\hat{\lambda}_E$									
P^*									
$\hat{\mu}_P$	0.0613	0.0732	0.0707	0.0900	0.1081	0.1162	0.1568	0.1239	0.1011
$\hat{\sigma}_P$	0.1431	0.1564	0.1584	0.1701	0.1803	0.1859	0.2093	0.2117	0.1733
$\hat{\lambda}_P$	0.2245	0.2813	0.2633	0.3648	0.4536	0.4926	0.6445	0.4815	0.4565
$\hat{\mu}_{P_{1y}}$	0.2882	0.3029	0.1942	0.2996	0.3887	0.4638	0.4674	0.4207	0.3408
$\hat{\sigma}_{P_{1y}}$	0.2315	0.2291	0.2202	0.2260	0.2270	0.2244	0.2091	0.2006	0.1779
$\hat{\lambda}_{P_{1y}}$	1.1188	1.1946	0.7499	1.2021	1.5968	1.9576	2.1307	1.9871	1.7921
$\hat{\mu}_{P_{3m}}$	0.4725	0.5186	0.3444	0.6075	0.5332	0.6258	0.4092	0.1355	-0.0524
$\hat{\sigma}_{P_{3m}}$	0.1735	0.1804	0.1485	0.1654	0.1995	0.2060	0.1521	0.1342	0.0961
$\hat{\lambda}_{P_{3m}}$	2.5558	2.7130	2.1237	3.5030	2.5410	2.9184	2.5455	0.8461	-0.7743