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Approximations to the Distribution of a Test Statistic for Auditing in Nuclear Materials Accountancy

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Abstract

The thesis work is about studying approximations to the distribution of a weighted sum of independent Chi-squared values (all centrally distributed and each with one degree of freedom). Knowing how to compute the upper tail of distribution will allow us to test a null hypothesis using a statistic which is a quadratic form in Gaussian variables. This statistic measures the agreement between a nuclear accountancy and the reality represented by this accountancy. The results of this study allow us to compute the significance level of an observed value or set the test threshold for a desired size. The coefficients (or weights) of the Chi-squared variables are determined by the variance matrix of the data used for judging the agreement between accountancy values and measured inspector values.

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Preface

This thesis work was done at the Institute for Protection and Security of the Citizen (IPSC) at the EU Joint Research Centre in Ispra, Italy. It was carried out under a year in-service training period from October 2003 at the Unit of Nuclear Safeguards (NUSAF). The NUSAF has a large research activity devoted to the development of tools and methodology for nuclear materials auditing.

First of all I would like to thank my supervisor Dr Michael Franklin at the NUSAF for introducing me to the subject of statistical methods in Nuclear Safeguards, for great guidance and excellent support during my whole trainee period. Grazie mille!

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Chapter 1. Introduction

1.1 Background

Every nation state having nuclear activities has legislation designed to ensure that the nuclear material on their territory is protected and that the material is only used for approved purposes [1,2,3,4]. This generally has the consequence that states insist on a high quality of accountancy of all their nuclear materials [5,6,7,8]. To maintain the assurance of high quality, most states have a state agency with responsibility for overseeing the quality of accountancy (e.g. SKI[1] for Sweden, NRC for USA, GAN for RF, IRSN for France). The role of such agencies includes the auditing of nuclear accounts to verify the agreement between accounts and the physical reality in facilities holding nuclear material. Verification basically consists of independently measuring selected material and comparing the results with the accounting values of the facility. It also includes checking the internal coherence of the accounts.

While this activity is driven by national legislation it also has a motivation coming from international agreements. The Non-Proliferation Treaty [9] provides an international framework in which nation states allow international inspectors (International Atomic Energy Agency, IAEA) to audit their accountancy. This is done in order to assure other states that the nuclear material is being used only for the purposes declared under international agreements. The IAEA auditing is designed to be able to detect "diversion" of any amount of material which has significance in terms of nuclear weapons. In addition the European Community has its own system of international inspections [10,11]. These international systems are often referred to as "international safeguards".

The existence of these state and international systems involving auditing of accounts means that the management of nuclear facilities must give a certain priority to accountancy needs. Facility management wishes to have state of the art accountancy because it must show excellence in meeting the requirements of the national laws and international agreements. The difficulty for facility management is to put in place the necessary procedures and technical means for ensuring that

[1] SKI Statens Kärnkraftsinspektion, NRC Nuclear Regulatory Commission, GAN Gosatomnadzor, IRSN Institut de Radioprotection et de Sureté Nucléaire

excellent accountancy is achieved. Excellence in accountancy means that big discrepancies should not occur between what the accounts say is in an item and the physical reality. For management, audits have the benefit that they are a mechanism for finding failures usually due to human error or poor procedures. Audits provide some mechanism for quantifying accountancy performance. Definitions of performance can be made and standards can be set for performance. After an audit, the achieved performance can be estimated and the difference between the desired performance and the achieved performance can be tested.

Statistical methods have been an important element of this auditing activity for more than 35 years. These methods have been mainly concerned with,

(a) characterizing the probability distribution of what are legitimate discrepancies between accounting values and reality (for example intrinsic measurement errors) and with
(b) identifying what types of statistical analysis of accounting and auditing data can be used to recognize anomalous situations and assess accountancy performance.

1.2 Contents of subsequent chapters

Chapter two presents an introduction to the notation of nuclear materials accountancy. The background to the problem of testing the estimated mean square error of accountancy discrepancy is presented. The chapter defines the main problem of the thesis, which is about finding approximate methods for computing the tail probabilities of the distribution of a certain test statistic. The chapter concludes with a study of the distribution of this test statistic.

Chapter three deals with typical measurement scenarios and what variance matrix structure comes from these scenarios. Different patterns of variance matrix correspond to different kinds of measurement methods. We specify reference variance matrices which are used as a benchmark for the methods studied in this.

Chapter four presents the different approximation methods used for computing the tail probabilities of the distribution of the test statistic. Five well known methods are considered. In addition to these five methods known in the literature, two crude and unprofessional methods are included in the study.

Chapter five presents a Monte Carlo simulation method. In order to assess the performance of a method, the results provided by the method are compared with the true values of the probability (significance level). A precise Monte Carlo simulation of the probability distribution provides the true values needed for these comparisons.

Chapter six presents the results of comparing the approximation methods. The results of this comparative study are then used to provide practical recommendations as to which method should be recommended to auditors for use, which is presented in chapter seven.

Appendix A contains a benchmark with numerical values for different types of reference variance matrices which are used in the calculations. Appendix B and C contain detailed tables of numerical results of the seven different approximating methods. Appendix D presents plots of the distributions of the different approximating methods. The last appendix, appendix E, contains the computer programs used in the report.

Chapter 2. Definition of the problem and some preparatory analyses

2.1 Accountancy performance

*Inspector measurements and facility accounted values***.** The objective of nuclear materials accountancy is to ensure that the location of all nuclear material is precisely known, and to ensure that the amount of each type of nuclear material, in each location, is precisely known. Consider the case when a nuclear materials accountancy is taken place in a nuclear facility. The nuclear material, such as plutonium or uranium, that needs to be accounted for, can be seen as a finite population of N items. The true values of mass of these items are denoted by $M_1, M_2, ..., M_N$. The inspector in charge of the accountancy generates a sample of n items by random sampling without replacement. The real mass of an item chosen at random in the sample will be denoted by $m_1, m_2, ..., m_n$. He or she uses his/hers own independent measurement system to measure the amount of nuclear material in each of the selected items. These measured values are denoted by $\hat{m}_1, \hat{m}_2, ..., \hat{m}_n$. The nuclear facility has information about the masses of plutonium and of uranium of each item in records. These facility accounted values are denoted by $Z_1, Z_2, ..., Z_N$ (in the population) and $z_1, z_2, ..., z_n$ (in the generated sample).

The inspector compares his/hers measured values with the facility accountancy values for each single item that he/she has verified. If the accountancy is a valid accountancy, the difference between the inspector's value and the accountancy declared value, should only be the sum of legitimate measurement errors. These are both the measurement errors in the accountancy values, and the measurement errors in his/hers own verification measurements. Apart from testing individual difference for each individual item, the inspector will carry out other tests using the entire set of differences. Studying the null hypothesis distribution of one of these tests has been the work of this thesis. Null hypotheses for testing the quality of accountancy, are expressed in terms of some joint distribution for the entire set of measurement errors in the accounts. The joint distribution of the measurement errors of the inspector is known.

The variance matrix of the differences. Let d_i represent the difference, for the i^{th} item, of the facility accountancy mass value and the inspector measured mass value, $i = 1, 2, ..., n,$ of a nuclear material

$$d_i = \underset{\substack{facility\\value}}{z_i} - \underset{\substack{inspector\\value}}{\hat{m}_i} \tag{2.1}$$

d_i also can be written as

$$d_i = \underset{\substack{facility\\dicrepancy}}{(z_i - m_i)} - \underset{\substack{inspector\\measurement\\error}}{(\hat{m}_i - m_i)} = l_i - \varepsilon_i \tag{2.2}$$

where m_i denotes the true mass value, l_i denotes the facility accountancy discrepancy and ε_i denotes the inspector measurement error in the generated sample. Let $\mathbf{V}_z$ denote the variance matrix of the n facility declared values and $\mathbf{V}_{\hat{m}}$ denote the variance matrix of the n inspector measurements. We assume that the facility and the inspector measurement systems are independent of each other. Then $\mathbf{V}_d$ denotes the variance matrix of the differences between the facility declared values and the inspector measurements, that is

$$\mathbf{V}_d = \mathbf{V}_z + \mathbf{V}_{\hat{m}} \tag{2.3}$$

At the moment of making a test, the variance matrix of $\mathbf{V}_z$ is assigned a value representing some null hypothesis about the desired behaviour of facility measurement errors. The inspector matrix $\mathbf{V}_{\hat{m}}$ is known and therefore $\mathbf{V}_d$ for the null hypothesis can be computed as above. In reality this means that there exist very good estimators of them which are unbiased and have little variance. In addition, we assume that the differences $d_i : s,\ i = 1, 2, ..., n,$ have a multivariate Gaussian distribution with zero mean and this "known" variance matrix $\mathbf{V}_d$, that is $\mathbf{d} \sim N(\mathbf{0}, \mathbf{V}_d)$.

Testing accountancy. The statistical tests the inspector performs are concerned with:

(1) Testing the single differences of the declared accountancy and inspector measured values and
(2) testing hypotheses about some criteria of accountancy performance.

(1) To test a single difference is straightforward and fairly simple. It is a statistical test of the difference between the accountancy declared value and the inspector measured value for each individual item. Remembering that d_i (2.1) represents the difference, for the i^{th} item, of the facility accountancy mass value and the inspector measured mass value, $i = 1, 2, ..., n,$ of a nuclear material. The test does not reject the hypothesis of legitimate measurement error as an explanation of d_i, if

$$|d_i| < 3\sigma_d \tag{2.4}$$

where σ_d is known and given by

$$\sigma_d^2 = \sigma_{fac}^2 + \sigma_{insp}^2 \tag{2.5}$$

σ_{fac}^2 denotes the variance of the facility declared value when the declared value only contains legitimate measurement errors and σ_{insp}^2 denotes to the inspector measurement error. The chosen value of $3\sigma_d$ will give a low frequency of false alarm probability.

(2) A more sophisticated test is related to the study of some overall criterion of accountancy performance. One criterion of performance which is studied in this regard is the mean squared error of the total inventory discrepancy. The total discrepancy is the total difference between the accountancy declared values and the true values and is a random variable denoted L_{tot}. It is defined as

$$L_{tot} = \sum_{i=1}^{N} L_i = \sum_{i=1}^{N} \left(Z_i - M_i \right) \tag{2.6}$$

The joint probability distribution of the random variables $L_i : s$ represents the sources of variation of measurement errors in the facility measurement system. The mean square error of L_{tot} is the parameter measuring performance of the accountancy. This is denoted as $MSE(L_{tot})$. Testing the performance of the accountancy involves testing an estimator of the mean square error of L_{tot}. In other words we wish to test an estimator of,

$$MSE(L_{tot}) = \left(E\left[\sum_{i=1}^{N} L_i \right] \right)^2 + Var\left(\sum_{i=1}^{N} L_i \right) \tag{2.7}$$

Estimator of the accountancy performance criterion. We will use the mean square error, $MSE(L_{tot})$, of the total inventory discrepancy as our criterion of "good" accountancy performance. In reality the variances and covariances would be derived from knowledge of the facility measurement methods and knowledge of which method were used for particular items. If the measurement system is properly calibrated, the means should be zero. If there is an unrecognised bias in some part of the measurement system, not all means will be zero.

We assume that a random sample of n items has been selected by the inspector and independently measured as verification of the accountancy. An unbiased estimator of $MSE(L_{tot})$ may be found by knowing **only** the measurement differences, $d_i : s$ (accountancy value minus inspector value) and the variance matrices of the n differences, $\mathbf{V}_{\hat{m}}$. Following Franklin, M.T. (1997), [13], it can be proved that an unbiased estimator is

$$M\hat{S}E(L_{tot}) = N^2\overline{d}^2 - \frac{N(N-n)}{n}\hat{S}^2 - \frac{N^2}{n^2}\hat{\sigma}^2 \qquad (2.8)$$

where

$$\begin{aligned} \hat{S}^2 &= \frac{1}{n-1}\left(\sum_{i=1}^{n}(d_i - \overline{d})^2 - \sum_{i=1}^{n}\hat{\sigma}^2(\hat{m}_i) + \frac{1}{n}\hat{\sigma}^2\right) \\ \hat{\sigma}^2(\hat{m}_i) &= V(\hat{m})_{i,i} \\ \hat{\sigma}^2 &= \sum_{i=1}^{n}\sum_{j=1}^{n}V(\hat{m})_{i,j} \end{aligned} \qquad (2.9)$$

The elements of $\mathbf{V}_{\hat{m}}$ are denoted by $V(\hat{m})_{i,i}$ and $V(\hat{m})_{i,j}$.

Note that in defining MSE we refer to expectation with respect to to (1) the process generating the $L_i : s$ (facility measurements), (2) the identities of the items randomly chosen for verification and (3) the measurement errors of the inspector, $\hat{m}_i - m_i$.

This gives

$$M\hat{S}E(L_{tot}) = N^2\overline{d}^2 - \frac{N(N-n)}{n(n-1)}\left(\sum_{i=1}^{n}(d_i - \overline{d})^2 - \sum_{i=1}^{n}\hat{\sigma}^2(\hat{m}_i) + \frac{1}{n}\hat{\sigma}^2\right) - \frac{N^2}{n^2}\hat{\sigma}^2$$

It can be proved that $M\hat{S}E(L_{tot})$ is an unbiased estimator *i.e.*

$$E[M\hat{S}E(L_{tot})] = MSE(L_{tot})$$

without making any particular assumptions about

$$E\left[\sum_{i=1}^{N} L_i\right] \text{ or } Var\left(\sum_{i=1}^{N} L_i\right)$$

Note that when proving that $M\hat{S}E(L_{tot})$ is unbiased, we have to assume (a) for the inspector values that we have an unbiased estimator for $\mathbf{V}_{\hat{m}}$ and (b) assume that the inspector measurement system is unbiased.

We want to test the null hypothesis that the expected value of the estimator of $MSE(L_{tot})$ is given by the mean square error of L_{tot}. The hypothesis is set up as

$$\mathbf{H_0} : E[M\hat{S}E(L_{tot})] = MSE_0(L_{tot}) \tag{2.10}$$

where the zero denotes expectation with respect to some specific null hypothesis distribution.

The estimator may be written as

$$E[N^2\bar{d}^2 - \frac{N(N-n)}{n(n-1)}\left(\sum_{i=1}^{n}(d_i - \bar{d})^2 - \sum_{i=1}^{n}\hat{\sigma}^2(\hat{m}_i) + \frac{1}{n}\hat{\sigma}^2\right) - \frac{N^2}{n^2}\hat{\sigma}^2] = \left(E_0\left[\sum_{i=1}^{N} L_i\right]\right)^2 + Var_0\left(\sum_{i=1}^{N} L_i\right) \tag{2.11}$$

The alternative hypothesis is set up as

$$\mathbf{H_1} : E[M\hat{S}E(L_{tot})] > MSE_0(L_{tot})$$

Another formulation of this hypothesis would be to say that the statistic

$$N^2\bar{d}^2 - \frac{N(N-n)}{n(n-1)}\left(\sum_{i=1}^{n}(d_i - \bar{d})^2\right) = N^2\bar{d}^2 - \frac{N(N-n)}{n} S_d^2$$

has expected value given by

$$\frac{N(N-n)}{n(n-1)}E_{\hat{m}}\left(-\sum_{i=1}^{n}\hat{\sigma}^2(\hat{m}_i)+\frac{1}{n}\hat{\sigma}^2\right)+\frac{N^2}{n^2}E_{\hat{m}}\left(\hat{\sigma}^2\right)+\left(E_0\left[\sum_{i=1}^{N}L_i\right]\right)^2+Var_0\left(\sum_{i=1}^{N}L_i\right) \tag{2.12}$$

This is written as

$$\mathbf{H_0}: E[N^2\overline{d}^2-\frac{N(N-n)}{n}S_d^2]=$$
$$\frac{N(N-n)}{n(n-1)}\left(-\sum_{i=1}^{n}\sigma^2(\hat{m}_i)+\frac{1}{n}\sigma^2\right)+\frac{N^2}{n^2}\sigma^2+\left(E_0\left[\sum_{i=1}^{N}L_i\right]\right)^2+Var_0\left(\sum_{i=1}^{N}L_i\right) \tag{2.13}$$

$$\mathbf{H_1}: E[N^2\overline{d}^2-\frac{N(N-n)}{n}S_d^2]>$$
$$\frac{N(N-n)}{n(n-1)}\left(-\sum_{i=1}^{n}\sigma^2(\hat{m}_i)+\frac{1}{n}\sigma^2\right)+\frac{N^2}{n^2}\sigma^2+\left(E_1\left[\sum_{i=1}^{N}L_i\right]\right)^2+Var_1\left(\sum_{i=1}^{N}L_i\right)$$

In this study, we are assuming that the inspector values $\mathbf{V}_{\hat{m}}$ are known values. Essentially we have put the inspector measurement variances to the other side of the equation.

A two test approach. In reality, however the hypothesis test mentioned above is performed in a slightly different way. In practise the test statistic $N^2\overline{d}^2-\frac{N(N-n)}{n}S_d^2$ is divided up into the two simple test statistics $\overline{d}^2$ and S_d^2. Then the two test statistics are tested separately with respect to the null hypothesis given by $\mathbf{V}_{d,0}$. We will therefore have a null hypothesis distribution for $\overline{d}^2$ and a null hypothesis distribution for S_d^2.

We can detect different things by testing them separately. The S_d^2 test is sensitive to anomalies affecting between-item variation in measurement error whereas the $\overline{d}^2$ test is sensitive affecting means and correlated error contributions (systematic factors).

If the two separate null hypotheses are true, then the null hypothesis for the mean square error is true. In practise the test of MSE is “bivariate” $(\bar{d}, S_d^2)$ in which if each separate null hypothesis is accepted, then the null hypothesis for MSE is accepted.

This thesis is only concerned with approximating the null hypothesis distribution of S_d^2 for the situation where the solution is not given by the Chi-squared distribution.

2.2 Statement of the problem

Inspectors need for their auditing work, a method for computing the one-sided significance level of an observed value of ${S_d}^2$ given the known variance matrix of the data. Too large a value of ${S_d}^2$ must be interpreted as indicating that achieved mean square error is not consistent with the null hypothesis represented by the variance matrix. The purpose of this study is to find methods for describing upper-tail probabilities of the null hypothesis distribution of the test statistic S_d^2 for different known variance matrices $\mathbf{V}_d$. The quadratic form ${S_d}^2$ must be tested in a situation in which the $d_i : s$ variables are not independent and not identically distributed but where the null hypothesis means and variances are known, see Franklin, M.T. (2002), [15].

2.3 Distribution of the test statistic

A transformed expression for the test statistic. Consider the test statistic

$$Q = (n-1)\, S_d^2 = \sum_{i=1}^{n} (d_i - \bar{d})^2 \tag{2.14}$$

which is a quadratic form in multivariate Gaussian variables, $\mathbf{d} \sim N(\mathbf{0}, \mathbf{V}_d)$ Define the matrix

$$\mathbf{B} = \mathbf{I} - \frac{1}{n}\mathbf{U} = \begin{pmatrix} 1-1/n & -1/n & \cdots & -1/n \\ -1/n & 1-1/n & \cdots & -1/n \\ \vdots & \vdots & \ddots & \vdots \\ -1/n & -1/n & \cdots & 1-1/n \end{pmatrix} \tag{2.15}$$

where **I** is the identity matrix and **U** is a matrix whose every element equals unity. **B** is a symmetric positive semi-definite matrix of rank *n-1*. We may write

$$Q = (n-1)S_d^2 = \sum_{i=1}^{n}(d_i - \overline{d})^2 = \sum_{i=1}^{n} d_i^2 - n\overline{d}^2 \tag{2.16}$$

where

$$\sum_{i=1}^{n} d_i^2 = \mathbf{d}'\mathbf{d} = \mathbf{d}'\begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}\mathbf{d}$$

and

$$n\overline{d}^2 = \mathbf{d}'\begin{pmatrix} 1/n & 1/n & \cdots & 1/n \\ 1/n & 1/n & \cdots & 1/n \\ \vdots & \vdots & \ddots & \vdots \\ 1/n & 1/n & \cdots & 1/n \end{pmatrix}\mathbf{d}$$

Hence

$$Q = (n-1)S_d^2 = \mathbf{d}'\begin{pmatrix} 1-1/n & -1/n & \cdots & -1/n \\ -1/n & 1-1/n & \cdots & -1/n \\ \vdots & \vdots & \ddots & \vdots \\ -1/n & -1/n & \cdots & 1-1/n \end{pmatrix}\mathbf{d} = \mathbf{d}'\mathbf{B}\mathbf{d} \tag{2.17}$$

The variance matrix $\mathbf{V}_d$ of the differences of the facility accountancy mass values and the inspector measured mass values is real, symmetric and of full rank (positive definite). According to Eigen Decomposition Theorem in linear algebra we may then write

$$\mathbf{V}_d = \mathbf{G}\sqrt{\mathbf{D}}\left(\mathbf{G}\sqrt{\mathbf{D}}\right)' \tag{2.18}$$

where **G** is a matrix formed from the eigenvectors of $\mathbf{V}_d$ and **D** is the diagonal matrix with the eigenvalues of $\mathbf{V}_d$.

Let

$$\mathbf{y} = \left(\mathbf{G}\sqrt{\mathbf{D}}\right)^{-1}\mathbf{d} \tag{2.19}$$

be a linear transformation of *d*-variables. Then the *y*-variables have zero mean and variance matrix

$$\mathbf{V}_y = \left(\mathbf{G}\sqrt{\mathbf{D}}\right)^{-1} \mathbf{V}_d \left(\left(\mathbf{G}\sqrt{\mathbf{D}}\right)^{-1}\right)' = \left(\mathbf{G}\sqrt{\mathbf{D}}\right)^{-1} \mathbf{G}\sqrt{\mathbf{D}}\left(\mathbf{G}\sqrt{\mathbf{D}}\right)'\left(\left(\mathbf{G}\sqrt{\mathbf{D}}\right)^{-1}\right)' = \mathbf{I} \qquad (2.20)$$

Hence, the *y*-variables are independent and identically Gaussian distributed.

By applying the transformation (2.19) we may write

$$Q = (n-1)S_d^2 = \mathbf{d}'\mathbf{B}\mathbf{d} = \mathbf{y}'\left(\mathbf{G}\sqrt{\mathbf{D}}\right)'\mathbf{B}\left(\mathbf{G}\sqrt{\mathbf{D}}\right)\mathbf{y} = \mathbf{y}'\mathbf{C}\mathbf{y} \qquad (2.21)$$

where

$$\mathbf{C} = \left(\mathbf{G}\sqrt{\mathbf{D}}\right)'\mathbf{B}\left(\mathbf{G}\sqrt{\mathbf{D}}\right) \qquad (2.22)$$

is a real, symmetric, positive semi-definite matrix of rank *n-1*. The matrix **C** depends on both the matrices **B** and $\mathbf{V}_d$. All eigenvalues of **C** are real and non-negative. One eigenvalue is zero. The other eigenvalues are real and positive.

Let **A** be an orthogonal transformation defined by

$$\mathbf{A}'\mathbf{C}\mathbf{A} = \mathbf{E} \qquad (2.23)$$

where **E** is a diagonal matrix. Further, let **Z** be defined by

$$\mathbf{Z} = \mathbf{A}'\mathbf{y} \qquad (2.24)$$

Then we may write

$$Q = (n-1)S_d^2 = \mathbf{Z}'\mathbf{E}\mathbf{Z} = \sum_{i=1}^{n} \lambda_i Z_i^2 \qquad \text{(one eigenvalue is zero)} \qquad (2.25)$$

where the λ_i are the eigenvalues of the matrix **C** and the Z_i^2's are chi-squared random variables, each with one degree of freedom.

We conclude that our test statistic may be written as a linear combination of chi-squared variables with known weights. Proof of the orthogonal transformation and of the eigenvalue representation can be found in Kendell, Ord and Stuart (1987), on pages 476 and 488, [17].

Moments of the distribution of the test statistic. A distribution is specified when all its moments are known. The eigen value representation allows us to find expressions for all the moments. The approximate methods we shall use for computing the distribution function, do not use more than the first four moments. By computing the eigen values, the λ_i 's, of $\mathbf{C} = \left(\mathbf{G}\sqrt{\mathbf{D}}\right)^{'} \mathbf{B}\left(\mathbf{G}\sqrt{\mathbf{D}}\right)$ we can determine the moments of (2.25), where Z_i is a standard Gaussian variable, $Z_i \sim N(0,1)$, and thus Z_i^2 is a chi-squared random variable with one degree of freedom, $Z_i^2 \sim \chi^2(1)$, and then The *k*th raw moment of the distribution or the *k*th moment about zero is denoted $\mu_k^{'}$, $\mu_k^{'} = E[X^k]$ of the random variable *X*. The *k*th moment about the mean or *k*th central moment is defined as $\mu_k = E[X - \mu_1^{'}]^k$. The first four moments may be found in the following tables 2.1 and 2.2.

Table 2.1 The first four moments about zero.

	Moment about zero (Raw moment)
First	$\mu_1^{'}(Q) = \sum_{i=1}^{n} \lambda_i$
Second	$\mu_2^{'}(Q) = 2\sum_{i=1}^{n} \lambda_i^2 + \left[\sum_{i=1}^{n} \lambda_i\right]^2$
Third	$\mu_3^{'}(Q) = 8\sum_{i=1}^{n} \lambda_i^3 + 6\sum_{i=1}^{n} \lambda_i \sum_{i=1}^{n} \lambda_i^2 + \left[\sum_{i=1}^{n} \lambda_i\right]^3$
Fourth	$\mu_4^{'}(Q) = 48\sum_{i=1}^{n} \lambda_i^4 + 12\left[\sum_{i=1}^{n} \lambda_i^2\right]^2 + 32\sum_{i=1}^{n} \lambda_i^3 \sum_{i=1}^{n} \lambda_i + 12\sum_{i=1}^{n} \lambda_i^2 \left[\sum_{i=1}^{n} \lambda_i\right]^2 + \left[\sum_{i=1}^{n} \lambda_i\right]^4$

Table 2.2 The first four moments about the mean.

	Moment about the mean (Central moment)
First (always zero)	$\mu_1(Q) = E[Z - \mu_1^{'}] = \mu_1^{'} - \mu_1^{'} = 0$
Second (variance)	$\sigma^2(Q) = \mu_2(Q) = \mu_2^{'} - \left[\mu_1^{'}\right]^2 = 2\sum_{i=1}^{n} \lambda_i^2$

Third (skewness)	$\mu_3(Q) = \mu_3^{'} - 3\mu_2^{'}\mu_1^{'} + 2\left[\mu_1^{'}\right]^3 = 8\sum_{i=1}^{n}\lambda_i^3$
Fourth (kurtosis)	$\mu_4(Q) = \mu_4^{'} - 4\mu_3^{'}\mu_1^{'} + 6\mu_2^{'}\left[\mu_1^{'}\right]^2 - 3\left[\mu_1^{'}\right]^4 = 48\sum_{i=1}^{n}\lambda_i^4 + 12\left[\sum_{i=1}^{n}\lambda_i^2\right]^2$

The moment generating function of the distribution of the test statistic. Let $\mathbf{Z^2} = (Z_1^2, Z_2^2, ..., Z_n^2)'$ be a random vector that comes from the χ^2-distribution with one degree of freedom. The moment generating function (m.g.f) of a random variable $\mathbf{Z^2} = \mathbf{X}$ is

$$\psi_i(t) = E[e^{tX}] = \int_0^\infty e^{tx}\frac{1}{\Gamma(1/2)}x^{-1/2}\frac{1}{2^{1/2}}e^{-x/2}dx = \frac{1}{2^{1/2}(1/2-t)^{1/2}}\int_0^\infty \frac{1}{\Gamma(1/2)}x^{-1/2}\left(\frac{1}{2}-t\right)^{1/2}e^{-x(1/2-t)}dx = \frac{1}{(1-2t)^{1/2}} \quad (2.26)$$

The m.g.f. of $\lambda_i Z_i^2 = \lambda_i X_i$, a constant multiple of a χ^2-distributed random variable with one degree of freedom is

$$\frac{1}{(1-2\lambda_i t)^{1/2}} \quad (2.27)$$

The m.g.f. of the random variable of (2.25) is given by

$$\varphi_Q(t) = \prod_{i=1}^{n}\frac{1}{(1-2\lambda_i t)^{1/2}} \quad (2.28)$$

The cumulant-generating function of the distribution of the test statistic. The cumulant-generating function (c.g.f) is defined as

$$\xi(t) = \ln\psi(t) \quad (2.29)$$

The c.g.f. of $\lambda_i Z_i^2$ is

$$\xi_i(t) = -\frac{1}{2}\ln(1-2\lambda_i t) \quad (2.30)$$

By the additive property of c.g.f's, the c.g.f of the random variable of (2.25) is

$$\xi_Q(t) = -\frac{1}{2}\sum_{i=1}^{n}\ln(1-2\lambda_i t) \tag{2.31}$$

The cumulants of Q are given by expanding the logarithm in series near $t = 0$, the cumulants are given by

$$\kappa_j = 2^{j-1}(j-1)!\sum_{i=1}^{n}\lambda_i^j \quad \text{or by} \tag{2.32}$$

$$\kappa_j = \frac{d^j}{dt^j}\xi_Q(t)\Big|_{t=0} \tag{2.33}$$

where d^j denotes the j^{th} derivative of $\xi_Q(t)$ with respect to t.

In this section we have thus shown how standard distribution theory allows us to represent the S_d^2 variable as a weighted sum of independent Chi-squared variables (each central and each with one degree of freedom). The weights in this sum are known since they can be calculated numerically from the known variance matrix $\mathbf{V}_d$ (the null hypothesis variance matrix of the d_i variables). In fact, the weights are the eigen values of the matrix representing S_d^2 as a quadratic form in standard independent and identically distributed Gaussian variables. The matrix of this quadratic form is a simple function of the known variance matrix. Using the representation of S_d^2 as a sum of Chi-squared variables, the moments of S_d^2 can be numerically calculated from the known eigen values. Knowing the values of moments becomes the basis for applying each of the approximate methods studied in this thesis.

Chapter 3. Measurement scenarios and the variance matrices of the differences between the facility declared values and the inspector measurements

3.1 Introduction to measurement methods

This thesis compares a number of methods for approximating the distribution function of S_d^2. These approximation methods are compared by applying them to a list of specific problems. The problems are defined in terms of variance matrices that represent typical measurement situations. The list of variance matrices is the benchmark in terms of which the approximations are defined as satisfactory or not. This chapter describes briefly the structure of each of the variance matrices. This description is prefaced by a brief overview in this section of how such variance matrices can in general be derived from the literature about methods for measurement of nuclear material.

A great many methods are used to measure nuclear material and these measurements can be a basis for accountancy information about chemical composition or isotopic composition or bulk mass. These methods include what are called destructive analysis (DA) methods in which a small representative sample of the material is destroyed in a chemical or isotopic analysis. Important DA methods include mass spectrometry, gravimetry, titrimetry and a variety of others, see Rogers, D.R. (1981), [22]. All chemical methods for the determination of uranium and plutonium require reference materials for calibration of the procedure. Any uncertainty about the values of these standard materials used for calibration, are sources of small covariances between measurements.

Non-destructive analysis (NDA) is the term used for methods that deduce some characteristic of the material (mass of fissile material or isotopic composition) from radiation emitted from the material. The radiation is usually neutrons or gamma radiation. The methods are classified as passive or active depending on whether the radiation is emitted spontaneously from the nuclear material (passive) or is the result of interrogating the material with an external source of radiation so as to create an emission. The variety of NDA methods is illustrated in Canada, T.R. (1984), [12]. NDA methods often make their measurement by interpreting the emitted radiation intensity through a calibration curve that has been empirically estimated using reference samples of nuclear material. Again the use of calibration curves which are themselves the result of experimental estimation (Ordinary Least Squares (OLS), etc.), becomes a source of small covariances between the measurement errors. Often the magnitude of the small covariances will vary with the masses being measured.

Balances or scales are also used to determine bulk weights of nuclear material products. Masses from balances are combined with DA results to determine mass of fissile material. Balances of course are regularly calibration using standard masses. In some balance systems, the calibration procedure creates a small systematic error whose standard deviation can be estimated. This error contribution then becomes a source of small covariance between mass measurements.

This description of measurement methods is intended to illustrate the idea that covariances are intrinsic to the measurement process. There are however additional causes of covariance between accountancy values. These occur when the measurement of some characteristics such as isotopic abundances for a batch of material, is applied to compute masses for a series of items produced from that batch. Any measurement error in the isotopic measurements (that are a common element of the different calculations) becomes a source of covariance between the accountancy masses.

For the statistical testing methods that are being studied here, the data (i.e. differences between accountancy values and inspector measured values) are usually the difference between an inspector NDA method and an accountancy value derived from DA analysis combined with a bulk mass measured on a balance. In many practical situations establishing the variance matrix of these differences may be a quite complex task. For a number of years, the activities of European Safeguards Research and Development Association (ESARDA) and IAEA working groups have been aimed at quantifying the variances and covariances of legitimate measurement errors in the accountancy values and in the inspector verification measurements, see Aigner et al. (2002), [24]. This type of information about achievable performance, provides a basic element for computing null hypothesis variance matrices for specific sets of differences. The variance matrices that are used as benchmark in this study, are typical of a combination of DA and NDA methods in which the covariances are small and vary in function of the pair of true values being measured and the variances are also function of the true value (heteroscedasticity). In addition, the measurement examples taken as the benchmark for this study were all cases in which correlations were postive.

3.2 Different types of variance matrices

In this section, we specify the structure of the different types and subtypes of variance matrices that are included in the benchmark. Seven main types of variance matrices have been identified. Including subtypes there are 15 cases of variance matrices. Appendix A contains this benchmark with numerical values for the different types and subtypes of reference variance matrices.

*Type 1. A symmetric distribution for the differences***.** In this first type the $\mathbf{V}_d$ variance matrix is from a symmetric distribution. This kind of variance matrix may occur when the sample of ‘n’ items of nuclear material comes from a single chemical batch and that the items have the same mass of material. The facility has measured and accounted the items in exactly the same way. In type 1, it is also assumed that the inspector has verified these items in the same way.

When the distribution is symmetric, each d_i, $i = 1,2,...,n,$ has the same mean and the same variance and every pair (d_i, d_j), $i \neq j$, has the same covariance. When the distribution is symmetric the elements of variance matrix depend on only two parameters. This can be expressed in either of two notations. In one notation, the diagonal elements, denoted as v_{ii}, consist of a systematic and random variance component and the off-diagonal elements, denoted as $v_{i,j}, i \neq j$, consist of the systematic variance component. In this notation, the $\mathbf{V}_d$ variance matrix is therefore of the form

$$v_{ii} = \sigma_{ran}^2 + \sigma_{sys}^2 = \sigma^2 \quad i=1,2,...,n \tag{3.1}$$

$$v_{i,j} = \sigma_{sys}^2 = \rho\sigma^2 \quad \forall i,j\ ,\ i \neq j \tag{3.2}$$

$$\mathbf{V}_d = \begin{pmatrix} \sigma_{sys}^2 + \sigma_{ran}^2 & \sigma_{sys}^2 & \cdots & \sigma_{sys}^2 \\ \sigma_{sys}^2 & \sigma_{sys}^2 + \sigma_{ran}^2 & \cdots & \sigma_{sys}^2 \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{sys}^2 & \sigma_{sys}^2 & \cdots & \sigma_{sys}^2 + \sigma_{ran}^2 \end{pmatrix} = \begin{pmatrix} \sigma^2 & \rho\sigma^2 & \cdots & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 & \cdots & \rho\sigma^2 \\ \vdots & \vdots & \ddots & \vdots \\ \rho\sigma^2 & \rho\sigma^2 & \cdots & \sigma^2 \end{pmatrix} \tag{3.3}$$

The final part of the above equation shows the alternative notation using the correlation coefficient as one of the two parameters.

When the $d_i : s$ have a $\mathbf{V}_d$ variance matrix of this form, an exact analytic solution may be found. The distribution of S_d^2 depends on *n* (the number of selected items in the random sample) and on $\sigma_{ran}^2 = (1-\rho)\sigma^2$. We have

$$\frac{(n-1)S_d^2}{\sigma_{ran}^2} \sim \chi_{v=n-1}^2 \tag{3.4}$$

i.e. has a chi-square distribution with (*n-1*) degrees of freedom.

A numerical example of this type of matrix is included in the benchmark set even though the exact distribution of S_d^2 is known. The values of σ^2 and ρ in the calculations for the reference matrix of type 1, are $\sigma^2 = 1$ and $\rho = 0.3$.

Type 2. In the next type, the $\mathbf{V}_d$ variance matrix is of the block diagonal form

$$\mathbf{V}_d = \begin{pmatrix} \mathbf{V}^{(1)} & \mathbf{0} \\ \mathbf{0} & \mathbf{V}^{(2)} \end{pmatrix}. \tag{3.5}$$

Where $\mathbf{V}^{(1)}$ and $\mathbf{V}^{(2)}$ are each a variance matrix of some symmetric distribution and the **0**:s correspond to blocks of covariances which are all zero. In this case, the sample of n verified items of nuclear material is be made up of two types (type may refer to size, container type or chemical composition), where n_1 comes from *type 1*and n_2 comes from *type 2*. The inspector uses two independent methods for measuring each type and each type is measured by the facility using two independent methods. We are here dealing with “two independent subsets” of differences and hence the variance matrix of the differences will have blocks of covariances which are zero.

Four numerical examples of this type of matrix are included in the benchmark set, see table 3.1. These four differ in the numerical values that are assigned to the four parameters of the matrix. The numerical values are described in the table below. The variance elements of $\mathbf{V}^{(1)}$ are denoted by σ_1^2 and the covariance elements are denoted $\rho_1\sigma_1^2$. The variances and covariances of $\mathbf{V}^{(2)}$ are denoted σ_2^2 and $\rho_2\sigma_2^2$.

Table 3.1 Four numerical examples of the type 2 variance matrix.

Case	Type 1 σ_1^2	Type 2 σ_2^2	Type 1 ρ_1	Type 2 ρ_2
a	1	1	0.3	0.3
b	1	0.25	0.3	0.1
c	1	0.25	0.3	0.3
d	1	0.25	0.3	0.5

Type 3. Even when items are measured with the same methods, the variances will often depend on the masses of the nuclear material of the object being measured. The covariance will often vary with the masses of the two objects being concerned. So this third type (and the other four following types of matrices we are dealing with), will have a more general variance matrix composition than type 1 and type 2. Consequently we are not anymore treating symmetric distributions of the differences.

In type 3, the $\mathbf{V}_d$ matrices will have varying values of the variances but with constant correlation coefficients. The covariances are computed as

$$v_{ij} = \rho\sqrt{v_{ii}}\sqrt{v_{jj}} \quad \forall i, j, i \neq j . \tag{3.6}$$

In defining numerical examples for the benchmark, we consider without loss of generality that, $v_{11} = 1.0;\ v_{11} \geq v_{22} \geq ... \geq v_{nn}$.

In type 3, we will have three subtypes

(3a) $v_{nn} = 0.1$ and v_{ii} are uniformly scattered in the interval [0.1 , 1.0] .

(3b) $v_{nn} = 0.25$ and v_{ii} are uniformly scattered in interval [0.25 , 1.0].

(3c) $v_{nn} = 0.6$ and v_{ii} are uniformly scattered in interval [0.6 , 1.0].

These represent different degrees of dispersion of the heteroscedastic variances.

Even when the $\{v_{ii}\}$ values are fixed, the type 3 variance matrix still has one free parameter, namely the correlation coefficient. The correlation coefficient ρ can be "small", "medium" or "large" depending on how important "systematic" sources of error are in the measurement. In the three numerical examples being used for the benchmark the correlation is considered to be medium represented by the specific value ρ = 0.3. One of the purposes of studying type 3 is to see how variation in the variances will affect the testing approximation methods.

Type 4. The $\mathbf{V}_d$ variance matrices in this forth type will have varying values of the variances and all correlations are very small or zero. The correlations however may vary. For Type 4 matrices, we set $v_{11} = 1.0$ and $v_{nn} = 0.25$ and the other v_{ii} are uniformly scattered in the interval [0.25 , 1.0]. The covariances are computed using

$$v_{ij} = \rho_{ij} \sqrt{v_{ii}} \sqrt{v_{jj}} \quad ; \quad \forall i, j, \quad i \neq j \, .$$

Within the variance matrices of type 4, we define three subtypes. These are,

(4a) all $\rho_{ij} = 0.0$

(4b) ρ_{ij} are uniformly scattered in [0.0 , 0.07]

(4c) ρ_{ij} are uniformly scattered in [0.0 , 0.1]

One example of each subtype is included in the benchmark set in appendix A.

Type 5. In this fifth type, the $\mathbf{V}_d$ variance matrices have the same variance but the correlations may be different. All the correlations are small. In defining this type, we put $v_{ii} = 1.0 \;\; i = 1, 2, ..., n$ and $v_{ij} = \rho_{ij} \;\; \forall i, j, i \neq j$.

We define two subtypes within variance matrices of type 5.

(5a) ρ_{ij} are uniformly scattered in [0.0 , 0.07]

(5b) ρ_{ij} are uniformly scattered in [0.0 , 0.1]

One example of each subtype is included in the benchmark set. The numerical values for these two subtypes are given in appendix A.

One of the purposes of studying type 4 and type 5 is to see the impact of variation in the covariances (impact on the quality of the approximation). In type 4 there is variation in the variances and in type 5 there is none.

*Type 6***.** The $\mathbf{V}_d$ variance matrices in this sixth type have varying values of both the variances and the correlation coefficients. The correlation coefficients ρ_{ij} are uniformly scattered in the interval [0.15 , 0.20]. In defining type 6 matrices, we put $v_{11} = 1.0$ and $v_{nn} = 0.25$ and the other v_{ii} are uniformly scattered in [0.25 , 1.0]. Given the variances and correlation coefficients, the covariances are computed using $v_{ij} = \rho_{ij}\sqrt{v_{ii}}\sqrt{v_{jj}} \quad \forall i, j, i \neq j$.

We do not define subtypes within type 6. One example of this type is included in the benchmark set in appendix A.

Type 7. The $\mathbf{V}_d$ variance matrices in this seventh type have varying values of the variances and the correlation coefficients ρ_{ij} vary to a large extent. The ρ_{ij} are uniformly scattered in the interval [0.0 , 0.3]. In defining type 7 matrices, we put $v_{11} = 1.0$ and $v_{nn} = 0.25$ and the other v_{ii} are uniformly scattered in [0.25, 1.0]. The covariances are computed using $v_{ij} = \rho_{ij}\sqrt{v_{ii}}\sqrt{v_{jj}} \quad \forall i, j, i \neq j$.

We do not define subtypes within type 7. One example of this type is included in the benchmark set in appendix A. One of the purposes of studying type 6 and type 7 is to see the impact of variation of magnitude of the covariances (impact on the quality of the approximation). In type 6, the covariances are not small but they are of similar order of magnitude, the ρ_{ij} :s are uniformly scattered in the interval [0.15 , 0.20]. In type 7, the covariances are on a broader scale, the ρ_{ij} :s are uniformly scattered in the interval [0.0 , 0.3].

The benchmark includes 45 matrices. The types and subtypes defined above imply that the benchmark consists of 15 reference variance matrices. In fact there are more than 15 matrices because we have not yet discussed the dimension of $\mathbf{V}_d$ variance matrices. The dimension of $\mathbf{V}_d$ variance matrix is the number of items that the inspector has verified (usually they are a random sample of the items in some population). An inspector will rarely verify more than 25 items in a population. Sometimes however he or she may verify as little as 10 items. For this reason, the benchmark has been designed to include variance matrices representing the three cases n = 10, n = 20 and n = 25. This means that the benchmark includes 45 different of variance matrices, see appendix A. For each value of ‘n’ the benchmark includes all the 15 different matrix types or subtypes mentioned above.

Chapter 4. Description of different approximation methods

4.1 Description of computing tail probabilities of the distribution of the test statistic

We are interested in comparing a number of different approximation methods for computing the upper tail probabilities of the distribution of the test statistic (2.14) *i.e.* for computing $P\left(Q \geq x \middle| \mathbf{V}_d\right)$, given a number of reference matrices $\mathbf{V}_d$ (see chapter 3 for description) and given values of x. Research in literature has been made to find suitable approximation methods. Five relevant methods have been found. The first one was invented in the mid-fifties and the last one was invented in the early nineties, this shows a broad time scale in distribution theory development. In addition to these five methods, two crude and simple methods are also included in the study. One of these two methods is being used today in inventory work. Seeing the poor effectiveness of these unprofessional methods, allows this study to document the necessity of having a good method even though a good method requires more computation. See below in table for the listed methods, authors and in what years they were developed.

Table 4.1 The seven approximation methods used in the study.

Method	Name	Author/s	Year
1	**Crude approximation method 1**	-	-
2	**Crude approximation method 2**	-	-
3	**Two moment approximation** (Distributions of quadratic forms and some applications)	Grad, A. and Solomon, H.	1955
4	**1st three moment approximation** (Computing the distribution of quadratic forms in normal variables)	Imhof, J.P.	1961
5	**Gaussian approximation** (A gaussian approximation to the distribution of a definite quadratic form)	Jensen, D.R. and Solomon, H.	1972
6	**2nd three moment approximation** (Distribution of a sum of weighted chi-square variables)	Solomon, H. and Stephens, M.A.	1977
7	**Saddlepoint approximation** (Tail areas of linear combinations of chi-squares and non-central chi-squares)	Field, C.	1993

The approximated computed values are denoted by $P_{M_i}\left(Q \geq x \middle| \mathbf{V}_d\right)$, $i = 1,2,...,7$ and the "true" values are denoted by $P_T\left(Q \geq x \middle| \mathbf{V}_d\right)$. The purpose of the study is to compare each $P_{M_i}\left(Q \geq x \middle| \mathbf{V}_d\right)$ with $P_T\left(Q \geq x \middle| \mathbf{V}_d\right)$ for different values of *x*. These *x* values of interest are those which are values of *Q* that correspond to reference levels of significance 0.10, 0.05, 0.02 and 0.01 in a one sided test when the variance matrix $\mathbf{V}_d$ is the null hypothesis.

In the next chapter, chapter five, we will talk about the computations to find the different values of *x* for each variance matrix $\mathbf{V}_d$ hence finding $P_T\left(Q \geq x \middle| \mathbf{V}_d\right)$. This is done by a simulation of the distribution of the test statistic *Q*. We need the simulated distribution as our reference distribution, in order to be able to assess the quality of the different approximating methods of this distribution. In this chapter however we will concentrate on a thorough study of the seven approximation methods.

4.2 Different approximation methods

*Method 1. Crude approximation method 1***.** Despite the fact that the differences d_i:s have a general variance matrix, method 1 consists in "pretending" that the d_i:s have a variance matrix from a symmetric distribution. An "approximation" to the real variance matrix is computed and the test is carried out using a chi-square distribution with *n-1* degrees of freedom

$$\frac{(n-1)S_d^2}{\sigma_{ran}^2} \sim \chi_{n-1}^2 \tag{4.1}$$

The value of σ_{ran}^2 for this method is computed as follows. Remember that we denote the diagonal elements of the variance matrix $\mathbf{V}_d$ as v_{ii} and the off-diagonal elements as v_{ij}, $i \neq j$, $i, j = 1,2,...,n$. We first compute the average of the variances, denoted as σ_{tot}^2, as

$$\sigma_{tot}^2 = \frac{1}{n}\sum_{i=1}^{n} v_{ii} \tag{4.2}$$

We then continue by computing the average of the covariances, which is the systematic variance component, as

$$\sigma_{sys}^2 = \frac{2}{n(n-1)} \sum_{i=1}^{n} \sum_{j=i+1}^{n} v_{ij} \tag{4.3}$$

To get the random variance component we just subtract the systematic variance component from the σ_{tot}^2, so

$$\sigma_{ran}^2 = \sigma_{tot}^2 - \sigma_{sys}^2 \tag{4.4}$$

We thus approximate the distribution of

$$\frac{(n-1)S_d^2}{\frac{1}{n}\sum_{i=1}^{n} v_{ii} - \frac{2}{n(n-1)} \sum_{i=1}^{n} \sum_{j=i+1}^{n} v_{ij}} = \frac{(n-1)S_d^2}{\sigma_{ran}^2} \tag{4.5}$$

as being a chi-square distribution with *n-1* of freedom, χ_{n-1}^2.

Method 2. Crude approximation method 2. This second approximation method is cruder than the first one. Here we add the limitation that we make no consideration of the covariances, just the variances. We compute the average of the variances as (4.2). Then we approximate the distribution of

$$\frac{(n-1)S_d^2}{\sigma_{tot}^2} \tag{4.6}$$

with a chi-square distribution with *n-1* degrees of freedom, χ_{n-1}^2.

Method 3. Two moment approximation. Grad and Solomon (1955), [27] approximates $Q = (n-1)S_d^2$ by the variable

$$A \cdot \chi_v^2 = A \cdot Y_v \tag{4.7}$$

where A is a constant to be calculated and Y_v is a central chi-squared variable with v degrees of freedom, where also v has to be calculated to give good approximation. If v is not an integer the probabilities will be computed using the Wilson-Hilferty approximation to be chi-squared or using interpolation of the chi-square variables. We start by calculating the first two moments of the distribution

$$Q = (n-1)S_d^2 = A \cdot Y_v$$

First moment about zero is as follows

$$E[Q] = E[A \cdot Y_v] = Av \tag{4.8}$$

At the end of section *2.3* in chapter *2,* we calculated the first moment about zero of Q as

$$\mu_1^{'}(Q) = \sum_{i=1}^{n} \lambda_i \tag{4.9}$$

Second moment about the mean *i.e.* the variance is

$$Var[Q] = Var[A \cdot Y_v] = A^2 2v \tag{4.10}$$

and for our statement of problem the variance can be written

$$\mu_2(Q) = 2\sum_{i=1}^{n} \lambda_i^2 \tag{4.11}$$

Equating (4.8) with (4.9) and (4.10) with (4.11), *i.e.* solving the two equations for A and v give

$$A = \frac{\sum_{i=1}^{n} \lambda_i^2}{\sum_{i=1}^{n} \lambda_i} \tag{4.12}$$

$$v = \frac{\left[\sum_{i=1}^{n} \lambda_i\right]^2}{\sum_{i=1}^{n} \lambda_i^2} \tag{4.13}$$

If v is not an integer, the distribution we are approximating with is still a special case of the Gamma distribution. For this distribution we can compute the percentiles of this distribution by two methods

(i) use the idea that this chi-square variable can be approximately transformed into a Gaussian variable using the Wilson-Hilferty approximation.

(ii) interpolate chi-squared values

(i) The Wilson-Hilferty, Wilson and Hilferty, (1931), approximation states that if Y_v is a chi-square variable with *v* degrees of freedom, then

$$\left(Y_v/v\right)^{1/3} \tag{4.14}$$

is approximately normally distributed with mean

$$1-\frac{2}{9v} \tag{4.15}$$

and variance

$$\frac{2}{9v} \tag{4.16}$$

that is

$$\left(Y_v/v\right)^{1/3} \sim N\left(1-\frac{2}{9v},\frac{2}{9v}\right). \tag{4.17}$$

Suppose we have a value of $Q=(n-1)S_d^2$ computed from real data and we wish to know what is the probability of getting a value larger than Q, i.e. $P\left(Q \geq x\middle|\mathbf{V}_d\right)$. We then wish to find the equivalent standardized Gaussian value. We have $Q=(n-1)S_d^2 = A\cdot Y_v$, so we compute a value of Y_v as

$$Y_v = \frac{(n-1)S_d^2}{A} \tag{4.18}$$

We then compute the standardized Gaussian variable as

$$\frac{\left(\frac{Y_v}{v}\right)^{1/3}-\left(1-\frac{2}{9v}\right)}{\sqrt{\frac{2}{9v}}} \tag{4.19}$$

hence we see how extreme our value of $Q=(n-1)S_d^2$ is by seeing what is the probability of a standard Gaussian larger than this value.

Method 4. First three moment approximation. Imhof (1961), [29]. Let Q as usual denote $(n-1)S_d^2$. We know the moments $\mu_1^{'}(Q)$ (first moment about zero), $\sigma^2(Q)$, $\mu_3(Q)$ and $\mu_4(Q)$ (second, third and fourth moment about the mean) as functions of the $\lambda_i : s$, see end of chapter 2 for the description of the moments. In this method we are going to look for an approximation based on a central chi-squared where the degrees of freedom v are to be chosen.

The idea is that we will consider the distribution of

$$\frac{\chi_v^2 - E\left[\chi_v^2\right]}{\sigma\left[\chi_v^2\right]} \equiv \frac{\chi_v^2 - v}{\sqrt{2v}} \tag{4.20}$$

as an approximation for the distribution of

$$\frac{Q - E[Q]}{\sqrt{Var[Q]}} \tag{4.21}$$

We choose v so that (4.20) and (4.21) have the same third central moment.

$$E\left(\frac{\chi_v^2 - v}{\sqrt{2v}}\right)^3 = E\left(\frac{\left(\chi_v^2 - v\right)^3}{\left(\sqrt{2v}\right)^3}\right) = \frac{1}{\left(\sqrt{2v}\right)^3} E\left(\chi_v^2 - v\right)^3 =$$
$$\frac{\mu_3\left(\chi_v^2\right)}{\left(2v\right)^{3/2}} = \frac{8v}{2v\sqrt{2v}} = \frac{4}{\sqrt{2v}} \tag{4.22}$$

$$E\left(\frac{Q - E[Q]}{\sqrt{Var[Q]}}\right)^3 = \frac{E\left(Q - E[Q]\right)^3}{\left(\sqrt{Var[Q]}\right)^3} = \frac{\mu_3(Q)}{\sigma(Q)^3} \tag{4.23}$$

Equating (4.22) with (4.23) give

$$\frac{\mu_3(Q)}{\sigma(Q)^3} = \frac{4}{\sqrt{2v}}$$

$$\sqrt{2v} = \frac{4\sigma(Q)^3}{\mu_3(Q)} = \frac{4\cdot 2\sum_{i=1}^{n}\lambda_i^2\left[2\sum_{i=1}^{n}\lambda_i^2\right]^{1/2}}{8\sum_{i=1}^{n}\lambda_i^3} = \frac{\sqrt{2}\left[\sum_{i=1}^{n}\lambda_i^2\right]^{3/2}}{\sum_{i=1}^{n}\lambda_i^3}$$

$$\sqrt{v} = \frac{\left[\sum_{i=1}^{n}\lambda_i^2\right]^{3/2}}{\sum_{i=1}^{n}\lambda_i^3}$$

$$v = \frac{\left[\sum_{i=1}^{n}\lambda_i^2\right]^{3}}{\left[\sum_{i=1}^{n}\lambda_i^3\right]^{2}} \tag{4.24}$$

Suppose we have a value of $Q=(n-1)S_d^2$ and we wish to know what is the probability of getting a value larger than Q. Return to the expression of $\frac{Q-E[Q]}{\sqrt{Var[Q]}}$ which is being approximated by $\frac{\chi_v^2 - v}{\sqrt{2v}}$ and compute

$$\sqrt{2v}\left[\frac{Q-E[Q]}{\sqrt{Var[Q]}}\right]+v=$$

$$\sqrt{2\left[\sum_{i=1}^{n}\lambda_i^2\right]^3 \Big/ \left[\sum_{i=1}^{n}\lambda_i^3\right]^2}\left[\frac{Q-\sum_{i=1}^{n}\lambda_i}{\sqrt{2\sum_{i=1}^{n}\lambda_i^2}}\right]+\left[\sum_{i=1}^{n}\lambda_i^2\right]^3 \Big/ \left[\sum_{i=1}^{n}\lambda_i^3\right]^2 \tag{4.25}$$

to get a chi-square, χ_v^2, value.

If v is not an integer we use the Wilson-Hilferty approximation to compute the probability or we use interpolation of the chi-square variable.

Method 5. Gaussian approximation. Jensen and Solomon (1972), [28]. Let Q denote $(n-1)S_d^2=\sum_{i=1}^{n}\lambda_i Z_i^2$, where the Z_i^2's as usual are chi-squared random variables, each with one degree of freedom. The cumulant generating function of Q is given by

$$\xi_Q(t) = -\frac{1}{2}\sum_{i=1}^{n}\ln(1-2\lambda_i t) \tag{4.26}$$

and the j^{th} cumulant is given by

$$\kappa_j = 2^{j-1}(j-1)!\sum_{i=1}^{n}\lambda_i^j \tag{4.27}$$

See section 2.3 for definition and derivation of the c.g.f. Let θ_j define $\sum_{i=1}^{k}\lambda_i^j$, this gives

$$\kappa_j = 2^{j-1}(j-1)!\theta_j \tag{4.28}$$

We use the relationships between cumulants and moments and calculate the first three moments of Q

$$\begin{aligned} \mu_1^{'}(Q) &= \theta_1 \\ \mu_2(Q) &= 2\theta_2 \\ \mu_3(Q) &= 8\theta_3 \end{aligned} \tag{4.29}$$

The distribution of Q tends to a Gaussian distribution as $\theta_1 \to \infty$. In this method we consider transformations of the type $(Q_k/\theta_1)^h$ which converges quickly for moderate values of θ_1. We need to calculate the moments of $(Q_k/\theta_1)^h$ expanded in powers of θ_1^{-1} and then choose h so that the leading term in the expression for the third moment disappears. The moments of $(Q_k/\theta_1)^h$ are denoted

$$\mu_r^{'}(h) = E\left[(Q_k/\theta_1)^{rh}\right] \tag{4.30}$$

We calculate $\mu_1^{'}(h),\ \mu_2^{'}(h),\ \mu_3^{'}(h)$ and the coefficient of skewness,

$$\gamma_1(h) = \mu_3(h)/(\mu_2(h))^{3/2} \tag{4.31}$$

We will choose h so that the leading terms of $\mu_3^{'}(h)$ and $\gamma_1(h)$ disappear. The solution is given by

$$h = 1 - \frac{2\theta_1\theta_3}{3\theta_2^2} \tag{4.32}$$

We are approximating the distribution of

$$\left(\frac{Q_k}{\theta_1}\right)^h$$

by a Gaussian distribution with mean

$$1 + \frac{\theta_2 h(h-1)}{\theta_1^2} \tag{4.33}$$

and variance

$$\frac{2\theta_2 h^2}{\theta_1^2} \tag{4.34}$$

The approximately standardized Gaussian variable will therefore be

$$Z = \frac{\theta_1\left[\left(\frac{Q_k}{\theta_1}\right)^h - 1 - \frac{\theta_2 h(h-1)}{\theta_1^2}\right]}{\left[h\sqrt{2\theta_2}\right]} \tag{4.35}$$

Suppose that we have a value x derived from real data and we wish to compute the significance level, $P(Q \geq x|\mathbf{V}_d)$. We proceed as follows, first we compute the values of $\theta_1, \theta_2, \theta_3$ and *h*. Then form the variable Y which is composed by taking the value x divided by the first moment about zero θ_1, namely $Y = \left(\frac{x}{\theta_1}\right)^h$.

Then we standardize the Y variable so the variable Z is formed, that is

$$Z = \frac{Y - \mu}{\sigma} \tag{4.36}$$

where

$$\mu = 1 + \frac{\theta_2 h(h-1)}{\theta_1^2} \qquad \text{and} \tag{4.37}$$

$$\sigma = \sqrt{\frac{2\theta_2 h^2}{\theta_1^2}} \tag{4.38}$$

Finally we calculate $P(Z \geq z) \quad \approx 1 - \Phi(z)$.

Method 6. Second three moment approximation. Solomon and Stephens (1977), [32]. Let Q as usual denote $(n-1)S_d^2 = \sum_{i=1}^{n} \lambda_i Z_i^2$, where the Z_i^2's are chi-squared random variables, each with one degree of freedom. The distribution of Q will be approximated by a constant A times the distribution of a chi-squared variable with v degrees of freedom to the power of r

$$A \cdot \left(\chi_v^2\right)^r \tag{4.39}$$

We will derive expressions for constants A, v and r as functions of the λ_i:s. We will equate the first three moments between the distribution of Q and $A \cdot \left(\chi_v^2\right)^r$.

The first three moments about zero of Q, see table 2.1, are

$$\mu_1^{'}(Q) = \sum_{i=1}^{n} \lambda_i$$

$$\mu_2^{'}(Q) = \sigma^2 + \left(\mu_1^{'}(Q)\right)^2 = 2\sum_{i=1}^{n} \lambda_i^2 + \left[\sum_{i=1}^{n} \lambda_i\right]^2$$

$$\mu_3^{'}(Q) = \mu_3(Q) + 3\mu_1^{'}(Q)\sigma^2 + \left(\mu_1^{'}(Q)\right)^3 = 8\sum_{i=1}^{n} \lambda_i^3 + 6\left[\sum_{i=1}^{n} \lambda_i\right]\left[\sum_{i=1}^{n} \lambda_i^2\right] + \left[\sum_{i=1}^{n} \lambda_i\right]^3$$

The derivation of the first three moments about zero of $\left(\chi_v^2\right)^r$

$$\mu_1^{'}((\chi_v^2)^r) = \int_0^\infty \frac{e^{-z/2} z^{v/2-1} z^r}{2^{v/2}\Gamma(v/2)} dz = \frac{1}{2^{v/2}\Gamma(v/2)} \int_0^\infty e^{-z/2} z^{r+v/2-1} dz =$$

$$\frac{2^{r+v/2-1}}{2^{v/2}\Gamma(v/2)} \int_0^\infty e^{-z/2} \left(\frac{z}{2}\right)^{r+v/2-1} dz = \frac{2^{r-1}}{\Gamma(v/2)} \cdot \frac{\Gamma(r+v/2)}{1/2} = \frac{2^r \Gamma(r+v/2)}{\Gamma(v/2)} \tag{4.40}$$

$$\mu_2^{'}((\chi_v^2)^r) = E[(\chi_v^2)^{2r}] = \int_0^{\infty} \frac{e^{-z/2} z^{v/2-1} z^{2r}}{2^{v/2}\Gamma(v/2)} dz = \frac{1}{2^{v/2}\Gamma(v/2)} \int_0^{\infty} e^{-z/2} z^{2r+v/2-1} dz =$$

$$\frac{2^{2r+v/2-1}}{2^{v/2}\Gamma(v/2)} \int_0^{\infty} e^{-z/2} \left(\frac{z}{2}\right)^{2r+v/2-1} dz = \frac{2^{2r-1}}{\Gamma(v/2)} \cdot \frac{\Gamma(2r+v/2)}{1/2} = \frac{2^{2r}\Gamma(2r+v/2)}{\Gamma(v/2)} \tag{4.41}$$

$$\mu_3^{'}((\chi_v^2)^r) = E[(\chi_v^2)^{3r}] = \int_0^{\infty} \frac{e^{-z/2} z^{v/2-1} z^{3r}}{2^{v/2}\Gamma(v/2)} dz = \frac{1}{2^{v/2}\Gamma(v/2)} \int_0^{\infty} e^{-z/2} z^{3r+v/2-1} dz =$$

$$\frac{2^{3r+v/2-1}}{2^{v/2}\Gamma(v/2)} \int_0^{\infty} e^{-z/2} \left(\frac{z}{2}\right)^{3r+v/2-1} dz = \frac{2^{3r-1}}{\Gamma(v/2)} \cdot \frac{\Gamma(3r+v/2)}{1/2} = \frac{2^{3r}\Gamma(3r+v/2)}{\Gamma(v/2)} \tag{4.42}$$

The first three moments about zero of $A \cdot \left(\chi_v^2\right)^r$ are

$$\mu_1^{'}\left(A \cdot \left(\chi_v^2\right)^r\right) = \frac{A 2^r \Gamma\left(r + v/2\right)}{\Gamma\left(v/2\right)}$$

$$\mu_2^{'}\left(A \cdot \left(\chi_v^2\right)^r\right) = \frac{A^2 2^{2r} \Gamma\left(2r + v/2\right)}{\Gamma\left(v/2\right)}$$

$$\mu_3^{'}\left(A \cdot \left(\chi_v^2\right)^r\right) = \frac{A^3 2^{3r} \Gamma\left(3r + v/2\right)}{\Gamma\left(v/2\right)} \tag{4.43}$$

Matching the first three moments of the distribution of Q and of the distribution of $A \cdot \left(\chi_v^2\right)^r$ gives the following three equations for solving for A, v and r

$$\mu_1^{'}(Q) = \frac{A 2^r \Gamma\left(r + v/2\right)}{\Gamma\left(v/2\right)} \tag{4.44}$$

$$\mu_2^{'}(Q) = \frac{A^2 2^{2r} \Gamma\left(2r + v/2\right)}{\Gamma\left(v/2\right)} \tag{4.46}$$

$$\mu_3^{'}(Q) = \frac{A^3 2^{3r} \Gamma\left(3r + v/2\right)}{\Gamma\left(v/2\right)} \tag{4.47}$$

Simplification of the three equations above gives the following two equations

$$\frac{\mu_2'(Q)}{\left[\mu_1'(Q)\right]^2} = \frac{\Gamma(2r+v/2)\Gamma(v/2)}{\left[\Gamma(r+v/2)\right]^2} \tag{4.48}$$

$$\frac{\mu_3'(Q)}{\left[\mu_1'(Q)\right]^3} = \frac{\Gamma(3r+v/2)\left[\Gamma(v/2)\right]^2}{\left[\Gamma(r+v/2)\right]^3} \tag{4.49}$$

These two equations have to be solved numerically for *r* and *v*. Then *A* can be calculated from the expression of $\mu_1'(Q)$, (4.44). In what follows these equations will be referred to as *F* = 0 and *G* = 0 where

$$F(r,v) = \frac{\Gamma(2r+v/2)\Gamma(v/2)}{\left[\Gamma(r+v/2)\right]^2} - \frac{\mu_2'(Q)}{\left[\mu_1'(Q)\right]^2} \tag{4.50}$$

$$G(r,v) = \frac{\Gamma(3r+v/2)\left[\Gamma(v/2)\right]^2}{\left[\Gamma(r+v/2)\right]^3} - \frac{\mu_3'(Q)}{\left[\mu_1'(Q)\right]^3} \tag{4.51}$$

Transforming the equations. Finding numerical values for r and *v* can be done by applying Newton-Raphson's method directly to these equations. Alternatively, the equations can be transformed before applying Newton-Raphson's method. A first step in transformation would be to take logarithms of both sides of the equations. This gives two simultaneous equations having terms involving $\ln(\Gamma(x))$. A second stage in transformation would then be to approximate the terms of the form $\ln(\Gamma(x))$ using a Stirling series of the form,

$$\ln(\Gamma(x)) = (x-\frac{1}{2})\ln(x) - x + \frac{1}{2}\ln(2\pi) + \frac{1}{12x} - \frac{1}{360x^3} + \frac{1}{1260x^5} - \frac{1}{1680x^7} + \frac{1}{1188x^9} - \ldots \tag{4.52}$$

Applying this approach provides an alternative approximate representation of the simultaneous equations in a form that is tractable for numerical methods. The two resulting equations are $F^*(r,v)=0$ and $G^*(r,v)=0$, where

$$F^*(r,v) = (2r + v/2 - 1/2)\ln(2r + v/2) + 1/(12\cdot(2r + v/2)) - 1/(360\cdot(2r + v/2)^3) + 1/(1260\cdot(2r + v/2)^5) - 1/(1680\cdot(2r + v/2)^7) + 1/(1188\cdot(2r + v/2)^9) + 1/2(v-1)\ln(v/2) + 1/(12\cdot(v/2)) - 1/(360\cdot(v/2)^3) + 1/(1260\cdot(v/2)^5) - 1/(1680\cdot(v/2)^7) + 1/(1188\cdot(v/2)^9) - 2\cdot[(r + v/2 - 1/2)\ln(r + v/2) + 1/(12\cdot(r + v/2)) - 1/(360\cdot(r + v/2)^3) + 1/(1260\cdot(r + v/2)^5) - 1/(1680\cdot(r + v/2)^7) + 1/(1188\cdot(r + v/2)^9)] - \ln(\frac{\mu_2'(Q)}{[\mu_1'(Q)]^2}) \quad (4.53)$$

$$G^*(r,v) = (3r + v/2 - 1/2)\ln(3r + v/2) + 1/(12\cdot(3r + v/2)) - 1/(360\cdot(3r + v/2)^3) + 1/(1260\cdot(3r + v/2)^5) - 1/(1680\cdot(3r + v/2)^7) + 1/(1188\cdot(3r + v/2)^9) + (v-1)\ln(v/2) + 2\cdot[1/(12\cdot(v/2)) - 1/(360\cdot(v/2)^3) + 1/(1260\cdot(v/2)^5) - 1/(1680\cdot(v/2)^7) + 1/(1188\cdot(v/2)^9) - 3\cdot[(r + v/2 - 1/2)\ln(r + v/2) + 1/(12\cdot(r + v/2)) - 1/(360\cdot(r + v/2)^3) + 1/(1260\cdot(r + v/2)^5) - 1/(1680\cdot(r + v/2)^7) + 1/(1188\cdot(r + v/2)^9)] - \ln(\frac{\mu_3'(Q)}{[\mu_1'(Q)]^3}) \quad (4.54)$$

In what follows, two methods for computing r and v are being used. The first is based on a numerical solution of the equations $F(r,v) = 0$ and $G(r,v) = 0$. This involves a direct computation of the gamma function by Matlab during the application of Newton-Raphson's method. This can be applied in the case when computing the gamma function is possible. But when the argument of the gamma function is too large, $\Gamma(x)$ cannot be computed by Matlab and hence for solving these equations, methods based on computing $\Gamma(x)$ cannot be used. In the second method, we apply the Newton-Raphson's method to the equations $F^*(r,v) = 0$ and $G^*(r,v) = 0$.

A Newton Raphson solution to the equations. We describe the application of Newton Raphson's method to either form of the equation system *i.e.* for the equation system $F(r,v) = 0$ and $G(r,v) = 0$ or for equation system $F^*(r,v) = 0$ and $G^*(r,v) = 0$. For simplicity in this numerical method description, the description is written using the notation $F(r,v)$ and $G(r,v)$ but is intended to be read also with $F^*(r,v)$ and $G^*(r,v)$ taking the place of $F(r,v)$ and $G(r,v)$.

Let r_0 and v_0 denote the initial values for the iterative approach for solving the simultaneous equations $F(r,v)=0$ and $G(r,v)=0$. The values r_0 and v_0 will be improved iteratively to give a sequence (r_i, v_i) converges to the solution of $F(r,v)=0$ and $G(r,v)=0$.
That is to construct iteratively

$$\begin{cases} r_{i+1} = r_{i+1}(r_i, v_i, F, G) \\ v_{i+1} = v_{i+1}(r_i, v_i, F, G) \end{cases} \tag{4.55}$$

The first order Taylor series approximations $\tilde{F}$ and $\tilde{G}$ to F and G in the neighbourhood of (r_i, v_i) are used for setting up the iteration step. The approximation functions $\tilde{F}$ and $\tilde{G}$ are defined by

$$\begin{aligned} \tilde{F}(r, v | r_i, v_i) &= F(r_i, v_i) + (r - r_i)\frac{\partial F}{\partial r}\Big|_{r_i v_i} + (v - v_i)\frac{\partial F}{\partial v}\Big|_{r_i v_i} \\ \tilde{G}(r, v | r_i, v_i) &= G(r_i, v_i) + (r - r_i)\frac{\partial G}{\partial r}\Big|_{r_i v_i} + (v - v_i)\frac{\partial G}{\partial v}\Big|_{r_i v_i} \end{aligned} \tag{4.56}$$

The iteration step consists of solving the simultaneous equations $\tilde{F} = 0$ and $\tilde{G} = 0$ for r and v and these solutions are taken as the values of r_{i+1}, v_{i+1}. The approximation simultaneous equations are

$$\begin{aligned} (r - r_i)\frac{\partial F}{\partial r}\Big|_{r_i v_i} + (v - v_i)\frac{\partial F}{\partial v}\Big|_{r_i v_i} &= -F(r_i, v_i) \\ (r - r_i)\frac{\partial G}{\partial r}\Big|_{r_i v_i} + (v - p_i)\frac{\partial G}{\partial v}\Big|_{r_i v_i} &= -G(r_i, v_i) \end{aligned} \tag{4.57}$$

In vector notation we may write

$$A_{r_i v_i} = \begin{pmatrix} \frac{\partial F}{\partial r}\Big|_{r_i v_i} & \frac{\partial F}{\partial v}\Big|_{r_i v_i} \\ \frac{\partial G}{\partial r}\Big|_{r_i v_i} & \frac{\partial G}{\partial v}\Big|_{r_i v_i} \end{pmatrix} \tag{4.58}$$

$$b_{r_i v_i} = \begin{pmatrix} -F(r_i, v_i) \\ -G(r_i, v_i) \end{pmatrix} \tag{4.59}$$

$$\vartriangle x_i = \begin{pmatrix} r - r_i \\ v - v_i \end{pmatrix} \tag{4.60}$$

$$A_{r_i v_i} \cdot \vartriangle x_i = b_{r_i v_i} \tag{4.61}$$

For a non singular matrix (4.58) the solution is

$$\vartriangle x_i = A_{r_i v_i}^{-1} \cdot b_{r_i v_i} \tag{4.62}$$

The next iteration point (r_{i+1}, v_{i+1}) is computed as

$$x_{i+1} = x_i + A_{r_i v_i}^{-1} \cdot b_{r_i v_i} \tag{4.63}$$

where $x_i = \begin{pmatrix} r_i \\ v_i \end{pmatrix}$.

The partial derivatives are computed as

$$\begin{aligned}
\left.\frac{\partial \hat{F}}{\partial r}\right|_{r'v'} &= \frac{F(r'+\triangle r, v') - F(r', v')}{\triangle r} \\
\left.\frac{\partial \hat{F}}{\partial v}\right|_{r'v'} &= \frac{F(r', v'+\triangle v) - F(r', v')}{\triangle v} \\
\left.\frac{\partial \hat{G}}{\partial r}\right|_{r'v'} &= \frac{G(r'+\triangle r, v') - G(r', v')}{\triangle r} \\
\left.\frac{\partial \hat{G}}{\partial v}\right|_{r'v'} &= \frac{G(r', v'+\triangle v) - G(r', v')}{\triangle v}
\end{aligned} \tag{4.64}$$

We specify the values of $\triangle r$ and $\triangle v$ by putting $\triangle r = \triangle v = 10^{-5}$.

The matrix $A_{r_i v_i}$ in the iteration equation (4.63), is computed using the numerical values as

$$\hat{A}_{r_i v_i} = \begin{pmatrix} \left.\frac{\partial \hat{F}}{\partial r}\right|_{r_i v_i} & \left.\frac{\partial \hat{F}}{\partial v}\right|_{r_i v_i} \\ \left.\frac{\partial \hat{G}}{\partial r}\right|_{r_i v_i} & \left.\frac{\partial \hat{G}}{\partial v}\right|_{r_i v_i} \end{pmatrix} \tag{4.65}$$

The vector $b_{r_i v_i}$ is computed as earlier formula (4.59).

*Choice of initial values***.** Solomon and Stephen's "three moment approximation" can be seen as an extension of Grad and Solomon's "two moment approximation". Since in the last mentioned method the distribution of $Q = (n-1)S_d^2$ is approximated by the variable $A \cdot \chi_v^2$ and in the previously mentioned the distribution is approximated by the variable $A \cdot \left(\chi_v^2\right)^r$. We can therefore choose the initial values (r_0, v_0) according to Grad and Solomon's method, with the following values

$$r_0 = 1 \tag{4.66}$$

$$v_0 = \frac{\left[\sum_{i=1}^{n} \lambda_i\right]^2}{\sum_{i=1}^{n} \lambda_i^2} \tag{4.67}$$

*Stopping rule***.** A suitable stopping rule for the iterative process, is to stop the process when

$$\frac{|r_i - r_{i+1}|}{|r_i|} < \in \qquad \frac{|v_i - v_{i+1}|}{|v_i|} < \in \tag{4.68}$$

where $\in$ equals 0.0001.

Once we have obtained values in the iteration process of (r_i, v_i), we need to check them so they satisfy the equations $F(r,v) = 0$ and $G(r,v) = 0$. The final values after "stopping" will be denoted (r^*, v^*). We therefore need to compute $\vartriangle F(r,v)$ and $\vartriangle G(r,v)$

$$\vartriangle F(r,v) = \frac{\Gamma(2r^* + v^*/2)\Gamma(v^*/2)}{\left[\Gamma(r^* + v^*/2)\right]^2} - \frac{\mu_2^{'}(Q)}{\left[\mu_1^{'}(Q)\right]^2} = 0 \tag{4.69}$$

$$\vartriangle G(r,v) = \frac{\Gamma(3r^* + v^*/2)\left[\Gamma(v^*/2)\right]^2}{\left[\Gamma(r^* + v^*/2)\right]^3} - \frac{\mu_3^{'}(Q)}{\left[\mu_1^{'}(Q)\right]^3} = 0 \tag{4.70}$$

Define $\vartriangle^*$ as $\vartriangle^* = Sup\{|\vartriangle F|, |\vartriangle G|\}$. The solution can be considered as satisfactory if

$$\vartriangle^* < 0.0001 \tag{4.71}$$

If (4.71) is not satisfied then reduce $\in$ in the stopping rule and continue the sequence of (r_{i+1}, v_{i+1}).

Computing the upper tail probability. Once the values of *r, v* and *A* are computed, we then calculate how extreme our value of *Q* is. Suppose we have a value of

$Q=(n-1)S_d^2$ computed from real data and we wish to know what is the probability of getting a value larger than Q, i.e. $P\left(Q \geq x \middle| \mathbf{V}_d\right)$.

We have $Q=(n-1)S_d^2 = A\cdot\left(\chi_v^2\right)^r = A\cdot\left(Y_v\right)^r$, so we compute a value of Y_v as

$$Y_v = \left(\frac{(n-1)S_d^2}{A}\right)^{1/r} \tag{4.72}$$

We then see how extreme our value of $Q=(n-1)S_d^2$ is by seeing what is the probability of a chi-square variable with v degrees of freedom larger than this value.

Method 7. Saddlepoint approximation. Method 7 uses a saddlepoint approximation method due to Field (1993), [27], for calculating tail probabilities of the sum of independent variables which are not identically distributed. The derivation of the approximation starts from a representation of the tail area coming from the Fourier inversion of the characteristic function. This integral is then transformed and the Jacobian is expanded in Taylor series. The method of Field is interesting as it is using a saddlepoint method for tail probabilities for a sum of non-iid random variables. As Field points out of course the use of the method can only be validated numerically. A very good general introduction to saddlepoint approximations is given in Kolassa (1994), [18].

Let Q as usual denote $(n-1)S_d^2 = \sum_{i=1}^{n}\lambda_i Z_i^2$ where $Z_i \sim N(0,1)$. The cumulant generating function of Q is given by

$$\xi_Q(t) = -\frac{1}{2}\sum_{i=1}^{n}\ln(1-2\lambda_i t) \tag{4.73}$$

Differentiating the c.g.f. $\xi_Q(t)$ twice with respect to t, gives the first derivative

$$\xi_Q^{'}(t) = \sum_{i=1}^{n}\frac{\lambda_i}{1-2\lambda_i t} \tag{4.74}$$

and the second derivative

$$\xi_Q''(t) = 2\sum_{i=1}^{n} \frac{\lambda_i^2}{(1-2\lambda_i t)^2} \tag{4.75}$$

Following Field we approximate the probability $P(Q \geq x)$, given the value x in three steps as follows

(i) Solve the equation $\xi_Q'(t) = \sum_{i=1}^{n} \frac{\lambda_i}{1-2\lambda_i t} = x$ to find a t value. (4.76)

(ii) Compute $f = \xi_Q(t) - tx = -\frac{1}{2}\sum_{i=1}^{n} \ln(1-2\lambda_i t) - tx$ (4.77)

(iii) Compute the approximate value for

$$P(Q \geq x) \approx 1 - \Phi\left(\sqrt{-2f}\right) + \frac{\exp(f)}{\sqrt{2\pi}}\left[\frac{1}{t\sqrt{\xi_Q''(t)}} - \frac{1}{\sqrt{-2f}}\right] \tag{4.78}$$

Chapter 5. Performance Evaluation Methodology based on Simulation

5.1 Introduction

We simulate the distribution $Q = (n-1)S_d^2$ defined by (2.25) in order to assess the quality of the seven different methods of approximating the distribution. Two of these methods are naïve methods included just to document the necessity for proper methods.

The simulations are carried out using the Matlab system version 6.5 and 7. The simulated distribution is used to estimate the true upper tail probabilities of the distribution. These estimated values are used for comparison of the seven approximation methods with the true distribution values of probabilities.

The following notations will be used

$P_{M_i}\left(Q \geq x \middle| \mathbf{V}_d\right)$	approximated computed upper tail probability from method i, $i = 1, 2, \ldots, 7$
$P_T\left(Q \geq x \middle| \mathbf{V}_d\right)$	"true" upper tail probability

The fact that the "true" probability values that are being used to judge performance, are in fact simulation estimates, means that performance of each method is being estimated (with "simulation noise") and the precision of this estimation must be taken into account in interpreting the results.

5.2 Generation of Chi-square Variable

In order to simulate the distribution of (2.25) we first need to be able to either generate independent standardized Gaussian variables or generate chi-square variables, since the distribution of (2.25) can be written as a weighted sum of independent chi-squared variables (each central and each with one degree of freedom).

The main Matlab program has a function that will generate numbers from a standard Gaussian distribution, the function is called *randn.* The additional toolbox program Matlab Statistics Toolbox has a function *chi2rnd*(ν) that will generate numbers from a chi-square distribution with ν degrees of freedom.

There are several general methods for generating continuous random variables, if we do not have access to this toolbox. The inverse transformation method and the rejection method are two of the most common ones that may be used. In this particular case of generating chi-square variables, the inverse transform method cannot be used directly, since we cannot find a simple closed form of the solution for its inverse of the distribution function. However we see the fact that the sum of t independent exponentials with the parameter 0.5 is a chi-square random variable with $2t$ degrees of freedom, see Ross (2000), [24]. Hence, we may apply the inverse transform method also in this case, since it is possible to find the inverse of the exponential distribution function.

5.3 Simulating the distribution Q

Simulating the distribution of (2.25) is here done by first simulating independent standardized Gaussian variables and then forming a linear combination of their squares multiplied by the eigen values λ_i of the matrix **C** defined by (2.22).

The simulation is used to estimate tail probabilities for Q. Hence each simulated result Q is classified as either smaller than a chosen value a_p or bigger than this value a_p. The quantile values used in such classification are denoted (a_p, b_p, c_p, d_p). These (a_p, b_p, c_p, d_p) values are values of Q that would be used for judging significance in a one sided test. We need to find such quantile values (a_p, b_p, c_p, d_p) for each specific variance matrix $\mathbf{V}_d$. For each variance matrix $\mathbf{V}_d$, the value of a_p (and of the other quantiles b_p, c_p, d_p) are defined respectively as the solutions of

$$P_T\left(Q \geq a_p \left|\mathbf{V}_d\right.\right) = 0.10, \quad P_T\left(Q \geq b_p \left|\mathbf{V}_d\right.\right) = 0.05, \quad P_T\left(Q \geq c_p \left|\mathbf{V}_d\right.\right) = 0.02 \quad \text{and}$$
$$P_T\left(Q \geq d_p \left|\mathbf{V}_d\right.\right) = 0.01 \tag{5.1}$$

For each variance matrix $\mathbf{V}_d$, the values of a_p, b_p, c_p, d_p have been estimated by simulation using order statistics theory as follows.

Estimation of the quantiles a_p, b_p, c_p, d_p**:** In what follows, we let *m* denote the number of independent values of Q in the simulation. In fact the number of repetitions in the simulation was $m = 10^6$.

Let $x_1, x_2, ..., x_m$ denote the simulation values of $Q_1 = (n-1)S_d^2$, $Q_2 = (n-1)S_d^2$, … , $Q_m = (n-1)S_d^2$. The values $x_1, x_2, ..., x_m$ are then ordered in ascending order and the notation of the order statistics is $x_{(1)}, x_{(2)}, ..., x_{(m)}$. Where $x_{(1)}$ is the smallest value, $x_{(m)}$ the largest and where x_i is the i^{th} order statistic based on the *m* simulated values. For each value of *P* =0.10, 0.05, 0.02 and 0.01 the parameter a_p (and similarly b_p, c_p, d_p) was estimated by $\hat{a}_p = x_{r(p)}$, where $r(p) = [m(1-p)]^{-}$ [#]. Used values of $r(p)$ are found in the table 5.1.

Table 5.1 Examples of relevant values of *P* and $r(p)$ with the number of replications *m* equal to 10^6.

P	$r(p) = [m(1-p)]^{-}$
0.10	900 000
0.05	950 000
0.02	980 000
0.01	990 000

5.4 Performance criteria

We will use three performance criteria for evaluating and ranking the approximation methods: (i) The delta criterion, (ii) the estimated percentage delta criterion (the relative error) and (iii) the estimated percentage signed delta criterion.

(i) The delta criterion, denoted δ_{VMi}, is defined by

$$\delta_{VMi} = \left| P_T\left(Q \geq a_p \,|\mathbf{V}_d\right) - P_{Mi}\left(Q \geq a_p \,|\mathbf{V}_d\right) \right| \tag{5.2}$$

[#] where $[\;...]^{-}$ denotes rounding down to the nearest integer value.

Using the simulated quantile $\hat{a}_p$, δ_{VMi} is estimated by

$$\hat{\delta}_{VMi} = \left| P_T - P_{Mi}\left(Q \geq \hat{a}_p \left| \mathbf{V}_d \right.\right) \right| \quad (5.3)$$

where $\hat{a}_p$ is the simulation estimator of a_p. This criterion is the primary performance criterion and measures how close the approximate method is to the true value.

(ii) The estimated percentage delta criterion (the relative error), denoted $\hat{\delta}_{\%VMi}$, is defined by

$$\hat{\delta}_{\%VMi} = \frac{\hat{\delta}_{VMi}}{P_T} \cdot 100 \quad (5.4)$$

(iii) The estimated percentage signed delta criterion. The criterion of "signed delta" is denoted $\operatorname{sgn} \delta_{VMi}$ and is defined as

$$\operatorname{sgn} \delta_{VMi} = P_T\left(Q \geq a_p \left| \mathbf{V}_d \right.\right) - P_{Mi}\left(Q \geq a_p \left| \mathbf{V}_d \right.\right) \quad (5.5)$$

and it is estimated by

$$\operatorname{sgn} \hat{\delta}_{VMi} = P_T - P_{Mi}\left(Q \geq \hat{a}_p \left| \mathbf{V}_d \right.\right) \quad (5.6)$$

where again $\hat{a}_p$ is the simulation estimator of a_p.

This can also be expressed as an estimated percent relative error denoted by $\operatorname{sgn} \hat{\delta}_{\%VMi}$

$$\operatorname{sgn} \hat{\delta}_{\%VMi} = \frac{\operatorname{sgn} \hat{\delta}_{VMi}}{P_T} \cdot 100. \quad (5.7)$$

The criterion of $\operatorname{sgn} \delta_{VMi}$ is important since it will enable us to see if a method has a consistent tendency to over estimate or underestimate the true value.

5.5 Estimating the standard deviation of the estimated percentage signed delta and the relative error

The distribution of $\hat{\delta}_{VMi}$ values is determined by the variation coming from the simulation. We have to use $\hat{\delta}_{VMi}$ to make inferences about the true value δ_{VMi}. As a first approximation for doing this, we assume that $\hat{\delta}_{VMi}$ is approximately Gaussian with mean δ_{VMi} and a standard deviation which is to be estimated from the distribution of $\hat{a}_p$.

Given that the simulated values are used to estimate the quantiles and hence compute $\operatorname{sgn}\hat{\delta}_{VMi}$, we have (5.6) with $\hat{a}_p = x_{r(p)}$.

Using first order Taylor series approximation, this gives

$$\sigma(\operatorname{sgn}\hat{\delta}_{VMi}) \simeq \left|\frac{\partial P_{Mi}}{\partial \hat{a}_p}\right| \sigma(\hat{a}_p) = \left|\frac{\partial P_{Mi}}{\partial \hat{a}_p}\right| \sigma(x_{r(p)}) \tag{5.8}$$

Now from order statistics theory, we have that the first order Taylor series approximation to $\sigma^2(x_{r(p)})$ is given by,

$$\sigma^2(x_{r(p)}) \simeq \frac{p(1-p)}{m\left[f(a_p)\right]^2} \tag{5.9}$$

where 'f' is the density function of Q evaluated at the percentile value a_p and m is the number of repetitions in the simulation ($m = 10^6$). If we consider that

$$\left(\frac{\partial P_{Mi}}{\partial \hat{a}_p}\right)_{a_p} \simeq f(a_p) \tag{5.10}$$

we have that

$$\sigma^2(\operatorname{sgn}\hat{\delta}_{VMi}) \simeq \frac{p(1-p)}{m} \tag{5.11}$$

Note that assuming the formula (5.10) is supposing that P_{Mi} is an approximation to P_T such that the gradient of P_{Mi} is an approximation to the gradient of P_T. In interpreting the results of this study, the formula (5.11) will be used in judging whether the $\operatorname{sgn}\hat{\delta}_{VMi}$ of an approximation method is or is not significantly different from zero. In doing this however, allowance is also made for the fact that a large number of significance tests are being made in this study (see below). The quantiles used in this study correspond to the probability values P_T=0.10, 0.05, 0.02 and 0.01. For these values of P_T and the values of $\sigma(\text{sgn}\,\hat{\delta}_{VMi})$ and $\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})$ are shown in table 5.2, where

$$\sigma^2(\text{sgn}\,\hat{\delta}_{\%VMi}) = \frac{\sigma^2(\text{sgn}\,\hat{\delta}_{VMi})}{P_T}\cdot 100 \tag{5.12}$$

Table 5.2 Examples of relevant values of P_T, $\sigma(\text{sgn}\,\hat{\delta}_{VMi})$ and $\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})$.

P_T	$\sigma(\text{sgn}\,\hat{\delta}_{VMi})$	$\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})$
0.10	0.000300	0.300
0.05	0.000218	0.436
0.02	0.000140	0.700
0.01	0.000099	0.990

In applying these standard deviations to results, the reader will notice that since the values of $\sigma(\text{sgn}\,\hat{\delta}_{VMi})$ and $\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})$ are different for the different quantiles (a_p, b_p, c_p, d_p), the δ_{VMi} values corresponding to different quantiles, will have different precisions of confidence interval. In other words, we have better relative precision for P_T = 0.10 than we have for P_T = 0.01.

5.6 Choosing the critical value for multiple testing

In this project, 808 significance tests are being made. There are 14 matrix types, 4 quantiles, 3 values of *n*, 5 methods (ignoring the two naïve methods). Of the possible 840 tests, 32 of these were not made because method 6 did not converge for the 4 type 2 matrices when *n*=20 and *n*=25. In interpreting the results, it is necessary to take account of the high risk of false alarms by having a test threshold for each individual test which is designed to control the overall distribution of false alarms. In principal this means considering the joint distribution of 808 possible false alarms. This can be readily done if the 808 tests are independent. In this case however the tests are not independent and we need to discuss the structure of dependence in order to elaborate an approach to the control of false alarms. The simulations for different variance matrices are independent (14 matrix types $\times 3$ values of n). In each simulation, the 4 quantiles are estimated by order statistics. The correlations between the order statistics are not negligible and the formula for them is given in Kendell, Ord and Stuart (1987), [17], section 10.10.

For any given quantile, the δ_{VM} of each of the 5 methods are tested (in some cases there are only 4 values) using the same value of the quantile estimator. Hence these 5 (or 4) δ_{VM} have strong correlation. In each simulation, the correlations between 2 δ_{VM} -values based on different (but quite correlated) order statistics are (to first order) equal to the correlations between the corresponding order statistics (values around 0.50). The accurate study of the false alarm distribution for this situation is extremely difficult. There are two simple models which can be studied as a basis for "controlling" the false alarm distribution. Each of them considers the 808 tests as partitioned into subgroups where the δ_{VM} in the same subgroup have correlation equal to one and the δ_{VM} in different subgroups have zero correlation. The difference between these two models depends on how the four order statistics in each simulation are treated. In each case they are treated as having correlations equal to one and in the second case they are treated as having correlations equal to zero. Both of these assumptions are false but the situation is somewhere between the two.

Hence we look at 2 models which are
Model A: This model consists of 42 groups (14 matrix types $\times$ 3 values of n) and with 20 values in each group (5 methods $\times$ 4 quantiles).

Model B: This model consists of 168 groups (14 matrix types$\times$ 3 values of n $\times$ 4 quantiles) and with 5 values in each group.

In specifying these models, we are ignoring for simplicity the fact that 32 tests could not be carried out. Having some variation in the numbers in a group, would be a slight complication and would provide little extra information in what is anyway a crude approximate approach.

We discuss the approach in general by saying that we have n groups and m δ_{VM} ´s in each group. If each individual test has size α, the number of groups giving false alarms will be Binomial with parameters n and α. Once a single group gives a false alarm, the number of tests giving false alarm will be m tests, since in model A or model B, all tests in the same group alarm or do not alarm simultaneously (when the null hypothesis is true). The number of individual tests giving false alarms is therefore one of the $n+1$ values 0, m, $2m$,…, nm. We must now choose α so that with high probability, this distribution produces a limited impact of false alarms. Since m is either 5 or 20 we express this desire for limited impact by choosing α for each case so that the impact is small. The probability of less than r groups giving false is

$$\gamma_{global} = \sum_{j=0}^{r-1} \left(\frac{n!}{(n-j)!\,j!} \right) \cdot \alpha^{j} (1-\alpha)^{n-j} \tag{5.13}$$

Now α can be chosen so that this probability is large for some desired value of r. A number of calculations based on this have been carried out separately for model A and model B. These are illustrated below.

Model A: For model A we have n=42 and m=20. We can express our risk by asking that

$$P\left(x = 0 \middle| n, \alpha \right) = (1-\alpha)^{n} \geq 0.85 \tag{5.14}$$

Where x is the number of groups giving false alarms. Solving the equation (5.14) gives $\alpha = 0.003862$ and this corresponds to a threshold of 2.88σ in a two sided test of zero mean. Our expression of risk here represents a fairly high aversion to even one group false alarm (since m=20).

Model B: For model B we have n=168 and m=5. we can express our risk in this context by asking that

$$\gamma_{global} = \sum_{j=0}^{2} \left(\frac{n!}{(n-j)!\,j!} \right) \cdot \alpha^{j} (1-\alpha)^{n-j} \geq 0.85 \tag{5.15}$$

We solve equation (5.15) using a Poisson approximation $\left(\lambda = 168\alpha \right)$ and using table 9.3 of Owen (1962), [21]. We look for λ such that

$$P\left(x \leq 2 \middle| \lambda = 168\alpha \right) \geq 0.85 \tag{5.16}$$

This gives $\lambda \approx 1.2717$ and $\alpha \approx 0.0076$. The corresponding threshold for the two sided test is 2.67σ. Our expression of risk here represents a fairly high aversion to more than 2 group false alarms (since m=5).

In both cases a number of other choices of r and γ_{global} were computed to provide a sensitivity analysis of the resulting threshold in function of how aversion to false alarm risk was formalized. In both cases the results varied from 2.5σ to 3.0σ, depending on how r and γ_{global} were chosen.

Both of these choices for false alarm risk are expression of risk aversion in a situation where the modeling of the risk is extremely approximate. In addition even if the probability modeling of the risk were accurate (in terms of the correlations), it is difficult to ensure that the expression if utility for different values of m are really "equivalent".

This approach could be made more sophisticated by considering the effect which the α value has on testing power. This involves assessing the relative desirability of specific values of detection probability (power) for specific values of the approximation bias (of the methods). This has not been studied in any detail. However taking the question of power into account and taking account of the calculations of size illustrated here and other similar calculations which give similar orders of magnitude for the threshold, it was decided for descriptive purposes to label as "significant", those values of $\operatorname{sgn}\delta_{VM}$ where $|\delta| > 2.7\sigma$.

Hence in displaying results in tables, we will refer to a value $\hat{\delta}_{VMi}$ as significant if

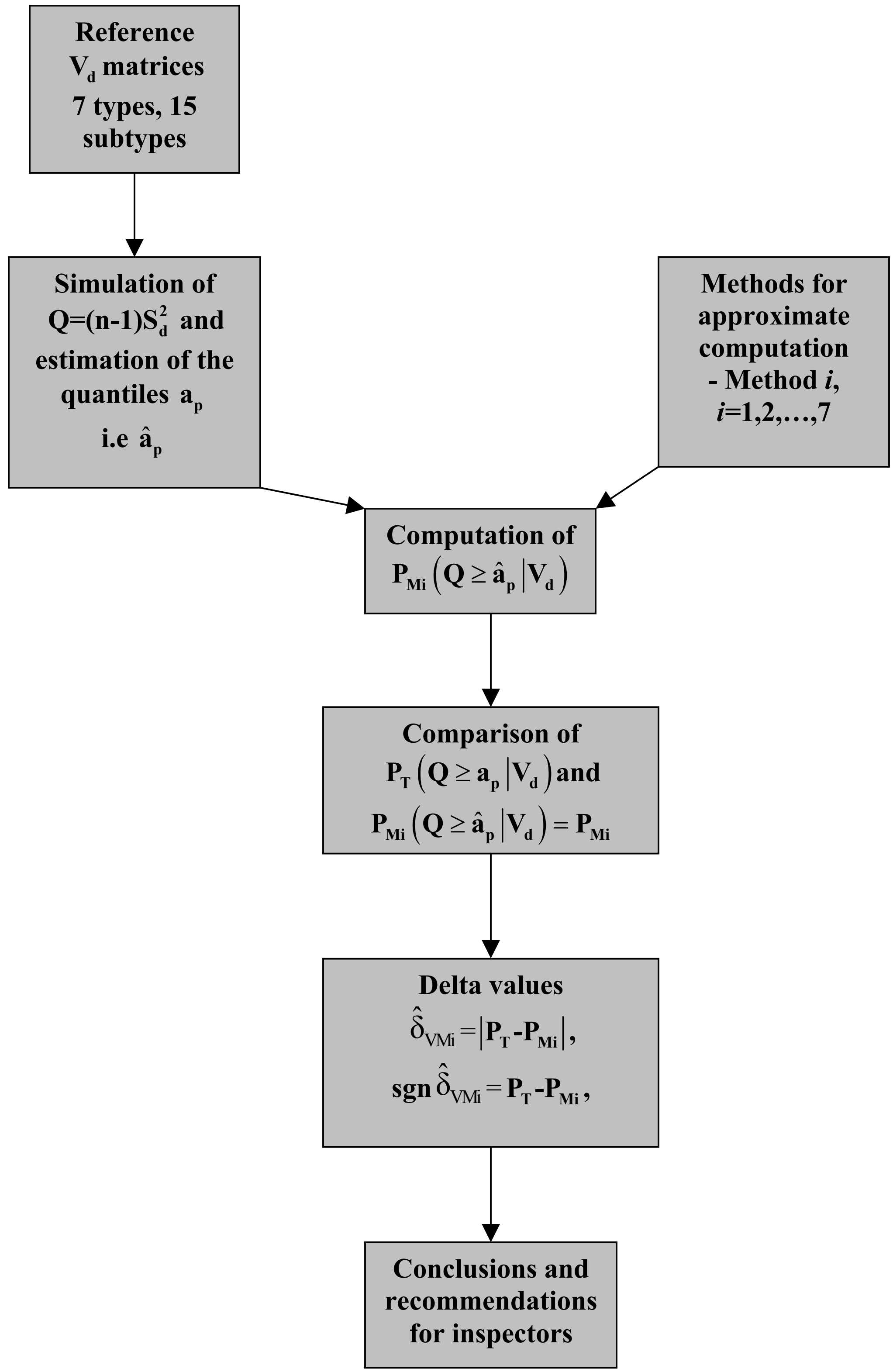
$$\operatorname{sgn}\hat{\delta}_{VMi} > 2.7 \cdot \sigma(\operatorname{sgn}\hat{\delta}_{VMi}) \tag{5.17}$$

Values not satisfying this condition might reasonably be considered as simulation noise.

5.7 Overview of the computations

Figure 5.1 gives an overview of the computations in the evaluation procedure ending in conclusions and recommendations for the inspectors.

Figure 5.1 Overview of the computations



Chapter 6. Results of comparing the approximation methods

6.1 Introduction

We would like to know which methods are good and how good they are for the different variance matrices. For the purpose of illustration we describe the obtained results in detail for one of the variance matrices, *viz.* variance matrix type 5a of size 20.

Consider Tables 6.1. The figures in this table are obtained as follows. We first estimate the quantiles $x = (a_p, b_p, c_p, d_p)$ by simulation of the distribution Q (chapter 5). These quantile-values are defined as the solutions of $P_T(Q \geq x|\mathbf{V}_d) = P$ where $P = 0.10, 0.05, 0.02$ and 0.01 (shaded rows). The approximated probability values for the methods, P_{Mi} (second column in the tables), are found as the solutions of $P_{M_i}\left(Q \geq \hat{x}|\mathbf{V}_d\right)$, for method i, $i = 1, 2, ..., 7$. Estimated delta values (5.3) were then obtained as the absolute differences between the "true probability values" and the approximated probability values (third columns in the tables). Estimated percentage delta values (5.4), (fourth columns), and then finally estimated percentage signed delta values (5.7), (fifth columns), are then calculated. Significant delta values, i.e. differences significantly different from zero at 1% level of false alarm risk (5.15).

Tables 6.1 reveal that in the case of variance matrix type 5a of size 20, methods 3 (Grad and Solomon), 4 (Imhof), 5 (Jensen and Solomon), 6 (Solomon and Stephens) and 7 (Field) perform extremely well. This is basically because the matrix of type 5a gives an easy approximation problem for any of the standard methods. Method 1 and especially method 2 are grossly inferior to the other methods. This to be expected because they are not appropriate methods as mentioned earlier.

Tables 6.1 Delta value at the considered quantile values for variance matrix of type 5a of size 20. Significant delta values at $2.7 \cdot \sigma$ level are given by bold face figures.

True tail area $P_T = 10\%$ **Quantile value** $\hat{a}_p = 26.3064$				
Method	Method tail area P_{Mi}	$\hat{\delta}_{VMi}$	$\hat{\delta}_{\%VMi}$	sgn $\hat{\delta}_{\%VMi}$
1	0.0994	0.0006	0.6	0.6
2	0.1219	**0.0219**	**21.9**	**-21.9**
3	0.0998	0.0002	0.2	0.2
4	0.1001	0.0001	0.1	-0.1
5	0.0999	0.0001	0.1	0.1
6	0.1001	0.0001	0.1	-0.1
7	0.1001	0.0001	0.1	-0.1

True tail area $P_T = 5\%$ **Quantile value** $\hat{b}_p = 29.1956$				
Method	Method tail area P_{Mi}	$\hat{\delta}_{VMi}$	$\hat{\delta}_{\%VMi}$	sgn $\hat{\delta}_{\%VMi}$
1	0.0490	**0.0010**	**2.0**	**2.0**
2	0.0630	**0.0130**	**26**	**-26**
3	0.0495	0.0005	1.0	1.0
4	0.0497	0.0003	0.6	0.6
5	0.0496	0.0004	0.8	0.8
6	0.0497	0.0003	0.6	0.6
7	0.0497	0.0003	0.6	0.6

True tail area $P_T = 2\%$ **Quantile value** $\hat{c}_p = 32.6458$				
Method	Method tail area P_{Mi}	$\hat{\delta}_{VMi}$	$\hat{\delta}_{\%VMi}$	sgn $\hat{\delta}_{\%VMi}$
1	0.0194	**0.0006**	**3.0**	**3.0**
2	0.0264	**0.0064**	**32**	**-32**
3	0.0198	0.0002	1.0	1.0
4	0.0199	0.0001	0.5	0.5
5	0.0199	0.0001	0.5	0.5
6	0.0199	0.0001	0.5	0.5
7	0.0199	0.0001	0.5	0.5

True tail area $P_T = 1\%$ **Quantile value** $\hat{d}_p = 35.0872$				
Method	Method tail area P_{Mi}	$\hat{\delta}_{VMi}$	$\hat{\delta}_{\%VMi}$	sgn $\hat{\delta}_{\%VMi}$
1	0.0096	**0.0004**	**4.0**	**4.0**
2	0.0136	**0.0036**	**36.0**	**-36.0**
3	0.0099	0.0001	1.0	1.0
4	0.0099	0.0001	1.0	1.0
5	0.0100	0	0.0	0.0
6	0.0099	0.0001	1.0	1.0
7	0.0099	0.0001	1.0	1.0

6.2 Overview of the results

We now go on to describe and comment on the results for all the variance matrix types. An overview of the results is given in the form of histograms presented in figure 6.1. Each histogram gives the distribution of the estimated percentage signed delta values for all 180 cases: 15 (variance matrices) × 3 (dimensions of variance matrices) × 4 (quantile levels). The X-axis shows the estimated percentage signed delta value. The Y-axis shows the frequency of considered quantity.

To be able to compare all methods and for graphical clearness, values of estimated percentage signed delta values greater than 50 are put equal to 50 and values less than -50 are put equal to -50. This gives a slight distortion of the histogram but gives a series of visually comparable histograms. Figure 6.1 shows that methods 4, 6 and 7 perform very well.

Inappropriate methods. The histograms show how really inferior the inappropriate crude methods 1 and 2 are. The results from these methods are in fact even "worse" than shown in the histograms (both methods have many results which are beyond the histogram limits of -50 and 50). Method 1 gives many estimates of percentage signed delta larger than 50 and method 2 gives a large number of estimates less than -50.

The worst overall approximation method is the second method, which uses a chi-square distribution with the degrees of freedom equal to the number of items in the sample minus one. This method is only considering the variance values of the variance matrix.

Caveat. Although method 6 (2^{nd} three moment approximation, Solomon and Stephens) provides excellent results when it works, the method was not applicable to all variance matrices, *i.e.* type 2 variance matrix for the dimensions 20 and 25. In these cases the algorithm for generating the estimates for this method did not converge. The histogram for method 6 is based on 32 fewer values than for the other histograms. 32 values = 4 (subtypes of type 2 variance matrix) × 4 (quantile values) × 2 (variance matrix dimensions 20 and 25).

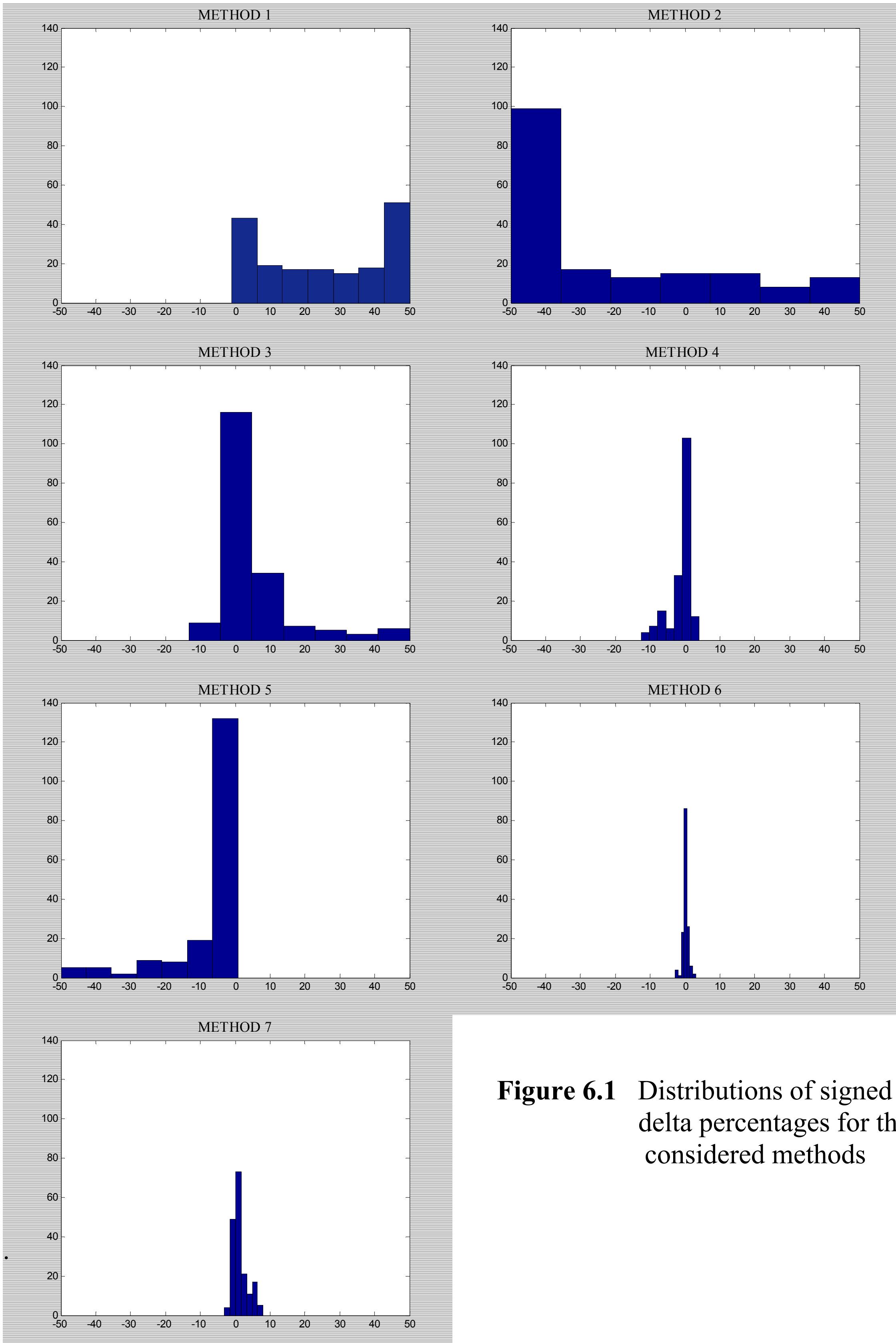


Figure 6.1 Distributions of signed delta percentages for the considered methods

A closer inspection of figure 6.1 and the results in appendix C may reveal whether an approximating method has a tendency to overestimate or underestimate the true value. It is important to bear in mind how we define $\operatorname{sgn}\hat{\delta}_{\%VMi}$ in (5.7). A positive estimated $\operatorname{sgn}\hat{\delta}_{\%VMi}$-value means in fact an underestimated true value and *vice versa*.

We find that the methods 1, 3 (two moment approximation, Grad and Solomon) and 7 (Field) in general underestimate the true value. Method 1 has 95% of underestimated results. This is worth mentioning because underestimation means a high rate of false alarms. The naïve method 1 would produce many false alarms and if the explanation were not understood, the auditing would be completely discredited. Method 3 has 77% of underestimated results and method 7 has 73% of underestimated results.

Methods 2 and 5 (Gaussian approximation, Jensen and Solomon) tend to overestimate the true value. Method 2 has 76% of overestimated results and method 5 has 81% of overestimated results.

6.3 Percentage delta values

By averaging the percentage delta values for each method and for each subtype of variance matrix, we may get another overview of how the different methods perform in comparison to each other, see Table 6.2. The table shows that method 6 (Solomon and Stephens) gives the best results, followed by methods 4 (Imhof) and 7 (Field). Methods 3 and 5 perform less well and 3 is inferior to 5. As already mentioned methods 1 and 2 perform worst

By looking at the three tables in appendix C we further see that that the worst method 2 performs unsatisfactory in all instances. In some cases of variance matrices of type 2 (particularly subtype 2c and 2d) and of type 4 the first crude method gives even poorer results than method 2 but the values of the last mentioned method are not satisfactory either. In the further discussion in this report we will leave out methods 1 and 2 since they have shown too poor results.

Table 6.2 Average percentage delta values by method and type of variance matrix. Missing values are denoted by stars.

Variance matrix	Size of variance matrix	Method 1	Method 2	Method 3	Method 4	Method 5	Method 6	Method 7
Type 1	10	0.18	472.03	0.55	0.18	0.55	0.18	0.00
	20	0.30	897.50	0.53	0.30	0.53	0.30	0.30
	25	0.45	989.80	0.70	0.45	0.70	0.45	0.45
Type 2	10	53.09	24.86	9.30	3.35	11.85	0.89	5.02
	20	71.70	76.36	18.30	5.96	23.45	***	4.74
	25	77.81	47.16	22.56	6.86	29.29	***	2.76
Type 3	10	19.02	353.40	2.32	0.41	1.89	0.20	0.46
	20	20.87	636.15	2.88	0.62	1.12	0.38	0.58
	25	22.53	759.9	3.33	0.78	0.82	0.67	0.98
Type 4	10	24.72	14.78	3.89	1.25	4.18	0.70	1.70
	20	25.76	11.83	4.29	1.17	0.65	0.98	1.52
	25	22.46	14.98	3.11	0.61	1.13	0.33	0.44
Type 5	10	1.30	26.34	0.76	0.34	0.46	0.30	0.31
	20	3.15	38.75	0.64	0.23	0.34	0.36	0.38
	25	2.95	43.01	0.33	0.55	0.53	0.54	0.50
Type 6&7	10	30.06	92.50	5.16	1.28	3.08	0.81	2.71
	20	34.64	148.68	5.00	0.85	1.74	0.56	1.14
	25	35.54	180.89	4.64	0.76	1.44	0.46	0.92

Looking at the column for method 6 we make again the observation that although method 6 performs very well when it works, it did not converge at all for type 2 matrices of dimension 20 and 25.

6.4 Impact of size of variance matrix

In this section we will further the discussion we begun in section 6.2. We will split up the general results into the three different matrix dimensions and see what impact the dimension of the variance matrix have on the results. We have for each variance matrix dimension (10, 20 and 25) produced a box plot, figures 6.2-6.4, showing the distributions of the estimated percentage signed delta values. For each of five methods 3 till 7 we have 15 (variance matrices) $\times$ 4 (quantile levels) = 60 values of estimated percentage signed delta (for each variance matrix dimension). In each plot, the lower and the upper lines of the box are the 25th and 75th percentile of the sample. The line in the middle of the box is the sample median. If the median is not centered in the box, this indicates a skewness of the distribution. The lines that are extending above and below the box, called 'whiskers', show the range of the rest of the sample (if there are no outliers). If there are no outliers then the maximum of the sample is the top of the upper whisker and the minimum of the sample is the bottom of the lower whisker. An outlier is presented as a plus sign at the top or the bottom of the plot and is defined as a value that is more then 1.5 times the interquartile range away from the the top or the bottom of the box. The interquartile range is the distance between the top and the bottom of the box.

By looking at the figures 6.2-6.4 we notice similar shapes of the distributions of the methods for the different variance matrix dimensions. The first striking difference between the different variance matrix dimensions is the spread of the outliers of method 3 (Grad and Solomon), of method 5 (Jensen and Solomon) and to some extent of method 4 (Imhof). For these methods, the higher dimension of variance matrix the larger spread of outliers. It is important to observe that the results for different variance matrix dimensions have different scales in the box plots. These outliers, and also the outliers from method 6 (Solomon and Stephens) and 7 (Field), come in most cases from the type 2 variance matrix types. For all methods, this kind of variance matrix (type 2) seems to present the greatest challenge and as we have already seen method 6 failed to converge for many type 2 variance matrices. So in figures 6.3 and 6.4 method 6 is without the results from type 2 matrices.

Judging in terms of the values in the interquartile range, we can say that for methods 4 and 7 the bigger variance matrix dimension the better the results gets. For method 5 we have a contrary effect, the bigger the variance matrix the less good the results.

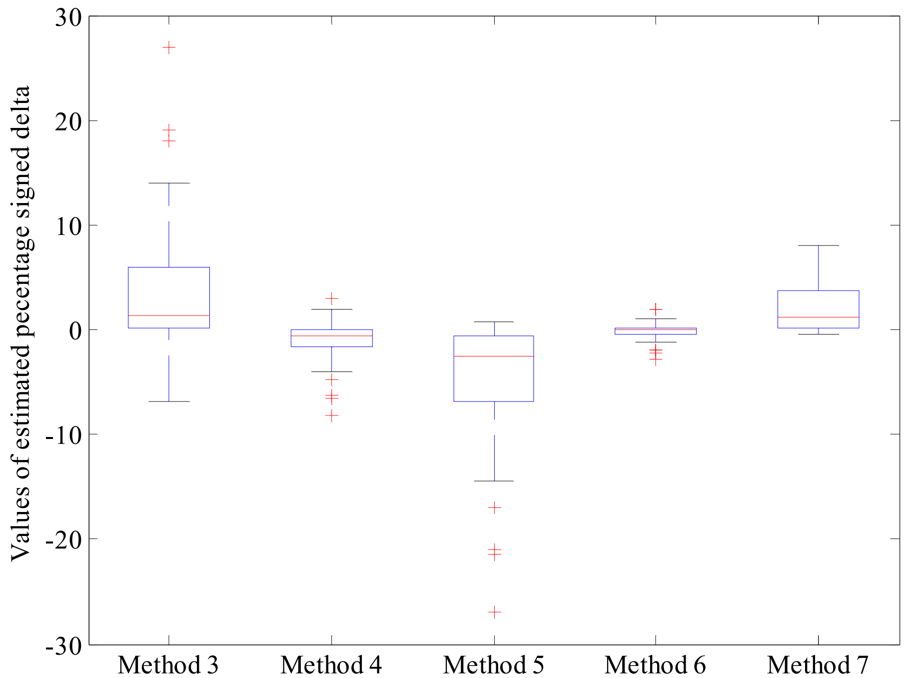


Figure 6.2 Box plot for variance matrices of size 10

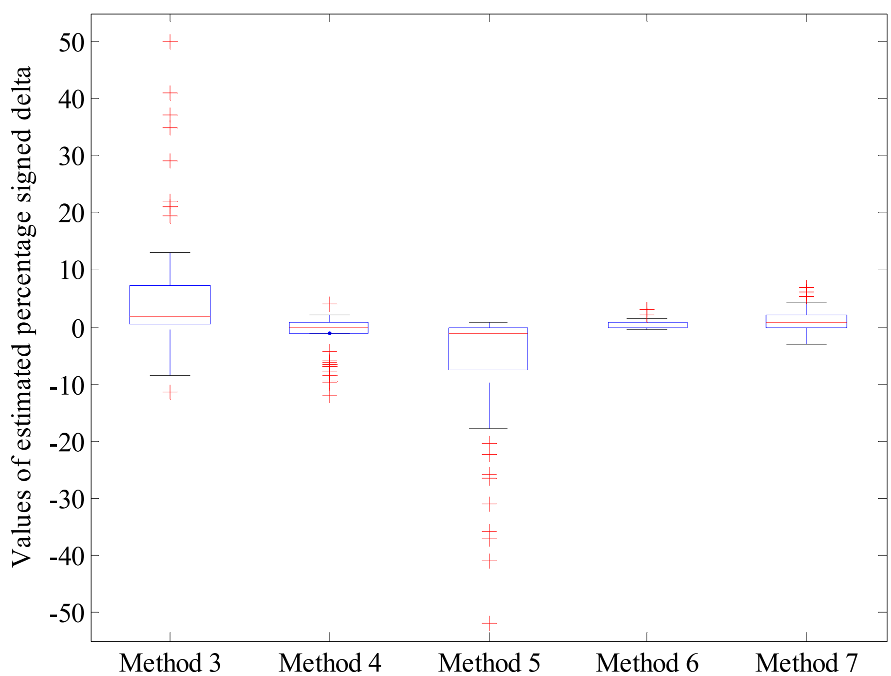


Figure 6.3 Box plot for variance matrices of size 20

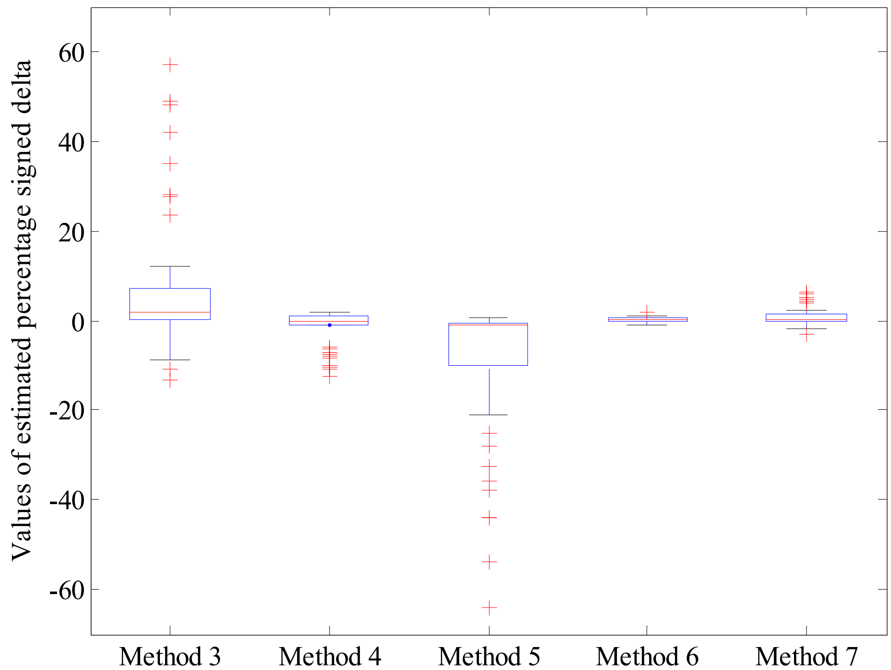


Figure 6.4 Box plot for variance matrices of size 25

6.5 Different performances at different quantiles

It is interesting to break the results down further in terms of the different quantile probability values and see whether the performances are influenced by the quantile probabilities.

Appendix D contains the box plots of the distributions of estimated percentage signed delta values of the methods at the quantile probabilities 90%, 95%, 98% and 99% for each variance matrix dimension. In total there are 12 box plots, four for each variance matrix dimension.

We may see in appendix D some general features valid for the three variance matrix dimensions. Method 3 (two moment approximation, Grad and Solomon) tends to overestimate the true value for the 90% quantile probability. This is not evident when the quantile probabilities are combined (figures 6.2, 6.3 and 6.4). For the 95% quantile probability, method 3 seem to estimate really well, albeit with a consistent slight underestimation. For the 98% and even more for the 99% quantile probability there seem to be an underestimation of the true value.

Roughly speaking, method 4 and 7 are approximating better the bigger the quantile gets. Method 5 on the other hand has the opposite effect, the bigger the quantile gets the bigger the spread of the distribution. Finally, method 6 performs equally well for all quantile probabilities. This method always seems to approximate very well when it works.

6.6 Significant delta values

In previous chapter, we introduced the concept of significant delta values. A delta value is considered significant if it has modulus greater than 2.7 times its standard deviation. This is equivalent to testing each individual delta for a discrepancy between the method value and the true value at 1% level of false alarm risk (5.15). Table 6.3 gives the number and percentage of significant delta values for each method by variance matrix dimension.

Table 6.3 Number and percentage of significant delta-values defined by (5.15) by method and by variance matrix dimension.

Size of variance matrix	Method 1	Method 2	Method 3	Method 4	Method 5	Method 6	Method 7
10	50 (83%)	57 (95%)	34 (57%)	21 (35%)	39 (65%)	4 (7%)	29 (48%)
20	55 (92%)	60 (100%)	36 (60%)	18 (30%)	24 (40%)	4 (7%)	18 (30%)
25	55 (92%)	60 (100%)	37 (62%)	13 (22%)	25 (42%)	0 (0%)	11 (18%)

The maximum number of significances is 60 for each method and variance matrix dimension, since we have 15 variance matrices with 4 quantile values in each for methods 3, 4, 5 and 7. For method 6 the maximum number of significances is 44 for variance matrix dimensions 20 and 25, since 16 values are missing and 60 for variance matrix dimension 10.

Table 6.3 shows that the method 6 gives almost no significant values at all in the cases when it worked. Methods 4 and 7 gave significant values in twenty to fifty percent of the cases. Methods 3 and 5 gave between forty to seventy percent significant delta-values. For the methods 1 and 2 almost all delta-values are significant. This is just another illustration of the fact that methods 1 and 2 are inappropriate.

6.7 Detailed numerical results

A great part of the detailed numerical results may be found in appendices B and C. These appendices contain tables of the following:

1. Quantiles for each variance matrix $\mathbf{V}_d$ for each value of P_T *i.e.* $(\hat{a}_p, \hat{b}_p, \hat{c}_p, \hat{d}_p)$

2. P_{Mi} *i.e.* $P_{M_i}\left(Q \geq \hat{a}_p \middle| \mathbf{V}_d\right)$ for each quantile, which are the computed approximate probability values of the seven approximating methods

3. Estimated delta for each quantile *i.e.* $\hat{\delta}_{VMi} = \left|P_T - P_{Mi}\left(Q \geq \hat{a}_p \middle| \mathbf{V}_d\right)\right|$

4. Estimated percentage delta for each quantile *i.e.* $\hat{\delta}_{\%VMi} = (\hat{\delta}_{VMi} \cdot 100) / P_T$

5. Estimated percentage signed deltas for each quantile *i.e.* $\operatorname{sgn}\hat{\delta}_{\%VMi} = (\operatorname{sgn}\hat{\delta}_{VMi} \cdot 100) / P_T$

6. Significance information for each quantile *i.e.* $\operatorname{sgn}\hat{\delta}_{VMi} / \sigma\left(\operatorname{sgn}\hat{\delta}_{VMi}\right)$

These results are given for the three dimensions n = 10, n = 20 and n = 25 of the 15 different variance matrices.

6.8 Difficulties with type 2 variance matrices

We have earlier in this chapter (section 6.4) mentioned that the type 2 variance matrices give maximum difficulty to all approximation methods. See table 6.2 for illustration. All the type 2 variance matrices have a spectrum of a zero eigenvalue with multiplicity one plus three (or in case a, only two) eigen values of which all but one have large multiplicity. See table 6.4 for eigenvalues and their multiplicities. Note that in each table we show three different matrices having different spectra but the spectra have many values in common.

Tables 6.4 Eigen values of type 2 variance matrices. Multiplicities are below the eigen values. '-' is used to denote that the value does not occur for this matrix.

Type 2 case a: Eigen values and their multiplicities					
Eigen value	**0**	**0.7**	**2.2**	**3.7**	**4.444**
Dimension 10	1	8	1	-	-
Dimension 20	1	18	-	1	-
Dimension 25	1	23	-	-	1

Type 2 case b: Eigen values and their multiplicities						
Eigen value	**0**	**0.225**	**0.7**	**1.275**	**2.0875**	**2.5**
Dimension 10	1	4	4	1	-	-
Dimension 20	1	9	9	-	1	-
Dimension 25	1	12	11	-	-	1

Type 2 case c: Eigen values and their multiplicities						
Eigen value	**0**	**0.175**	**0.7**	**1.375**	**2.3125**	**2.788**
Dimension 10	1	4	4	1	-	-
Dimension 20	1	9	9	-	1	-
Dimension 25	1	12	11	-	-	1

Type 2 case d: Eigen values and their multiplicities						
Eigen value	**0**	**0.125**	**0.7**	**0.475**	**2.5375**	**3.076**
Dimension 10	1	4	4	1	-	-
Dimension 20	1	9	9	-	1	-
Dimension 25	1	12	11	-	-	1

We may write (2.25), $Q = \sum_{i=1}^{n} \lambda_i Z_i^2$ (lambda = eigenvalue of matrix **C**) for variance matrix type 2 case a as

$$Q = \lambda_1 Z_1^2 + \lambda_2 \sum_{i=2}^{r} Z_i^2 + \lambda_3 Z_s^2 = [\text{where } \lambda_1 \text{ is } 0] = \lambda_2 \sum_{i=1}^{r} Z_i^2 + \lambda_3 Z_s^2$$

(r=9, 19, 24 and s=10, 20, 25)

and for the other three cases, b, c and d, as

$$Q = \lambda_1 Z_1^2 + \lambda_2 \sum_{i=2}^{p+1} Z_i^2 + \lambda_3 \sum_{j=p+2}^{p+2+q} Z_j^2 + \lambda_4 Z_t^2 = \text{[where } \lambda_1 \text{ is 0]} = \lambda_2 \sum_{i=1}^{p+1} Z_i^2 + \lambda_3 \sum_{j=p+2}^{p+2+q} Z_j^2 + \lambda_4 Z_t^2$$

(p=4, 9, 12, q=4, 9, 11 and t= 10, 20, 25)

We see that Q can thus be written as a sum of three (or two) central chi-square variables (with different degrees of freedom) each multiplied by a very different eigenvalue. That the eigenvalues of type 2 variance matrices are so different in magnitude and so few in spectra is the reason why all the methods are having such difficulties in approximating it.

All methods apart from the second crude method gave satisfactory results (*i.e.* not significantly different from zero) for type 1 matrices. Q is here a central chi-square and all methods (apart from method 2) are well able to approximate a central Chi-square variable. All other matrices (types 3 – 7) are cases in which all eigen values have multiplicity one and are not clustered.

6.9 Difficulties with method 6

We have seen that when method 6 works, it provides excellent results. But in the cases of type 2 variance matrices of the dimensions 20 and 25 the algorithm for generating the estimates for this method did not converge (section 6.2). The argument of the gamma function is too large in these cases to compute it directly and the second method for implementing Newton-Raphson's method is used. But the algorithms are not converging in these cases and they are also giving complex numbers (negative valued natural logarithms) for the two parameters, r and v (4.39) in the iteration process. Maybe our chosen initial values for the iterating process can be chosen differently but we have not had time for deeper investigation of this. We would therefore recommend that a closer study of the numerical procedure used for method 6 is made so that accurate estimates may be obtained in all cases.

Chapter 7. Conclusions and recommendations for the inspectors

7.1 Conclusions

The objective of the study is to have a reliable computational method to be applied for testing sets of verification measurements to see if the magnitudes of the differences are compatible with the hypothesis that the variance matrix $\mathbf{V}_d$ is the true variance matrix. The particular form of the objective is to test $\bar{d}$ and S_d^2 using the variance matrix $\mathbf{V}_d$ as the null hypothesis distribution since these two separate tests represent testing the estimator of mean square error of the total accounting discrepancy. This thesis is focused on one of the tests, the one of computing upper tail probabilities for the null hypothesis distribution of the test statistic $Q=(n-1)S_d^2$ for different known variance matrices $\mathbf{V}_d$ when $\mathbf{d} \sim N(\mathbf{0},\mathbf{V}_d)$. We have used five methods for approximating this test statistic and also included two inappropriate methods in the study.

When judging whether an approximating method is sufficiently good we must take into account that for auditing applications we do not need meticulous precision in computing the upper tail probabilities. A relative error of 5% is perfectly acceptable when computing a significance level somewhere between 0.10 and 0.01. The significance level is only intended to be an alarm leading to an investigation which may or may not confirm that there is an anomaly. Having an unknown error of 5% in the target false alarm rate is not a serious difficulty. For academic studies of these methods (*e.g.* Imhof, Field) on the other hand, the authors have been interested in great precision of the extreme tails. Consequently in auditing applications there is another object with the results than with the studied articles.

Since this study provides the error values only for a specific set of variance matrices, caution is needed in conjecturing that one method may be globally superior to another. We can say that for the cases studied one method is superior to another, but it purely is a conjecture that this pattern of performance would be maintained if further variance matrices were considered. Even when we say that one method is better than another for one case which is studied, the conclusion remains at the level of a statistical inference since the computed delta values are estimators involving simulation variation throughout this study when giving conclusions about performances, these conclusions have taken into account the inference questions involved in any comparisons (see chapter 5).

One may draw the following conclusions based on the findings reported in previous chapter:

- Method 6 is the overall best method when applicable. However there is no a priori way of knowing when it will be applicable. It has to be tried and in some cases the algorithm for generating the estimates for this method may not converge. If the algorithm converges the result is good.

- Methods 4 and 7 are second best methods.

- Methods 3 and 5 are sometimes working very well and sometimes not at all.

- Methods 1 and 2 are inferior to the other methods.

One can also make the observation that it is easy to perform the approximation computations for all the considered methods using the numerical procedures available in the Matlab program package provided with its Statistical Toolbox.

7.2 Recommendations for the inspectors

Based on the findings it is recommended that the inspectors in the field perform estimations for method 6. It is further recommended that estimations are performed for methods 4 and 7 in those cases when method 6 does not work. If these methods give the same result the inspector may be confident in the assessments made also in these cases.

To achieve this computer program package may developed and used by the inspectors. This program package may be based on the programs used in the present work.

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Appendices

Appendix A

Reference variance matrices

This appendix contains the numerical values of the benchmark of the variance matrices used in the calculations. That is 15 variance matrices from each variance dimension 10, 20 and 25.

The lambda vector that follows every matrix type is the eigenvalue vector of matrix **C**, (2.22). These eigenvalues are the known weights when we write the test statistic (2.25) as a linear combination of chi-squared variables with these known weights.

Reference variance matrices of dimension 10

V matrix 1

V =

[1.0	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
0.3	1.0	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
0.3	0.3	1.0	0.3	0.3	0.3	0.3	0.3	0.3	0.3
0.3	0.3	0.3	1.0	0.3	0.3	0.3	0.3	0.3	0.3
0.3	0.3	0.3	0.3	1.0	0.3	0.3	0.3	0.3	0.3
0.3	0.3	0.3	0.3	0.3	1.0	0.3	0.3	0.3	0.3
0.3	0.3	0.3	0.3	0.3	0.3	1.0	0.3	0.3	0.3
0.3	0.3	0.3	0.3	0.3	0.3	0.3	1.0	0.3	0.3
0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	1.0	0.3
0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	1.0]

lambda =
[0.7 0.0 0.7 0.7 0.7 0.7 0.7 0.7 0.7 0.7]

V matrix 2 case a

V =

[1.0	0.3	0.3	0.3	0.3	0.0	0.0	0.0	0.0	0.0
0.3	1.0	0.3	0.3	0.3	0.0	0.0	0.0	0.0	0.0
0.3	0.3	1.0	0.3	0.3	0.0	0.0	0.0	0.0	0.0
0.3	0.3	0.3	1.0	0.3	0.0	0.0	0.0	0.0	0.0
0.3	0.3	0.3	0.3	1.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	1.0	0.3	0.3	0.3	0.3
0.0	0.0	0.0	0.0	0.0	0.3	1.0	0.3	0.3	0.3
0.0	0.0	0.0	0.0	0.0	0.3	0.3	1.0	0.3	0.3
0.0	0.0	0.0	0.0	0.0	0.3	0.3	0.3	1.0	0.3
0.0	0.0	0.0	0.0	0.0	0.3	0.3	0.3	0.3	1.0]

lambda =
[0.7 0.0 2.2 0.7 0.7 0.7 0.7 0.7 0.7 0.7]

V matrix 2 case b

V=

[1.000	0.300	0.300	0.300	0.300	0.000	0.000	0.000	0.000	0.000
0.300	1.000	0.300	0.300	0.300	0.000	0.000	0.000	0.000	0.000

0.300 0.300 1.000 0.300 0.300 0.000 0.000 0.000 0.000 0.000
0.300 0.300 0.300 1.000 0.300 0.000 0.000 0.000 0.000 0.000
0.300 0.300 0.300 0.300 1.000 0.000 0.000 0.000 0.000 0.000
0.000 0.000 0.000 0.000 0.000 0.250 0.025 0.025 0.025 0.025
0.000 0.000 0.000 0.000 0.000 0.025 0.250 0.025 0.025 0.025
0.000 0.000 0.000 0.000 0.000 0.025 0.025 0.250 0.025 0.025
0.000 0.000 0.000 0.000 0.000 0.025 0.025 0.025 0.250 0.025
0.000 0.000 0.000 0.000 0.000 0.025 0.025 0.025 0.025 0.250]

lambda =
[0.225 1.275 0.000 0.225 0.225 0.225 0.700 0.700 0.700 0.700]

V matrix 2 case c

V =
[1.000 0.300 0.300 0.300 0.300 0.000 0.000 0.000 0.000 0.000
0.300 1.000 0.300 0.300 0.300 0.000 0.000 0.000 0.000 0.000
0.300 0.300 1.000 0.300 0.300 0.000 0.000 0.000 0.000 0.000
0.300 0.300 0.300 1.000 0.300 0.000 0.000 0.000 0.000 0.000
0.300 0.300 0.300 0.300 1.000 0.000 0.000 0.000 0.000 0.000
0.000 0.000 0.000 0.000 0.000 0.250 0.075 0.075 0.075 0.075
0.000 0.000 0.000 0.000 0.000 0.075 0.250 0.075 0.075 0.075
0.000 0.000 0.000 0.000 0.000 0.075 0.075 0.250 0.075 0.075
0.000 0.000 0.000 0.000 0.000 0.075 0.075 0.075 0.250 0.075
0.000 0.000 0.000 0.000 0.000 0.075 0.075 0.075 0.075 0.250]

lambda =
[0.175 1.375 0.000 0.700 0.175 0.175 0.175 0.700 0.700 0.700]

V matrix 2 case d

V =
[1.000 0.300 0.300 0.300 0.300 0.000 0.000 0.000 0.000 0.000
0.300 1.000 0.300 0.300 0.300 0.000 0.000 0.000 0.000 0.000
0.300 0.300 1.000 0.300 0.300 0.000 0.000 0.000 0.000 0.000
0.300 0.300 0.300 1.000 0.300 0.000 0.000 0.000 0.000 0.000
0.300 0.300 0.300 0.300 1.000 0.000 0.000 0.000 0.000 0.000
0.000 0.000 0.000 0.000 0.000 0.250 0.125 0.125 0.125 0.125
0.000 0.000 0.000 0.000 0.000 0.125 0.250 0.125 0.125 0.125
0.000 0.000 0.000 0.000 0.000 0.125 0.125 0.250 0.125 0.125
0.000 0.000 0.000 0.000 0.000 0.125 0.125 0.125 0.250 0.125
0.000 0.000 0.000 0.000 0.000 0.125 0.125 0.125 0.125 0.250]

lambda =
[0.125 0.475 0.000 0.125 0.125 0.125 0.700 0.700 0.700 0.700]

V matrix 3 case a

V =
[1.0000 0.2932 0.2849 0.2660 0.2412 0.2199 0.2144 0.1665 0.1025 0.0949
0.2932 0.9551 0.2785 0.2599 0.2357 0.2149 0.2095 0.1627 0.1001 0.0927
0.2849 0.2785 0.9022 0.2526 0.2291 0.2089 0.2037 0.1581 0.0973 0.0901
0.2660 0.2599 0.2526 0.7859 0.2138 0.1950 0.1901 0.1476 0.0908 0.0841
0.2412 0.2357 0.2291 0.2138 0.6462 0.1768 0.1724 0.1338 0.0824 0.0763
0.2199 0.2149 0.2089 0.1950 0.1768 0.5374 0.1572 0.1221 0.0751 0.0695
0.2144 0.2095 0.2037 0.1901 0.1724 0.1572 0.5108 0.1190 0.0732 0.0678
0.1665 0.1627 0.1581 0.1476 0.1338 0.1221 0.1190 0.3080 0.0569 0.0527
0.1025 0.1001 0.0973 0.0908 0.0824 0.0751 0.0732 0.0569 0.1167 0.0324
0.0949 0.0927 0.0901 0.0841 0.0763 0.0695 0.0678 0.0527 0.0324 0.1000]

lambda =
[0.0000 0.0759 0.1749 0.6942 0.6600 0.6173 0.5332 0.3043 0.4351 0.3678]

V matrix 3 case b

V =
[1.0000 0.2911 0.2792 0.2756 0.2689 0.2531 0.2292 0.2233 0.1855 0.1500
0.2911 0.9414 0.2709 0.2674 0.2609 0.2455 0.2223 0.2167 0.1799 0.1455
0.2792 0.2709 0.8661 0.2565 0.2503 0.2355 0.2133 0.2079 0.1726 0.1396
0.2756 0.2674 0.2565 0.8440 0.2471 0.2325 0.2105 0.2052 0.1704 0.1378
0.2689 0.2609 0.2503 0.2471 0.8037 0.2269 0.2054 0.2002 0.1663 0.1345
0.2531 0.2455 0.2355 0.2325 0.2269 0.7116 0.1933 0.1884 0.1565 0.1265
0.2292 0.2223 0.2133 0.2105 0.2054 0.1933 0.5835 0.1706 0.1417 0.1146
0.2233 0.2167 0.2079 0.2052 0.2002 0.1884 0.1706 0.5543 0.1381 0.1117
0.1855 0.1799 0.1726 0.1704 0.1663 0.1565 0.1417 0.1381 0.3822 0.0927
0.1500 0.1455 0.1396 0.1378 0.1345 0.1265 0.1146 0.1117 0.0927 0.2500]

lambda =
[0.0000 0.2131 0.3176 0.3983 0.4641 0.6910 0.6473 0.5322 0.5770 0.6001]

V matrix 3 case c

V =
[1.0000 0.2961 0.2950 0.2935 0.2886 0.2622 0.2583 0.2368 0.2331 0.2324
0.2961 0.9742 0.2911 0.2897 0.2848 0.2588 0.2549 0.2337 0.2301 0.2294
0.2950 0.2911 0.9668 0.2886 0.2837 0.2578 0.2539 0.2329 0.2292 0.2285
0.2935 0.2897 0.2886 0.9575 0.2824 0.2566 0.2527 0.2317 0.2281 0.2274
0.2886 0.2848 0.2837 0.2824 0.9253 0.2523 0.2484 0.2278 0.2243 0.2235
0.2622 0.2588 0.2578 0.2566 0.2523 0.7641 0.2258 0.2070 0.2038 0.2031
0.2583 0.2549 0.2539 0.2527 0.2484 0.2258 0.7411 0.2039 0.2007 0.2001
0.2368 0.2337 0.2329 0.2317 0.2278 0.2070 0.2039 0.6232 0.1840 0.1834
0.2331 0.2301 0.2292 0.2281 0.2243 0.2038 0.2007 0.1840 0.6039 0.1806
0.2324 0.2294 0.2285 0.2274 0.2235 0.2031 0.2001 0.1834 0.1806 0.6000]

lambda =
[0.0000 0.4213 0.4309 0.4822 0.5273 0.5963 0.6556 0.6957 0.6731 0.6798]

V matrix 4 case a

V =
[1.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.0000 0.8101 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.0000 0.0000 0.7028 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.0000 0.0000 0.0000 0.4541 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.0000 0.0000 0.0000 0.0000 0.4021 0.0000 0.0000 0.0000 0.0000 0.0000
0.0000 0.0000 0.0000 0.0000 0.0000 0.3991 0.0000 0.0000 0.0000 0.0000
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.3990 0.0000 0.0000 0.0000
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.3542 0.0000 0.0000
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.2615 0.0000
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.2500]

lambda =
[0.9560 0.0000 0.7733 0.6448 0.2551 0.2923 0.4423 0.3656 0.4011 0.3991]

V matrix 4 case b

V =
[1.0000 0.0013 0.0449 0.0231 0.0467 0.0273 0.0379 0.0225 0.0135 0.0066
0.0013 0.9489 0.0194 0.0321 0.0083 0.0368 0.0197 0.0440 0.0369 0.0202
0.0449 0.0194 0.8847 0.0369 0.0432 0.0337 0.0172 0.0143 0.0142 0.0176
0.0231 0.0321 0.0369 0.7541 0.0181 0.0331 0.0254 0.0203 0.0268 0.0189
0.0467 0.0083 0.0432 0.0181 0.6439 0.0426 0.0116 0.0106 0.0312 0.0207
0.0273 0.0368 0.0337 0.0331 0.0426 0.5995 0.0118 0.0191 0.0022 0.0268
0.0379 0.0197 0.0172 0.0254 0.0116 0.0118 0.5838 0.0305 0.0180 0.0171
0.0225 0.0440 0.0143 0.0203 0.0106 0.0191 0.0305 0.5640 0.0201 0.0013
0.0135 0.0369 0.0142 0.0268 0.0312 0.0022 0.0180 0.0201 0.4020 0.0111
0.0066 0.0202 0.0176 0.0189 0.0207 0.0268 0.0171 0.0013 0.0111 0.2500]

lambda =
[0.0000 0.2751 0.9809 0.8807 0.4176 0.7908 0.6917 0.6112 0.5405 0.5722]

V matrix 4 case c

V =

[1.0000	0.0539	0.0394	0.0037	0.0023	0.0263	0.0008	0.0244	0.0404	0.0046
0.0539	0.8794	0.0498	0.0013	0.0013	0.0150	0.0352	0.0034	0.0204	0.0296
0.0394	0.0498	0.7625	0.0337	0.0328	0.0259	0.0086	0.0376	0.0361	0.0318
0.0037	0.0013	0.0337	0.7224	0.0517	0.0637	0.0149	0.0138	0.0435	0.0099
0.0023	0.0013	0.0328	0.0517	0.7216	0.0056	0.0348	0.0103	0.0424	0.0074
0.0263	0.0150	0.0259	0.0637	0.0056	0.7054	0.0211	0.0316	0.0060	0.0016
0.0008	0.0352	0.0086	0.0149	0.0348	0.0211	0.4094	0.0263	0.0366	0.0213
0.0244	0.0034	0.0376	0.0138	0.0103	0.0316	0.0263	0.4053	0.0130	0.0053
0.0404	0.0204	0.0361	0.0435	0.0424	0.0060	0.0366	0.0130	0.3503	0.0122
0.0046	0.0296	0.0318	0.0099	0.0074	0.0016	0.0213	0.0053	0.0122	0.2500]

lambda =

[0.0000 0.9530 0.2685 0.8516 0.3313 0.3814 0.5184 0.6428 0.7235 0.7089]

V matrix 5 case a

V =

[1.0000	0.0435	0.0459	0.0416	0.0492	0.0673	0.0525	0.0611	0.0476	0.0554
0.0435	1.0000	0.0274	0.0396	0.0339	0.0041	0.0518	0.0359	0.0037	0.0570
0.0459	0.0274	1.0000	0.0502	0.0080	0.0252	0.0302	0.0513	0.0250	0.0469
0.0416	0.0396	0.0502	1.0000	0.0465	0.0384	0.0444	0.0296	0.0349	0.0141
0.0492	0.0339	0.0080	0.0465	1.0000	0.0183	0.0562	0.0673	0.0304	0.0191
0.0673	0.0041	0.0252	0.0384	0.0183	1.0000	0.0059	0.0050	0.0394	0.0438
0.0525	0.0518	0.0302	0.0444	0.0562	0.0059	1.0000	0.0387	0.0432	0.0376
0.0611	0.0359	0.0513	0.0296	0.0673	0.0050	0.0387	1.0000	0.0079	0.0042
0.0476	0.0037	0.0250	0.0349	0.0304	0.0394	0.0432	0.0079	1.0000	0.0062
0.0554	0.0570	0.0469	0.0141	0.0191	0.0438	0.0376	0.0042	0.0062	1.0000]

lambda =

[0.0000 1.0447 0.8932 1.0269 0.9092 0.9944 0.9838 0.9654 0.9291 0.9361]

V matrix 5 case b

V =

[1.0000	0.0657	0.0674	0.0192	0.0471	0.0241	0.0069	0.0866	0.0792	0.0608
0.0657	1.0000	0.0957	0.0716	0.0149	0.0978	0.0853	0.0254	0.0153	0.0176
0.0674	0.0957	1.0000	0.0251	0.0136	0.0641	0.0180	0.0569	0.0833	0.0002
0.0192	0.0716	0.0251	1.0000	0.0532	0.0230	0.0032	0.0159	0.0192	0.0790
0.0471	0.0149	0.0136	0.0532	1.0000	0.0681	0.0734	0.0594	0.0639	0.0514
0.0241	0.0978	0.0641	0.0230	0.0681	1.0000	0.0537	0.0331	0.0669	0.0213
0.0069	0.0853	0.0180	0.0032	0.0734	0.0537	1.0000	0.0659	0.0772	0.0103
0.0866	0.0254	0.0569	0.0159	0.0594	0.0331	0.0659	1.0000	0.0380	0.0157
0.0792	0.0153	0.0833	0.0192	0.0639	0.0669	0.0772	0.0380	1.0000	0.0408
0.0608	0.0176	0.0002	0.0790	0.0514	0.0213	0.0103	0.0157	0.0408	1.0000]

lambda =

[0.0000 0.8070 1.0770 1.0459 1.0342 0.9775 0.8760 0.8999 0.9287 0.9390]

V matrix 6

V =

[1.0000	0.1528	0.1651	0.1384	0.1304	0.1340	0.1296	0.1220	0.0918	0.0862
0.1528	0.9317	0.1468	0.1454	0.1281	0.1302	0.1406	0.1172	0.1057	0.0766
0.1651	0.1468	0.6979	0.1107	0.1078	0.0980	0.1076	0.0909	0.0971	0.0829
0.1384	0.1454	0.1107	0.6972	0.1111	0.1190	0.1185	0.1013	0.1011	0.0700
0.1304	0.1281	0.1078	0.1111	0.6086	0.1130	0.1114	0.0925	0.0766	0.0595
0.1340	0.1302	0.0980	0.1190	0.1130	0.6055	0.0919	0.0906	0.0878	0.0731
0.1296	0.1406	0.1076	0.1185	0.1114	0.0919	0.5568	0.1013	0.0800	0.0727
0.1220	0.1172	0.0909	0.1013	0.0925	0.0906	0.1013	0.4967	0.0844	0.0579
0.0918	0.1057	0.0971	0.1011	0.0766	0.0878	0.0800	0.0844	0.3711	0.0495
0.0862	0.0766	0.0829	0.0700	0.0595	0.0731	0.0727	0.0579	0.0495	0.2500]

lambda =

[0.0000 0.8247 0.7254 0.2409 0.3275 0.4243 0.5878 0.5484 0.4766 0.4985]

V matrix 7

V =

[1.0000	0.1531	0.0287	0.1899	0.0046	0.1389	0.1678	0.0402	0.1289	0.1362
0.1531	0.9556	0.0181	0.2644	0.2117	0.1364	0.0024	0.0893	0.0513	0.0229
0.0287	0.0181	0.8883	0.2040	0.1548	0.1434	0.0560	0.0765	0.1006	0.0173
0.1899	0.2644	0.2040	0.8858	0.1394	0.0398	0.1403	0.1172	0.1553	0.1077
0.0046	0.2117	0.1548	0.1394	0.7764	0.1294	0.0518	0.0909	0.0741	0.0954
0.1389	0.1364	0.1434	0.0398	0.1294	0.5913	0.1364	0.0693	0.1219	0.0752
0.1678	0.0024	0.0560	0.1403	0.0518	0.1364	0.4070	0.0084	0.0883	0.0722
0.0402	0.0893	0.0765	0.1172	0.0909	0.0693	0.0084	0.3482	0.0906	0.0587
0.1289	0.0513	0.1006	0.1553	0.0741	0.1219	0.0883	0.0906	0.3108	0.0739
0.1362	0.0229	0.0173	0.1077	0.0954	0.0752	0.0722	0.0587	0.0739	0.2500]

lambda =

[0.0000 0.9935 0.9252 0.7478 0.5964 0.5250 0.3836 0.3019 0.1942 0.2097]

Reference variance matrices of dimension 20

V matrix 1

V=

[1.0000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000
0.3000	1.0000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000
0.3000	0.3000	1.0000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000
0.3000	0.3000	0.3000	1.0000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000
0.3000	0.3000	0.3000	0.3000	1.0000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000
0.3000	0.3000	0.3000	0.3000	0.3000	1.0000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000
0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	1.0000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000
0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	1.0000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000
0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	1.0000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000
0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	1.0000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000
0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	1.0000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000
0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	1.0000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000
0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	1.0000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000
0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	1.0000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000
0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	1.0000	0.3000	0.3000	0.3000	0.3000	0.3000
0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	1.0000	0.3000	0.3000	0.3000	0.3000
0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	1.0000	0.3000	0.3000	0.3000
0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	1.0000	0.3000	0.3000
0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	1.0000	0.3000
0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	1.0000]

lambda=

[0.7000 0.0000 0.7000 0.7000 0.7000 0.7000 0.7000 0.7000 0.7000 0.7000 0.7000 0.7000 0.7000 0.7000 0.7000 0.7000 0.7000 0.7000 0.7000 0.7000]

V matrix 2 case 1

V=

[1.0000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.3000	1.0000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.3000	0.3000	1.0000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.3000	0.3000	0.3000	1.0000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.3000	0.3000	0.3000	0.3000	1.0000	0.3000	0.3000	0.3000	0.3000	0.3000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.3000	0.3000	0.3000	0.3000	0.3000	1.0000	0.3000	0.3000	0.3000	0.3000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	1.0000	0.3000	0.3000	0.3000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	1.0000	0.3000	0.3000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	1.0000	0.3000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.3000	1.0000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000

0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.3000 0.3000 1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.3000 0.3000 0.3000 1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000
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0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.3000 0.3000 0.3000 0.3000 0.3000 1.0000 0.3000 0.3000 0.3000 0.3000
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 1.0000 0.3000 0.3000 0.3000
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 1.0000 0.3000 0.3000
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 1.0000 0.3000
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 1.0000]

lambda=
[0.7000 3.7000 0.0000 0.7000 0.7000 0.7000 0.7000 0.7000 0.7000 0.7000 0.7000 0.7000 0.7000 0.7000 0.7000 0.7000 0.7000 0.7000 0.7000 0.7000]

V matrix 2 case b

V=
[1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 0.3000 1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 0.3000 0.3000 1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 0.3000 0.3000 0.3000 1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 0.3000 0.3000 0.3000 0.3000 1.0000 0.3000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 1.0000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
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0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 1.0000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 1.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.2500 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0250 0.2500 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0250 0.0250 0.2500 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0250 0.0250 0.0250 0.2500 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0250 0.0250 0.0250 0.0250 0.2500 0.0250 0.0250 0.0250 0.0250 0.0250
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0250 0.0250 0.0250 0.0250 0.0250 0.2500 0.0250 0.0250 0.0250 0.0250
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.2500 0.0250 0.0250 0.0250
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.2500 0.0250 0.0250
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.2500 0.0250
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.2500]

lambda=
[0.2250 2.0875 0.2250 0.0000 0.2250 0.2250 0.7000 0.7000 0.2250 0.2250 0.2250 0.2250 0.2250 0.7000 0.7000 0.7000 0.7000 0.7000 0.7000 0.7000]

V matrix 2 case c

V =
[1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 0.3000 1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
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0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0750 0.2500 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0750 0.0750 0.2500 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0750 0.0750 0.0750 0.2500 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750
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0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0750 0.0750 0.0750 0.0750 0.0750 0.2500 0.0750 0.0750 0.0750 0.0750
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0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.2500 0.0750 0.0750
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.2500 0.0750
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.2500]

lambda=
[0.1750 2.3125 0.0000 0.1750 0.1750 0.7000 0.7000 0.1750 0.1750 0.7000 0.7000 0.7000 0.1750 0.1750 0.1750 0.1750 0.7000 0.7000 0.7000 0.7000]

V matrix 2 case d

V=
[1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
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0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.1250 0.1250 0.1250 0.2500 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.1250 0.1250 0.1250 0.1250 0.2500 0.1250 0.1250 0.1250 0.1250 0.1250

0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1250	0.1250	0.1250	0.1250	0.1250	0.2500	0.1250	0.1250	0.1250	0.1250
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	0.2500	0.1250	0.1250	0.1250
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	0.2500	0.1250	0.1250
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	0.2500	0.1250
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	0.1250	0.2500]

lambda=

[0.1250 2.5375 0.0000 0.1250 0.1250 0.7000 0.7000 0.1250 0.1250 0.1250 0.7000 0.7000 0.7000 0.1250 0.1250 0.1250 0.7000 0.7000 0.7000 0.7000]

V matrix 3 case a

V=

[1.0000	0.2932	0.2912	0.2893	0.2886	0.2849	0.2748	0.2705	0.2660	0.2623	0.2426	0.2412	0.2199	0.2144	0.2122	0.2046	0.1665	0.1526	0.1025	0.0949
0.2932	0.9551	0.2845	0.2827	0.2820	0.2785	0.2686	0.2643	0.2599	0.2563	0.2371	0.2357	0.2149	0.2095	0.2074	0.2000	0.1627	0.1491	0.1001	0.0927
0.2912	0.2845	0.9419	0.2807	0.2801	0.2765	0.2667	0.2625	0.2581	0.2546	0.2354	0.2340	0.2134	0.2081	0.2059	0.1986	0.1616	0.1481	0.0994	0.0921
0.2893	0.2827	0.2807	0.9296	0.2782	0.2747	0.2650	0.2608	0.2564	0.2529	0.2339	0.2325	0.2120	0.2067	0.2046	0.1973	0.1605	0.1471	0.0988	0.0915
0.2886	0.2820	0.2801	0.2782	0.9252	0.2741	0.2644	0.2601	0.2558	0.2523	0.2333	0.2320	0.2115	0.2062	0.2041	0.1968	0.1602	0.1468	0.0986	0.0913
0.2849	0.2785	0.2765	0.2747	0.2741	0.9022	0.2610	0.2569	0.2526	0.2491	0.2304	0.2291	0.2089	0.2037	0.2015	0.1943	0.1581	0.1449	0.0973	0.0901
0.2748	0.2686	0.2667	0.2650	0.2644	0.2610	0.8393	0.2478	0.2436	0.2403	0.2222	0.2209	0.2015	0.1964	0.1944	0.1874	0.1525	0.1398	0.0939	0.0869
0.2705	0.2643	0.2625	0.2608	0.2601	0.2569	0.2478	0.8127	0.2398	0.2365	0.2187	0.2174	0.1983	0.1933	0.1913	0.1845	0.1501	0.1375	0.0924	0.0855
0.2660	0.2599	0.2581	0.2564	0.2558	0.2526	0.2436	0.2398	0.7859	0.2325	0.2151	0.2138	0.1950	0.1901	0.1881	0.1814	0.1476	0.1353	0.0908	0.0841
0.2623	0.2563	0.2546	0.2529	0.2523	0.2491	0.2403	0.2365	0.2325	0.7644	0.2121	0.2108	0.1923	0.1875	0.1855	0.1789	0.1456	0.1334	0.0896	0.0829
0.2426	0.2371	0.2354	0.2339	0.2333	0.2304	0.2222	0.2187	0.2151	0.2121	0.6539	0.1950	0.1778	0.1734	0.1716	0.1654	0.1346	0.1234	0.0829	0.0767
0.2412	0.2357	0.2340	0.2325	0.2320	0.2291	0.2209	0.2174	0.2138	0.2108	0.1950	0.6462	0.1768	0.1724	0.1706	0.1645	0.1338	0.1226	0.0824	0.0763
0.2199	0.2149	0.2134	0.2120	0.2115	0.2089	0.2015	0.1983	0.1950	0.1923	0.1778	0.1768	0.5374	0.1572	0.1555	0.1500	0.1221	0.1118	0.0751	0.0695
0.2144	0.2095	0.2081	0.2067	0.2062	0.2037	0.1964	0.1933	0.1901	0.1875	0.1734	0.1724	0.1572	0.5108	0.1516	0.1462	0.1190	0.1090	0.0732	0.0678
0.2122	0.2074	0.2059	0.2046	0.2041	0.2015	0.1944	0.1913	0.1881	0.1855	0.1716	0.1706	0.1555	0.1516	0.5002	0.1447	0.1178	0.1079	0.0725	0.0671
0.2046	0.2000	0.1986	0.1973	0.1968	0.1943	0.1874	0.1845	0.1814	0.1789	0.1654	0.1645	0.1500	0.1462	0.1447	0.4651	0.1136	0.1041	0.0699	0.0647
0.1665	0.1627	0.1616	0.1605	0.1602	0.1581	0.1525	0.1501	0.1476	0.1456	0.1346	0.1338	0.1221	0.1190	0.1178	0.1136	0.3080	0.0847	0.0569	0.0527
0.1526	0.1491	0.1481	0.1471	0.1468	0.1449	0.1398	0.1375	0.1353	0.1334	0.1234	0.1226	0.1118	0.1090	0.1079	0.1041	0.0847	0.2586	0.0521	0.0482
0.1025	0.1001	0.0994	0.0988	0.0986	0.0973	0.0939	0.0924	0.0908	0.0896	0.0829	0.0824	0.0751	0.0732	0.0725	0.0699	0.0569	0.0521	0.1167	0.0324
0.0949	0.0927	0.0921	0.0915	0.0913	0.0901	0.0869	0.0855	0.0841	0.0829	0.0767	0.0763	0.0695	0.0678	0.0671	0.0647	0.0527	0.0482	0.0324	0.1000]

lambda=

[0.0000 0.0758 0.1428 0.2013 0.2876 0.3368 0.3542 0.3711 0.4400 0.4555 0.7051 0.5295 0.5470 0.5666 0.5859 0.6710 0.6315 0.6607 0.6517 0.6482]

V matrix 3 case b

V=

[1.0000	0.2922	0.2878	0.2782	0.2700	0.2515	0.2323	0.2292	0.2253	0.2240	0.2152	0.2022	0.1902	0.1895	0.1895	0.1785	0.1625	0.1534	0.1522	0.1500
0.2922	0.9489	0.2803	0.2710	0.2630	0.2450	0.2263	0.2233	0.2195	0.2182	0.2096	0.1969	0.1853	0.1846	0.1846	0.1739	0.1583	0.1494	0.1483	0.1461
0.2878	0.2803	0.9202	0.2669	0.2590	0.2413	0.2228	0.2199	0.2161	0.2149	0.2065	0.1939	0.1825	0.1818	0.1818	0.1713	0.1559	0.1472	0.1460	0.1439
0.2782	0.2710	0.2669	0.8599	0.2504	0.2332	0.2154	0.2126	0.2089	0.2077	0.1996	0.1875	0.1764	0.1757	0.1757	0.1656	0.1507	0.1422	0.1411	0.1391
0.2700	0.2630	0.2590	0.2504	0.8101	0.2264	0.2091	0.2063	0.2028	0.2016	0.1937	0.1820	0.1712	0.1706	0.1706	0.1607	0.1463	0.1381	0.1370	0.1350
0.2515	0.2450	0.2413	0.2332	0.2264	0.7028	0.1947	0.1922	0.1889	0.1878	0.1804	0.1695	0.1595	0.1589	0.1589	0.1497	0.1362	0.1286	0.1276	0.1258

0.2323	0.2263	0.2228	0.2154	0.2091	0.1947	0.5995	0.1775	0.1744	0.1735	0.1666	0.1565	0.1473	0.1467	0.1467	0.1382	0.1258	0.1188	0.1178	0.1161
0.2292	0.2233	0.2199	0.2126	0.2063	0.1922	0.1775	0.5838	0.1721	0.1712	0.1644	0.1545	0.1453	0.1448	0.1448	0.1364	0.1242	0.1172	0.1163	0.1146
0.2253	0.2195	0.2161	0.2089	0.2028	0.1889	0.1744	0.1721	0.5640	0.1683	0.1616	0.1518	0.1429	0.1423	0.1423	0.1341	0.1220	0.1152	0.1143	0.1126
0.2240	0.2182	0.2149	0.2077	0.2016	0.1878	0.1735	0.1712	0.1683	0.5577	0.1607	0.1510	0.1421	0.1415	0.1415	0.1333	0.1214	0.1146	0.1137	0.1120
0.2152	0.2096	0.2065	0.1996	0.1937	0.1804	0.1666	0.1644	0.1616	0.1607	0.5147	0.1450	0.1365	0.1360	0.1360	0.1281	0.1166	0.1100	0.1092	0.1076
0.2022	0.1969	0.1939	0.1875	0.1820	0.1695	0.1565	0.1545	0.1518	0.1510	0.1450	0.4541	0.1282	0.1277	0.1277	0.1203	0.1095	0.1034	0.1026	0.1011
0.1902	0.1853	0.1825	0.1764	0.1712	0.1595	0.1473	0.1453	0.1429	0.1421	0.1365	0.1282	0.4021	0.1202	0.1202	0.1132	0.1030	0.0973	0.0965	0.0951
0.1895	0.1846	0.1818	0.1757	0.1706	0.1589	0.1467	0.1448	0.1423	0.1415	0.1360	0.1277	0.1202	0.3991	0.1197	0.1128	0.1027	0.0969	0.0962	0.0948
0.1895	0.1846	0.1818	0.1757	0.1706	0.1589	0.1467	0.1448	0.1423	0.1415	0.1360	0.1277	0.1202	0.1197	0.3990	0.1128	0.1027	0.0969	0.0961	0.0948
0.1785	0.1739	0.1713	0.1656	0.1607	0.1497	0.1382	0.1364	0.1341	0.1333	0.1281	0.1203	0.1132	0.1128	0.1128	0.3542	0.0967	0.0913	0.0906	0.0893
0.1625	0.1583	0.1559	0.1507	0.1463	0.1362	0.1258	0.1242	0.1220	0.1214	0.1166	0.1095	0.1030	0.1027	0.1027	0.0967	0.2934	0.0831	0.0824	0.0813
0.1534	0.1494	0.1472	0.1422	0.1381	0.1286	0.1188	0.1172	0.1152	0.1146	0.1100	0.1034	0.0973	0.0969	0.0969	0.0913	0.0831	0.2615	0.0778	0.0767
0.1522	0.1483	0.1460	0.1411	0.1370	0.1276	0.1178	0.1163	0.1143	0.1137	0.1092	0.1026	0.0965	0.0962	0.0961	0.0906	0.0824	0.0778	0.2574	0.0761
0.1500	0.1461	0.1439	0.1391	0.1350	0.1258	0.1161	0.1146	0.1126	0.1120	0.1076	0.1011	0.0951	0.0948	0.0948	0.0893	0.0813	0.0767	0.0761	0.2500]

lambda=
[0.0000 0.7114 0.6698 0.6473 0.6033 0.5670 0.4893 0.1769 0.1817 0.1979 0.4172 0.4053 0.3929 0.3805 0.3518 0.2305 0.3116 0.2600 0.2808 0.2794]

V matrix 3 case c

V=

[1.0000	0.2906	0.2901	0.2897	0.2820	0.2803	0.2802	0.2796	0.2711	0.2700	0.2685	0.2635	0.2601	0.2549	0.2548	0.2476	0.2469	0.2466	0.2339	0.2324
0.2906	0.9385	0.2811	0.2807	0.2732	0.2715	0.2715	0.2709	0.2626	0.2616	0.2601	0.2553	0.2520	0.2469	0.2468	0.2398	0.2392	0.2389	0.2266	0.2251
0.2901	0.2811	0.9352	0.2802	0.2727	0.2711	0.2710	0.2704	0.2622	0.2611	0.2597	0.2548	0.2516	0.2465	0.2464	0.2394	0.2388	0.2385	0.2262	0.2247
0.2897	0.2807	0.2802	0.9327	0.2724	0.2707	0.2706	0.2701	0.2618	0.2608	0.2593	0.2545	0.2512	0.2462	0.2460	0.2391	0.2385	0.2382	0.2259	0.2244
0.2820	0.2732	0.2727	0.2724	0.8838	0.2635	0.2634	0.2629	0.2549	0.2538	0.2524	0.2477	0.2445	0.2396	0.2395	0.2327	0.2321	0.2319	0.2199	0.2185
0.2803	0.2715	0.2711	0.2707	0.2635	0.8729	0.2618	0.2613	0.2533	0.2523	0.2509	0.2462	0.2430	0.2381	0.2380	0.2313	0.2307	0.2304	0.2185	0.2171
0.2802	0.2715	0.2710	0.2706	0.2634	0.2618	0.8725	0.2612	0.2532	0.2522	0.2508	0.2461	0.2430	0.2381	0.2380	0.2313	0.2306	0.2304	0.2185	0.2171
0.2796	0.2709	0.2704	0.2701	0.2629	0.2613	0.2612	0.8689	0.2527	0.2517	0.2503	0.2456	0.2425	0.2376	0.2375	0.2308	0.2301	0.2299	0.2180	0.2166
0.2711	0.2626	0.2622	0.2618	0.2549	0.2533	0.2532	0.2527	0.8167	0.2440	0.2427	0.2381	0.2351	0.2303	0.2302	0.2237	0.2231	0.2229	0.2114	0.2100
0.2700	0.2616	0.2611	0.2608	0.2538	0.2523	0.2522	0.2517	0.2440	0.8101	0.2417	0.2372	0.2341	0.2294	0.2293	0.2228	0.2222	0.2220	0.2105	0.2091
0.2685	0.2601	0.2597	0.2593	0.2524	0.2509	0.2508	0.2503	0.2427	0.2417	0.8011	0.2359	0.2328	0.2281	0.2280	0.2216	0.2210	0.2207	0.2093	0.2080
0.2635	0.2553	0.2548	0.2545	0.2477	0.2462	0.2461	0.2456	0.2381	0.2372	0.2359	0.7716	0.2285	0.2239	0.2238	0.2175	0.2169	0.2166	0.2054	0.2041
0.2601	0.2520	0.2516	0.2512	0.2445	0.2430	0.2430	0.2425	0.2351	0.2341	0.2328	0.2285	0.7518	0.2210	0.2209	0.2147	0.2141	0.2138	0.2028	0.2015
0.2549	0.2469	0.2465	0.2462	0.2396	0.2381	0.2381	0.2376	0.2303	0.2294	0.2281	0.2239	0.2210	0.7218	0.2164	0.2103	0.2098	0.2095	0.1987	0.1974
0.2548	0.2468	0.2464	0.2460	0.2395	0.2380	0.2380	0.2375	0.2302	0.2293	0.2280	0.2238	0.2209	0.2164	0.7211	0.2102	0.2097	0.2094	0.1986	0.1973
0.2476	0.2398	0.2394	0.2391	0.2327	0.2313	0.2313	0.2308	0.2237	0.2228	0.2216	0.2175	0.2147	0.2103	0.2102	0.6811	0.2038	0.2035	0.1930	0.1918
0.2469	0.2392	0.2388	0.2385	0.2321	0.2307	0.2306	0.2301	0.2231	0.2222	0.2210	0.2169	0.2141	0.2098	0.2097	0.2038	0.6774	0.2030	0.1925	0.1913
0.2466	0.2389	0.2385	0.2382	0.2319	0.2304	0.2304	0.2299	0.2229	0.2220	0.2207	0.2166	0.2138	0.2095	0.2094	0.2035	0.2030	0.6759	0.1923	0.1910
0.2339	0.2266	0.2262	0.2259	0.2199	0.2185	0.2185	0.2180	0.2114	0.2105	0.2093	0.2054	0.2028	0.1987	0.1986	0.1930	0.1925	0.1923	0.6079	0.1812
0.2324	0.2251	0.2247	0.2244	0.2185	0.2171	0.2171	0.2166	0.2100	0.2091	0.2080	0.2041	0.2015	0.1974	0.1973	0.1918	0.1913	0.1910	0.1812	0.6000]

lambda=
[0.0000 0.6969 0.4224 0.4377 0.6561 0.6538 0.6449 0.4736 0.4757 0.4907 0.5050 0.5185 0.5340 0.5508 0.5638 0.5698 0.5943 0.6169 0.6091 0.6109]

V matrix 4 case a

V =

[1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.9248	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.8950	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.8902	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.8662	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.8635	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.7953	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.7734	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.7452	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.7337	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.6952	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.6506	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.6224	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.5338	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.5065	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.5059	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.4820	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.4673	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.3632	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2500]

lambda=

[0.0000 0.2674 0.3779 0.9892 0.4721 0.4895 0.5243 0.5062 0.5742 0.6339 0.6692 0.7094 0.9184 0.7391 0.7610 0.7863 0.8929 0.8802 0.8648 0.8299]

V matrix 4 case b

V=

[1.0000	0.0575	0.0183	0.0519	0.0429	0.0139	0.0383	0.0368	0.0400	0.0109	0.0365	0.0094	0.0258	0.0298	0.0307	0.0280	0.0348	0.0005	0.0111	0.0273
0.0575	0.9996	0.0472	0.0051	0.0046	0.0008	0.0144	0.0314	0.0278	0.0416	0.0334	0.0281	0.0036	0.0092	0.0170	0.0114	0.0306	0.0113	0.0228	0.0345
0.0183	0.0472	0.9891	0.0629	0.0073	0.0525	0.0574	0.0095	0.0074	0.0449	0.0411	0.0358	0.0359	0.0315	0.0393	0.0112	0.0166	0.0076	0.0013	0.0028
0.0519	0.0051	0.0629	0.9685	0.0146	0.0552	0.0411	0.0533	0.0291	0.0579	0.0211	0.0289	0.0085	0.0236	0.0186	0.0268	0.0263	0.0338	0.0068	0.0038
0.0429	0.0046	0.0073	0.0146	0.8645	0.0314	0.0114	0.0405	0.0141	0.0514	0.0073	0.0268	0.0399	0.0419	0.0139	0.0168	0.0174	0.0050	0.0045	0.0173
0.0139	0.0008	0.0525	0.0552	0.0314	0.8625	0.0142	0.0552	0.0361	0.0126	0.0363	0.0342	0.0060	0.0010	0.0109	0.0045	0.0026	0.0284	0.0060	0.0011
0.0383	0.0144	0.0574	0.0411	0.0114	0.0142	0.8249	0.0140	0.0313	0.0086	0.0309	0.0166	0.0286	0.0375	0.0231	0.0368	0.0285	0.0050	0.0271	0.0061
0.0368	0.0314	0.0095	0.0533	0.0405	0.0552	0.0140	0.7546	0.0001	0.0407	0.0256	0.0102	0.0043	0.0065	0.0158	0.0146	0.0018	0.0294	0.0047	0.0117
0.0400	0.0278	0.0074	0.0291	0.0141	0.0361	0.0313	0.0001	0.7496	0.0269	0.0318	0.0008	0.0347	0.0332	0.0270	0.0165	0.0028	0.0255	0.0060	0.0135
0.0109	0.0416	0.0449	0.0579	0.0514	0.0126	0.0086	0.0407	0.0269	0.7159	0.0299	0.0320	0.0207	0.0289	0.0195	0.0212	0.0325	0.0250	0.0096	0.0174
0.0365	0.0334	0.0411	0.0211	0.0073	0.0363	0.0309	0.0256	0.0318	0.0299	0.6702	0.0445	0.0362	0.0220	0.0239	0.0261	0.0035	0.0000	0.0159	0.0002
0.0094	0.0281	0.0358	0.0289	0.0268	0.0342	0.0166	0.0102	0.0008	0.0320	0.0445	0.6238	0.0214	0.0290	0.0276	0.0158	0.0252	0.0134	0.0057	0.0160
0.0258	0.0036	0.0359	0.0085	0.0399	0.0060	0.0286	0.0043	0.0347	0.0207	0.0362	0.0214	0.4679	0.0043	0.0067	0.0030	0.0038	0.0112	0.0193	0.0067
0.0298	0.0092	0.0315	0.0236	0.0419	0.0010	0.0375	0.0065	0.0332	0.0289	0.0220	0.0290	0.0043	0.4661	0.0036	0.0048	0.0076	0.0136	0.0119	0.0228
0.0307	0.0170	0.0393	0.0186	0.0139	0.0109	0.0231	0.0158	0.0270	0.0195	0.0239	0.0276	0.0067	0.0036	0.4090	0.0091	0.0151	0.0141	0.0001	0.0220

0.0280 0.0114 0.0112 0.0268 0.0168 0.0045 0.0368 0.0146 0.0165 0.0212 0.0261 0.0158 0.0030 0.0048 0.0091 0.3482 0.0001 0.0122 0.0157 0.0167
0.0348 0.0306 0.0166 0.0263 0.0174 0.0026 0.0285 0.0018 0.0028 0.0325 0.0035 0.0252 0.0038 0.0076 0.0151 0.0001 0.3216 0.0063 0.0062 0.0143
0.0005 0.0113 0.0076 0.0338 0.0050 0.0284 0.0050 0.0294 0.0255 0.0250 0.0000 0.0134 0.0112 0.0136 0.0141 0.0122 0.0063 0.2630 0.0138 0.0068
0.0111 0.0228 0.0013 0.0068 0.0045 0.0060 0.0271 0.0047 0.0060 0.0096 0.0159 0.0057 0.0193 0.0119 0.0001 0.0157 0.0062 0.0138 0.2611 0.0014
0.0273 0.0345 0.0028 0.0038 0.0173 0.0011 0.0061 0.0117 0.0135 0.0174 0.0002 0.0160 0.0067 0.0228 0.0220 0.0167 0.0143 0.0068 0.0014 0.2500]

lambda=
[0.0000 0.2415 0.2463 0.2743 0.3262 0.3666 0.4276 0.4618 0.5021 1.0092 0.9813 0.6041 0.9276 0.9023 0.8371 0.8322 0.6562 0.6876 0.7376 0.7545]

V matrix 4 case c

V=
[1.0000 0.0061 0.0255 0.0178 0.0872 0.0116 0.0012 0.0294 0.0551 0.0694 0.0439 0.0115 0.0208 0.0151 0.0005 0.0241 0.0385 0.0048 0.0397 0.0485
0.0061 0.9935 0.0230 0.0186 0.0249 0.0669 0.0863 0.0505 0.0429 0.0660 0.0592 0.0491 0.0439 0.0529 0.0604 0.0295 0.0483 0.0362 0.0158 0.0329
0.0255 0.0230 0.9456 0.0084 0.0623 0.0471 0.0191 0.0561 0.0052 0.0732 0.0149 0.0115 0.0407 0.0000 0.0204 0.0163 0.0026 0.0052 0.0206 0.0397
0.0178 0.0186 0.0084 0.9094 0.0660 0.0856 0.0554 0.0563 0.0259 0.0658 0.0245 0.0042 0.0450 0.0590 0.0093 0.0241 0.0053 0.0243 0.0428 0.0187
0.0872 0.0249 0.0623 0.0660 0.9035 0.0744 0.0396 0.0477 0.0044 0.0402 0.0321 0.0605 0.0534 0.0290 0.0467 0.0379 0.0000 0.0071 0.0243 0.0018
0.0116 0.0669 0.0471 0.0856 0.0744 0.8834 0.0701 0.0138 0.0377 0.0332 0.0226 0.0263 0.0547 0.0462 0.0519 0.0261 0.0445 0.0071 0.0032 0.0176
0.0012 0.0863 0.0191 0.0554 0.0396 0.0701 0.7840 0.0115 0.0340 0.0343 0.0552 0.0526 0.0499 0.0349 0.0313 0.0329 0.0054 0.0079 0.0189 0.0248
0.0294 0.0505 0.0561 0.0563 0.0477 0.0138 0.0115 0.6334 0.0586 0.0161 0.0120 0.0029 0.0317 0.0281 0.0048 0.0308 0.0209 0.0030 0.0154 0.0099
0.0551 0.0429 0.0052 0.0259 0.0044 0.0377 0.0340 0.0586 0.6220 0.0336 0.0166 0.0216 0.0033 0.0278 0.0412 0.0070 0.0080 0.0030 0.0335 0.0053
0.0694 0.0660 0.0732 0.0658 0.0402 0.0332 0.0343 0.0161 0.0336 0.6097 0.0524 0.0386 0.0493 0.0082 0.0365 0.0178 0.0210 0.0017 0.0227 0.0145
0.0439 0.0592 0.0149 0.0245 0.0321 0.0226 0.0552 0.0120 0.0166 0.0524 0.5474 0.0100 0.0179 0.0439 0.0238 0.0041 0.0322 0.0002 0.0230 0.0354
0.0115 0.0491 0.0115 0.0042 0.0605 0.0263 0.0526 0.0029 0.0216 0.0386 0.0100 0.5433 0.0363 0.0060 0.0020 0.0166 0.0303 0.0390 0.0182 0.0048
0.0208 0.0439 0.0407 0.0450 0.0534 0.0547 0.0499 0.0317 0.0033 0.0493 0.0179 0.0363 0.4308 0.0320 0.0400 0.0160 0.0051 0.0269 0.0305 0.0056
0.0151 0.0529 0.0000 0.0590 0.0290 0.0462 0.0349 0.0281 0.0278 0.0082 0.0439 0.0060 0.0320 0.4175 0.0031 0.0283 0.0293 0.0240 0.0270 0.0121
0.0005 0.0604 0.0204 0.0093 0.0467 0.0519 0.0313 0.0048 0.0412 0.0365 0.0238 0.0020 0.0400 0.0031 0.3902 0.0233 0.0290 0.0001 0.0257 0.0200
0.0241 0.0295 0.0163 0.0241 0.0379 0.0261 0.0329 0.0308 0.0070 0.0178 0.0041 0.0166 0.0160 0.0283 0.0233 0.3697 0.0087 0.0278 0.0293 0.0039
0.0385 0.0483 0.0026 0.0053 0.0000 0.0445 0.0054 0.0209 0.0080 0.0210 0.0322 0.0303 0.0051 0.0293 0.0290 0.0087 0.3514 0.0295 0.0214 0.0091
0.0048 0.0362 0.0052 0.0243 0.0071 0.0071 0.0079 0.0030 0.0030 0.0017 0.0002 0.0390 0.0269 0.0240 0.0001 0.0278 0.0295 0.3197 0.0030 0.0253
0.0397 0.0158 0.0206 0.0428 0.0243 0.0032 0.0189 0.0154 0.0335 0.0227 0.0230 0.0182 0.0305 0.0270 0.0257 0.0293 0.0214 0.0030 0.2663 0.0067
0.0485 0.0329 0.0397 0.0187 0.0018 0.0176 0.0248 0.0099 0.0053 0.0145 0.0354 0.0048 0.0056 0.0121 0.0200 0.0039 0.0091 0.0253 0.0067 0.2500]

lambda=
[0.0000 1.0316 0.9520 0.9351 0.8390 0.7699 0.8049 0.7066 0.6231 0.2321 0.2513 0.5703 0.5462 0.5362 0.4665 0.2980 0.3287 0.3897 0.3980 0.3483]

V matrix 5 case a

V=
[1.0000 0.0137 0.0534 0.0401 0.0172 0.0194 0.0026 0.0643 0.0092 0.0450 0.0114 0.0580 0.0419 0.0519 0.0528 0.0256 0.0677 0.0301 0.0151 0.0262
0.0137 1.0000 0.0459 0.0130 0.0328 0.0405 0.0219 0.0546 0.0591 0.0226 0.0386 0.0223 0.0155 0.0088 0.0539 0.0544 0.0548 0.0242 0.0637 0.0677
0.0534 0.0459 1.0000 0.0517 0.0348 0.0576 0.0572 0.0229 0.0163 0.0018 0.0368 0.0417 0.0279 0.0311 0.0566 0.0240 0.0287 0.0640 0.0101 0.0434
0.0401 0.0130 0.0517 1.0000 0.0052 0.0659 0.0164 0.0517 0.0047 0.0249 0.0644 0.0547 0.0211 0.0415 0.0380 0.0542 0.0670 0.0112 0.0145 0.0469
0.0172 0.0328 0.0348 0.0052 1.0000 0.0311 0.0411 0.0306 0.0232 0.0521 0.0309 0.0024 0.0443 0.0445 0.0340 0.0103 0.0138 0.0272 0.0595 0.0319

0.0194	0.0405	0.0576	0.0659	0.0311	1.0000	0.0647	0.0414	0.0403	0.0209	0.0031	0.0600	0.0661	0.0373	0.0309	0.0104	0.0371	0.0677	0.0499	0.0587
0.0026	0.0219	0.0572	0.0164	0.0411	0.0647	1.0000	0.0080	0.0096	0.0127	0.0675	0.0478	0.0004	0.0146	0.0439	0.0447	0.0353	0.0047	0.0056	0.0576
0.0643	0.0546	0.0229	0.0517	0.0306	0.0414	0.0080	1.0000	0.0607	0.0291	0.0009	0.0561	0.0572	0.0472	0.0054	0.0065	0.0338	0.0222	0.0573	0.0379
0.0092	0.0591	0.0163	0.0047	0.0232	0.0403	0.0096	0.0607	1.0000	0.0607	0.0386	0.0642	0.0484	0.0169	0.0381	0.0650	0.0264	0.0089	0.0244	0.0065
0.0450	0.0226	0.0018	0.0249	0.0521	0.0209	0.0127	0.0291	0.0607	1.0000	0.0654	0.0272	0.0687	0.0504	0.0146	0.0340	0.0364	0.0081	0.0114	0.0207
0.0114	0.0386	0.0368	0.0644	0.0309	0.0031	0.0675	0.0009	0.0386	0.0654	1.0000	0.0330	0.0049	0.0280	0.0189	0.0114	0.0571	0.0115	0.0378	0.0422
0.0580	0.0223	0.0417	0.0547	0.0024	0.0600	0.0478	0.0561	0.0642	0.0272	0.0330	1.0000	0.0031	0.0276	0.0204	0.0193	0.0146	0.0066	0.0646	0.0128
0.0419	0.0155	0.0279	0.0211	0.0443	0.0661	0.0004	0.0572	0.0484	0.0687	0.0049	0.0031	1.0000	0.0288	0.0576	0.0092	0.0272	0.0372	0.0306	0.0513
0.0519	0.0088	0.0311	0.0415	0.0445	0.0373	0.0146	0.0472	0.0169	0.0504	0.0280	0.0276	0.0288	1.0000	0.0288	0.0016	0.0032	0.0534	0.0378	0.0384
0.0528	0.0539	0.0566	0.0380	0.0340	0.0309	0.0439	0.0054	0.0381	0.0146	0.0189	0.0204	0.0576	0.0288	1.0000	0.0162	0.0213	0.0292	0.0489	0.0109
0.0256	0.0544	0.0240	0.0542	0.0103	0.0104	0.0447	0.0065	0.0650	0.0340	0.0114	0.0193	0.0092	0.0016	0.0162	1.0000	0.0570	0.0585	0.0001	0.0648
0.0677	0.0548	0.0287	0.0670	0.0138	0.0371	0.0353	0.0338	0.0264	0.0364	0.0571	0.0146	0.0272	0.0032	0.0213	0.0570	1.0000	0.0563	0.0586	0.0142
0.0301	0.0242	0.0640	0.0112	0.0272	0.0677	0.0047	0.0222	0.0089	0.0081	0.0115	0.0066	0.0372	0.0534	0.0292	0.0585	0.0563	1.0000	0.0623	0.0006
0.0151	0.0637	0.0101	0.0145	0.0595	0.0499	0.0056	0.0573	0.0244	0.0114	0.0378	0.0646	0.0306	0.0378	0.0489	0.0001	0.0586	0.0623	1.0000	0.0482
0.0262	0.0677	0.0434	0.0469	0.0319	0.0587	0.0576	0.0379	0.0065	0.0207	0.0422	0.0128	0.0513	0.0384	0.0109	0.0648	0.0142	0.0006	0.0482	1.0000]

lambda=
[0.0000 0.8094 0.8288 1.1086 0.8590 0.8700 0.8865 1.0833 1.0687 1.0596 1.0503 1.0305 1.0208 1.0024 0.9870 0.9135 0.9630 0.9497 0.9247 0.9379]

V matrix 5 case b

V=

[1.0000	0.0893	0.0566	0.0366	0.0211	0.0928	0.0224	0.0235	0.0816	0.0991	0.0257	0.0982	0.0400	0.0943	0.0527	0.0978	0.0952	0.0718	0.0635	0.0717
0.0893	1.0000	0.0870	0.0094	0.0591	0.0167	0.1000	0.0128	0.0395	0.0184	0.0278	0.0412	0.0252	0.0089	0.0807	0.0861	0.0719	0.0870	0.0232	0.0859
0.0566	0.0870	1.0000	0.0335	0.0837	0.0387	0.0261	0.0413	0.0826	0.0559	0.0288	0.0348	0.0092	0.0543	0.0393	0.0014	0.0779	0.0872	0.0616	0.0186
0.0366	0.0094	0.0335	1.0000	0.0888	0.0486	0.0230	0.0635	0.0338	0.0487	0.0735	0.0668	0.0602	0.0903	0.0962	0.0486	0.0618	0.0762	0.0269	0.0462
0.0211	0.0591	0.0837	0.0888	1.0000	0.0811	0.0525	0.0182	0.0197	0.0397	0.0497	0.0444	0.0832	0.0861	0.0030	0.0416	0.0649	0.0669	0.0991	0.0903
0.0928	0.0167	0.0387	0.0486	0.0811	1.0000	0.0561	0.0403	0.0510	0.0493	0.0055	0.0840	0.0293	0.0631	0.0954	0.0773	0.0756	0.0902	0.0760	0.0022
0.0224	0.1000	0.0261	0.0230	0.0525	0.0561	1.0000	0.0689	0.0210	0.0969	0.0191	0.0530	0.0466	0.0262	0.0714	0.0488	0.0148	0.0821	0.0482	0.0760
0.0235	0.0128	0.0413	0.0635	0.0182	0.0403	0.0689	1.0000	0.0580	0.0827	0.0725	0.0257	0.0046	0.0379	0.0647	0.0523	0.0599	0.0833	0.0945	0.0796
0.0816	0.0395	0.0826	0.0338	0.0197	0.0510	0.0210	0.0580	1.0000	0.0128	0.0974	0.0156	0.0086	0.0337	0.0447	0.0782	0.0899	0.0640	0.0361	0.0314
0.0991	0.0184	0.0559	0.0487	0.0397	0.0493	0.0969	0.0827	0.0128	1.0000	0.0704	0.0558	0.0513	0.0315	0.0175	0.0591	0.0172	0.0156	0.0084	0.0236
0.0257	0.0278	0.0288	0.0735	0.0497	0.0055	0.0191	0.0725	0.0974	0.0704	1.0000	0.0961	0.0715	0.0947	0.0835	0.0126	0.0819	0.0783	0.0949	0.0494
0.0982	0.0412	0.0348	0.0668	0.0444	0.0840	0.0530	0.0257	0.0156	0.0558	0.0961	1.0000	0.0035	0.0535	0.0970	0.0110	0.0069	0.0731	0.0691	0.0685
0.0400	0.0252	0.0092	0.0602	0.0832	0.0293	0.0466	0.0046	0.0086	0.0513	0.0715	0.0035	1.0000	0.0702	0.0135	0.0663	0.0956	0.0565	0.0616	0.0945
0.0943	0.0089	0.0543	0.0903	0.0861	0.0631	0.0262	0.0379	0.0337	0.0315	0.0947	0.0535	0.0702	1.0000	0.0251	0.0997	0.0317	0.0723	0.0895	0.0439
0.0527	0.0807	0.0393	0.0962	0.0030	0.0954	0.0714	0.0647	0.0447	0.0175	0.0835	0.0970	0.0135	0.0251	1.0000	0.0346	0.0005	0.0240	0.0013	0.0497
0.0978	0.0861	0.0014	0.0486	0.0416	0.0773	0.0488	0.0523	0.0782	0.0591	0.0126	0.0110	0.0663	0.0997	0.0346	1.0000	0.0760	0.0490	0.0301	0.0099
0.0952	0.0719	0.0779	0.0618	0.0649	0.0756	0.0148	0.0599	0.0899	0.0172	0.0819	0.0069	0.0956	0.0317	0.0005	0.0760	1.0000	0.0424	0.0965	0.0554
0.0718	0.0870	0.0872	0.0762	0.0669	0.0902	0.0821	0.0833	0.0640	0.0156	0.0783	0.0731	0.0565	0.0723	0.0240	0.0490	0.0424	1.0000	0.0102	0.0436
0.0635	0.0232	0.0616	0.0269	0.0991	0.0760	0.0482	0.0945	0.0361	0.0084	0.0949	0.0691	0.0616	0.0895	0.0013	0.0301	0.0965	0.0102	1.0000	0.0181
0.0717	0.0859	0.0186	0.0462	0.0903	0.0022	0.0760	0.0796	0.0314	0.0236	0.0494	0.0685	0.0945	0.0439	0.0497	0.0099	0.0554	0.0436	0.0181	1.0000]

lambda=
[0.0000 0.7182 1.1556 0.7579 0.7766 1.1217 1.1037 0.8248 0.8407 1.0751 1.0526 1.0368 0.8590 0.8898 0.9086 1.0127 0.9952 0.9664 0.9480 0.9430]

V matrix 6

V=
[1.0000 0.1652 0.1500 0.1841 0.1860 0.1479 0.1594 0.1602 0.1403 0.1657 0.1580 0.1351 0.1234 0.1097 0.1138 0.1290 0.1177 0.0818 0.0930 0.0992
0.1652 0.9989 0.1742 0.1525 0.1690 0.1727 0.1459 0.1545 0.1434 0.1515 0.1616 0.1102 0.1157 0.1209 0.1127 0.1171 0.1042 0.0918 0.0961 0.0874
0.1500 0.1742 0.9871 0.1849 0.1682 0.1723 0.1668 0.1781 0.1516 0.1610 0.1580 0.1326 0.1358 0.1066 0.1184 0.1342 0.1139 0.0886 0.0941 0.0844
0.1841 0.1525 0.1849 0.9854 0.1577 0.1776 0.1728 0.1486 0.1479 0.1600 0.1367 0.1119 0.1170 0.1394 0.1242 0.1055 0.0950 0.0866 0.0993 0.0990
0.1860 0.1690 0.1682 0.1577 0.9708 0.1721 0.1498 0.1694 0.1598 0.1641 0.1464 0.1279 0.1066 0.1279 0.1172 0.1114 0.0948 0.0767 0.0884 0.0928
0.1479 0.1727 0.1723 0.1776 0.1721 0.9503 0.1594 0.1657 0.1419 0.1275 0.1400 0.1215 0.1387 0.1239 0.1277 0.1224 0.1128 0.0838 0.0820 0.0971
0.1594 0.1459 0.1668 0.1728 0.1498 0.1594 0.8596 0.1655 0.1238 0.1227 0.1208 0.1168 0.1151 0.1179 0.0976 0.1182 0.1020 0.0948 0.0783 0.0751
0.1602 0.1545 0.1781 0.1486 0.1694 0.1657 0.1655 0.8316 0.1509 0.1387 0.1223 0.1095 0.1173 0.1227 0.1171 0.1180 0.1076 0.0839 0.0910 0.0790
0.1403 0.1434 0.1516 0.1479 0.1598 0.1419 0.1238 0.1509 0.7680 0.1253 0.1071 0.0970 0.1100 0.1226 0.1144 0.1194 0.0900 0.0850 0.0853 0.0665
0.1657 0.1515 0.1610 0.1600 0.1641 0.1275 0.1227 0.1387 0.1253 0.7545 0.1059 0.1015 0.1233 0.1049 0.1194 0.1088 0.0964 0.0810 0.0693 0.0812
0.1580 0.1616 0.1580 0.1367 0.1464 0.1400 0.1208 0.1223 0.1071 0.1059 0.6581 0.1027 0.0940 0.1097 0.0956 0.1050 0.0883 0.0779 0.0659 0.0781
0.1351 0.1102 0.1326 0.1119 0.1279 0.1215 0.1168 0.1095 0.0970 0.1015 0.1027 0.5155 0.0940 0.0857 0.0763 0.0904 0.0645 0.0582 0.0583 0.0631
0.1234 0.1157 0.1358 0.1170 0.1066 0.1387 0.1151 0.1173 0.1100 0.1233 0.0940 0.0940 0.5093 0.0858 0.0838 0.0784 0.0709 0.0627 0.0567 0.0561
0.1097 0.1209 0.1066 0.1394 0.1279 0.1239 0.1179 0.1227 0.1226 0.1049 0.1097 0.0857 0.0858 0.4941 0.0894 0.0797 0.0643 0.0555 0.0587 0.0664
0.1138 0.1127 0.1184 0.1242 0.1172 0.1277 0.0976 0.1171 0.1144 0.1194 0.0956 0.0763 0.0838 0.0894 0.4839 0.0919 0.0632 0.0695 0.0593 0.0600
0.1290 0.1171 0.1342 0.1055 0.1114 0.1224 0.1182 0.1180 0.1194 0.1088 0.1050 0.0904 0.0784 0.0797 0.0919 0.4799 0.0773 0.0683 0.0628 0.0548
0.1177 0.1042 0.1139 0.0950 0.0948 0.1128 0.1020 0.1076 0.0900 0.0964 0.0883 0.0645 0.0709 0.0643 0.0632 0.0773 0.3522 0.0506 0.0579 0.0483
0.0818 0.0918 0.0886 0.0866 0.0767 0.0838 0.0948 0.0839 0.0850 0.0810 0.0779 0.0582 0.0627 0.0555 0.0695 0.0683 0.0506 0.2646 0.0474 0.0480
0.0930 0.0961 0.0941 0.0993 0.0884 0.0820 0.0783 0.0910 0.0853 0.0693 0.0659 0.0583 0.0567 0.0587 0.0593 0.0628 0.0579 0.0474 0.2505 0.0383
0.0992 0.0874 0.0844 0.0990 0.0928 0.0971 0.0751 0.0790 0.0665 0.0812 0.0781 0.0631 0.0561 0.0664 0.0600 0.0548 0.0483 0.0480 0.0383 0.2500]

lambda=
[0.0000 0.2056 0.2098 0.2605 0.8580 0.8479 0.8130 0.8012 0.7768 0.7461 0.7071 0.6616 0.6333 0.5936 0.3313 0.4956 0.3845 0.4054 0.4254 0.4181]

V matrix 7

V=
[1.0000 0.2414 0.0772 0.2193 0.0069 0.2022 0.2071 0.1508 0.1791 0.1527 0.0951 0.1820 0.1471 0.1286 0.1665 0.1020 0.1277 0.1030 0.0703 0.0924
0.2414 0.9701 0.2153 0.0383 0.2029 0.0262 0.0513 0.0720 0.1778 0.1183 0.0310 0.1154 0.1864 0.0922 0.1918 0.1134 0.1273 0.0247 0.0126 0.0883
0.0772 0.2153 0.9316 0.1199 0.2441 0.1207 0.1251 0.0251 0.0161 0.1363 0.0685 0.1257 0.0039 0.1743 0.0782 0.0983 0.1756 0.0758 0.1194 0.0992
0.2193 0.0383 0.1199 0.9222 0.2569 0.2611 0.0763 0.1804 0.1884 0.0762 0.1039 0.1560 0.0880 0.1483 0.1511 0.0265 0.1144 0.0102 0.1493 0.1403
0.0069 0.2029 0.2441 0.2569 0.9056 0.2567 0.0369 0.2085 0.1061 0.1623 0.1126 0.0629 0.1723 0.0933 0.0598 0.0590 0.0832 0.1403 0.0168 0.0400
0.2022 0.0262 0.1207 0.2611 0.2567 0.8610 0.2344 0.1984 0.0481 0.0258 0.1664 0.2121 0.0335 0.0970 0.1610 0.1568 0.1148 0.0961 0.0660 0.0242
0.2071 0.0513 0.1251 0.0763 0.0369 0.2344 0.7632 0.1236 0.1580 0.1071 0.1939 0.0454 0.0618 0.1710 0.0395 0.0743 0.1633 0.1247 0.0596 0.1208
0.1508 0.0720 0.0251 0.1804 0.2085 0.1984 0.1236 0.7532 0.1233 0.2036 0.1326 0.1007 0.1240 0.1243 0.0820 0.0101 0.0699 0.1598 0.0812 0.0339
0.1791 0.1778 0.0161 0.1884 0.1061 0.0481 0.1580 0.1233 0.6726 0.0606 0.1824 0.0817 0.1098 0.0266 0.1134 0.1426 0.1528 0.0136 0.1332 0.0461
0.1527 0.1183 0.1363 0.0762 0.1623 0.0258 0.1071 0.2036 0.0606 0.6174 0.0856 0.0939 0.0097 0.0371 0.0966 0.0395 0.0778 0.1049 0.0916 0.0760
0.0951 0.0310 0.0685 0.1039 0.1126 0.1664 0.1939 0.1326 0.1824 0.0856 0.6138 0.0085 0.1543 0.0219 0.0424 0.0442 0.0812 0.1446 0.0289 0.0372
0.1820 0.1154 0.1257 0.1560 0.0629 0.2121 0.0454 0.1007 0.0817 0.0939 0.0085 0.5961 0.1273 0.0919 0.1114 0.0339 0.0559 0.0154 0.0097 0.0223
0.1471 0.1864 0.0039 0.0880 0.1723 0.0335 0.0618 0.1240 0.1098 0.0097 0.1543 0.1273 0.5699 0.1408 0.0220 0.0876 0.1224 0.0638 0.1097 0.0264
0.1286 0.0922 0.1743 0.1483 0.0933 0.0970 0.1710 0.1243 0.0266 0.0371 0.0219 0.0919 0.1408 0.5480 0.0836 0.0820 0.0771 0.0182 0.0569 0.0244
0.1665 0.1918 0.0782 0.1511 0.0598 0.1610 0.0395 0.0820 0.1134 0.0966 0.0424 0.1114 0.0220 0.0836 0.5369 0.1245 0.1141 0.0955 0.1187 0.0087

0.1020	0.1134	0.0983	0.0265	0.0590	0.1568	0.0743	0.0101	0.1426	0.0395	0.0442	0.0339	0.0876	0.0820	0.1245	0.4280	0.0607	0.0371	0.1178	0.0122
0.1277	0.1273	0.1756	0.1144	0.0832	0.1148	0.1633	0.0699	0.1528	0.0778	0.0812	0.0559	0.1224	0.0771	0.1141	0.0607	0.4052	0.0076	0.0463	0.0513
0.1030	0.0247	0.0758	0.0102	0.1403	0.0961	0.1247	0.1598	0.0136	0.1049	0.1446	0.0154	0.0638	0.0182	0.0955	0.0371	0.0076	0.3806	0.0382	0.0637
0.0703	0.0126	0.1194	0.1493	0.0168	0.0660	0.0596	0.0812	0.1332	0.0916	0.0289	0.0097	0.1097	0.0569	0.1187	0.1178	0.0463	0.0382	0.3663	0.0654
0.0924	0.0883	0.0992	0.1403	0.0400	0.0242	0.1208	0.0339	0.0461	0.0760	0.0372	0.0223	0.0264	0.0244	0.0087	0.0122	0.0513	0.0637	0.0654	0.2500]

lambda=
[1.1092 1.0795 0.0000 0.9106 0.8875 0.7705 0.7312 0.1167 0.1610 0.6538 0.6318 0.2126 0.2434 0.5385 0.5112 0.4571 0.4519 0.3713 0.3452 0.3456]

Reference variance matrices of dimension 25

V matrix 1

V=

[1.0000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000
0.3000	1.0000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000
0.3000	0.3000	1.0000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000
0.3000	0.3000	0.3000	1.0000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000
0.3000	0.3000	0.3000	0.3000	1.0000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000
0.3000	0.3000	0.3000	0.3000	0.3000	1.0000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000
0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	1.0000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000
0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	1.0000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000
0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	1.0000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000
0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	1.0000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000
0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	1.0000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000
0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	1.0000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000
0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	1.0000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000
0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	1.0000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000
0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	1.0000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000
0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	1.0000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000
0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	1.0000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000
0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	1.0000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000
0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	1.0000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000
0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	1.0000	0.3000	0.3000	0.3000	0.3000	0.3000
0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	1.0000	0.3000	0.3000	0.3000	0.3000
0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	1.0000	0.3000	0.3000	0.3000
0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	1.0000	0.3000	0.3000
0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	1.0000	0.3000
0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000	1.0000]

lambda=
[0.0000 0.7000]

V matrix 2 case a

V=
[1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 0.3000 1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 0.3000 0.3000 1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 0.3000 0.3000 0.3000 1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 0.3000 0.3000 0.3000 0.3000 1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 1.0000 0.3000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 1.0000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 1.0000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 1.0000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 1.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.3000 1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.3000 0.3000 1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.3000 0.3000 0.3000 1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.3000 0.3000 0.3000 0.3000 1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.3000 0.3000 0.3000 0.3000 0.3000 1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 1.0000 0.3000 0.3000 0.3000 0.3000 0.3000
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 1.0000 0.3000 0.3000 0.3000 0.3000
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 1.0000 0.3000 0.3000 0.3000
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 1.0000 0.3000 0.3000
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 1.0000 0.3000
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 1.0000]

lambda=
[0.7000 4.4440 0.0000 0.7000]

V matrix 2 case b

V=
[1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 0.3000 1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 0.3000 0.3000 1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 0.3000 0.3000 0.3000 1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 0.3000 0.3000 0.3000 0.3000 1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 1.0000 0.3000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 1.0000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 1.0000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 1.0000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 1.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.2500 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0250 0.2500 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0250 0.0250 0.2500 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0250 0.0250 0.0250 0.2500 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250

0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0250 0.0250 0.0250 0.0250 0.2500 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0250 0.0250 0.0250 0.0250 0.0250 0.2500 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.2500 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.2500 0.0250 0.0250 0.0250 0.0250 0.0250
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.2500 0.0250 0.0250 0.0250 0.0250
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.2500 0.0250 0.0250 0.0250
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.2500 0.0250 0.0250
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.2500 0.0250
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.0250 0.2500]

lambda=
[0.2250 2.5000 0.2250 0.0000 0.7000 0.7000 0.2250 0.2250 0.2250 0.7000 0.7000 0.2250 0.2250 0.7000 0.7000 0.7000 0.2250 0.2250 0.7000 0.7000 0.2250 0.2250 0.2250 0.7000 0.7000]

V matrix 2 case c

V=
[1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 0.3000 1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 0.3000 0.3000 1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 0.3000 0.3000 0.3000 1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 0.3000 0.3000 0.3000 0.3000 1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 1.0000 0.3000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 1.0000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 1.0000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 1.0000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 1.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.2500 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0750 0.2500 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0750 0.0750 0.2500 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0750 0.0750 0.0750 0.2500 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0750 0.0750 0.0750 0.0750 0.2500 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0750 0.0750 0.0750 0.0750 0.0750 0.2500 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.2500 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.2500 0.0750 0.0750 0.0750 0.0750 0.0750
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.2500 0.0750 0.0750 0.0750 0.0750
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.2500 0.0750 0.0750 0.0750
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.2500 0.0750 0.0750
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.2500 0.0750
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.0750 0.2500]

lambda=
[0.1750 0.1750 2.7880 0.0000 0.7000 0.7000 0.7000 0.1750 0.1750 0.1750 0.1750 0.1750 0.1750 0.1750 0.1750 0.7000 0.7000 0.7000 0.7000 0.7000 0.7000 0.1750 0.1750 0.7000 0.7000]

V matrix 2 case d

V=
[1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 0.3000 1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000

0.3000 0.3000 0.3000 0.3000 1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 0.3000 0.3000 0.3000 0.3000 1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 1.0000 0.3000 0.3000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 1.0000 0.3000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 1.0000 0.3000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 1.0000 0.3000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 1.0000 0.3000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 0.3000 1.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.2500 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.1250 0.2500 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.1250 0.1250 0.2500 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.1250 0.1250 0.1250 0.2500 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.1250 0.1250 0.1250 0.1250 0.2500 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.1250 0.1250 0.1250 0.1250 0.1250 0.2500 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.2500 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.2500 0.1250 0.1250 0.1250 0.1250 0.1250
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.2500 0.1250 0.1250 0.1250 0.1250
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.2500 0.1250 0.1250 0.1250
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.2500 0.1250 0.1250
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.2500 0.1250
0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.1250 0.2500]

lambda=
[0.1250 3.0760 0.0000 0.1250 0.1250 0.7000 0.7000 0.1250 0.1250 0.7000 0.7000 0.7000 0.1250 0.1250 0.1250 0.1250 0.7000 0.7000 0.7000 0.7000 0.1250 0.1250 0.1250 0.7000 0.7000]

V matrix 3 case a

V=
[1.0000 0.2906 0.2785 0.2773 0.2764 0.2636 0.2533 0.2519 0.2406 0.2270 0.2230 0.2162 0.2123 0.2071 0.1993 0.1762 0.1594 0.1594 0.1584 0.1584 0.1423 0.1029 0.1012 0.0990 0.0949
0.2906 0.9386 0.2698 0.2686 0.2677 0.2554 0.2454 0.2440 0.2331 0.2199 0.2160 0.2095 0.2056 0.2007 0.1931 0.1707 0.1545 0.1545 0.1535 0.1535 0.1379 0.0997 0.0980 0.0959 0.0919
0.2785 0.2698 0.8616 0.2574 0.2565 0.2447 0.2352 0.2338 0.2234 0.2107 0.2070 0.2007 0.1970 0.1923 0.1850 0.1636 0.1480 0.1480 0.1471 0.1470 0.1321 0.0955 0.0939 0.0919 0.0881
0.2773 0.2686 0.2574 0.8543 0.2554 0.2437 0.2342 0.2328 0.2224 0.2098 0.2061 0.1998 0.1962 0.1915 0.1843 0.1629 0.1474 0.1473 0.1464 0.1464 0.1315 0.0951 0.0935 0.0915 0.0877
0.2764 0.2677 0.2565 0.2554 0.8486 0.2428 0.2334 0.2320 0.2217 0.2091 0.2054 0.1992 0.1955 0.1908 0.1836 0.1623 0.1469 0.1469 0.1460 0.1459 0.1311 0.0948 0.0932 0.0912 0.0874
0.2636 0.2554 0.2447 0.2437 0.2428 0.7721 0.2226 0.2213 0.2114 0.1995 0.1959 0.1900 0.1865 0.1820 0.1752 0.1548 0.1401 0.1401 0.1392 0.1392 0.1250 0.0904 0.0889 0.0870 0.0834
0.2533 0.2454 0.2352 0.2342 0.2334 0.2226 0.7131 0.2127 0.2032 0.1917 0.1883 0.1826 0.1792 0.1749 0.1683 0.1488 0.1347 0.1346 0.1338 0.1338 0.1202 0.0869 0.0854 0.0836 0.0801
0.2519 0.2440 0.2338 0.2328 0.2320 0.2213 0.2127 0.7049 0.2020 0.1906 0.1872 0.1815 0.1782 0.1739 0.1674 0.1479 0.1339 0.1338 0.1330 0.1330 0.1195 0.0864 0.0849 0.0831 0.0797
0.2406 0.2331 0.2234 0.2224 0.2217 0.2114 0.2032 0.2020 0.6434 0.1821 0.1789 0.1734 0.1703 0.1662 0.1599 0.1413 0.1279 0.1279 0.1271 0.1271 0.1141 0.0825 0.0812 0.0794 0.0761
0.2270 0.2199 0.2107 0.2098 0.2091 0.1995 0.1917 0.1906 0.1821 0.5726 0.1687 0.1636 0.1606 0.1568 0.1508 0.1333 0.1207 0.1206 0.1199 0.1199 0.1077 0.0779 0.0766 0.0749 0.0718
0.2230 0.2160 0.2070 0.2061 0.2054 0.1959 0.1883 0.1872 0.1789 0.1687 0.5525 0.1607 0.1578 0.1540 0.1482 0.1310 0.1185 0.1185 0.1178 0.1178 0.1058 0.0765 0.0752 0.0736 0.0705
0.2162 0.2095 0.2007 0.1998 0.1992 0.1900 0.1826 0.1815 0.1734 0.1636 0.1607 0.5194 0.1530 0.1493 0.1437 0.1270 0.1149 0.1149 0.1142 0.1142 0.1026 0.0742 0.0729 0.0713 0.0684
0.2123 0.2056 0.1970 0.1962 0.1955 0.1865 0.1792 0.1782 0.1703 0.1606 0.1578 0.1530 0.5006 0.1466 0.1410 0.1247 0.1128 0.1128 0.1121 0.1121 0.1007 0.0728 0.0716 0.0700 0.0671
0.2071 0.2007 0.1923 0.1915 0.1908 0.1820 0.1749 0.1739 0.1662 0.1568 0.1540 0.1493 0.1466 0.4768 0.1376 0.1217 0.1101 0.1101 0.1094 0.1094 0.0983 0.0711 0.0699 0.0684 0.0655
0.1993 0.1931 0.1850 0.1843 0.1836 0.1752 0.1683 0.1674 0.1599 0.1508 0.1482 0.1437 0.1410 0.1376 0.4415 0.1171 0.1060 0.1059 0.1053 0.1053 0.0946 0.0684 0.0672 0.0658 0.0630
0.1762 0.1707 0.1636 0.1629 0.1623 0.1548 0.1488 0.1479 0.1413 0.1333 0.1310 0.1270 0.1247 0.1217 0.1171 0.3450 0.0937 0.0936 0.0931 0.0930 0.0836 0.0604 0.0594 0.0581 0.0557
0.1594 0.1545 0.1480 0.1474 0.1469 0.1401 0.1347 0.1339 0.1279 0.1207 0.1185 0.1149 0.1128 0.1101 0.1060 0.0937 0.2825 0.0847 0.0842 0.0842 0.0756 0.0547 0.0538 0.0526 0.0504
0.1594 0.1545 0.1480 0.1473 0.1469 0.1401 0.1346 0.1338 0.1279 0.1206 0.1185 0.1149 0.1128 0.1101 0.1059 0.0936 0.0847 0.2824 0.0842 0.0842 0.0756 0.0547 0.0538 0.0526 0.0504
0.1584 0.1535 0.1471 0.1464 0.1460 0.1392 0.1338 0.1330 0.1271 0.1199 0.1178 0.1142 0.1121 0.1094 0.1053 0.0931 0.0842 0.0842 0.2789 0.0837 0.0752 0.0544 0.0534 0.0523 0.0501
0.1584 0.1535 0.1470 0.1464 0.1459 0.1392 0.1338 0.1330 0.1271 0.1199 0.1178 0.1142 0.1121 0.1094 0.1053 0.0930 0.0842 0.0842 0.0837 0.2788 0.0751 0.0543 0.0534 0.0523 0.0501
0.1423 0.1379 0.1321 0.1315 0.1311 0.1250 0.1202 0.1195 0.1141 0.1077 0.1058 0.1026 0.1007 0.0983 0.0946 0.0836 0.0756 0.0756 0.0752 0.0751 0.2250 0.0488 0.0480 0.0470 0.0450
0.1029 0.0997 0.0955 0.0951 0.0948 0.0904 0.0869 0.0864 0.0825 0.0779 0.0765 0.0742 0.0728 0.0711 0.0684 0.0604 0.0547 0.0547 0.0544 0.0543 0.0488 0.1177 0.0347 0.0340 0.0325
0.1012 0.0980 0.0939 0.0935 0.0932 0.0889 0.0854 0.0849 0.0812 0.0766 0.0752 0.0729 0.0716 0.0699 0.0672 0.0594 0.0538 0.0538 0.0534 0.0534 0.0480 0.0347 0.1137 0.0334 0.0320
0.0990 0.0959 0.0919 0.0915 0.0912 0.0870 0.0836 0.0831 0.0794 0.0749 0.0736 0.0713 0.0700 0.0684 0.0658 0.0581 0.0526 0.0526 0.0523 0.0523 0.0470 0.0340 0.0334 0.1089 0.0313
0.0949 0.0919 0.0881 0.0877 0.0874 0.0834 0.0801 0.0797 0.0761 0.0718 0.0705 0.0684 0.0671 0.0655 0.0630 0.0557 0.0504 0.0504 0.0501 0.0501 0.0450 0.0325 0.0320 0.0313 0.1000]

lambda=
[0.7668 0.6742 0.0000 0.6245 0.6007 0.5956 0.5452 0.5031 0.4950 0.4514 0.0721 0.0778 0.0813 0.4002 0.3857 0.3619 0.3480 0.3304 0.3044 0.1384 0.2371 0.1724 0.1965 0.1977 0.1952]

V matrix 3 case b

V=
[1.0000 0.2885 0.2838 0.2831 0.2792 0.2788 0.2653 0.2638 0.2618 0.2590 0.2570 0.2501 0.2430 0.2367 0.2268 0.2192 0.2135 0.2134 0.2075 0.2072 0.2051 0.1886 0.1879 0.1808 0.1500
0.2885 0.9248 0.2729 0.2722 0.2685 0.2681 0.2551 0.2537 0.2518 0.2490 0.2471 0.2405 0.2337 0.2276 0.2181 0.2108 0.2053 0.2052 0.1996 0.1993 0.1972 0.1813 0.1807 0.1739 0.1443
0.2838 0.2729 0.8950 0.2678 0.2641 0.2637 0.2510 0.2496 0.2477 0.2450 0.2431 0.2366 0.2299 0.2239 0.2146 0.2074 0.2020 0.2019 0.1963 0.1960 0.1940 0.1784 0.1778 0.1710 0.1419
0.2831 0.2722 0.2678 0.8902 0.2634 0.2630 0.2503 0.2489 0.2470 0.2443 0.2425 0.2360 0.2293 0.2233 0.2140 0.2068 0.2014 0.2013 0.1958 0.1955 0.1935 0.1779 0.1773 0.1706 0.1415
0.2792 0.2685 0.2641 0.2634 0.8662 0.2595 0.2469 0.2456 0.2437 0.2410 0.2392 0.2328 0.2262 0.2203 0.2111 0.2040 0.1987 0.1986 0.1931 0.1929 0.1909 0.1755 0.1749 0.1683 0.1396
0.2788 0.2681 0.2637 0.2630 0.2595 0.8635 0.2465 0.2452 0.2433 0.2406 0.2388 0.2324 0.2258 0.2199 0.2108 0.2037 0.1984 0.1983 0.1928 0.1925 0.1906 0.1752 0.1746 0.1680 0.1394
0.2653 0.2551 0.2510 0.2503 0.2469 0.2465 0.7821 0.2333 0.2315 0.2290 0.2273 0.2212 0.2149 0.2093 0.2006 0.1938 0.1888 0.1887 0.1835 0.1833 0.1814 0.1668 0.1662 0.1599 0.1327
0.2638 0.2537 0.2496 0.2489 0.2456 0.2452 0.2333 0.7734 0.2303 0.2277 0.2260 0.2200 0.2137 0.2081 0.1995 0.1928 0.1878 0.1877 0.1825 0.1822 0.1804 0.1658 0.1652 0.1590 0.1319
0.2618 0.2518 0.2477 0.2470 0.2437 0.2433 0.2315 0.2303 0.7617 0.2260 0.2243 0.2183 0.2121 0.2066 0.1980 0.1913 0.1863 0.1862 0.1811 0.1808 0.1790 0.1646 0.1640 0.1578 0.1309
0.2590 0.2490 0.2450 0.2443 0.2410 0.2406 0.2290 0.2277 0.2260 0.7452 0.2218 0.2159 0.2098 0.2043 0.1958 0.1892 0.1843 0.1842 0.1791 0.1789 0.1770 0.1628 0.1622 0.1561 0.1295
0.2570 0.2471 0.2431 0.2425 0.2392 0.2388 0.2273 0.2260 0.2243 0.2218 0.7337 0.2143 0.2082 0.2027 0.1943 0.1877 0.1829 0.1828 0.1777 0.1775 0.1757 0.1615 0.1609 0.1549 0.1285
0.2501 0.2405 0.2366 0.2360 0.2328 0.2324 0.2212 0.2200 0.2183 0.2159 0.2143 0.6952 0.2026 0.1973 0.1891 0.1827 0.1780 0.1779 0.1730 0.1728 0.1710 0.1572 0.1567 0.1507 0.1251
0.2430 0.2337 0.2299 0.2293 0.2262 0.2258 0.2149 0.2137 0.2121 0.2098 0.2082 0.2026 0.6563 0.1917 0.1838 0.1776 0.1730 0.1729 0.1681 0.1679 0.1661 0.1528 0.1522 0.1465 0.1215
0.2367 0.2276 0.2239 0.2233 0.2203 0.2199 0.2093 0.2081 0.2066 0.2043 0.2027 0.1973 0.1917 0.6224 0.1790 0.1729 0.1684 0.1683 0.1637 0.1635 0.1618 0.1488 0.1482 0.1426 0.1183
0.2268 0.2181 0.2146 0.2140 0.2111 0.2108 0.2006 0.1995 0.1980 0.1958 0.1943 0.1891 0.1838 0.1790 0.5717 0.1657 0.1614 0.1613 0.1569 0.1567 0.1551 0.1426 0.1421 0.1367 0.1134
0.2192 0.2108 0.2074 0.2068 0.2040 0.2037 0.1938 0.1928 0.1913 0.1892 0.1877 0.1827 0.1776 0.1729 0.1657 0.5338 0.1560 0.1559 0.1516 0.1514 0.1498 0.1378 0.1373 0.1321 0.1096
0.2135 0.2053 0.2020 0.2014 0.1987 0.1984 0.1888 0.1878 0.1863 0.1843 0.1829 0.1780 0.1730 0.1684 0.1614 0.1560 0.5065 0.1519 0.1477 0.1475 0.1459 0.1342 0.1337 0.1287 0.1068
0.2134 0.2052 0.2019 0.2013 0.1986 0.1983 0.1887 0.1877 0.1862 0.1842 0.1828 0.1779 0.1729 0.1683 0.1613 0.1559 0.1519 0.5059 0.1476 0.1474 0.1459 0.1341 0.1336 0.1286 0.1067
0.2075 0.1996 0.1963 0.1958 0.1931 0.1928 0.1835 0.1825 0.1811 0.1791 0.1777 0.1730 0.1681 0.1637 0.1569 0.1516 0.1477 0.1476 0.4785 0.1433 0.1419 0.1304 0.1300 0.1251 0.1038
0.2072 0.1993 0.1960 0.1955 0.1929 0.1925 0.1833 0.1822 0.1808 0.1789 0.1775 0.1728 0.1679 0.1635 0.1567 0.1514 0.1475 0.1474 0.1433 0.4771 0.1416 0.1302 0.1298 0.1249 0.1036
0.2051 0.1972 0.1940 0.1935 0.1909 0.1906 0.1814 0.1804 0.1790 0.1770 0.1757 0.1710 0.1661 0.1618 0.1551 0.1498 0.1459 0.1459 0.1419 0.1416 0.4673 0.1289 0.1284 0.1236 0.1025
0.1886 0.1813 0.1784 0.1779 0.1755 0.1752 0.1668 0.1658 0.1646 0.1628 0.1615 0.1572 0.1528 0.1488 0.1426 0.1378 0.1342 0.1341 0.1304 0.1302 0.1289 0.3951 0.1181 0.1136 0.0943
0.1879 0.1807 0.1778 0.1773 0.1749 0.1746 0.1662 0.1652 0.1640 0.1622 0.1609 0.1567 0.1522 0.1482 0.1421 0.1373 0.1337 0.1336 0.1300 0.1298 0.1284 0.1181 0.3922 0.1132 0.0939
0.1808 0.1739 0.1710 0.1706 0.1683 0.1680 0.1599 0.1590 0.1578 0.1561 0.1549 0.1507 0.1465 0.1426 0.1367 0.1321 0.1287 0.1286 0.1251 0.1249 0.1236 0.1136 0.1132 0.3632 0.0904
0.1500 0.1443 0.1419 0.1415 0.1396 0.1394 0.1327 0.1319 0.1309 0.1295 0.1285 0.1251 0.1215 0.1183 0.1134 0.1096 0.1068 0.1067 0.1038 0.1036 0.1025 0.0943 0.0939 0.0904 0.2500]

lambda=
[0.0000 0.7040 0.1939 0.6483 0.6266 0.6231 0.6035 0.6059 0.5462 0.5394 0.5306 0.5190 0.5082 0.4818 0.4543 0.4294 0.2603 0.2756 0.2991 0.3950 0.3696 0.3294 0.3344 0.3462 0.3543]

V matrix 3 case c

V=
[1.0000 0.2988 0.2974 0.2927 0.2924 0.2901 0.2874 0.2838 0.2832 0.2816 0.2811 0.2763 0.2729 0.2714 0.2706 0.2698 0.2646 0.2595 0.2552 0.2525 0.2512 0.2454 0.2427 0.2333 0.2324
0.2988 0.9919 0.2962 0.2915 0.2913 0.2890 0.2863 0.2826 0.2820 0.2805 0.2799 0.2752 0.2717 0.2703 0.2695 0.2687 0.2635 0.2584 0.2542 0.2515 0.2501 0.2444 0.2417 0.2323 0.2314
0.2974 0.2962 0.9827 0.2902 0.2899 0.2876 0.2849 0.2813 0.2807 0.2792 0.2786 0.2740 0.2705 0.2691 0.2683 0.2675 0.2623 0.2572 0.2530 0.2503 0.2490 0.2433 0.2406 0.2313 0.2304
0.2927 0.2915 0.2902 0.9521 0.2854 0.2831 0.2805 0.2769 0.2763 0.2748 0.2743 0.2696 0.2662 0.2648 0.2640 0.2633 0.2582 0.2532 0.2490 0.2464 0.2451 0.2395 0.2368 0.2276 0.2267
0.2924 0.2913 0.2899 0.2854 0.9503 0.2828 0.2802 0.2767 0.2760 0.2745 0.2740 0.2694 0.2660 0.2646 0.2638 0.2630 0.2579 0.2530 0.2488 0.2462 0.2448 0.2392 0.2366 0.2274 0.2265
0.2901 0.2890 0.2876 0.2831 0.2828 0.9354 0.2780 0.2745 0.2739 0.2724 0.2718 0.2673 0.2639 0.2625 0.2617 0.2610 0.2559 0.2510 0.2468 0.2442 0.2429 0.2374 0.2348 0.2256 0.2247
0.2874 0.2863 0.2849 0.2805 0.2802 0.2780 0.9179 0.2719 0.2713 0.2698 0.2693 0.2648 0.2614 0.2601 0.2593 0.2585 0.2535 0.2486 0.2445 0.2419 0.2406 0.2351 0.2325 0.2235 0.2226
0.2838 0.2826 0.2813 0.2769 0.2767 0.2745 0.2719 0.8949 0.2679 0.2664 0.2659 0.2614 0.2581 0.2568 0.2560 0.2553 0.2503 0.2455 0.2414 0.2389 0.2376 0.2322 0.2296 0.2207 0.2198
0.2832 0.2820 0.2807 0.2763 0.2760 0.2739 0.2713 0.2679 0.8908 0.2658 0.2653 0.2608 0.2575 0.2562 0.2554 0.2547 0.2497 0.2449 0.2409 0.2384 0.2371 0.2316 0.2291 0.2202 0.2193
0.2816 0.2805 0.2792 0.2748 0.2745 0.2724 0.2698 0.2664 0.2658 0.8811 0.2638 0.2594 0.2561 0.2548 0.2540 0.2533 0.2484 0.2436 0.2396 0.2370 0.2358 0.2304 0.2278 0.2190 0.2181
0.2811 0.2799 0.2786 0.2743 0.2740 0.2718 0.2693 0.2659 0.2653 0.2638 0.8778 0.2589 0.2556 0.2543 0.2535 0.2528 0.2479 0.2431 0.2391 0.2366 0.2353 0.2299 0.2274 0.2186 0.2177
0.2763 0.2752 0.2740 0.2696 0.2694 0.2673 0.2648 0.2614 0.2608 0.2594 0.2589 0.8485 0.2513 0.2500 0.2493 0.2486 0.2437 0.2390 0.2351 0.2326 0.2314 0.2261 0.2236 0.2149 0.2141
0.2729 0.2717 0.2705 0.2662 0.2660 0.2639 0.2614 0.2581 0.2575 0.2561 0.2556 0.2513 0.8272 0.2469 0.2461 0.2454 0.2407 0.2360 0.2321 0.2297 0.2284 0.2232 0.2208 0.2122 0.2114

0.2714	0.2703	0.2691	0.2648	0.2646	0.2625	0.2601	0.2568	0.2562	0.2548	0.2543	0.2500	0.2469	0.8186	0.2448	0.2441	0.2394	0.2348	0.2309	0.2285	0.2272	0.2220	0.2196	0.2111	0.2103
0.2706	0.2695	0.2683	0.2640	0.2638	0.2617	0.2593	0.2560	0.2554	0.2540	0.2535	0.2493	0.2461	0.2448	0.8136	0.2434	0.2387	0.2341	0.2302	0.2278	0.2266	0.2214	0.2189	0.2104	0.2096
0.2698	0.2687	0.2675	0.2633	0.2630	0.2610	0.2585	0.2553	0.2547	0.2533	0.2528	0.2486	0.2454	0.2441	0.2434	0.8090	0.2380	0.2334	0.2296	0.2271	0.2259	0.2207	0.2183	0.2098	0.2090
0.2646	0.2635	0.2623	0.2582	0.2579	0.2559	0.2535	0.2503	0.2497	0.2484	0.2479	0.2437	0.2407	0.2394	0.2387	0.2380	0.7780	0.2289	0.2251	0.2227	0.2215	0.2165	0.2141	0.2058	0.2050
0.2595	0.2584	0.2572	0.2532	0.2530	0.2510	0.2486	0.2455	0.2449	0.2436	0.2431	0.2390	0.2360	0.2348	0.2341	0.2334	0.2289	0.7482	0.2208	0.2184	0.2172	0.2123	0.2099	0.2018	0.2010
0.2552	0.2542	0.2530	0.2490	0.2488	0.2468	0.2445	0.2414	0.2409	0.2396	0.2391	0.2351	0.2321	0.2309	0.2302	0.2296	0.2251	0.2208	0.7237	0.2148	0.2137	0.2088	0.2065	0.1985	0.1977
0.2525	0.2515	0.2503	0.2464	0.2462	0.2442	0.2419	0.2389	0.2384	0.2370	0.2366	0.2326	0.2297	0.2285	0.2278	0.2271	0.2227	0.2184	0.2148	0.7086	0.2114	0.2066	0.2043	0.1964	0.1956
0.2512	0.2501	0.2490	0.2451	0.2448	0.2429	0.2406	0.2376	0.2371	0.2358	0.2353	0.2314	0.2284	0.2272	0.2266	0.2259	0.2215	0.2172	0.2137	0.2114	0.7009	0.2055	0.2032	0.1953	0.1946
0.2454	0.2444	0.2433	0.2395	0.2392	0.2374	0.2351	0.2322	0.2316	0.2304	0.2299	0.2261	0.2232	0.2220	0.2214	0.2207	0.2165	0.2123	0.2088	0.2066	0.2055	0.6692	0.1986	0.1908	0.1901
0.2427	0.2417	0.2406	0.2368	0.2366	0.2348	0.2325	0.2296	0.2291	0.2278	0.2274	0.2236	0.2208	0.2196	0.2189	0.2183	0.2141	0.2099	0.2065	0.2043	0.2032	0.1986	0.6546	0.1887	0.1880
0.2333	0.2323	0.2313	0.2276	0.2274	0.2256	0.2235	0.2207	0.2202	0.2190	0.2186	0.2149	0.2122	0.2111	0.2104	0.2098	0.2058	0.2018	0.1985	0.1964	0.1953	0.1908	0.1887	0.6047	0.1807
0.2324	0.2314	0.2304	0.2267	0.2265	0.2247	0.2226	0.2198	0.2193	0.2181	0.2177	0.2141	0.2114	0.2103	0.2096	0.2090	0.2050	0.2010	0.1977	0.1956	0.1946	0.1901	0.1880	0.1807	0.6000]

lambda=

[0.0000 0.4215 0.4351 0.4623 0.4764 0.4931 0.5019 0.5165 0.5347 0.6987 0.6926 0.6851 0.5526 0.6659 0.6617 0.6517 0.5677 0.5715 0.5770 0.5899 0.6395 0.6057 0.6254 0.6156 0.6210]

V matrix 4 case a

V=

[1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.9913	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.9204	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.8375	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.8203	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.7606	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.7461	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.7304	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.6871	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.6849	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.6474	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.6366	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.6019	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.5958	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.5747	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.5676	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.5349	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.5005	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.4740	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.4633	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.4195	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.4068	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.3994	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2986	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2500]

lambda=

[0.0000 0.9963 0.9718 0.2600 0.9063 0.3113 0.8314 0.8063 0.4021 0.4127 0.4338 0.4678 0.4859 0.7556 0.7392 0.7162 0.5136 0.6860 0.6677 0.5449 0.6424 0.6219 0.5708 0.5990 0.5845]

V matrix 4 case b

V=

[1.0000	0.0651	0.0473	0.0147	0.0559	0.0405	0.0086	0.0132	0.0382	0.0393	0.0222	0.0337	0.0260	0.0024	0.0014	0.0164	0.0007	0.0188	0.0331	0.0044	0.0016	0.0233	0.0228	0.0006	0.0006
0.0651	0.9926	0.0040	0.0252	0.0419	0.0460	0.0443	0.0053	0.0285	0.0275	0.0211	0.0090	0.0387	0.0385	0.0386	0.0250	0.0289	0.0059	0.0218	0.0335	0.0399	0.0103	0.0095	0.0309	0.0081
0.0473	0.0040	0.9781	0.0163	0.0033	0.0050	0.0407	0.0120	0.0525	0.0107	0.0101	0.0577	0.0250	0.0186	0.0166	0.0189	0.0204	0.0287	0.0057	0.0018	0.0203	0.0327	0.0347	0.0094	0.0055
0.0147	0.0252	0.0163	0.9701	0.0436	0.0552	0.0006	0.0086	0.0508	0.0264	0.0525	0.0425	0.0390	0.0188	0.0087	0.0080	0.0099	0.0204	0.0408	0.0227	0.0360	0.0172	0.0169	0.0159	0.0142
0.0559	0.0419	0.0033	0.0436	0.9058	0.0398	0.0601	0.0335	0.0240	0.0118	0.0357	0.0409	0.0206	0.0005	0.0213	0.0376	0.0395	0.0429	0.0389	0.0165	0.0265	0.0264	0.0069	0.0308	0.0190
0.0405	0.0460	0.0050	0.0552	0.0398	0.8457	0.0231	0.0367	0.0405	0.0228	0.0228	0.0353	0.0443	0.0189	0.0208	0.0287	0.0272	0.0323	0.0228	0.0336	0.0202	0.0065	0.0242	0.0323	0.0260
0.0086	0.0443	0.0407	0.0006	0.0601	0.0231	0.8416	0.0480	0.0389	0.0572	0.0529	0.0032	0.0190	0.0278	0.0128	0.0287	0.0024	0.0257	0.0311	0.0415	0.0309	0.0258	0.0149	0.0208	0.0258
0.0132	0.0053	0.0120	0.0086	0.0335	0.0367	0.0480	0.8260	0.0242	0.0545	0.0039	0.0295	0.0153	0.0431	0.0163	0.0324	0.0025	0.0159	0.0219	0.0186	0.0229	0.0213	0.0039	0.0292	0.0240
0.0382	0.0285	0.0525	0.0508	0.0240	0.0405	0.0389	0.0242	0.8084	0.0152	0.0220	0.0250	0.0470	0.0296	0.0158	0.0226	0.0281	0.0071	0.0361	0.0404	0.0240	0.0010	0.0274	0.0196	0.0221
0.0393	0.0275	0.0107	0.0264	0.0118	0.0228	0.0572	0.0545	0.0152	0.7950	0.0411	0.0232	0.0318	0.0469	0.0304	0.0116	0.0165	0.0082	0.0212	0.0172	0.0185	0.0207	0.0024	0.0100	0.0190
0.0222	0.0211	0.0101	0.0525	0.0357	0.0228	0.0529	0.0039	0.0220	0.0411	0.7326	0.0207	0.0141	0.0186	0.0229	0.0324	0.0137	0.0047	0.0184	0.0188	0.0006	0.0216	0.0233	0.0086	0.0078
0.0337	0.0090	0.0577	0.0425	0.0409	0.0353	0.0032	0.0295	0.0250	0.0232	0.0207	0.7022	0.0274	0.0213	0.0199	0.0039	0.0194	0.0150	0.0123	0.0336	0.0285	0.0302	0.0176	0.0004	0.0175
0.0260	0.0387	0.0250	0.0390	0.0206	0.0443	0.0190	0.0153	0.0470	0.0318	0.0141	0.0274	0.6759	0.0343	0.0392	0.0123	0.0108	0.0376	0.0052	0.0364	0.0259	0.0265	0.0065	0.0134	0.0023
0.0024	0.0385	0.0186	0.0188	0.0005	0.0189	0.0278	0.0431	0.0296	0.0469	0.0186	0.0213	0.0343	0.6237	0.0158	0.0341	0.0019	0.0231	0.0363	0.0107	0.0315	0.0030	0.0019	0.0066	0.0258
0.0014	0.0386	0.0166	0.0087	0.0213	0.0208	0.0128	0.0163	0.0158	0.0304	0.0229	0.0199	0.0392	0.0158	0.5790	0.0393	0.0007	0.0306	0.0229	0.0200	0.0083	0.0238	0.0075	0.0205	0.0176
0.0164	0.0250	0.0189	0.0080	0.0376	0.0287	0.0287	0.0324	0.0226	0.0116	0.0324	0.0039	0.0123	0.0341	0.0393	0.5615	0.0363	0.0249	0.0027	0.0025	0.0004	0.0065	0.0145	0.0123	0.0184
0.0007	0.0289	0.0204	0.0099	0.0395	0.0272	0.0024	0.0025	0.0281	0.0165	0.0137	0.0194	0.0108	0.0019	0.0007	0.0363	0.5590	0.0057	0.0044	0.0268	0.0242	0.0185	0.0212	0.0177	0.0231
0.0188	0.0059	0.0287	0.0204	0.0429	0.0323	0.0257	0.0159	0.0071	0.0082	0.0047	0.0150	0.0376	0.0231	0.0306	0.0249	0.0057	0.4900	0.0170	0.0139	0.0207	0.0179	0.0252	0.0048	0.0027
0.0331	0.0218	0.0057	0.0408	0.0389	0.0228	0.0311	0.0219	0.0361	0.0212	0.0184	0.0123	0.0052	0.0363	0.0229	0.0027	0.0044	0.0170	0.4787	0.0173	0.0225	0.0105	0.0093	0.0071	0.0210
0.0044	0.0335	0.0018	0.0227	0.0165	0.0336	0.0415	0.0186	0.0404	0.0172	0.0188	0.0336	0.0364	0.0107	0.0200	0.0025	0.0268	0.0139	0.0173	0.4510	0.0268	0.0221	0.0064	0.0137	0.0037
0.0016	0.0399	0.0203	0.0360	0.0265	0.0202	0.0309	0.0229	0.0240	0.0185	0.0006	0.0285	0.0259	0.0315	0.0083	0.0004	0.0242	0.0207	0.0225	0.0268	0.4105	0.0052	0.0025	0.0036	0.0091
0.0233	0.0103	0.0327	0.0172	0.0264	0.0065	0.0258	0.0213	0.0010	0.0207	0.0216	0.0302	0.0265	0.0030	0.0238	0.0065	0.0185	0.0179	0.0105	0.0221	0.0052	0.2944	0.0167	0.0037	0.0085
0.0228	0.0095	0.0347	0.0169	0.0069	0.0242	0.0149	0.0039	0.0274	0.0024	0.0233	0.0176	0.0065	0.0019	0.0075	0.0145	0.0212	0.0252	0.0093	0.0064	0.0025	0.0167	0.2877	0.0128	0.0003
0.0006	0.0309	0.0094	0.0159	0.0308	0.0323	0.0208	0.0292	0.0196	0.0100	0.0086	0.0004	0.0134	0.0066	0.0205	0.0123	0.0177	0.0048	0.0071	0.0137	0.0036	0.0037	0.0128	0.2613	0.0068
0.0006	0.0081	0.0055	0.0142	0.0190	0.0260	0.0258	0.0240	0.0221	0.0190	0.0078	0.0175	0.0023	0.0258	0.0176	0.0184	0.0231	0.0027	0.0210	0.0037	0.0091	0.0085	0.0003	0.0068	0.2500]

lambda=

[0.0000 0.2423 0.2605 0.2747 0.3252 1.0017 0.9960 0.9544 0.9237 0.3904 0.4317 0.4555 0.4821 0.5274 0.8690 0.5671 0.5953 0.6271 0.6652 0.6891 0.7140 0.8268 0.7630 0.7928 0.7740]

V matrix 4 case c

V=

[1.0000	0.0911	0.0812	0.0674	0.0287	0.0509	0.0555	0.0652	0.0046	0.0097	0.0700	0.0137	0.0029	0.0244	0.0624	0.0177	0.0435	0.0666	0.0223	0.0139	0.0241	0.0622	0.0438	0.0343	0.0391
0.0911	0.9971	0.0316	0.0770	0.0590	0.0601	0.0468	0.0778	0.0661	0.0275	0.0610	0.0347	0.0470	0.0089	0.0243	0.0468	0.0661	0.0693	0.0014	0.0075	0.0267	0.0062	0.0179	0.0361	0.0489
0.0812	0.0316	0.9934	0.0823	0.0216	0.0061	0.0288	0.0486	0.0116	0.0729	0.0271	0.0387	0.0342	0.0203	0.0748	0.0725	0.0699	0.0324	0.0224	0.0005	0.0109	0.0526	0.0392	0.0171	0.0013
0.0674	0.0770	0.0823	0.9561	0.0378	0.0774	0.0532	0.0046	0.0335	0.0249	0.0123	0.0240	0.0003	0.0334	0.0505	0.0606	0.0547	0.0115	0.0365	0.0347	0.0596	0.0275	0.0027	0.0503	0.0007
0.0287	0.0590	0.0216	0.0378	0.8671	0.0798	0.0367	0.0140	0.0487	0.0407	0.0281	0.0572	0.0613	0.0232	0.0424	0.0548	0.0023	0.0575	0.0430	0.0505	0.0566	0.0230	0.0269	0.0044	0.0431
0.0509	0.0601	0.0061	0.0774	0.0798	0.8329	0.0093	0.0463	0.0171	0.0307	0.0227	0.0460	0.0685	0.0004	0.0569	0.0475	0.0662	0.0046	0.0028	0.0457	0.0495	0.0152	0.0393	0.0144	0.0038
0.0555	0.0468	0.0288	0.0532	0.0367	0.0093	0.7586	0.0393	0.0154	0.0495	0.0174	0.0499	0.0277	0.0268	0.0274	0.0098	0.0566	0.0014	0.0544	0.0372	0.0039	0.0351	0.0401	0.0357	0.0352
0.0652	0.0778	0.0486	0.0046	0.0140	0.0463	0.0393	0.7192	0.0210	0.0588	0.0289	0.0355	0.0620	0.0251	0.0414	0.0111	0.0493	0.0223	0.0446	0.0196	0.0435	0.0080	0.0077	0.0289	0.0040
0.0046	0.0661	0.0116	0.0335	0.0487	0.0171	0.0154	0.0210	0.7186	0.0496	0.0269	0.0323	0.0652	0.0519	0.0265	0.0611	0.0124	0.0323	0.0289	0.0277	0.0212	0.0281	0.0423	0.0094	0.0165
0.0097	0.0275	0.0729	0.0249	0.0407	0.0307	0.0495	0.0588	0.0496	0.7106	0.0112	0.0260	0.0648	0.0044	0.0204	0.0080	0.0072	0.0100	0.0054	0.0303	0.0426	0.0224	0.0431	0.0362	0.0004
0.0700	0.0610	0.0271	0.0123	0.0281	0.0227	0.0174	0.0289	0.0269	0.0112	0.6841	0.0107	0.0355	0.0597	0.0276	0.0337	0.0427	0.0001	0.0468	0.0497	0.0011	0.0052	0.0189	0.0426	0.0256
0.0137	0.0347	0.0387	0.0240	0.0572	0.0460	0.0499	0.0355	0.0323	0.0260	0.0107	0.6320	0.0584	0.0126	0.0005	0.0413	0.0236	0.0217	0.0486	0.0507	0.0250	0.0009	0.0399	0.0024	0.0036
0.0029	0.0470	0.0342	0.0003	0.0613	0.0685	0.0277	0.0620	0.0652	0.0648	0.0355	0.0584	0.6316	0.0057	0.0178	0.0492	0.0584	0.0039	0.0523	0.0496	0.0505	0.0285	0.0462	0.0135	0.0106
0.0244	0.0089	0.0203	0.0334	0.0232	0.0004	0.0268	0.0251	0.0519	0.0044	0.0597	0.0126	0.0057	0.6105	0.0194	0.0212	0.0575	0.0529	0.0229	0.0494	0.0257	0.0484	0.0458	0.0300	0.0253
0.0624	0.0243	0.0748	0.0505	0.0424	0.0569	0.0274	0.0414	0.0265	0.0204	0.0276	0.0005	0.0178	0.0194	0.5832	0.0037	0.0348	0.0346	0.0091	0.0211	0.0357	0.0209	0.0098	0.0241	0.0320
0.0177	0.0468	0.0725	0.0606	0.0548	0.0475	0.0098	0.0111	0.0611	0.0080	0.0337	0.0413	0.0492	0.0212	0.0037	0.5674	0.0045	0.0504	0.0085	0.0197	0.0243	0.0389	0.0187	0.0319	0.0007
0.0435	0.0661	0.0699	0.0547	0.0023	0.0662	0.0566	0.0493	0.0124	0.0072	0.0427	0.0236	0.0584	0.0575	0.0348	0.0045	0.5474	0.0299	0.0413	0.0144	0.0124	0.0113	0.0249	0.0085	0.0246

```
 0.0666  0.0693  0.0324  0.0115  0.0575  0.0046  0.0014  0.0223  0.0323  0.0100  0.0001  0.0217  0.0039  0.0529  0.0346  0.0504  0.0299  0.5194  0.0096  0.0195  0.0074  0.0030  0.0053  0.0011  0.0312
 0.0223  0.0014  0.0224  0.0365  0.0430  0.0028  0.0544  0.0446  0.0289  0.0054  0.0468  0.0486  0.0523  0.0229  0.0091  0.0085  0.0413  0.0096  0.4576  0.0198  0.0309  0.0221  0.0410  0.0066  0.0189
 0.0139  0.0075  0.0005  0.0347  0.0505  0.0457  0.0372  0.0196  0.0277  0.0303  0.0497  0.0507  0.0496  0.0494  0.0211  0.0197  0.0144  0.0195  0.0198  0.4570  0.0329  0.0214  0.0023  0.0069  0.0245
 0.0241  0.0267  0.0109  0.0596  0.0566  0.0495  0.0039  0.0435  0.0212  0.0426  0.0011  0.0250  0.0505  0.0257  0.0357  0.0243  0.0124  0.0074  0.0309  0.0329  0.4388  0.0236  0.0389  0.0336  0.0243
 0.0622  0.0062  0.0526  0.0275  0.0230  0.0152  0.0351  0.0080  0.0281  0.0224  0.0052  0.0009  0.0285  0.0484  0.0209  0.0389  0.0113  0.0030  0.0221  0.0214  0.0236  0.4063  0.0313  0.0176  0.0160
 0.0438  0.0179  0.0392  0.0027  0.0269  0.0393  0.0401  0.0077  0.0423  0.0431  0.0189  0.0399  0.0462  0.0458  0.0098  0.0187  0.0249  0.0053  0.0410  0.0023  0.0389  0.0313  0.3738  0.0263  0.0120
 0.0343  0.0361  0.0171  0.0503  0.0044  0.0144  0.0357  0.0289  0.0094  0.0362  0.0426  0.0024  0.0135  0.0300  0.0241  0.0319  0.0085  0.0011  0.0066  0.0069  0.0336  0.0176  0.0263  0.2840  0.0139
 0.0391  0.0489  0.0013  0.0007  0.0431  0.0038  0.0352  0.0040  0.0165  0.0004  0.0256  0.0036  0.0106  0.0253  0.0320  0.0007  0.0246  0.0312  0.0189  0.0245  0.0243  0.0160  0.0120  0.0139  0.2500]

lambda=
 [0.0000  1.0185  0.9885  0.9292  0.9002  0.2357  0.8323  0.2737  0.7997  0.3317  0.7512  0.3737  0.7106  0.3935  0.4236  0.4485  0.4625  0.6763  0.6518  0.6331  0.6045  0.4955  0.5687  0.5472  0.5557]
```

V matrix 5 case a

```
V=
 [1.0000  0.0534  0.0324  0.0381  0.0072  0.0013  0.0010  0.0333  0.0132  0.0359  0.0225  0.0556  0.0067  0.0321  0.0431  0.0271  0.0282  0.0126  0.0253  0.0595  0.0394  0.0519  0.0365  0.0049  0.0560
 0.0534  1.0000  0.0632  0.0215  0.0148  0.0356  0.0038  0.0158  0.0426  0.0273  0.0034  0.0047  0.0386  0.0172  0.0432  0.0039  0.0462  0.0398  0.0262  0.0362  0.0343  0.0143  0.0236  0.0607  0.0545
 0.0324  0.0632  1.0000  0.0471  0.0596  0.0609  0.0517  0.0673  0.0147  0.0651  0.0544  0.0217  0.0005  0.0581  0.0581  0.0263  0.0200  0.0327  0.0691  0.0101  0.0109  0.0164  0.0459  0.0206  0.0650
 0.0381  0.0215  0.0471  1.0000  0.0520  0.0559  0.0445  0.0215  0.0698  0.0198  0.0387  0.0238  0.0088  0.0212  0.0685  0.0460  0.0673  0.0275  0.0621  0.0544  0.0456  0.0629  0.0276  0.0387  0.0508
 0.0072  0.0148  0.0596  0.0520  1.0000  0.0559  0.0652  0.0545  0.0012  0.0539  0.0047  0.0572  0.0039  0.0023  0.0306  0.0491  0.0404  0.0019  0.0130  0.0313  0.0509  0.0551  0.0070  0.0548  0.0067
 0.0013  0.0356  0.0609  0.0559  0.0559  1.0000  0.0323  0.0627  0.0308  0.0509  0.0347  0.0003  0.0534  0.0691  0.0592  0.0064  0.0677  0.0239  0.0171  0.0116  0.0003  0.0128  0.0323  0.0049  0.0315
 0.0010  0.0038  0.0517  0.0445  0.0652  0.0323  1.0000  0.0178  0.0104  0.0640  0.0585  0.0346  0.0050  0.0336  0.0278  0.0664  0.0360  0.0683  0.0173  0.0180  0.0636  0.0354  0.0389  0.0443  0.0684
 0.0333  0.0158  0.0673  0.0215  0.0545  0.0627  0.0178  1.0000  0.0435  0.0074  0.0323  0.0429  0.0503  0.0422  0.0294  0.0374  0.0605  0.0404  0.0122  0.0511  0.0463  0.0043  0.0212  0.0029  0.0678
 0.0132  0.0426  0.0147  0.0698  0.0012  0.0308  0.0104  0.0435  1.0000  0.0622  0.0546  0.0262  0.0098  0.0361  0.0624  0.0637  0.0158  0.0539  0.0234  0.0043  0.0035  0.0095  0.0442  0.0017  0.0452
 0.0359  0.0273  0.0651  0.0198  0.0539  0.0509  0.0640  0.0074  0.0622  1.0000  0.0344  0.0139  0.0431  0.0684  0.0242  0.0217  0.0170  0.0438  0.0284  0.0079  0.0317  0.0016  0.0367  0.0505  0.0065
 0.0225  0.0034  0.0544  0.0387  0.0047  0.0347  0.0585  0.0323  0.0546  0.0344  1.0000  0.0627  0.0330  0.0173  0.0566  0.0507  0.0629  0.0513  0.0437  0.0570  0.0115  0.0609  0.0216  0.0620  0.0540
 0.0556  0.0047  0.0217  0.0238  0.0572  0.0003  0.0346  0.0429  0.0262  0.0139  0.0627  1.0000  0.0357  0.0468  0.0696  0.0268  0.0496  0.0105  0.0629  0.0323  0.0329  0.0430  0.0324  0.0656  0.0283
 0.0067  0.0386  0.0005  0.0088  0.0039  0.0534  0.0050  0.0503  0.0098  0.0431  0.0330  0.0357  1.0000  0.0103  0.0547  0.0629  0.0683  0.0290  0.0475  0.0636  0.0576  0.0237  0.0093  0.0647  0.0196
 0.0321  0.0172  0.0581  0.0212  0.0023  0.0691  0.0336  0.0422  0.0361  0.0684  0.0173  0.0468  0.0103  1.0000  0.0596  0.0048  0.0322  0.0048  0.0092  0.0627  0.0595  0.0480  0.0533  0.0529  0.0159
 0.0431  0.0432  0.0581  0.0685  0.0306  0.0592  0.0278  0.0294  0.0624  0.0242  0.0566  0.0696  0.0547  0.0596  1.0000  0.0185  0.0519  0.0619  0.0331  0.0145  0.0675  0.0152  0.0094  0.0264  0.0101
 0.0271  0.0039  0.0263  0.0460  0.0491  0.0064  0.0664  0.0374  0.0637  0.0217  0.0507  0.0268  0.0629  0.0048  0.0185  1.0000  0.0468  0.0582  0.0677  0.0612  0.0569  0.0162  0.0302  0.0584  0.0065
 0.0282  0.0462  0.0200  0.0673  0.0404  0.0677  0.0360  0.0605  0.0158  0.0170  0.0629  0.0496  0.0683  0.0322  0.0519  0.0468  1.0000  0.0221  0.0349  0.0166  0.0185  0.0310  0.0601  0.0481  0.0492
 0.0126  0.0398  0.0327  0.0275  0.0019  0.0239  0.0683  0.0404  0.0539  0.0438  0.0513  0.0105  0.0290  0.0048  0.0619  0.0582  0.0221  1.0000  0.0278  0.0479  0.0144  0.0234  0.0327  0.0389  0.0477
 0.0253  0.0262  0.0691  0.0621  0.0130  0.0171  0.0173  0.0122  0.0234  0.0284  0.0437  0.0629  0.0475  0.0092  0.0331  0.0677  0.0349  0.0278  1.0000  0.0672  0.0653  0.0187  0.0200  0.0060  0.0320
 0.0595  0.0362  0.0101  0.0544  0.0313  0.0116  0.0180  0.0511  0.0043  0.0079  0.0570  0.0323  0.0636  0.0627  0.0145  0.0612  0.0166  0.0479  0.0672  1.0000  0.0275  0.0528  0.0046  0.0388  0.0373
 0.0394  0.0343  0.0109  0.0456  0.0509  0.0003  0.0636  0.0463  0.0035  0.0317  0.0115  0.0329  0.0576  0.0595  0.0675  0.0569  0.0185  0.0144  0.0653  0.0275  1.0000  0.0291  0.0460  0.0476  0.0209
 0.0519  0.0143  0.0164  0.0629  0.0551  0.0128  0.0354  0.0043  0.0095  0.0016  0.0609  0.0430  0.0237  0.0480  0.0152  0.0162  0.0310  0.0234  0.0187  0.0528  0.0291  1.0000  0.0441  0.0299  0.0282
 0.0365  0.0236  0.0459  0.0276  0.0070  0.0323  0.0389  0.0212  0.0442  0.0367  0.0216  0.0324  0.0093  0.0533  0.0094  0.0302  0.0601  0.0327  0.0200  0.0046  0.0460  0.0441  1.0000  0.0114  0.0107
 0.0049  0.0607  0.0206  0.0387  0.0548  0.0049  0.0443  0.0029  0.0017  0.0505  0.0620  0.0656  0.0647  0.0529  0.0264  0.0584  0.0481  0.0389  0.0060  0.0388  0.0476  0.0299  0.0114  1.0000  0.0553
 0.0560  0.0545  0.0650  0.0508  0.0067  0.0315  0.0684  0.0678  0.0452  0.0065  0.0540  0.0283  0.0196  0.0159  0.0101  0.0065  0.0492  0.0477  0.0320  0.0373  0.0209  0.0282  0.0107  0.0553  1.0000]

lambda=
 [0.0000  0.7833  0.8002  0.8121  1.1355  0.8495  0.8726  0.8824  0.8926  0.8981  1.0956  1.0913  0.9218  0.9318  1.0766  0.9474  0.9662  1.0632  1.0536  1.0438  0.9849  0.9970  1.0220  1.0157  1.0090]
```

V matrix 5 case b

```
V=
 [1.0000  0.0364  0.0646  0.0939  0.0562  0.0631  0.0840  0.0784  0.0953  0.0995  0.0800  0.0689  0.0744  0.0329  0.0430  0.0237  0.0295  0.0878  0.0428  0.0014  0.0270  0.0309  0.0419  0.0055  0.0591
 0.0364  1.0000  0.0124  0.0666  0.0701  0.0591  0.0240  0.0149  0.0624  0.0090  0.0423  0.0720  0.0489  0.0261  0.0049  0.0728  0.0632  0.0728  0.0833  0.0790  0.0935  0.0824  0.0119  0.0142  0.0023
 0.0646  0.0124  1.0000  0.0598  0.0285  0.0054  0.0022  0.0773  0.0458  0.0721  0.0892  0.0939  0.0650  0.0388  0.0905  0.0259  0.0531  0.0853  0.0171  0.0900  0.0221  0.0687  0.0519  0.0408  0.0182
 0.0939  0.0666  0.0598  1.0000  0.0987  0.0869  0.0148  0.0520  0.0648  0.0997  0.0992  0.0008  0.0042  0.0897  0.0978  0.0470  0.0871  0.0178  0.0441  0.0002  0.0190  0.0530  0.0972  0.0778  0.0467
```

0.0562 0.0701 0.0285 0.0987 1.0000 0.0728 0.0476 0.0587 0.0979 0.0108 0.0401 0.0582 0.0335 0.0767 0.0658 0.0858 0.0584 0.0540 0.0752 0.0516 0.0281 0.0178 0.0329 0.0163 0.0613
0.0631 0.0591 0.0054 0.0869 0.0728 1.0000 0.0878 0.0519 0.0421 0.0457 0.0341 0.0556 0.0433 0.0244 0.0727 0.0833 0.0603 0.0472 0.0615 0.0624 0.0674 0.0897 0.0339 0.0026 0.0140
0.0840 0.0240 0.0022 0.0148 0.0476 0.0878 1.0000 0.0141 0.0976 0.0133 0.0317 0.0689 0.0099 0.0455 0.0489 0.0476 0.0488 0.0732 0.0254 0.0265 0.0598 0.0810 0.0802 0.0851 0.0212
0.0784 0.0149 0.0773 0.0520 0.0587 0.0519 0.0141 1.0000 0.0519 0.0704 0.0364 0.0761 0.0681 0.0272 0.0686 0.0340 0.0943 0.0981 0.0350 0.0399 0.0389 0.0389 0.0160 0.0253 0.0990
0.0953 0.0624 0.0458 0.0648 0.0979 0.0421 0.0976 0.0519 1.0000 0.0306 0.0616 0.0824 0.0027 0.0423 0.0363 0.0542 0.0347 0.0667 0.0613 0.0978 0.0936 0.0479 0.0159 0.0395 0.0330
0.0995 0.0090 0.0721 0.0997 0.0108 0.0457 0.0133 0.0704 0.0306 1.0000 0.0597 0.0528 0.0321 0.0699 0.0401 0.0562 0.0200 0.0683 0.0338 0.0304 0.0827 0.0607 0.0164 0.0687 0.0426
0.0800 0.0423 0.0892 0.0992 0.0401 0.0341 0.0317 0.0364 0.0616 0.0597 1.0000 0.0342 0.0052 0.0817 0.0514 0.0925 0.0098 0.0875 0.0636 0.0567 0.0051 0.0652 0.0914 0.0871 0.0146
0.0689 0.0720 0.0939 0.0008 0.0582 0.0556 0.0689 0.0761 0.0824 0.0528 0.0342 1.0000 0.0370 0.0825 0.0046 0.0274 0.0652 0.0508 0.0517 0.0216 0.0828 0.0763 0.0775 0.0681 0.0546
0.0744 0.0489 0.0650 0.0042 0.0335 0.0433 0.0099 0.0681 0.0027 0.0321 0.0052 0.0370 1.0000 0.0403 0.0991 0.0296 0.0606 0.0468 0.0520 0.0135 0.0388 0.0448 0.0430 0.0139 0.0092
0.0329 0.0261 0.0388 0.0897 0.0767 0.0244 0.0455 0.0272 0.0423 0.0699 0.0817 0.0825 0.0403 1.0000 0.0310 0.0613 0.0811 0.0245 0.0626 0.0979 0.0446 0.0643 0.0259 0.0577 0.0435
0.0430 0.0049 0.0905 0.0978 0.0658 0.0727 0.0489 0.0686 0.0363 0.0401 0.0514 0.0046 0.0991 0.0310 1.0000 0.0891 0.0462 0.0566 0.0949 0.0214 0.0009 0.0281 0.0622 0.0684 0.0084
0.0237 0.0728 0.0259 0.0470 0.0858 0.0833 0.0476 0.0340 0.0542 0.0562 0.0925 0.0274 0.0296 0.0613 0.0891 1.0000 0.0094 0.0761 0.0223 0.0136 0.0280 0.0443 0.0985 0.0316 0.0555
0.0295 0.0632 0.0531 0.0871 0.0584 0.0603 0.0488 0.0943 0.0347 0.0200 0.0098 0.0652 0.0606 0.0811 0.0462 0.0094 1.0000 0.0345 0.0995 0.0053 0.0292 0.0248 0.0637 0.0512 0.0018
0.0878 0.0728 0.0853 0.0178 0.0540 0.0472 0.0732 0.0981 0.0667 0.0683 0.0875 0.0508 0.0468 0.0245 0.0566 0.0761 0.0345 1.0000 0.0877 0.0265 0.0430 0.0152 0.0233 0.0446 0.0644
0.0428 0.0833 0.0171 0.0441 0.0752 0.0615 0.0254 0.0350 0.0613 0.0338 0.0636 0.0517 0.0520 0.0626 0.0949 0.0223 0.0995 0.0877 1.0000 0.0757 0.0701 0.0207 0.0784 0.0520 0.0392
0.0014 0.0790 0.0900 0.0002 0.0516 0.0624 0.0265 0.0399 0.0978 0.0304 0.0567 0.0216 0.0135 0.0979 0.0214 0.0136 0.0053 0.0265 0.0757 1.0000 0.0010 0.0914 0.0003 0.0366 0.0327
0.0270 0.0935 0.0221 0.0190 0.0281 0.0674 0.0598 0.0389 0.0936 0.0827 0.0051 0.0828 0.0388 0.0446 0.0009 0.0280 0.0292 0.0430 0.0701 0.0010 1.0000 0.0583 0.0797 0.0652 0.0265
0.0309 0.0824 0.0687 0.0530 0.0178 0.0897 0.0810 0.0389 0.0479 0.0607 0.0652 0.0763 0.0448 0.0643 0.0281 0.0443 0.0248 0.0152 0.0207 0.0914 0.0583 1.0000 0.0208 0.0872 0.0510
0.0419 0.0119 0.0519 0.0972 0.0329 0.0339 0.0802 0.0160 0.0159 0.0164 0.0914 0.0775 0.0430 0.0259 0.0622 0.0985 0.0637 0.0233 0.0784 0.0003 0.0797 0.0208 1.0000 0.0868 0.0456
0.0055 0.0142 0.0408 0.0778 0.0163 0.0026 0.0851 0.0253 0.0395 0.0687 0.0871 0.0681 0.0139 0.0577 0.0684 0.0316 0.0512 0.0446 0.0520 0.0366 0.0652 0.0872 0.0868 1.0000 0.0810
0.0591 0.0023 0.0182 0.0467 0.0613 0.0140 0.0212 0.0990 0.0330 0.0426 0.0146 0.0546 0.0092 0.0435 0.0084 0.0555 0.0018 0.0644 0.0392 0.0327 0.0265 0.0510 0.0456 0.0810 1.0000]

lambda=
[0.0000 0.7050 1.1807 0.7516 0.7747 1.1435 1.1397 1.1173 1.0973 0.7993 1.0595 1.0487 1.0311 1.0238 0.8269 0.8341 0.8431 0.8727 0.8595 0.8944 0.9884 0.9631 0.9557 0.9442 0.9295]

V matrix 6

V=
[1.0000 0.1957 0.1497 0.1620 0.1496 0.1400 0.1793 0.1311 0.1392 0.1434 0.1430 0.1569 0.1502 0.1248 0.1304 0.1300 0.1293 0.1057 0.1144 0.0931 0.1100 0.1004 0.1134 0.09535 0.0859
0.1957 0.9972 0.1726 0.1820 0.1787 0.1475 0.1357 0.1463 0.1648 0.1310 0.1284 0.1360 0.1584 0.1402 0.1448 0.1140 0.1336 0.1234 0.1265 0.1193 0.1116 0.1040 0.1165 0.0920 0.0827
0.1497 0.1726 0.9623 0.1489 0.1575 0.1398 0.1436 0.1369 0.1297 0.1465 0.1224 0.1543 0.1528 0.1214 0.1291 0.1266 0.1104 0.1090 0.1219 0.0985 0.1074 0.1136 0.0973 0.0782 0.0957
0.1620 0.1820 0.1489 0.9045 0.1510 0.1414 0.1421 0.1278 0.1324 0.1273 0.1270 0.1266 0.1482 0.1404 0.1371 0.1240 0.1218 0.1301 0.0906 0.1122 0.1139 0.0887 0.1091 0.0825 0.0879
0.1496 0.1787 0.1575 0.1510 0.8798 0.1607 0.1315 0.1390 0.1409 0.1339 0.1476 0.1315 0.1437 0.1228 0.1225 0.1095 0.1316 0.1098 0.1170 0.0971 0.1002 0.0955 0.0949 0.0887 0.0881
0.1400 0.1475 0.1398 0.1414 0.1607 0.8229 0.1526 0.1285 0.1544 0.1214 0.1439 0.1478 0.1434 0.1255 0.1064 0.1100 0.1124 0.1097 0.0932 0.0954 0.1063 0.1048 0.0972 0.0919 0.0741
0.1793 0.1357 0.1436 0.1421 0.1315 0.1526 0.8065 0.1494 0.1187 0.1383 0.1175 0.1482 0.1185 0.1308 0.1106 0.1302 0.1213 0.1085 0.1135 0.0938 0.1074 0.1007 0.0925 0.0880 0.0811
0.1311 0.1463 0.1369 0.1278 0.1390 0.1285 0.1494 0.7451 0.1281 0.1382 0.1304 0.1341 0.1142 0.1048 0.1298 0.1245 0.1116 0.1067 0.1014 0.1049 0.0950 0.0974 0.1006 0.0763 0.0685
0.1392 0.1648 0.1297 0.1324 0.1409 0.1544 0.1187 0.1281 0.7424 0.1182 0.1283 0.1292 0.1146 0.1341 0.1193 0.1015 0.0937 0.0998 0.1019 0.0948 0.1014 0.0836 0.0983 0.0776 0.0679
0.1434 0.1310 0.1465 0.1273 0.1339 0.1214 0.1383 0.1382 0.1182 0.6983 0.1113 0.1058 0.1303 0.1324 0.0986 0.1164 0.0919 0.0895 0.1032 0.0816 0.0817 0.0941 0.0888 0.0783 0.0770
0.1430 0.1284 0.1224 0.1270 0.1476 0.1439 0.1175 0.1304 0.1283 0.1113 0.6904 0.1149 0.1346 0.1061 0.1024 0.0994 0.1030 0.1032 0.0906 0.0908 0.0882 0.0805 0.0814 0.0701 0.0668
0.1569 0.1360 0.1543 0.1266 0.1315 0.1478 0.1482 0.1341 0.1292 0.1058 0.1149 0.6861 0.1108 0.1044 0.1022 0.1155 0.1013 0.1005 0.1046 0.1004 0.0855 0.0860 0.0801 0.0753 0.0779
0.1502 0.1584 0.1528 0.1482 0.1437 0.1434 0.1185 0.1142 0.1146 0.1303 0.1346 0.1108 0.6855 0.1065 0.0997 0.1035 0.1180 0.0999 0.0993 0.0930 0.0865 0.0754 0.0945 0.0747 0.0635
0.1248 0.1402 0.1214 0.1404 0.1228 0.1255 0.1308 0.1048 0.1341 0.1324 0.1061 0.1044 0.1065 0.6389 0.1168 0.1060 0.1012 0.0864 0.0901 0.0737 0.0853 0.0845 0.0727 0.0678 0.0647
0.1304 0.1448 0.1291 0.1371 0.1225 0.1064 0.1106 0.1298 0.1193 0.0986 0.1024 0.1022 0.0997 0.1168 0.6053 0.1011 0.1095 0.0890 0.0789 0.0810 0.0733 0.0793 0.0750 0.0678 0.0652
0.1300 0.1140 0.1266 0.1240 0.1095 0.1100 0.1302 0.1245 0.1015 0.1164 0.0994 0.1155 0.1035 0.1060 0.1011 0.5362 0.0813 0.0884 0.0820 0.0809 0.0801 0.0856 0.0825 0.0631 0.0660
0.1293 0.1336 0.1104 0.1218 0.1316 0.1124 0.1213 0.1116 0.0937 0.0919 0.1030 0.1013 0.1180 0.1012 0.1095 0.0813 0.5100 0.0920 0.0702 0.0712 0.0659 0.0679 0.0779 0.0626 0.0609
0.1057 0.1234 0.1090 0.1301 0.1098 0.1097 0.1085 0.1067 0.0998 0.0895 0.1032 0.1005 0.0999 0.0864 0.0890 0.0884 0.0920 0.4736 0.0751 0.0808 0.0631 0.0624 0.0733 0.0646 0.0622
0.1144 0.1265 0.1219 0.0906 0.1170 0.0932 0.1135 0.1014 0.1019 0.1032 0.0906 0.1046 0.0993 0.0901 0.0789 0.0820 0.0702 0.0751 0.4023 0.0638 0.0688 0.0683 0.0607 0.0643 0.0544
0.0931 0.1193 0.0985 0.1122 0.0971 0.0954 0.0938 0.1049 0.0948 0.0816 0.0908 0.1004 0.0930 0.0737 0.0810 0.0809 0.0712 0.0808 0.0638 0.3734 0.0580 0.0654 0.0667 0.0533 0.0522
0.1100 0.1116 0.1074 0.1139 0.1002 0.1063 0.1074 0.0950 0.1014 0.0817 0.0882 0.0855 0.0865 0.0853 0.0733 0.0801 0.0659 0.0631 0.0688 0.0580 0.3727 0.0656 0.0695 0.0626 0.0567
0.1004 0.1040 0.1136 0.0887 0.0955 0.1048 0.1007 0.0974 0.0836 0.0941 0.0805 0.0860 0.0754 0.0845 0.0793 0.0856 0.0679 0.0624 0.0683 0.0654 0.0656 0.3597 0.0616 0.0596 0.0581
0.1134 0.1165 0.0973 0.1091 0.0949 0.0972 0.0925 0.1006 0.0983 0.0888 0.0814 0.0801 0.0945 0.0727 0.0750 0.0825 0.0779 0.0733 0.0607 0.0667 0.0695 0.0616 0.3424 0.0515 0.0477
0.0953 0.0920 0.0782 0.0825 0.0887 0.0919 0.0880 0.0763 0.0776 0.0783 0.0701 0.0753 0.0747 0.0678 0.0678 0.0631 0.0626 0.0646 0.0643 0.0533 0.0626 0.0596 0.0515 0.2801 0.0401
0.0859 0.0827 0.0957 0.0879 0.0881 0.0741 0.0811 0.0685 0.0679 0.0770 0.0668 0.0779 0.0635 0.0647 0.0652 0.0660 0.0609 0.0622 0.0544 0.0522 0.0567 0.0581 0.0477 0.0401 0.2500]

lambda=
[0.0000 0.8511 0.2167 0.8273 0.8040 0.2385 0.2779 0.2933 0.3250 0.3112 0.7549 0.7235 0.3769 0.3999 0.4357 0.6809 0.6558 0.4683 0.4981 0.6069 0.6128 0.5832 0.5637 0.5481 0.5417]

V matrix 7

V=
[1.0000 0.1519 0.1345 0.2204 0.2783 0.2047 0.1604 0.1904 0.0278 0.0511 0.2572 0.1981 0.2289 0.1318 0.0931 0.1756 0.1724 0.0856 0.1491 0.0245 0.1942 0.0732 0.1009 0.0114 0.1126
0.1519 0.9882 0.1567 0.2404 0.2634 0.1028 0.0216 0.1660 0.0162 0.2531 0.0058 0.1129 0.1860 0.0619 0.0029 0.0798 0.0580 0.1045 0.1248 0.0762 0.1765 0.1157 0.1772 0.0082 0.1118
0.1345 0.1567 0.9735 0.1982 0.0475 0.2296 0.2601 0.1844 0.1874 0.2101 0.0507 0.1407 0.0250 0.2033 0.0417 0.1222 0.1507 0.0855 0.1681 0.0927 0.0378 0.0985 0.1645 0.0899 0.0301
0.2204 0.2404 0.1982 0.8825 0.0980 0.1336 0.0115 0.2008 0.1338 0.1711 0.2310 0.2343 0.0173 0.1704 0.1682 0.0057 0.1426 0.0439 0.0405 0.1566 0.0170 0.1613 0.0536 0.0215 0.1087
0.2783 0.2634 0.0475 0.0980 0.8684 0.2376 0.1399 0.2408 0.1684 0.0354 0.2202 0.1890 0.1458 0.1604 0.1488 0.0460 0.1923 0.1236 0.0008 0.1521 0.0995 0.0336 0.0848 0.0907 0.0519
0.2047 0.1028 0.2296 0.1336 0.2376 0.8630 0.2176 0.2292 0.1374 0.1724 0.0392 0.1536 0.0364 0.0051 0.0550 0.1132 0.1109 0.2162 0.0086 0.1928 0.1398 0.1328 0.0228 0.0299 0.1141
0.1604 0.0216 0.2601 0.0115 0.1399 0.2176 0.8340 0.2215 0.1014 0.1255 0.1071 0.1999 0.1999 0.1995 0.1960 0.1900 0.2128 0.1159 0.1265 0.1845 0.1137 0.0015 0.1471 0.0179 0.0512
0.1904 0.1660 0.1844 0.2008 0.2408 0.2292 0.2215 0.8196 0.1557 0.2173 0.1343 0.0421 0.0970 0.1619 0.0603 0.0514 0.0193 0.1808 0.0191 0.0926 0.0605 0.1100 0.1409 0.0588 0.0615
0.0278 0.0162 0.1874 0.1338 0.1684 0.1374 0.1014 0.1557 0.7922 0.0508 0.1104 0.0126 0.1986 0.1015 0.0939 0.0077 0.0900 0.0325 0.1885 0.0612 0.1437 0.1748 0.0742 0.0097 0.0505
0.0511 0.2531 0.2101 0.1711 0.0354 0.1724 0.1255 0.2173 0.0508 0.7566 0.0245 0.0714 0.2115 0.1063 0.0216 0.0038 0.1576 0.1458 0.0746 0.0741 0.0949 0.1586 0.0693 0.1378 0.0725
0.2572 0.0058 0.0507 0.2310 0.2202 0.0392 0.1071 0.1343 0.1104 0.0245 0.7489 0.0396 0.0779 0.0826 0.1809 0.0401 0.1797 0.1511 0.0162 0.0254 0.1328 0.1511 0.0217 0.0810 0.0382
0.1981 0.1129 0.1407 0.2343 0.1890 0.1536 0.1999 0.0421 0.0126 0.0714 0.0396 0.7167 0.1432 0.0781 0.1805 0.0418 0.0732 0.0378 0.1555 0.0276 0.0884 0.1330 0.0765 0.0020 0.0597
0.2289 0.1860 0.0250 0.0173 0.1458 0.0364 0.1999 0.0970 0.1986 0.2115 0.0779 0.1432 0.6844 0.0326 0.0033 0.1794 0.1845 0.1180 0.1168 0.1691 0.0721 0.1337 0.0951 0.0915 0.0762
0.1318 0.0619 0.2033 0.1704 0.1604 0.0051 0.1995 0.1619 0.1015 0.1063 0.0826 0.0781 0.0326 0.6497 0.0995 0.1069 0.1015 0.0031 0.0628 0.1125 0.0493 0.0675 0.1210 0.1065 0.1192
0.0931 0.0029 0.0417 0.1682 0.1488 0.0550 0.1960 0.0603 0.0939 0.0216 0.1809 0.1805 0.0033 0.0995 0.6342 0.1654 0.1720 0.1644 0.0642 0.0824 0.0386 0.0468 0.0398 0.1119 0.0513
0.1756 0.0798 0.1222 0.0057 0.0460 0.1132 0.1900 0.0514 0.0077 0.0038 0.0401 0.0418 0.1794 0.1069 0.1654 0.6114 0.0887 0.0235 0.0190 0.0446 0.1388 0.0201 0.0246 0.0420 0.1032
0.1724 0.0580 0.1507 0.1426 0.1923 0.1109 0.2128 0.0193 0.0900 0.1576 0.1797 0.0732 0.1845 0.1015 0.1720 0.0887 0.6064 0.1193 0.0691 0.1676 0.0373 0.1471 0.1088 0.0749 0.0385
0.0856 0.1045 0.0855 0.0439 0.1236 0.2162 0.1159 0.1808 0.0325 0.1458 0.1511 0.0378 0.1180 0.0031 0.1644 0.0235 0.1193 0.6018 0.1561 0.1121 0.0758 0.0700 0.0851 0.1098 0.1110
0.1491 0.1248 0.1681 0.0405 0.0008 0.0086 0.1265 0.0191 0.1885 0.0746 0.0162 0.1555 0.1168 0.0628 0.0642 0.0190 0.0691 0.1561 0.5459 0.0743 0.1437 0.0841 0.0825 0.0238 0.0635
0.0245 0.0762 0.0927 0.1566 0.1521 0.1928 0.1845 0.0926 0.0612 0.0741 0.0254 0.0276 0.1691 0.1125 0.0824 0.0446 0.1676 0.1121 0.0743 0.5275 0.1005 0.1200 0.0464 0.0417 0.0433
0.1942 0.1765 0.0378 0.0170 0.0995 0.1398 0.1137 0.0605 0.1437 0.0949 0.1328 0.0884 0.0721 0.0493 0.0386 0.1388 0.0373 0.0758 0.1437 0.1005 0.4954 0.0990 0.1051 0.0529 0.0155
0.0732 0.1157 0.0985 0.1613 0.0336 0.1328 0.0015 0.1100 0.1748 0.1586 0.1511 0.1330 0.1337 0.0675 0.0468 0.0201 0.1471 0.0700 0.0841 0.1200 0.0990 0.4319 0.0940 0.0331 0.0274
0.1009 0.1772 0.1645 0.0536 0.0848 0.0228 0.1471 0.1409 0.0742 0.0693 0.0217 0.0765 0.0951 0.1210 0.0398 0.0246 0.1088 0.0851 0.0825 0.0464 0.1051 0.0940 0.4169 0.0089 0.0731
0.0114 0.0082 0.0899 0.0215 0.0907 0.0299 0.0179 0.0588 0.0097 0.1378 0.0810 0.0020 0.0915 0.1065 0.1119 0.0420 0.0749 0.1098 0.0238 0.0417 0.0529 0.0331 0.0089 0.2905 0.0408
0.1126 0.1118 0.0301 0.1087 0.0519 0.1141 0.0512 0.0615 0.0505 0.0725 0.0382 0.0597 0.0762 0.1192 0.0513 0.1032 0.0385 0.1110 0.0635 0.0433 0.0155 0.0274 0.0731 0.0408 0.2500]

lambda=
[1.1854 1.1426 0.0000 0.0579 1.0253 0.9878 0.9485 0.1347 0.1438 0.1934 0.2070 0.2615 0.2881 0.3611 0.4019 0.4287 0.8428 0.8079 0.7703 0.7008 0.6440 0.6529 0.5026 0.5469 0.5370]

Appendix B

This appendix contains the computational results of the following:

1. Quantiles for each variance matrix V_d for each value of P_T *i.e.* $(\hat{a}_p, \hat{b}_p, \hat{c}_p, \hat{d}_p)$

2. P_{Mi} *i.e.* $P_{M_i}\left(Q \geq \hat{a}_p \middle| V_d\right)$ for each quantile, which are the computed approximate probability values of the seven approximating methods

3. Estimated delta for each quantile *i.e.* $\hat{\delta}_{VMi} = \left| P_T - P_{Mi}\left(Q \geq \hat{a}_p \middle| V_d\right)\right|$

4. Estimated percentage delta for each quantile *i.e.* $\hat{\delta}_{\%VMi} = (\hat{\delta}_{VMi} \cdot 100) / P_T$

5. Estimated percentage signed deltas for each quantile *i.e.* $\operatorname{sgn} \hat{\delta}_{\%VMi} = (\operatorname{sgn} \hat{\delta}_{VMi} \cdot 100) / P_T$

6. Significance information for each quantile *i.e.* $\operatorname{sgn} \hat{\delta}_{VMi} / \sigma\left(\operatorname{sgn} \hat{\delta}_{VMi}\right)$

The abbreviations of the different methods correspond to the following:

M1	Crude approximation method 1
M2	Crude approximation method 2
M3	Two moment approximation , Grad, A., and Solomon, H. (1955)
M4	Three moment approximation, Imhof, J.P. (1961)
M5	Gaussian approximation, Jensen, D.R., and Solomon, H. (1972)
M6	Three moment approximation, Solomon, H., and Stephens, M.A. (1977)
M7	Saddle point approximation, Field, C. (1993)

The results are given for the three dimensions n = 10, n = 20 and n = 25 of the 15 different variance matrices.

Variance matrix dimension 10

Table $P\left(Q \geq 10.2788^{1} \middle| V_1\right)$

V matrix type 1
$P_T = 0.10$

	2.	3.	4.	5.	6.
Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.1000	0	0.0	0.0	0.0
M2	0.3284	0.2284	228.4	-228.4	-761.3
M3	0.0994	0.0006	0.6	0.6	2.0
M4	0.1000	0	0.0	0.0	0.0
M5	0.0994	0.0006	0.6	0.6	2.0
M6	0.1000	0	0.0	0.0	0.0
M7	0.1000	0	0.0	0.0	0.0

Table $P\left(Q \geq 11.8469 \middle| V_1\right)$

V matrix type 1
$P_T = 0.05$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0499	0.0001	0.2	0.2	0.5
M2	0.2221	0.1721	344.2	-344.2	-789.5
M3	0.0497	0.0003	0.6	0.60	1.4
M4	0.0499	0.0001	0.2	0.2	0.5
M5	0.0497	0.0003	0.6	0.6	1.4
M6	0.0499	0.0001	0.2	0.2	0.5
M7	0.0500	0	0.0	0.0	0.0

Table $P\left(Q \geq 13.7818 \middle| V_1\right)$

V matrix type 1
$P_T = 0.02$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0199	0.0001	0.5	0.5	0.7
M2	0.1303	0.1103	551.5	-551.5	-787.9
M3	0.0200	0	0.0	0.0	0.0
M4	0.0199	0.0001	0.5	0.5	0.7
M5	0.0200	0	0.0	0.0	0.0
M6	0.0199	0.0001	0.5	0.5	0.7
M7	0.0200	0	0.0	0.0	0.0

Table $P\left(Q \geq 15.1671 \middle| V_1\right)$

V matrix type 1

$P_T = 0.01$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0100	0	0.0	0.0	0.0
M2	0.0864	0.0764	764.0	-764.0	-771.7
M3	0.0101	0.0001	1.0	-1.0	-1.0
M4	0.0100	0	0.0	0.0	0
M5	0.0101	0.0001	1.0	-1.0	-1.0
M6	0.0100	0	0.0	0.0	0
M7	0.0100	0	0.0	0.0	0

Table $P\left(Q \geq 13.1612 \middle| V_2\right)$

V matrix type 2a

$P_T = 0.10$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0860	0.0140	14.0	14.0	46.7
M2	0.1554	0.0554	55.4	-55.4	-184.7
M3	0.1068	0.0068	6.8	-6.8	-22.7
M4	0.1062	0.0062	6.2	-6.2	-20.7
M5	0.1071	0.0071	7.1	-7.1	-23.7
M6	0.1023	0.0023	2.3	-2.3	-7.7
M7	0.0938	0.0062	6.2	6.2	20.7

Table $P\left(Q \geq 15.6606 \middle| V_2\right)$

V matrix type 2a

$P_T = 0.05$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0344	0.0156	31.2	31.2	71.6
M2	0.0743	0.0243	48.6	-48.6	-111.5
M3	0.0504	0.0004	0.8	-0.8	-1.8
M4	0.0541	0.0041	8.2	-8.2	-18.8
M5	0.0565	0.0065	13	-13	-29.8
M6	0.0514	0.0014	2.8	-2.8	-6.4
M7	0.0475	0.0025	5.0	5.0	11.5

Table $P\left(Q \geq 18.9997 \middle| V_2\right)$

V matrix type 2a

$P_T = 0.02$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0091	0.0109	54.5	54.5	77.86

M2	0.0252	0.0052	26.0	-26.0	-37.14
M3	0.0174	0.0026	13.0	13.0	18.57
M4	0.0213	0.0013	6.5	-6.5	-9.29
M5	0.0243	0.0043	21.5	-21.5	-30.71
M6	0.0204	0.0004	2.0	-2.0	-2.86
M7	0.0197	0.0003	1.5	1.5	2.14

Table $P\left(Q \geq 21.6149 \middle| V_2\right)$

V matrix type 2a

$P_T = 0.01$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \left\|P_T - P_{Mi}\right\|$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn}\hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn}\hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn}\hat{\delta}_{\%VMi})}$
M1	0.0030	0.0070	70.0	70.0	70.7
M2	0.0102	0.0002	2.0	-2.0	-2.0
M3	0.0073	0.0027	27.0	27.0	27.3
M4	0.0101	0.0001	1.0	-1.0	-1.0
M5	0.0127	0.0027	27.0	-27.0	-27.3
M6	0.0100	0	0.0	0.0	0.0
M7	0.0100	0	0.0	0.0	0.0

Table $P\left(Q \geq 8.5969 \middle| V_2\right)$

V matrix type 2b

$P_T = 0.10$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \left\|P_T - P_{Mi}\right\|$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn}\hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn}\hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn}\hat{\delta}_{\%VMi})}$
M1	0.0768	0.0232	23.2	23.2	77.3
M2	0.1313	0.0313	31.3	-31.3	-104.3
M3	0.1022	0.0022	2.2	-2.2	-7.3
M4	0.1026	0.0026	2.6	-2.6	-8.7
M5	0.1028	0.0028	2.8	-2.8	-9.3
M6	0.1001	0.0001	0.1	-0.1	-0.3
M7	0.0954	0.0046	4.6	4.6	15.3

Table $P\left(Q \geq 10.2312 \middle| V_2\right)$

V matrix type 2b

$P_T = 0.05$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \left\|P_T - P_{Mi}\right\|$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn}\hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn}\hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn}\hat{\delta}_{\%VMi})}$
M1	0.0297	0.0203	40.6	40.6	93.1
M2	0.0595	0.0095	19.0	-19.0	-43.6
M3	0.0487	0.0013	2.6	2.6	6.0
M4	0.0512	0.0012	2.4	-2.4	-5.5
M5	0.0525	0.0025	5.0	-5.0	-11.5
M6	0.0497	0.0003	0.6	0.6	1.4
M7	0.0467	0.0033	6.6	6.6	15.4

Table $P\left(Q \geq 12.3248 \middle| V_2\right)$

V matrix type 2b

$P_T = 0.02$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0080	0.0120	60.0	60.0	85.7
M2	0.0197	0.0003	1.5	1.5	2.1
M3	0.0179	0.0021	10.5	10.5	15.0
M4	0.0203	0.0003	1.5	-1.5	-2.1
M5	0.0219	0.0019	9.5	-9.5	-13.6
M6	0.0199	0.0001	0.5	0.5	0.7
M7	0.0186	0.0014	7.0	7.0	10.0

Table $P\left(Q \geq 13.9233 \middle| V_2\right)$

V matrix type 2b

$P_T = 0.01$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0028	0.0072	72.0	72.0	72.7
M2	0.0080	0.0020	20.0	20.0	20.2
M3	0.0081	0.0019	19.0	19.0	19.2
M4	0.0098	0.0002	2.0	2.0	2.0
M5	0.0113	0.0013	13.0	-13.0	-13.1
M6	0.0098	0.0002	2.0	2.0	2.0
M7	0.0092	0.0008	8.0	8.0	8.1

Table $P\left(Q \geq 8.5633 \middle| V_2\right)$

V matrix type 2c

$P_T = 0.10$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0710	0.0290	29.0	29.0	96.7
M2	0.1334	0.0334	33.4	-33.4	-111.3
M3	0.1030	0.0030	3.0	-3.0	-10.0
M4	0.1031	0.0031	3.1	-3.1	-10.3
M5	0.1034	0.0034	3.4	-3.4	-11.3
M6	0.1003	0.0003	0.3	-0.3	-1.0
M7	0.0949	0.0051	5.1	5.1	17.0

Table $P\left(Q \geq 10.2554 \middle| V_2\right)$

V matrix type 2c

$P_T = 0.05$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0258	0.0242	48.4	48.4	111.0
M2	0.0588	0.0088	17.6	-17.6	-40.4

M3	0.0492	0.0008	1.6	1.6	3.7
M4	0.0520	0.0020	4.0	-4.0	-9.2
M5	0.0534	0.0034	6.8	-6.8	-15.6
M6	0.0501	0.0001	0.2	-0.2	-0.5
M7	0.0470	0.0030	6.0	6.0	13.8

Table $P\left(Q \geq 12.4734 \middle| V_2\right)$

V matrix type 2c

$P_T = 0.02$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0061	0.0139	69.5	69.5	99.3
M2	0.0182	0.0018	9.0	9.0	12.9
M3	0.0178	0.0022	11.0	11.0	15.7
M4	0.0205	0.0005	2.5	-2.5	-3.6
M5	0.0224	0.0024	12.0	-12	-17.1
M6	0.0200	0	0.0	0.0	0.0
M7	0.0188	0.0012	6.0	6.0	8.6

Table $P\left(Q \geq 14.1512 \middle| V_2\right)$

V matrix type 2c

$P_T = 0.01$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0019	0.0081	81.0	81.0	81.8
M2	0.0071	0.0029	29.0	29.0	29.3
M3	0.0081	0.0019	19.0	19.0	19.2
M4	0.0100	0	0.0	0.0	0.0
M5	0.0117	0.0017	17.0	-17.0	-17.2
M6	0.0099	0.0001	1.0	1.0	1.0
M7	0.0095	0.0005	5.0	5.0	5.1

Table $P\left(Q \geq 8.5512 \middle| V_2\right)$

V matrix type 2d

$P_T = 0.10$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0645	0.0355	35.5	35.5	118.3
M2	0.1341	0.0341	34.1	-34.1	-113.7
M3	0.1035	0.0035	3.5	-3.5	-11.7
M4	0.1033	0.0033	3.3	-3.3	-11.0
M5	0.1036	0.0036	3.6	-3.6	-12.0
M6	0.1003	0.0003	0.3	-0.3	-1.0
M7	0.0945	0.0055	5.5	5.5	18.33

Table $P\left(Q \geq 12.6440 \middle| V_2\right)$

V matrix type 2d

$P_T = 0.05$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\dfrac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0217	0.0283	56.6	56.6	129.8
M2	0.0572	0.0072	14.4	-14.4	-33.0
M3	0.0496	0.0004	0.8	0.8	1.8
M4	0.0524	0.0024	4.8	-4.8	-11.0
M5	0.0540	0.0040	8.0	-8.0	-18.4
M6	0.0503	0.0003	0.6	-0.6	-1.4
M7	0.0471	0.0029	5.8	5.8	13.3

Table $P\left(Q \geq 12.6440 \middle| V_2\right)$

V matrix type 2d

$P_T = 0.02$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\dfrac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0046	0.0154	77.0	77.0	110.0
M2	0.0165	0.0035	17.5	17.5	25.0
M3	0.0180	0.0020	10.0	10.0	14.3
M4	0.0207	0.0007	3.5	-3.5	-5.0
M5	0.0229	0.0029	14.5	-14.5	-20.7
M6	0.0201	0.0001	0.5	-0.5	-0.7
M7	0.0190	0.0010	5.0	5.0	7.1

Table $P\left(Q \geq 14.4025 \middle| V_2\right)$

V matrix type 2d

$P_T = 0.01$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\dfrac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0013	0.0087	87.0	87.0	87.9
M2	0.0061	0.0039	39.0	39.0	39.4
M3	0.0082	0.0018	18.0	18.0	18.2
M4	0.0102	0.0002	2.0	-2.0	-2.0
M5	0.0121	0.0021	21.0	-21.0	-21.2
M6	0.0101	0.0001	1.0	-1.0	-1.0
M7	0.0097	0.0003	3.0	3.0	3.0

Table $P\left(Q \geq 6.5547 \middle| V_3\right)$

V matrix type 3a

$P_T = 0.10$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\dfrac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0837	0.0163	16.3	16.3	54.3
M2	0.2635	0.1635	163.5	-163.5	-545.0
M3	0.1001	0.0001	0.1	-0.1	-0.3

M4	0.1008	0.0008	0.8	-0.8	-2.7
M5	0.1005	0.0005	0.5	-0.5	-1.7
M6	0.0998	0.0002	0.2	0.2	0.7
M7	0.0992	0.0008	0.8	0.8	2.7

Table $P\left(Q \geq 7.6825 \middle| V_3\right)$

V matrix type 3a

$P_T = 0.05$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0364	0.0136	27.2	27.2	62.4
M2	0.1579	0.1079	215.8	-215.8	-495.0
M3	0.0491	0.0009	1.8	1.8	4.1
M4	0.0505	0.0005	1.0	-1.0	-2.3
M5	0.0508	0.0008	1.6	-1.6	-3.7
M6	0.0499	0.0001	0.2	0.2	0.5
M7	0.0496	0.0004	0.8	0.8	1.8

Table $P\left(Q \geq 9.0984 \middle| V_3\right)$

V matrix type 3a

$P_T = 0.02$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0118	0.0082	41.0	41.0	58.6
M2	0.0776	0.0576	288.0	-288.0	-411.4
M3	0.0191	0.0009	4.5	4.5	6.4
M4	0.0202	0.0002	1.0	-1.0	-1.4
M5	0.0209	0.0009	4.5	-4.5	-6.4
M6	0.0201	0.0001	0.5	-0.5	-0.7
M7	0.0200	0	0.0	0.0	0.0

Table $P\left(Q \geq 10.1529 \middle| V_3\right)$

V matrix type 3a

$P_T = 0.01$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0049	0.0051	51.0	51.0	51.5
M2	0.0439	0.0339	339.0	-339.0	-342.4
M3	0.0092	0.0008	8.0	8.0	8.1
M4	0.0100	0	0.0	0.0	0.0
M5	0.0106	0.0006	6.0	-6.0	-6.1
M6	0.0100	0	0.0	0.0	0.0
M7	0.0099	0.0001	1.0	1.0	1.0

Table $P\left(Q \geq 7.3655 \middle| V_3\right)$

V matrix type 3b

$P_T = 0.10$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0930	0.0070	7.0	7.0	23.3
M2	0.3028	0.2028	202.8	-202.8	-676.0
M3	0.0997	0.0003	0.3	0.3	1.0
M4	0.1005	0.0005	0.5	-0.5	-1.7
M5	0.1000	0	0.0	0.0	0.0
M6	0.0999	0.0001	0.1	0.1	0.3
M7	0.0994	0.0006	0.6	0.6	2.0

Table $P\left(Q \geq 8.5551 \middle| V_3\right)$

V matrix type 3b

$P_T = 0.05$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0437	0.0063	12.6	12.6	28.9
M2	0.1952	0.1452	290.4	-290.4	-666.1
M3	0.0493	0.0007	1.4	1.4	3.2
M4	0.0502	0.0002	0.4	-0.4	-0.9
M5	0.0503	0.0003	0.6	-0.6	-1.4
M6	0.0499	0.0001	0.2	0.2	0.5
M7	0.0496	0.0004	0.8	0.8	1.8

Table $P\left(Q \geq 10.0353 \middle| V_3\right)$

V matrix type 3b

$P_T = 0.02$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0159	0.0041	20.5	20.5	29.3
M2	0.1067	0.0867	433.5	-433.5	-619.3
M3	0.0194	0.0006	3.0	3.0	4.3
M4	0.0201	0.0001	0.5	-0.5	-0.7
M5	0.0205	0.0005	2.5	-2.5	-3.6
M6	0.0200	0	0.0	0.0	0.0
M7	0.0199	0.0001	0.5	0.5	0.7

Table $P\left(Q \geq 11.1081 \middle| V_3\right)$

V matrix type 3b

$P_T = 0.01$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0074	0.0026	26.0	26.0	26.3
M2	0.0666	0.0566	566.0	-566.0	-571.7
M3	0.0096	0.0004	4.0	4.0	4.0

M4	0.0100	0	0.0	0.0	0.0
M5	0.0104	0.0004	4.0	-4.0	-4.0
M6	0.0101	0.0001	1.0	-1.0	-1.0
M7	0.0100	0	0.0	0.0	0.0

Table $P\left(Q \geq 8.4674 \middle| V_3\right)$

V matrix type 3c

$P_T = 0.10$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0977	0.0023	2.3	2.3	7.7
M2	0.3205	0.2205	220.5	-220.5	-735.0
M3	0.0998	0.0002	0.2	0.2	0.7
M4	0.1005	0.0005	0.5	-0.5	-1.7
M5	0.1000	0	0.0	0.0	0.0
M6	0.1002	0.0002	0.2	-0.2	-0.7
M7	0.0999	0.0001	0.1	0.1	0.3

Table $P\left(Q \geq 9.7936 \middle| V_3\right)$

V matrix type 3c

$P_T = 0.05$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0476	0.0024	4.8	4.8	11.0
M2	0.2129	0.1629	325.8	325.8	747.3
M3	0.0495	0.0005	1.0	1.0	2.3
M4	0.0501	0.0001	0.2	0.2	0.5
M5	0.0500	0	0.0	0.0	0.0
M6	0.0500	0	0.0	0.0	0.0
M7	0.0498	0.0002	0.4	0.4	0.9

Table $P\left(Q \geq 11.4321 \middle| V_3\right)$

V matrix type 3c

$P_T = 0.02$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0183	0.0017	8.5	8.5	12.1
M2	0.1217	0.1017	508.5	-508.5	-726.4
M3	0.0197	0.0003	1.5	1.5	2.1
M4	0.0200	0	0.0	0.0	0.0
M5	0.0202	0.0002	1.0	-1.0	-1.4
M6	0.0200	0	0.0	0.0	0.0
M7	0.0199	0.0001	0.5	0.5	0.7

Table $P\left(Q \geq 12.6204 \middle| V_3\right)$

V matrix type 3c

$P_T = 0.01$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0089	0.0011	11.0	11.0	11.1
M2	0.0787	0.0687	687.0	-687.0	-693.9
M3	0.0098	0.0002	2.0	2.0	2.0
M4	0.0100	0	0.0	0.0	0.0
M5	0.0102	0.0002	2.0	-2.0	-2.0
M6	0.0100	0	0.0	0.0	0.0
M7	0.0100	0	0.0	0.0	0.0

Table $P\left(Q \geq 7.6096 \middle| V_4\right)$

V matrix type 4a

$P_T = 0.10$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0877	0.0123	12.3	12.3	41.0
M2	0.0877	0.0123	12.3	12.3	41.0
M3	0.1020	0.0020	2.0	-2.0	-6.7
M4	0.1026	0.0026	2.6	-2.6	-8.7
M5	0.1026	0.0026	2.6	-2.6	-8.7
M6	0.1008	0.0008	0.8	-0.8	-2.7
M7	0.0979	0.0021	2.1	2.1	7.0

Table $P\left(Q \geq 8.9284 \middle| V_4\right)$

V matrix type 4a

$P_T = 0.05$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0383	0.0117	23.4	23.4	53.7
M2	0.0383	0.0117	23.4	23.4	53.7
M3	0.0496	0.0004	0.8	0.8	1.8
M4	0.0517	0.0017	3.4	-3.4	-7.8
M5	0.0524	0.0024	4.8	-4.8	-11.0
M6	0.0506	0.0006	1.2	-1.2	-2.8
M7	0.0488	0.0012	2.4	2.4	5.5

Table $P\left(Q \geq 10.6339 \middle| V_3\right)$

V matrix type 4a

$P_T = 0.02$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0121	0.0079	39.5	39.5	56.43
M2	0.0121	0.0079	39.5	39.5	56.43
M3	0.0185	0.0015	7.5	7.5	10.71

M4	0.0203	0.0003	1.5	-1.5	-2.14
M5	0.0215	0.0015	7.5	-7.5	-10.71
M6	0.0201	0.0001	0.5	-0.5	-0.71
M7	0.0193	0.0007	3.5	3.5	5

Table $P\left(Q \geq 11.8778 \middle| V_4\right)$

V matrix type 4a

$P_T = 0.01$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0050	0.0050	50.0	50.0	50.5
M2	0.0050	0.0050	50.0	50.0	50.5
M3	0.0087	0.0013	13.0	13.0	13.1
M4	0.0100	0	0.0	0.0	0.0
M5	0.0111	0.0011	11.0	-11.0	-11.1
M6	0.0101	0.0001	1.0	-1.0	-1.0
M7	0.0097	0.0003	3.0	3.0	3.0

Table $P\left(Q \geq 9.5619 \middle| V_4\right)$

V matrix type 4b

$P_T = 0.10$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0926	0.0074	7.4	7.4	24.7
M2	0.1081	0.0081	8.1	-8.1	-27.0
M3	0.1006	0.0006	0.6	-0.6	-2.0
M4	0.1013	0.0013	1.3	-1.3	-4.3
M5	0.1010	0.0010	1.0	-1.0	-3.3
M6	0.1005	0.0005	0.5	-0.5	-1.7
M7	0.0994	0.0006	0.6	0.6	2.0

Table $P\left(Q \geq 11.1226 \middle| V_4\right)$

V matrix type 4b

$P_T = 0.05$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0431	0.0069	13.8	13.8	31.7
M2	0.0524	0.0024	4.8	-4.8	-11.0
M3	0.0497	0.0003	0.6	0.6	1.4
M4	0.0508	0.0008	1.6	-1.6	-3.7
M5	0.0510	0.0010	2.0	-2.0	-4.6
M6	0.0504	0.0004	0.8	-0.8	-1.8
M7	0.0496	0.0004	0.8	0.8	1.8

Table $P\left(Q \geq 13.1127 \middle| V_4\right)$

V matrix type 4b

$P_T = 0.02$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn}\hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn}\hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn}\hat{\delta}_{\%VMi})}$
M1	0.0151	0.0049	24.5	24.5	35.0
M2	0.0194	0.0006	3.0	3.0	4.3
M3	0.0191	0.0009	4.5	4.5	6.4
M4	0.0200	0	0	0	0
M5	0.0205	0.0005	2.5	-2.5	-3.6
M6	0.0199	0.0001	0.5	0.5	0.7
M7	0.0195	0.0005	2.5	2.5	3.6

Table $P\left(Q \geq 14.5319 \middle| V_4\right)$

V matrix type 4b

$P_T = 0.01$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn}\hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn}\hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn}\hat{\delta}_{\%VMi})}$
M1	0.0069	0.0031	31.0	31.0	31.3
M2	0.0092	0.0008	8.0	8.0	8.1
M3	0.0094	0.0006	6.0	6.0	6.1
M4	0.0100	0	0	0	0
M5	0.0105	0.0005	5.0	-5.0	-5.1
M6	0.0100	0	0	0	0
M7	0.0098	0.0002	2.0	2.0	2.0

Table $P\left(Q \geq 8.9921 \middle| V_4\right)$

V matrix type 4c

$P_T = 0.10$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn}\hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn}\hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn}\hat{\delta}_{\%VMi})}$
M1	0.0897	0.0103	10.3	10.3	34.3
M2	0.1060	0.0060	6.0	-6.0	-20.0
M3	0.1001	0.0001	0.1	-0.1	-0.3
M4	0.1009	0.0009	0.9	-0.9	-3.0
M5	0.1006	0.0006	0.6	-0.6	-2.0
M6	0.0999	0.0001	0.1	0.1	0.3
M7	0.0989	0.0011	1.1	1.1	3.7

Table $P\left(Q \geq 10.4868 \middle| V_4\right)$

V matrix type 4c

$P_T = 0.05$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn}\hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn}\hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn}\hat{\delta}_{\%VMi})}$
M1	0.0408	0.0092	18.4	18.4	42.2
M2	0.0504	0.0004	0.8	-0.8	-1.8
M3	0.0492	0.0008	1.6	1.6	3.7

M4	0.0506	0.0006	1.2	-1.2	-2.8
M5	0.0508	0.0008	1.6	-1.6	-3.7
M6	0.0500	0	0	0	0
M7	0.0493	0.0007	1.4	1.4	3.2

Table $P\left(Q \geq 12.3541 \middle| V_4\right)$

V matrix type 4c

$P_T = 0.02$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \left\|P_T - P_{Mi}\right\|$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0142	0.0058	29.0	29.0	41.43
M2	0.0185	0.0015	7.5	7.5	10.71
M3	0.0192	0.0008	4.0	4.0	5.71
M4	0.0203	0.0003	1.5	-1.5	-2.14
M5	0.0209	0.0009	4.5	-4.5	-6.43
M6	0.0202	0.0002	1.0	-1.0	-1.43
M7	0.0198	0.0002	1.0	1.0	1.43

Table $P\left(Q \geq 13.7141 \middle| V_4\right)$

V matrix type 4c

$P_T = 0.01$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \left\|P_T - P_{Mi}\right\|$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0063	0.0037	37.0	37.0	37.4
M2	0.0086	0.0014	14.0	14.0	14.1
M3	0.0094	0.0006	6.0	6.0	6.1
M4	0.0101	0.0001	1.0	-1.0	-1.0
M5	0.0107	0.0007	7.0	-7.0	-7.1
M6	0.0102	0.0002	2.0	-2.0	-2.0
M7	0.0100	0	0	0	0

Table $P\left(Q \geq 14.1684 \middle| V_5\right)$

V matrix type 5a

$P_T = 0.10$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \left\|P_T - P_{Mi}\right\|$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0999	0.0001	0.1	0.1	0.3
M2	0.1165	0.0165	16.5	-16.5	-55.0
M3	0.0995	0.0005	0.5	0.5	1.7
M4	0.1002	0.0002	0.2	-0.2	-0.7
M5	0.0995	0.0005	0.5	0.5	1.7
M6	0.1001	0.0001	0.1	-0.1	-0.3
M7	0.1001	0.0001	0.1	-0.1	-0.3

Table $P\left(Q \geq 16.3393 \middle| V_5\right)$

V matrix type 5a

$P_T = 0.05$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn}\hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn}\hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn}\hat{\delta}_{\%VMi})}$
M1	0.0497	0.0003	0.6	0.6	1.4
M2	0.0601	0.0101	20.2	-20.2	-46.3
M3	0.0496	0.0004	0.8	0.8	1.8
M4	0.0499	0.0001	0.2	0.2	0.5
M5	0.0497	0.0003	0.6	0.6	1.4
M6	0.0499	0.0001	0.2	0.2	0.5
M7	0.0499	0.0001	0.2	0.2	0.5

Table $P\left(Q \geq 18.9954 \middle| V_5\right)$

V matrix type 5a

$P_T = 0.02$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn}\hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn}\hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn}\hat{\delta}_{\%VMi})}$
M1	0.0199	0.0001	0.5	0.5	0.7
M2	0.0252	0.0052	26	-26	-37.1
M3	0.0201	0.0001	0.5	-0.5	-0.7
M4	0.0201	0.0001	0.5	-0.5	-0.7
M5	0.0201	0.0001	0.5	-0.5	-0.7
M6	0.0201	0.0001	0.5	-0.5	-0.7
M7	0.0201	0.0001	0.5	-0.5	-0.7

Table $P\left(Q \geq 20.9251 \middle| V_5\right)$

V matrix type 5a

$P_T = 0.01$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn}\hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn}\hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn}\hat{\delta}_{\%VMi})}$
M1	0.0099	0.0001	1.0	1.0	1.0
M2	0.0130	0.0030	30.0	-30.0	-30.3
M3	0.0101	0.0001	1.0	-1.0	-1.0
M4	0.0100	0	0	0	0
M5	0.0101	0.0001	1.0	-1.0	-1.0
M6	0.0100	0	0	0	0
M7	0.0100	0	0	0	0

Table $P\left(Q \geq 14.0305 \middle| V_5\right)$

V matrix type 5b

$P_T = 0.10$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn}\hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn}\hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn}\hat{\delta}_{\%VMi})}$
M1	0.0993	0.0007	0.7	0.7	2.3
M2	0.1212	0.0212	21.2	-21.2	-70.7
M3	0.0993	0.0007	0.7	0.7	2.3

M4	0.0999	0.0001	0.1	0.1	0.3
M5	0.0993	0.0007	0.7	0.7	2.3
M6	0.0999	0.0001	0.1	0.1	0.3
M7	0.0998	0.0002	0.2	0.2	0.7

Table $P\left(Q \geq 16.1715 \middle| V_5\right)$

V matrix type 5b

$P_T = 0.05$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \left\|P_T - P_{Mi}\right\|$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0495	0.0005	1.0	1.0	2.3
M2	0.0634	0.0134	26.8	-26.8	-61.5
M3	0.0497	0.0003	0.6	0.6	1.4
M4	0.0501	0.0001	0.2	-0.2	-0.5
M5	0.0498	0.0002	0.4	0.4	0.9
M6	0.0500	0	0	0	0
M7	0.0500	0	0	0	0

Table $P\left(Q \geq 18.8428 \middle| V_5\right)$

V matrix type 5b

$P_T = 0.02$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \left\|P_T - P_{Mi}\right\|$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0195	0.0005	2.5	2.5	3.6
M2	0.0266	0.0066	33.0	-33.0	-47.1
M3	0.0198	0.0002	1.0	1.0	1.4
M4	0.0199	0.0001	0.5	0.5	0.7
M5	0.0200	0	0	0	0
M6	0.0199	0.0001	0.5	0.5	0.7
M7	0.0199	0.0001	0.5	0.5	0.7

Table $P\left(Q \geq 20.7745 \middle| V_5\right)$

V matrix type 5b

$P_T = 0.01$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \left\|P_T - P_{Mi}\right\|$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0096	0.0004	4.0	4.0	4.0
M2	0.0137	0.0037	37.0	-37.0	-37.4
M3	0.0099	0.0001	1.0	1.0	1.0
M4	0.0099	0.0001	1.0	1.0	1.0
M5	0.0100	0	0	0	0
M6	0.0099	0.0001	1.0	1.0	1.0
M7	0.0099	0.0001	1.0	1.0	1.0

Table $P\left(Q \geq 7.7355 \middle| V_6\right)$

V matrix type 6

$P_T = 0.10$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0921	0.0079	7.9	7.9	26.3
M2	0.1893	0.0893	89.3	-89.3	-297.7
M3	0.1003	0.0003	0.3	-0.3	-1.0
M4	0.1010	0.0010	1.0	-1.0	-3.3
M5	0.1007	0.0007	0.7	-0.7	-2.3
M6	0.1001	0.0001	0.1	-0.1	-0.3
M7	0.0986	0.0014	1.4	1.4	4.7

Table $P\left(Q \geq 9.0058 \middle| V_6\right)$

V matrix type 6

$P_T = 0.05$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0426	0.0074	14.8	14.8	33.9
M2	0.1060	0.0560	112.0	-112.0	-256.9
M3	0.0493	0.0007	1.4	1.4	3.2
M4	0.0506	0.0006	1.2	-1.2	2.8
M5	0.0508	0.0008	1.6	-1.6	3.7
M6	0.0501	0.0001	0.2	-0.2	-0.5
M7	0.0490	0.0010	2.0	2.0	4.6

Table $P\left(Q \geq 10.6127 \middle| V_6\right)$

V matrix type 6

$P_T = 0.02$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0149	0.0051	25.5	25.5	36.4
M2	0.0476	0.0276	138.0	-138.0	-197.1
M3	0.0190	0.0010	5.0	5.0	7.1
M4	0.0200	0	0	0	0
M5	0.0205	0.0005	2.5	-2.5	-3.6
M6	0.0199	0.0001	0.5	0.5	0.7
M7	0.0193	0.0007	3.5	3.5	5.0

Table $P\left(Q \geq 11.7964 \middle| V_6\right)$

V matrix type 6

$P_T = 0.01$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0066	0.0034	34.0	34.0	34.3
M2	0.0254	0.0154	154.0	-154.0	-155.6
M3	0.0091	0.0009	9.0	9.0	9.1

M4	0.0098	0.0002	2.0	2.0	2.0
M5	0.0103	0.0003	3.0	-3.0	-3.0
M6	0.0098	0.0002	2.0	2.0	2.0
M7	0.0095	0.0005	5.0	5.0	5.1

Table $P\left(Q \geq 8.3179 \middle| V_7\right)$

V matrix type 7

$P_T = 0.10$

Method	P_{Mi}	$\hat{\delta}_{VMi} =$ $\lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0818	0.0182	18.2	18.2	60.7
M2	0.1640	0.0640	64.0	-64.0	-213.3
M3	0.1007	0.0007	0.7	-0.7	-2.3
M4	0.1014	0.0014	1.4	-1.4	-4.7
M5	0.1013	0.0013	1.3	-1.3	-4.3
M6	0.0997	0.0003	0.3	0.3	1.0
M7	0.0985	0.0015	1.5	1.5	5.0

Table $P\left(Q \geq 9.7988 \middle| V_7\right)$

V matrix type 7

$P_T = 0.05$

Method	P_{Mi}	$\hat{\delta}_{VMi} =$ $\lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0342	0.0158	31.6	31.6	72.5
M2	0.0836	0.0336	67.2	-67.2	-154.1
M3	0.0488	0.0012	2.4	2.4	5.5
M4	0.0508	0.0008	1.6	-1.6	-3.7
M5	0.0515	0.0015	3.0	-3.0	-6.9
M6	0.0498	0.0002	0.4	0.4	0.9
M7	0.0491	0.0009	1.8	1.8	4.1

Table $P\left(Q \geq 11.6991 \middle| V_7\right)$

V matrix type 7

$P_T = 0.02$

Method	P_{Mi}	$\hat{\delta}_{VMi} =$ $\lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0103	0.0097	48.5	48.5	69.3
M2	0.0325	0.0125	62.5	-62.5	-89.3
M3	0.0183	0.0017	8.5	8.5	12.1
M4	0.0200	0	0	0	0
M5	0.0211	0.0011	5.5	-5.5	-7.9
M6	0.0198	0.0002	1.0	1.0	1.4
M7	0.0195	0.0005	2.5	2.5	3.6

Table $P\left(Q \geq 13.1161 \middle| V_7\right)$

V matrix type 7

$P_T = 0.01$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0040	0.0060	60.0	60.0	60.6
M2	0.0153	0.0053	53.0	-53.0	-53.5
M3	0.0086	0.0014	14.0	14.0	14.1
M4	0.0097	0.0003	3.0	3.0	3.0
M5	0.0107	0.0007	7.0	-7.0	-7.1
M6	0.0098	0.0002	2.0	2.0	2.0
M7	0.0096	0.0004	4.0	4.0	4.0

Variance matrix dimension 20

Table $P\left(Q \geq 19.0405 \middle| V_1\right)$

V matrix type 1

$P_T = 0.10$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.1001	0.0001	0.1	-0.1	-0.3
M2	0.3907	0.2907	290.7	-290.7	-969.0
M3	0.0998	0.0002	0.2	0.2	0.7
M4	0.1001	0.0001	0.1	-0.1	-0.3
M5	0.0998	0.0002	0.2	0.2	0.7
M6	0.1001	0.0001	0.1	-0.1	-0.3
M7	0.1001	0.0001	0.1	-0.1	-0.3

Table $P\left(Q \geq 21.0851 \middle| V_1\right)$

V matrix type 1

$P_T = 0.05$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0503	0.0003	0.6	-0.6	-1.4
M2	0.3084	0.2584	516.8	-516.8	-1185.3
M3	0.0502	0.0002	0.4	-0.4	-0.9
M4	0.0503	0.0003	0.6	-0.6	-1.4
M5	0.0502	0.0002	0.4	-0.4	-0.9
M6	0.0503	0.0003	0.6	-0.6	-1.4
M7	0.0503	0.0003	0.6	-0.6	-1.4

Table $P\left(Q \geq 23.5695 \middle| V_1\right)$

V matrix type 1

$P_T = 0.02$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0201	0.0001	0.5	-0.5	-0.7
M2	0.2257	0.2057	1028.5	-1028.5	-1469.3

M3	0.0201	0.0001	0.5	-0.5	-0.7
M4	0.0201	0.0001	0.5	-0.5	-0.7
M5	0.0201	0.0001	0.5	-0.5	-0.7
M6	0.0201	0.0001	0.5	-0.5	-0.7
M7	0.0201	0.0001	0.5	-0.5	-0.7

Table $P\left(Q \geq 25.3353 \middle| V_1\right)$

V matrix type 1

$P_T = 0.01$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0100	0	0	0	0
M2	0.1782	0.1682	1682.0	-1682.0	-1699.0
M3	0.0101	0.0001	1.0	-1.0	-1.0
M4	0.0100	0	0	0	0
M5	0.0101	0.0001	1.0	-1.0	-1.0
M6	0.0100	0	0	0	0
M7	0.0100	0	0	0	0

Table $P\left(Q \geq 24.6965 \middle| V_2\right)$

V matrix type 2a

$P_T = 0.10$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0694	0.0306	30.6	30.6	102.0
M2	0.1944	0.0944	94.4	-94.4	-314.7
M3	0.1113	0.0113	11.3	-11.3	-37.7
M4	0.1094	0.0094	9.4	-9.4	-31.3
M5	0.1120	0.0120	12.0	-12.0	-40.0
M6	***				
M7	0.0948	0.0052	5.2	5.2	17.3

Table $P\left(Q \geq 28.7194 \middle| V_2\right)$

V matrix type 2a

$P_T = 0.05$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0212	0.0288	57.6	57.6	132.1
M2	0.1101	0.0601	120.2	-120.2	-275.7
M3	0.0493	0.0007	1.4	1.4	3.2
M4	0.0560	0.0060	12.0	-12.0	-27.5
M5	0.0612	0.0112	22.4	-22.4	-51.4
M6	***				
M7	0.0489	0.0011	2.2	2.2	5.1

Table $P\left(Q \geq 34.3295 \middle| V_2\right)$

V matrix type 2a

$P_T = 0.02$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0033	0.0167	83.5	83.5	119.3
M2	0.0462	0.0262	131.0	-131.0	-187.1
M3	0.0142	0.0058	29.0	29.0	41.4
M4	0.0214	0.0014	7.0	-7.0	-10.0
M5	0.0274	0.0074	37.0	-37.0	-52.9
M6	***				
M7	0.0202	0.0002	1.0	-1.0	-1.4

Table $P\left(Q \geq 38.7173 \middle| V_2\right)$

V matrix type 2a

$P_T = 0.01$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	6.5712e-004	0.0093	93.0	93.0	93.9
M2	0.0223	0.0123	123.0	-123.0	-124.2
M3	0.0050	0.0050	50.0	50.0	50.5
M4	0.0099	0.0001	1.0	1.0	1.0
M5	0.0152	0.0052	52.0	-52.0	-52.5
M6	***				
M7	0.0103	0.0003	3.0	-3.0	-3.0

Table $P\left(Q \geq 15.9579 \middle| V_2\right)$

V matrix type 2b

$P_T = 0.10$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0641	0.0359	35.9	35.9	119.7
M2	0.1735	0.0735	73.5	-73.5	-245.0
M3	0.1057	0.0057	5.7	-5.7	-19.0
M4	0.1059	0.0059	5.9	-5.9	-19.7
M5	0.1073	0.0073	7.3	-7.3	-24.3
M6	***				
M7	0.0932	0.0068	6.8	6.8	22.7

Table $P\left(Q \geq 18.3794 \middle| V_2\right)$

V matrix type 2b

$P_T = 0.05$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0208	0.0292	58.4	58.4	133.9
M2	0.0994	0.0493	98.6	-98.6	-226.2
M3	0.0490	0.0010	2.0	2.0	4.6

M4	0.0539	0.0039	7.8	-7.8	-17.9
M5	0.0567	0.0067	13.4	-13.4	-30.7
M6	***				
M7	0.0469	0.0031	6.2	6.2	14.2

Table $P\left(Q \geq 21.6179 \middle| V_2\right)$
V matrix type 2b
$P_T = 0.02$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0039	0.0161	80.5	80.5	115.0
M2	0.0443	0.0243	121.5	-121.5	-173.6
M3	0.0161	0.0039	19.5	19.5	27.9
M4	0.0209	0.0009	4.5	-4.5	-6.4
M5	0.0241	0.0041	20.5	-20.5	-29.3
M6	***				
M7	0.0191	0.0009	4.5	4.5	6.4

Table $P\left(Q \geq 24.0891 \middle| V_2\right)$
V matrix type 2b
$P_T = 0.01$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	9.5767e-004	0.0090	90.0	90.0	90.9
M2	0.0230	0.0130	130.0	-130.0	-131.3
M3	0.0065	0.0035	35.0	35.0	35.4
M4	0.0099	0.0001	1.0	1.0	1.0
M5	0.0126	0.0026	26.0	-26.0	-26.3
M6	***				
M7	0.0098	0.0002	2.0	2.0	2.0

Table $P\left(Q \geq 15.9399 \middle| V_2\right)$
V matrix type 2c
$P_T = 0.10$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0554	0.0446	44.6	44.6	148.7
M2	0.1446	0.0466	44.6	-44.6	-148.7
M3	0.1067	0.0067	6.7	-6.7	-22.3
M4	0.1062	0.0062	6.2	-6.2	-20.7
M5	0.1079	0.0079	7.9	-7.9	-26.3
M6	***				
M7	0.0931	0.0069	6.9	6.9	23.0

Table $P\left(Q \geq 18.5541 \middle| V_2\right)$
V matrix type 2c

$P_T = 0.05$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	sgn $\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0156	0.0344	68.8	68.8	157.8
M2	0.0559	0.0059	11.8	-11.8	-27.1
M3	0.0489	0.0011	2.2	2.2	5.1
M4	0.0542	0.0042	8.4	-8.4	-19.3
M5	0.0576	0.0076	15.2	-15.2	-34.9
M6	***				
M7	0.0473	0.0027	5.4	5.4	12.4

Table $P\left(Q \geq 22.0473 \middle| V_2\right)$

V matrix type 2c

$P_T = 0.02$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	sgn $\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0023	0.0177	88.5	88.5	126.4
M2	0.0129	0.0071	35.5	35.5	50.7
M3	0.0158	0.0042	21.0	21.0	30.0
M4	0.0213	0.0013	6.5	-6.5	-9.3
M5	0.0253	0.0053	26.5	-26.5	-37.9
M6	***				
M7	0.0196	0.0004	2.0	2.0	2.9

Table $P\left(Q \geq 24.7591 \middle| V_2\right)$

V matrix type 2c

$P_T = 0.01$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	sgn $\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	4.6791e-004	0.0095	95.0	95.0	96.0
M2	0.0037	0.0063	63.0	63.0	63.6
M3	0.0063	0.0037	37.0	37.0	37.4
M4	0.0101	0.0001	1.0	-1.0	-1.0
M5	0.0136	0.0036	36.0	-36.0	-36.4
M6	***				
M7	0.0101	0.0001	1.0	-1.0	-1.0

Table $P\left(Q \geq 15.9218 \middle| V_2\right)$

V matrix type 2d

$P_T = 0.10$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	sgn $\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0473	0.0527	52.7	52.7	175.7
M2	0.1455	0.0455	45.5	-45.5	-151.7
M3	0.1084	0.0084	8.4	-8.4	-28.0

M4	0.1070	0.0070	7.0	-7.0	-23.3
M5	0.1090	0.0090	9.0	-9.0	-30.0
M6	***				
M7	0.0939	0.0061	6.1	6.1	20.3

Table $P\left(Q \geq 18.7239 \middle| V_2\right)$

V matrix type 2d

$P_T = 0.05$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0115	0.0385	77.0	77.0	176.6
M2	0.0523	0.0023	4.6	-4.6	-10.6
M3	0.0496	0.0004	0.8	0.8	1.8
M4	0.0549	0.0049	9.8	-9.8	-22.5
M5	0.0590	0.0090	18.0	-18.0	-41.3
M6	***				
M7	0.0482	0.0018	3.6	3.6	8.3

Table $P\left(Q \geq 22.5591 \middle| V_2\right)$

V matrix type 2d

$P_T = 0.02$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0013	0.0187	93.5	93.5	133.6
M2	0.0103	0.0097	48.5	48.5	69.3
M3	0.0156	0.0044	22.0	22.0	31.4
M4	0.0214	0.0014	7.0	-7.0	-10.0
M5	0.0262	0.0062	31.0.	-31.0.	-44.3
M6	***				
M7	0.0200	0	0	0	0

Table $P\left(Q \geq 25.6227 \middle| V_2\right)$

V matrix type 2d

$P_T = 0.01$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	1.9207e-004	0.0098	98.0	98.0	99.0
M2	0.0024	0.0076	76.0	76.0	76.8
M3	0.0059	0.0041	41.0	41.0	41.4
M4	0.0099	0.0001	1.0	1.0	1.0
M5	0.0141	0.0041	41.0	-41.0	-41.4
M6	***				
M7	0.0101	0.0001	1.0	-1.0	-1.0

Table $P\left(Q \geq 13.0061 \middle| V_3\right)$

V matrix type 3a

$P_T = 0.10$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0857	0.0143	14.3	14.3	47.7
M2	0.3897	0.2897	289.7	-289.7	-965.7
M3	0.0990	0.0010	1.0	1.0	3.3
M4	0.0996	0.0004	0.4	0.4	1.3
M5	0.0995	0.0005	0.5	0.5	1.7
M6	0.0992	0.0008	0.8	0.8	2.7
M7	0.0991	0.0009	0.9	0.9	3.0

Table $P\left(Q \geq 14.5315 \middle| V_3\right)$

V matrix type 3a

$P_T = 0.05$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0388	0.0112	22.4	22.4	51.4
M2	0.2631	0.2131	426.2	-426.2	-977.5
M3	0.0490	0.0010	2.0	2.0	4.6
M4	0.0499	0.0001	0.2	0.2	0.5
M5	0.0501	0.0001	0.2	-0.2	-0.5
M6	0.0497	0.0003	0.6	0.6	1.4
M7	0.0497	0.0003	0.6	0.6	1.4

Table $P\left(Q \geq 16.3992 \middle| V_3\right)$

V matrix type 3a

$P_T = 0.02$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0134	0.0066	33.0	33.0	47.1
M2	0.1503	0.1303	651.5	-651.5	-930.7
M3	0.0192	0.0008	4.0	4.0	5.7
M4	0.0199	0.0001	0.5	0.5	0.7
M5	0.0202	0.0002	1.0	-1.0	-1.4
M6	0.0199	0.0001	0.5	0.5	0.7
M7	0.0199	0.0001	0.5	0.5	0.7

Table $P\left(Q \geq 17.7239 \middle| V_3\right)$

V matrix type 3a

$P_T = 0.01$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0059	0.0041	41.0	41.0	41.4
M2	0.0964	0.0864	864.0	-864.0	-872.7
M3	0.0095	0.0005	5.0	5.0	5.1

M4	0.0099	0.0001	1.0	1.0	1.0
M5	0.0102	0.0002	2.0	-2.0	-2.0
M6	0.0100	0	0	0	0
M7	0.0100	0	0	0	0

Table $P\left(Q \geq 11.0869 \middle| V_3\right)$

V matrix type 3b

$P_T = 0.10$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0857	0.0143	14.3	14.3	47.7
M2	0.3995	0.2995	299.5	-299.5	-998.3
M3	0.1004	0.0004	0.4	-0.4	-1.3
M4	0.1011	0.0011	1.1	-1.1	-3.7
M5	0.1011	0.0011	1.1	-1.1	-3.7
M6	0.1003	0.0003	0.3	-0.3	-1.0
M7	0.0997	0.0003	0.3	0.3	1.0

Table $P\left(Q \geq 12.4240 \middle| V_3\right)$

V matrix type 3b

$P_T = 0.05$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0379	0.0121	24.2	24.2	55.5
M2	0.2686	0.2186	437.2	-437.2	-1002.8
M3	0.0491	0.0009	1.8	1.8	4.1
M4	0.0506	0.0006	1.2	-1.2	-2.8
M5	0.0509	0.0009	1.8	-1.8	-4.1
M6	0.0502	0.0002	0.4	-0.4	-0.9
M7	0.0498	0.0002	0.4	0.4	0.9

Table $P\left(Q \geq 14.0760 \middle| V_3\right)$

V matrix type 3b

$P_T = 0.02$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0125	0.0075	37.5	37.5	53.6
M2	0.1513	0.1313	656.5	-656.5	-937.9
M3	0.0187	0.0013	6.5	6.5	9.3
M4	0.0200	0	0	0	0
M5	0.0205	0.0005	2.5	-2.5	-3.6
M6	0.0200	0	0	0	0
M7	0.0198	0.0002	1.0	1.0	1.4

Table $P\left(Q \geq 15.2614 \middle| V_3\right)$

V matrix type 3b

$P_T = 0.01$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0053	0.0047	47.0	47.0	47.5
M2	0.0953	0.0853	853.0	-853.0	-861.6
M3	0.0089	0.0011	11.0	11.0	11.1
M4	0.0099	0.0001	1.0	1.0	1.0
M5	0.0104	0.0004	4.0	-4.0	-4.0
M6	0.0100	0	0	0	0
M7	0.0099	0.0001	1.0	1.0	1.0

Table $P\left(Q \geq 15.2481 \middle| V_3\right)$

V matrix type 3c

$P_T = 0.10$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0986	0.0014	1.4	1.4	4.7
M2	0.4485	0.3485	348.5	-348.5	-1161.7
M3	0.1000	0	0	0	0
M4	0.1004	0.0004	0.4	-0.4	-1.3
M5	0.1001	0.0001	0.1	-0.1	-0.3
M6	0.1003	0.0003	0.3	-0.3	-1.0
M7	0.1002	0.0002	0.2	-0.2	-0.7

Table $P\left(Q \geq 16.9227 \middle| V_3\right)$

V matrix type 3c

$P_T = 0.05$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0486	0.0014	2.8	2.8	6.4
M2	0.3241	0.2741	548.2	-548.2	-1257.3
M3	0.0498	0.0002	0.4	0.4	0.9
M4	0.0501	0.0001	0.2	-0.2	-0.5
M5	0.0501	0.0001	0.2	-0.2	-0.5
M6	0.0501	0.0001	0.2	-0.2	-0.5
M7	0.0500	0	0	0	0

Table $P\left(Q \geq 18.9592 \middle| V_3\right)$

V matrix type 3c

$P_T = 0.02$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0189	0.0011	5.5	5.5	7.9
M2	0.2045	0.1845	922.5	-922.5	-1317.9
M3	0.0197	0.0003	1.5	1.5	2.1

M4	0.0199	0.0001	0.5	0.5	0.7
M5	0.0200	0	0	0	0
M6	0.0199	0.0001	0.5	0.5	0.7
M7	0.0198	0.0002	1.0	1.0	1.4

Table $P\left(Q \geq 20.3848 \middle| V_3\right)$

V matrix type 3c

$P_T = 0.01$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0093	0.0007	7.0	7.0	7.1
M2	0.1425	0.1325	1325.0	-1325.0	-1338.4
M3	0.0099	0.0001	1.0	1.0	1.0
M4	0.0099	0.0001	1.0	1.0	1.0
M5	0.0100	0	0.0	0.0	0
M6	0.0099	0.0001	1.0	1.0	1.0
M7	0.0099	0.0001	1.0	1.0	1.0

Table $P\left(Q \geq 18.6689 \middle| V_4\right)$

V matrix type 4a

$P_T = 0.10$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0930	0.0070	7.0	7.0	23.3
M2	0.0930	0.0070	7.0	7.0	23.3
M3	0.1001	0.0001	0.1	-0.1	-0.3
M4	0.1006	0.0006	0.6	-0.6	-2.0
M5	0.1004	0.0004	0.4	-0.4	-0.3
M6	0.1003	0.0003	0.3	-0.3	-1.0
M7	0.1000	0	0	0	0

Table $P\left(Q \geq 20.7985 \middle| V_4\right)$

V matrix type 4a

$P_T = 0.05$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0439	0.0061	12.2	12.2	28.0
M2	0.0439	0.0061	12.2	12.2	28.0
M3	0.0495	0.0005	1.0	1.0	2.3
M4	0.0502	0.0002	0.4	-0.4	-0.9
M5	0.0503	0.0003	0.6	-0.6	-1.4
M6	0.0501	0.0001	0.2	-0.2	-0.5
M7	0.0499	0.0001	0.2	0.2	0.5

Table $P\left(Q \geq 23.3862 \middle| V_3\right)$

V matrix type 4a

$P_T = 0.02$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0161	0.0039	19.5	19.5	27.9
M2	0.0161	0.0039	19.5	19.5	27.9
M3	0.0194	0.0006	3.0	3.0	4.3
M4	0.0200	0	0	0	0
M5	0.0202	0.0002	1.0	-1.0	-1.4
M6	0.0200	0	0	0	0
M7	0.0199	0.0001	0.5	0.5	0.7

Table $P\left(Q \geq 25.2329 \middle| V_4\right)$

V matrix type 4a

$P_T = 0.01$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0075	0.0025	25.0	25.0	25.3
M2	0.0075	0.0025	25.0	25.0	25.3
M3	0.0096	0.0004	4.0	4.0	4.0
M4	0.0099	0.0001	1.0	1.0	1.0
M5	0.0102	0.0002	2.0	-2.0	-2.0
M6	0.0100	0	0	0	0
M7	0.0099	0.0001	1.0	1.0	1.0

Table $P\left(Q \geq 17.2824 \middle| V_4\right)$

V matrix type 4b

$P_T = 0.10$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0857	0.0143	14.3	14.3	47.7
M2	0.1051	0.0051	5.1	-5.1	-17.0
M3	0.0995	0.0005	0.5	0.5	1.7
M4	0.1001	0.0001	0.1	-0.1	-0.3
M5	0.1001	0.0001	0.1	-0.1	-0.3
M6	0.0996	0.0004	0.4	0.4	1.3
M7	0.0993	0.0007	0.7	0.7	2.3

Table $P\left(Q \geq 19.3384 \middle| V_4\right)$

V matrix type 4b

$P_T = 0.05$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0384	0.0116	23.2	23.2	53.2
M2	0.0494	0.0006	1.2	1.2	2.6
M3	0.0489	0.0011	2.2	2.2	5.1

M4	0.0500	0	0	0	0
M5	0.0503	0.0003	0.6	-0.6	-1.4
M6	0.0498	0.0002	0.4	0.4	0.9
M7	0.0496	0.0004	0.8	0.8	1.8

Table $P\left(Q \geq 21.8946 \middle| V_4\right)$

V matrix type 4b

$P_T = 0.02$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0128	0.0072	36.0	36.0	51.4
M2	0.0175	0.0025	12.5	12.5	17.9
M3	0.0186	0.0014	7.0	7.0	10.0
M4	0.0196	0.0004	2.0	2.0	2.9
M5	0.0200	0	0	0	0
M6	0.0196	0.0004	2.0	2.0	2.9
M7	0.0195	0.0005	2.5	2.5	3.6

Table $P\left(Q \geq 23.7301 \middle| V_4\right)$

V matrix type 4b

$P_T = 0.01$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0055	0.0045	45.0	45.0	45.5
M2	0.0078	0.0022	22.0	22.0	22.2
M3	0.0089	0.0011	11.0	11.0	11.1
M4	0.0096	0.0004	4.0	4.0	4.0
M5	0.0100	0	0	0	0
M6	0.0097	0.0003	3.0	3.0	3.0
M7	0.0096	0.0004	4.0	4.0	4.0

Table $P\left(Q \geq 16.2150 \middle| V_4\right)$

V matrix type 4c

$P_T = 0.10$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0846	0.0154	15.4	15.4	51.3
M2	0.1132	0.0132	13.2	-13.2	-44.0
M3	0.0994	0.0006	0.6	0.6	2.0
M4	0.1002	0.0002	0.2	-0.2	-0.7
M5	0.1001	0.0001	0.1	-0.1	-0.3
M6	0.0994	0.0006	0.6	0.6	2.0
M7	0.0989	0.0011	1.1	1.1	3.7

Table $P\left(Q \geq 18.1595 \middle| V_4\right)$

V matrix type 4c

$P_T = 0.05$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0375	0.0125	25.0	25.0	57.3
M2	0.0539	0.0039	7.8	-7.8	-17.9
M3	0.0487	0.0013	2.6	2.6	6.0
M4	0.0501	0.0001	0.2	-0.2	-0.5
M5	0.0505	0.0005	1.0	-1.0	-2.3
M6	0.0498	0.0002	0.4	0.4	0.9
M7	0.0495	0.0005	1.0	1.0	2.3

Table $P\left(Q \geq 20.5798 \middle| V_4\right)$

V matrix type 4c

$P_T = 0.02$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0123	0.0077	38.5	38.5	55.0
M2	0.0193	0.0007	3.5	3.5	50
M3	0.0185	0.0015	7.5	7.5	10.7
M4	0.0197	0.0003	1.5	1.5	2.1
M5	0.0202	0.0002	1.0	-1.0	-1.4
M6	0.0197	0.0003	1.5	1.5	2.1
M7	0.0195	0.0005	2.5	2.5	3.6

Table $P\left(Q \geq 22.3282 \middle| V_4\right)$

V matrix type 4c

$P_T = 0.01$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0052	0.0048	48.0	48.0	48.5
M2	0.0087	0.0013	13.0	13.0	13.1
M3	0.0088	0.0012	12.0	12.0	12.1
M4	0.0096	0.0004	4.0	4.0	4.0
M5	0.0101	0.0001	1.0	-1.0	-1.0
M6	0.0097	0.0003	3.0	3.0	3.0
M7	0.0096	0.0004	4.0	4.0	4.0

Table $P\left(Q \geq 26.3064 \middle| V_5\right)$

V matrix type 5a

$P_T = 0.10$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0994	0.0006	0.6	0.6	2.0
M2	0.1219	0.0219	21.9	-21.9	-73.0
M3	0.0998	0.0002	0.2	0.2	0.7

M4	0.1001	0.0001	0.1	-0.1	-0.3
M5	0.0999	0.0001	0.1	0.1	0.3
M6	0.1001	0.0001	0.1	-0.1	-0.3
M7	0.1001	0.0001	0.1	-0.1	-0.3

Table $P\left(Q \geq 29.1956 \middle| V_5\right)$

V matrix type 5a

$P_T = 0.05$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0490	0.0010	2.0	2.0	4.6
M2	0.0630	0.0130	26.0	-26.0	59.6
M3	0.0495	0.0005	1.0	1.0	2.3
M4	0.0497	0.0003	0.6	0.6	1.4
M5	0.0496	0.0004	0.8	0.8	1.8
M6	0.0497	0.0003	0.6	0.6	1.4
M7	0.0497	0.0003	0.6	0.6	1.4

Table $P\left(Q \geq 32.6458 \middle| V_5\right)$

V matrix type 5a

$P_T = 0.02$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0194	0.0006	3.0	3.0	4.3
M2	0.0264	0.0064	32.0	-32.0	-45.7
M3	0.0198	0.0002	1.0	1.0	1.4
M4	0.0199	0.0001	0.5	0.5	0.7
M5	0.0199	0.0001	0.5	0.5	0.7
M6	0.0199	0.0001	0.5	0.5	0.7
M7	0.0199	0.0001	0.5	0.5	0.7

Table $P\left(Q \geq 35.0872 \middle| V_5\right)$

V matrix type 5a

$P_T = 0.01$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0096	0.0004	4.0	4.0	4.0
M2	0.0136	0.0036	36.0	-36.0	-36.4
M3	0.0099	0.0001	1.0	1.0	1.0
M4	0.0099	0.0001	1.0	1.0	1.0
M5	0.0100	0	0	0	0
M6	0.0099	0.0001	1.0	1.0	1.0
M7	0.0099	0.0001	1.0	1.0	1.0

Table $P\left(Q \geq 25.8350 \middle| V_5\right)$

V matrix type 5b

$P_T = 0.10$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \|P_T - P_{Mi}\|$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn}\hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn}\hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn}\hat{\delta}_{\%VMi})}$
M1	0.0981	0.0019	1.9	1.9	6.3
M2	0.1349	0.0349	34.9	-34.9	-116.3
M3	0.0994	0.0006	0.6	0.6	2.0
M4	0.0997	0.0003	0.3	0.3	1.0
M5	0.0995	0.0005	0.5	0.5	1.7
M6	0.0997	0.0003	0.3	0.3	1.0
M7	0.0996	0.0004	0.4	0.4	1.3

Table $P\left(Q \geq 28.6566 \middle| V_5\right)$

V matrix type 5b

$P_T = 0.05$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \|P_T - P_{Mi}\|$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn}\hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn}\hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn}\hat{\delta}_{\%VMi})}$
M1	0.0484	0.0016	3.2	3.2	7.3
M2	0.0716	0.0216	43.2	-43.2	-99.1
M3	0.0496	0.0004	0.8	0.8	1.8
M4	0.0499	0.0001	0.2	0.2	0.5
M5	0.0498	0.0002	0.4	0.4	0.9
M6	0.0498	0.0002	0.4	0.4	0.9
M7	0.0498	0.0002	0.4	0.4	0.9

Table $P\left(Q \geq 32.0578 \middle| V_5\right)$

V matrix type 5b

$P_T = 0.02$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \|P_T - P_{Mi}\|$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn}\hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn}\hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn}\hat{\delta}_{\%VMi})}$
M1	0.0191	0.0009	4.5	4.5	6.4
M2	0.0308	0.0108	54	-54	-77.1
M3	0.0199	0.0001	0.5	0.5	0.7
M4	0.0200	0	0	0	0
M5	0.0201	0.0001	0.5	-0.5	-0.7
M6	0.0200	0	0	0	0
M7	0.0200	0	0	0	0

Table $P\left(Q \geq 34.4617 \middle| V_5\right)$

V matrix type 5b

$P_T = 0.01$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \|P_T - P_{Mi}\|$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn}\hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn}\hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn}\hat{\delta}_{\%VMi})}$
M1	0.0094	0.0006	6.0	6.0	6.1
M2	0.0162	0.0062	62.0	-62.0	-62.6
M3	0.0100	0	0	0	0

M4	0.0100	0	0	0	0
M5	0.0101	0.0001	1.0	-1.0	-1.0
M6	0.0100	0	0	0	0
M7	0.0100	0	0	0	0

Table $P\left(Q \geq 15.4871 \middle| V_6\right)$

V matrix type 6

$P_T = 0.10$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0868	0.0132	13.2	13.2	44.0
M2	0.2296	0.1296	129.6	-129.6	-432.0
M3	0.0996	0.0004	0.4	0.4	1.3
M4	0.1002	0.0002	0.2	-0.2	-0.7
M5	0.1001	0.0001	0.1	-0.1	-0.3
M6	0.0997	0.0003	0.3	0.3	1.0
M7	0.0995	0.0005	0.5	0.5	1.7

Table $P\left(Q \geq 17.3220 \middle| V_6\right)$

V matrix type 6

$P_T = 0.05$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0391	0.0109	21.8	21.8	50.0
M2	0.1324	0.0824	164.8	-164.8	-378.0
M3	0.0489	0.0011	2.2	2.2	5.1
M4	0.0500	0	0	0	0
M5	0.0502	0.0002	0.4	-0.4	-0.9
M6	0.0497	0.0003	0.6	0.6	1.4
M7	0.0496	0.0004	0.8	0.8	1.8

Table $P\left(Q \geq 19.5626 \middle| V_6\right)$

V matrix type 6

$P_T = 0.02$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0134	0.0066	33.0	33.0	47.1
M2	0.0618	0.0418	209.0	-209.0	-298.6
M3	0.0190	0.0010	5.0	5.0	7.1
M4	0.0198	0.0002	1.0	1.0	1.4
M5	0.0202	0.0002	1.0	-1.0	-1.4
M6	0.0198	0.0002	1.0	1.0	1.4
M7	0.0198	0.0002	1.0	1.0	1.4

Table $P\left(Q \geq 21.1599 \middle| V_6\right)$

V matrix type 6

$P_T = 0.01$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0059	0.0041	41.0	41.0	41.4
M2	0.0341	0.0241	241.0	-241.0	-243.4
M3	0.0092	0.0008	8.0	8.0	8.1
M4	0.0099	0.0001	1.0	1.0	1.0
M5	0.0102	0.0002	2.0	-2.0	-2.0
M6	0.0099	0.0001	1.0	1.0	1.0
M7	0.0099	0.0001	1.0	1.0	1.0

Table $P\left(Q \geq 15.6609 \middle| V_7\right)$

V matrix type 7

$P_T = 0.10$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0785	0.0215	21.5	21.5	71.7
M2	0.1990	0.0990	99.0	-99.0	-330.0
M3	0.1003	0.0003	0.3	-0.3	-1.0
M4	0.1011	0.0011	1.1	-1.1	-3.7
M5	0.1012	0.0012	1.2	-1.2	-4.0
M6	0.1001	0.0001	0.1	-0.1	-0.3
M7	0.0992	0.0008	0.8	0.8	2.7

Table $P\left(Q \geq 17.6357 \middle| V_7\right)$

V matrix type 7

$P_T = 0.05$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0327	0.0173	34.6	34.6	79.4
M2	0.1060	0.0560	112.0	-112.0	-256.9
M3	0.0487	0.0013	2.6	2.6	6.0
M4	0.0505	0.0005	1.0	-1.0	-2.3
M5	0.0511	0.0011	2.2	-2.2	-5.1
M6	0.0500	0	0	0	0
M7	0.0495	0.0005	1.0	1.0	2.3

Table $P\left(Q \geq 20.0944 \middle| V_7\right)$

V matrix type 7

$P_T = 0.02$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0098	0.0102	51.0	51.0	72.9
M2	0.0436	0.0236	118.0	-118.0	-168.6
M3	0.0183	0.0017	8.5	8.5	12.1

M4	0.0199	0.0001	0.5	0.5	0.7
M5	0.0206	0.0006	3.0	-3.0	-4.3
M6	0.0199	0.0001	0.5	0.5	0.7
M7	0.0196	0.0004	20	20	2.9

Table $P\left(Q \geq 21.8599 \middle| V_7\right)$

V matrix type 7

$P_T = 0.01$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0039	0.0061	61.0	61.0	61.6
M2	0.0216	0.0116	116.0	-116.0	-117.2
M3	0.0087	0.0013	13.0	13.0	13.1
M4	0.0098	0.0002	2.0	2.0	2.0
M5	0.0104	0.0004	4.0	-4.0	-4.0
M6	0.0099	0.0001	1.0	1.0	1.0
M7	0.0098	0.0002	2.0	2.0	2.0

Variance matrix dimension 25

Table $P\left(Q \geq 23.2406 \middle| V_1\right)$

V matrix type 1

$P_T = 0.10$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0999	0.0001	0.1	0.1	0.3
M2	0.5056	0.4056	405.6	-405.6	-1352.0
M3	0.0997	0.0003	0.3	0.3	1.0
M4	0.0999	0.0001	0.1	0.1	0.3
M5	0.0997	0.0003	0.3	0.3	1.0
M6	0.0999	0.0001	0.1	0.1	0.3
M7	0.0999	0.0001	0.1	0.1	0.3

Table $P\left(Q \geq 25.4853 \middle| V_1\right)$

V matrix type 1

$P_T = 0.05$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0501	0.0001	0.2	-0.2	-0.5
M2	0.3798	0.3298	659.6	-659.6	-1512.8
M3	0.0500	0	0	0	0
M4	0.0501	0.0001	0.2	-0.2	-0.5
M5	0.0500	0	0	0	0
M6	0.0501	0.0001	0.2	-0.2	-0.5
M7	0.0501	0.0001	0.2	-0.2	-0.5

Table $P\left(Q \geq 28.1783 \middle| V_1\right)$

V matrix type 1

$P_T = 0.02$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0201	0.0001	0.5	-0.5	-0.7
M2	0.2526	0.2326	1163.0	-1163.0	-1661.4
M3	0.0201	0.0001	0.5	-0.5	-0.7
M4	0.0201	0.0001	0.5	-0.5	-0.7
M5	0.0201	0.0001	0.5	-0.5	-0.7
M6	0.0201	0.0001	0.5	-0.5	-0.7
M7	0.0201	0.0001	0.5	-0.5	-0.7

Table $P\left(Q \geq 30.0504 \middle| V_1\right)$

V matrix type 1

$P_T = 0.01$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0101	0.0001	1.0	-1.0	-1.0
M2	0.1831	0.1731	1731.0	-1731.0	-1748.5
M3	0.0102	0.0002	2.0	-2.0	-2.0
M4	0.0101	0.0001	1.0	-1.0	-1.0
M5	0.0102	0.0002	2.0	-2.0	-2.0
M6	0.0101	0.0001	1.0	-1.0	-1.0
M7	0.0101	0.0001	1.0	-1.0	-1.0

Table $P\left(Q \geq 30.2930 \middle| V_2\right)$

V matrix type 2a

$P_T = 0.10$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0628	0.0372	37.2	37.2	124.0
M2	0.1752	0.0752	75.2	-75.2	-250.7
M3	0.1133	0.0133	13.3	-13.3	-44.3
M4	0.1106	0.0106	10.6	-10.6	-35.3
M5	0.1139	0.0139	13.9	-13.9	-46.3
M6	***				
M7	0.0959	0.0041	4.1	4.1	13.7

Table $P\left(Q \geq 35.1296 \middle| V_2\right)$

V matrix type 2a

$P_T = 0.05$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0165	0.0335	67.0	67.0	153.7
M2	0.0665	0.0165	33.0	-33.0	-75.7

M3	0.0485	0.0015	3.0	3.0	6.9
M4	0.0563	0.0063	12.6	-12.6	-28.9
M5	0.0626	0.0126	25.2	-25.2	-57.8
M6	***				
M7	0.0493	0.0007	1.4	1.4	3.2

Table $P\left(Q \geq 41.8941 \middle| V_2\right)$

V matrix type 2a

$P_T = 0.02$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0019	0.0181	90.5	90.5	129.3
M2	0.0133	0.0067	33.5	33.5	47.9
M3	0.0130	0.0070	35.0	35.0	50.0
M4	0.0213	0.0013	6.5	-6.5	-9.3
M5	0.0288	0.0088	44	-44	-62.9
M6	***				
M7	0.0203	0.0003	1.5	-1.5	-2.1

Table $P\left(Q \geq 47.1735 \middle| V_2\right)$

V matrix type 2a

$P_T = 0.01$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	3.0260e-004	0.0097	97.0	97.0	98.0
M2	0.0032	0.0068	68.0	68.0	68.7
M3	0.0043	0.0057	57.0	57.0	57.6
M4	0.0099	0.0001	1.0	1.0	1.0
M5	0.0164	0.0064	64.0	-64.0	-64.7
M6	***				
M7	0.0103	0.0003	3.0	-3.0	-3.0

Table $P\left(Q \geq 19.2094 \middle| V_2\right)$

V matrix type 2b

$P_T = 0.10$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0582	0.0418	41.8	41.8	139.3
M2	0.1401	0.0401	40.1	-40.1	-133.7
M3	0.1079	0.0079	7.9	-7.9	-26.3
M4	0.1077	0.0077	7.7	-7.7	-25.7
M5	0.1097	0.0097	9.7	-9.7	-32.3
M6	***				
M7	0.0937	0.0063	6.3	6.3	21.0

Table $P\left(Q \geq 22.0286 \middle| V_2\right)$

V matrix type 2b

$P_T = 0.05$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \|P_T - P_{Mi}\|$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0167	0.0333	66.6	66.6	152.8
M2	0.0535	0.0035	7.0	-7.0	-16.1
M3	0.0490	0.0010	2.0	2.0	4.6
M4	0.0550	0.0050	10.0	-10.0	-22.9
M5	0.0587	0.0087	17.4	-17.4	-39.9
M6	***				
M7	0.0476	0.0024	4.8	4.8	11.0

Table $P\left(Q \geq 25.8185 \middle| V_2\right)$

V matrix type 2b

$P_T = 0.02$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \|P_T - P_{Mi}\|$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0025	0.0175	87.5	87.5	125.0
M2	0.0119	0.0081	40.5	40.5	57.9
M3	0.0153	0.0047	23.5	23.5	33.6
M4	0.0214	0.0014	7.0	-7.0	-10.0
M5	0.0256	0.0056	28.0	-28.0	-40.0
M6	***				
M7	0.0197	0.0003	1.5	1.5	2.1

Table $P\left(Q \geq 28.7844 \middle| V_2\right)$

V matrix type 2b

$P_T = 0.01$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \|P_T - P_{Mi}\|$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	4.8888e-004	0.0095	95.0	95.0	96.0
M2	0.0032	0.0068	68.0	68.0	68.7
M3	0.0058	0.0042	42.0	42.0	42.4
M4	0.0099	0.0001	1.0	1.0	1.0
M5	0.0136	0.0036	36.0	-36.0	-36.4
M6	***				
M7	0.0100	0	0	0	0

Table $P\left(Q \geq 19.1826 \middle| V_2\right)$

V matrix type 2c

$P_T = 0.10$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \|P_T - P_{Mi}\|$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0482	0.0518	51.8	51.8	172.7
M2	0.1413	0.0413	41.3	-41.3	-137.7
M3	0.1089	0.0089	8.9	-8.9	-29.7

M4	0.1078	0.0078	7.8	-7.8	-26.0
M5	0.1102	0.0102	10.2	-10.2	-34.0
M6	***				
M7	0.0939	0.0061	6.1	6.1	20.3

Table $P\left(Q \geq 22.2701 \middle| V_2\right)$

V matrix type 2c

$P_T = 0.05$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0115	0.0385	77.0	77.0	176.6
M2	0.0490	0.0010	2.0	2.0	4.6
M3	0.0486	0.0014	2.8	2.8	6.4
M4	0.0551	0.0051	10.2	-10.2	-23.4
M5	0.0597	0.0097	19.4	-19.4	-44.5
M6	***				
M7	0.0480	0.0020	4.0	4.0	9.2

Table $P\left(Q \geq 26.5019 \middle| V_2\right)$

V matrix type 2c

$P_T = 0.02$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0012	0.0188	94.0	94.0	134.3
M2	0.0088	0.0112	56.0	56.0	80.0
M3	0.0145	0.0055	27.5	27.5	39.3
M4	0.0212	0.0012	6.0	-6.0	-8.6
M5	0.0265	0.0065	32.5	-32.5	-46.4
M6	***				
M7	0.0198	0.0002	1.0	1.0	1.4

Table $P\left(Q \geq 29.8629 \middle| V_2\right)$

V matrix type 2c

$P_T = 0.01$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	1.7067e-004	0.0098	98.0	98.0	99.0
M2	0.0019	0.0081	81.0	81.0	81.8
M3	0.0052	0.0048	48.0	48.0	48.5
M4	0.0098	0.0002	2.0	2.0	2.0
M5	0.0144	0.0044	44.0	-44.0	-44.4
M6	***				
M7	0.0100	0	0	0	0

Table $P\left(Q \geq 19.1514 \middle| V_2\right)$

V matrix type 2d

$P_T = 0.10$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	sgn $\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0395	0.0605	60.5	60.5	201.7
M2	0.1427	0.0427	42.7	-42.7	-142.3
M3	0.1109	0.0109	10.9	-10.9	-36.3
M4	0.1085	0.0085	8.5	-8.5	-28.3
M5	0.1112	0.0112	11.2	-11.2	-37.3
M6	***				
M7	0.0950	0.0050	5.0	5.0	16.7

Table $P\left(Q \geq 22.5301 \middle| V_2\right)$

V matrix type 2d

$P_T = 0.05$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	sgn $\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0075	0.0425	85.0	85.0	195.0
M2	0.0444	0.0056	11.2	11.2	25.7
M3	0.0489	0.0011	2.2	2.2	5.1
M4	0.0554	0.0054	10.8	-10.8	-27.8
M5	0.0606	0.0106	21.2	-21.2	-48.6
M6	***				
M7	0.0488	0.0012	2.4	2.4	5.5

Table $P\left(Q \geq 27.1834 \middle| V_2\right)$

V matrix type 2d

$P_T = 0.02$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	sgn $\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	5.5326e-004	0.0194	97.0	97.0	138.6
M2	0.0066	0.0134	67.0	67.0	95.7
M3	0.0144	0.0056	28.0	28.0	40.0
M4	0.0214	0.0014	7.0	-7.0	-10.0
M5	0.0276	0.0076	38.0	-38.0	-54.3
M6	***				
M7	0.0202	0.0002	1.0	-1.0	-1.4

Table $P\left(Q \geq 30.8846 \middle| V_2\right)$

V matrix type 2d

$P_T = 0.01$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	sgn $\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	5.6435e-005	0.0099	99.0	99.0	100.0
M2	0.0012	0.0088	88.0	88.0	88.9
M3	0.0051	0.0049	49.0	49.0	49.5

M4	0.0099	0.0001	1.0	1.0	1.0
M5	0.0154	0.0054	54.0	-54.0	-54.6
M6	***				
M7	0.0102	0.0002	2.0	-2.0	-2.0

Table $P\left(Q \geq 12.5821 \middle| V_3\right)$

V matrix type 3a

$P_T = 0.10$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0763	0.0237	23.7	23.7	79.0
M2	0.4004	0.3004	300.4	-300.4	-1001.3
M3	0.1001	0.0001	0.1	-0.1	-0.3
M4	0.1009	0.0009	0.9	-0.9	-3.0
M5	0.1009	0.0009	0.9	-0.9	-3.0
M6	0.1001	0.0001	0.1	-0.1	-0.3
M7	0.0996	0.0004	0.4	0.4	1.3

Table $P\left(Q \geq 14.0143 \middle| V_3\right)$

V matrix type 3a

$P_T = 0.05$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0314	0.0186	37.2	37.2	85.3
M2	0.2619	0.2119	423.8	-423.8	-972.0
M3	0.0487	0.0013	2.6	2.6	6.0
M4	0.0503	0.0003	0.6	-0.6	-1.4
M5	0.0507	0.0007	1.4	-1.4	-3.2
M6	0.0499	0.0001	0.2	0.2	0.5
M7	0.0496	0.0004	0.8	0.8	1.8

Table $P\left(Q \geq 15.7743 \middle| V_3\right)$

V matrix type 3a

$P_T = 0.02$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0093	0.0107	53.5	53.5	76.4
M2	0.1408	0.1208	604.0	-604.0	-862.9
M3	0.0185	0.0015	7.5	7.5	10.7
M4	0.0198	0.0002	1.0	1.0	1.4
M5	0.0204	0.0004	2.0	-2.0	-2.9
M6	0.0198	0.0002	1.0	1.0	1.4
M7	0.0196	0.0004	2.0	2.0	2.8

Table $P\left(Q \geq 17.0244 \middle| V_3\right)$

V matrix type 3a

$P_T = 0.01$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0037	0.0063	63.0	63.0	63.64
M2	0.0854	0.0754	754.0.	-754.0.	-761.62
M3	0.0088	0.0012	12.0	12.0	12.12
M4	0.0098	0.0002	2.0	2.0	2.02
M5	0.0103	0.0003	3.0	-3.0	-3.03
M6	0.0099	0.0001	1.0	1.0	1.01
M7	0.0098	0.0002	2.0	2.0	2.02

Table $P\left(Q \geq 15.3814 \middle| V_3\right)$

V matrix type 3b

$P_T = 0.10$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0915	0.0085	8.5	8.5	28.3
M2	0.4726	0.3726	372.6	-372.6	-1242.0
M3	0.0995	0.0005	0.5	0.5	1.7
M4	0.1000	0	0	0	0
M5	0.0999	0.0001	0.1	0.1	0.3
M6	0.0997	0.0003	0.3	0.3	1.0
M7	0.0995	0.0005	0.5	0.5	1.7

Table $P\left(Q \geq 16.9638 \middle| V_3\right)$

V matrix type 3b

$P_T = 0.05$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0429	0.0071	14.2	14.2	32.6
M2	0.3402	0.2902	580.4	-580.4	-1331.2
M3	0.0491	0.0009	1.8	1.8	4.1
M4	0.0499	0.0001	0.2	0.2	0.5
M5	0.0500	0	0	0	0
M6	0.0497	0.0003	0.6	0.6	1.4
M7	0.0496	0.0004	0.8	0.8	1.8

Table $P\left(Q \geq 18.8810 \middle| V_3\right)$

V matrix type 3b

$P_T = 0.02$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0155	0.0045	22.5	22.5	32.1
M2	0.2118	0.1918	959.0	-959.0	-1370.0
M3	0.0191	0.0009	4.5	4.5	6.4

M4	0.0198	0.0002	1.0	1.0	1.4
M5	0.0200	0	0	0	0
M6	0.0198	0.0002	1.0	1.0	1.4
M7	0.0197	0.0003	1.5	1.5	2.1

Table $P\left(Q \geq 20.2429 \middle| V_3\right)$

V matrix type 3b

$P_T = 0.01$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0071	0.0029	29.0	29.0	29.3
M2	0.1444	0.1344	1344.0	-1344.0	-1357.6
M3	0.0093	0.0007	7.0	7.0	7.1
M4	0.0098	0.0002	2.0	2.0	2.0
M5	0.0100	0	0	0	0
M6	0.0098	0.0002	2.0	2.0	2.0
M7	0.0098	0.0002	2.0	2.0	2.0

Table $P\left(Q \geq 19.2385 \middle| V_3\right)$

V matrix type 3c

$P_T = 0.10$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0978	0.0022	2.2	2.2	7.3
M2	0.4980	0.3980	398.0	-398.0	-1326.7
M3	0.0994	0.0006	0.6	0.6	2.0
M4	0.0997	0.0003	0.3	0.3	1.0
M5	0.0995	0.0005	0.5	0.5	1.7
M6	0.0996	0.0004	0.4	0.4	1.3
M7	0.0996	0.0004	0.4	0.4	1.3

Table $P\left(Q \geq 21.1277 \middle| V_3\right)$

V matrix type 3c

$P_T = 0.05$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0482	0.0018	3.6	3.6	8.3
M2	0.3703	0.3203	640.6	-640.6	-1469.3
M3	0.0496	0.0004	0.8	0.8	1.8
M4	0.0498	0.0002	0.4	0.4	0.9
M5	0.0498	0.0002	0.4	0.4	0.9
M6	0.0498	0.0002	0.4	0.4	0.9
M7	0.0498	0.0002	0.4	0.4	0.9

Table $P\left(Q \geq 23.4098 \middle| V_3\right)$

V matrix type 3c

$P_T = 0.02$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0188	0.0012	6.0	6.0	8.6
M2	0.2420	0.2220	1110.0	-1110.0	-1585.7
M3	0.0197	0.0003	1.5	1.5	2.1
M4	0.0198	0.0002	1.0	1.0	1.4
M5	0.0199	0.0001	0.5	0.5	0.7
M6	0.0198	0.0002	1.0	1.0	1.4
M7	0.0198	0.0002	1.0	1.0	1.4

Table $P\left(Q \geq 24.9895 \middle| V_3\right)$

V matrix type 3c

$P_T = 0.01$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0093	0.0007	7.0	7.0	7.1
M2	0.1732	0.1632	1632.0	-1632.0	-1648.5
M3	0.0099	0.0001	1.0	1.0	1.0
M4	0.0100	0	0	0	0
M5	0.0101	0.0001	1.0	-1.0	-1.0
M6	0.0100	0	0	0	0
M7	0.0100	0	0	0	0

Table $P\left(Q \geq 20.9107 \middle| V_4\right)$

V matrix type 4a

$P_T = 0.10$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0917	0.0083	8.3	8.3	27.7
M2	0.0917	0.0083	8.3	8.3	27.7
M3	0.1000	0	0	0	0
M4	0.1006	0.0006	0.6	-0.6	-2.0
M5	0.1005	0.0005	0.5	-0.5	-1.7
M6	0.1003	0.0003	0.3	-0.3	-1.0
M7	0.0999	0.0001	0.1	0.1	0.3

Table $P\left(Q \geq 23.0652 \middle| V_4\right)$

V matrix type 4a

$P_T = 0.05$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0429	0.0071	14.2	14.2	32.6
M2	0.0429	0.0071	14.2	14.2	32.6
M3	0.0494	0.0006	1.2	1.2	2.8

M4	0.0503	0.0003	0.6	-0.6	-1.4
M5	0.0504	0.0004	0.8	-0.8	-1.8
M6	0.0501	0.0001	0.2	-0.2	-0.5
M7	0.0499	0.0001	0.2	0.2	0.5

Table $P\left(Q \geq 25.6786 \middle| V_3\right)$

V matrix type 4a

$P_T = 0.02$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0155	0.0045	22.5	22.5	32.1
M2	0.0155	0.0045	22.5	22.5	32.1
M3	0.0193	0.0007	3.5	3.5	5.0
M4	0.0200	0	0	0	0
M5	0.0203	0.0003	1.5	-1.5	-2.1
M6	0.0200	0	0	0	0
M7	0.0199	0.0001	0.5	0.5	0.7

Table $P\left(Q \geq 27.5240 \middle| V_4\right)$

V matrix type 4a

$P_T = 0.01$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0071	0.0029	29.0	29.0	29.3
M2	0.0071	0.0029	29.0	29.0	29.3
M3	0.0095	0.0005	5.0	5.0	5.1
M4	0.0099	0.0001	1.0	1.0	1.0
M5	0.0102	0.0002	2.0	-2.0	-2.0
M6	0.0100	0	0	0	0
M7	0.0100	0	0	0	0

Table $P\left(Q \geq 21.3448 \middle| V_4\right)$

V matrix type 4b

$P_T = 0.10$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0880	0.0120	12.0	12.0	40.0
M2	0.1107	0.0107	10.7	-10.7	-35.7
M3	0.0998	0.0002	0.2	0.2	0.7
M4	0.1004	0.0004	0.4	-0.4	-1.3
M5	0.1003	0.0003	0.3	-0.3	-1.0
M6	0.1000	0	0	0	0
M7	0.0997	0.0003	0.3	0.3	1.0

Table $P\left(Q \geq 23.5915 \middle| V_4\right)$

V matrix type 4b

$P_T = 0.05$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \|P_T - P_{Mi}\|$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn}\hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn}\hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn}\hat{\delta}_{\%VMi})}$
M1	0.0401	0.0099	19.8	19.8	45.4
M2	0.0532	0.0032	6.4	-6.4	-14.7
M3	0.0491	0.0009	1.8	1.8	4.1
M4	0.0501	0.0001	0.2	-0.2	-0.5
M5	0.0503	0.0003	0.6	-0.6	-1.4
M6	0.0499	0.0001	0.2	0.2	0.5
M7	0.0498	0.0002	0.4	0.4	0.9

Table $P\left(Q \geq 26.3365 \middle| V_4\right)$

V matrix type 4b

$P_T = 0.02$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \|P_T - P_{Mi}\|$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn}\hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn}\hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn}\hat{\delta}_{\%VMi})}$
M1	0.0139	0.0061	30.5	30.5	43.6
M2	0.0196	0.0004	2.0	2.0	2.9
M3	0.0190	0.0010	5.0	5.0	7.1
M4	0.0198	0.0002	1.0	1.0	1.4
M5	0.0201	0.0001	0.5	-0.5	-0.7
M6	0.0198	0.0002	1.0	1.0	1.4
M7	0.0197	0.0003	1.5	1.5	2.1

Table $P\left(Q \geq 28.2591 \middle| V_4\right)$

V matrix type 4b

$P_T = 0.01$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \|P_T - P_{Mi}\|$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn}\hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn}\hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn}\hat{\delta}_{\%VMi})}$
M1	0.0062	0.0038	38.0	38.0	38.4
M2	0.0092	0.0008	8.0	8.0	8.1
M3	0.0093	0.0007	7.0	7.0	7.1
M4	0.0098	0.0002	2.0	2.0	2.0
M5	0.0101	0.0001	1.0	-1.0	-1.0
M6	0.0099	0.0001	1.0	1.0	1.0
M7	0.0099	0.0001	1.0	1.0	1.0

Table $P\left(Q \geq 20.5496 \middle| V_4\right)$

V matrix type 4c

$P_T = 0.10$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \|P_T - P_{Mi}\|$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn}\hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn}\hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn}\hat{\delta}_{\%VMi})}$
M1	0.0889	0.0111	11.1	11.1	37.0
M2	0.1242	0.0242	24.2	-24.2	-80.7
M3	0.1002	0.0002	0.2	-0.2	-0.7

M4	0.1002	0.0002	0.2	-0.2	0.7
M5	0.1007	0.0007	0.7	-0.7	-2.3
M6	0.1003	0.0003	0.3	-0.3	-1.0
M7	0.0999	0.0001	0.1	0.1	0.3

Table $P\left(Q \geq 22.7080 \middle| V_4\right)$

V matrix type 4c

$P_T = 0.05$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0407	0.0093	18.6	18.6	42.7
M2	0.0615	0.0115	23.0	-23.0	-52.8
M3	0.0493	0.0007	1.4	1.4	3.2
M4	0.0504	0.0004	0.8	-0.8	-1.8
M5	0.0506	0.0006	1.2	-1.2	-2.8
M6	0.0502	0.0002	0.4	-0.4	-0.9
M7	0.0499	0.0001	0.2	0.2	0.5

Table $P\left(Q \geq 25.3453 \middle| V_4\right)$

V matrix type 4c

$P_T = 0.02$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0141	0.0059	29.5	29.5	42.1
M2	0.0235	0.0035	17.5	-17.5	-25.0
M3	0.0190	0.0010	5	5	7.1
M4	0.0199	0.0001	0.5	0.5	0.7
M5	0.0203	0.0003	1.5	-1.5	-2.1
M6	0.0199	0.0001	0.5	0.5	0.7
M7	0.0198	0.0002	1	1	1.4

Table $P\left(Q \geq 27.1856 \middle| V_4\right)$

V matrix type 4c

$P_T = 0.01$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0064	0.0036	36.0	36.0	48.5
M2	0.0114	0.0014	14.0	-14.0	-14.1
M3	0.0093	0.0007	7.0	7.0	7.1
M4	0.0100	0	0	0	0
M5	0.0103	0.0003	3.0	-3.0	-3.0
M6	0.0100	0	0	0	0
M7	0.0100	0	0	0	0

Table $P\left(Q \geq 32.0275 \middle| V_5\right)$

V matrix type 5a

$P_T = 0.10$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0997	0.0003	0.3	0.3	1.0
M2	0.1263	0.0263	26.3	-26.3	-87.7
M3	0.1005	0.0005	0.5	-0.5	-1.7
M4	0.1008	0.0008	0.8	-0.8	-2.7
M5	0.1006	0.0006	0.6	-0.6	-2.0
M6	0.1007	0.0007	0.7	-0.7	-2.3
M7	0.1007	0.0007	0.7	-0.7	-2.3

Table $P\left(Q \geq 35.1680 \middle| V_5\right)$

V matrix type 5a

$P_T = 0.05$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0494	0.0006	1.2	1.2	2.8
M2	0.0659	0.0159	31.8	-31.8	-72.9
M3	0.0501	0.0001	0.2	-0.2	-0.5
M4	0.0503	0.0003	0.6	-0.6	-1.4
M5	0.0503	0.0003	0.6	-0.6	-1.4
M6	0.0503	0.0003	0.6	-0.6	-1.4
M7	0.0503	0.0003	0.6	-0.6	-1.4

Table $P\left(Q \geq 38.9415 \middle| V_5\right)$

V matrix type 5a

$P_T = 0.02$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0195	0.0005	2.5	2.5	3.6
M2	0.0277	0.0077	38.5	-38.5	-55.0
M3	0.0200	0	0	0	0
M4	0.0200	0	0	0	0
M5	0.0201	0.0001	0.5	-0.5	-0.7
M6	0.0200	0	0	0	0
M7	0.0200	0	0	0	0

Table $P\left(Q \geq 41.5676 \middle| V_5\right)$

V matrix type 5a

$P_T = 0.01$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0097	0.0003	3.0	3.0	3.0
M2	0.0144	0.0044	44.0	-44.0	-44.4
M3	0.0100	0	0	0	0

M4	0.0101	0.0001	1.0	-1.0	-1.0
M5	0.0101	0.0001	1.0	-1.0	-1.0
M6	0.0101	0.0001	1.0	-1.0	-1.0
M7	0.0101	0.0001	1.0	-1.0	-1.0

Table $P\left(Q \geq 31.5977 \middle| V_5\right)$

V matrix type 5b

$P_T = 0.10$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0982	0.0018	1.8	1.8	6.0
M2	0.1373	0.0373	37.3	-37.3	-124.3
M3	0.0998	0.0002	0.2	0.2	0.7
M4	0.1001	0.0001	0.1	-0.1	-0.3
M5	0.0999	0.0001	0.1	0.1	0.3
M6	0.1001	0.0001	0.1	-0.1	-0.3
M7	0.1000	0	0	0	0

Table $P\left(Q \geq 34.6913 \middle| V_5\right)$

V matrix type 5b

$P_T = 0.05$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0486	0.0014	2.8	2.8	6.4
M2	0.0731	0.0231	46.2	-46.2	-106.0
M3	0.0499	0.0001	0.2	0.2	0.5
M4	0.0502	0.0002	0.4	-0.4	-0.9
M5	0.0502	0.0002	0.4	-0.4	-0.9
M6	0.0502	0.0002	0.4	-0.4	-0.9
M7	0.0501	0.0001	0.2	-0.2	-0.5

Table $P\left(Q \geq 38.4155 \middle| V_5\right)$

V matrix type 5b

$P_T = 0.02$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0190	0.0010	5.0	5.0	7.1
M2	0.0314	0.0114	57.0	-57.0	-81.4
M3	0.0199	0.0001	0.5	0.5	0.7
M4	0.0201	0.0001	0.5	-0.5	-0.7
M5	0.0202	0.0002	1.0	-1.0	-1.4
M6	0.0201	0.0001	0.5	-0.5	-0.7
M7	0.0201	0.0001	0.5	-0.5	-0.7

Table $P\left(Q \geq 41.0767 \middle| V_5\right)$

V matrix type 5b

$P_T = 0.01$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0093	0.0007	7.0	7.0	7.1
M2	0.0163	0.0063	63.0	-63.0	-63.6
M3	0.0099	0.0001	1.0	1.0	1.0
M4	0.0099	0.0001	1.0	1.0	1.0
M5	0.0100	0	0	0	0
M6	0.0099	0.0001	1.0	1.0	1.0
M7	0.0099	0.0001	1.0	1.0	1.0

Table $P\left(Q \geq 17.7293 \middle| V_6\right)$

V matrix type 6

$P_T = 0.10$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0886	0.0114	11.4	11.4	38.0
M2	0.2553	0.1553	155.3	-155.3	-517.7
M3	0.0998	0.0002	0.2	0.2	0.7
M4	0.1004	0.0004	0.4	-0.4	-1.3
M5	0.1004	0.0004	0.4	-0.4	-1.3
M6	0.1000	0	0	0	0
M7	0.0997	0.0003	0.3	0.3	1.0

Table $P\left(Q \geq 19.6024 \middle| V_6\right)$

V matrix type 6

$P_T = 0.05$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0404	0.0096	19.2	19.2	44.0
M2	0.1514	0.1014	202.8	-202.8	-465.1
M3	0.0489	0.0011	2.2	2.2	5.1
M4	0.0499	0.0001	0.2	0.2	0.5
M5	0.0501	0.0001	0.2	-0.2	-0.5
M6	0.0497	0.0003	0.6	0.6	1.4
M7	0.0495	0.0005	1.0	1.0	2.3

Table $P\left(Q \geq 21.8508 \middle| V_6\right)$

V matrix type 6

$P_T = 0.02$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\text{sgn}\,\hat{\delta}_{\%VMi}$	$\frac{\text{sgn}\,\hat{\delta}_{\%VMi}}{\sigma(\text{sgn}\,\hat{\delta}_{\%VMi})}$
M1	0.0142	0.0058	29.0	29.0	41.4
M2	0.0738	0.0538	269.0	-269.0	-384.3

M3	0.0191	0.0009	4.5	4.5	6.4
M4	0.0199	0.0001	0.5	0.5	0.7
M5	0.0202	0.0002	1.0	-1.0	-1.4
M6	0.0199	0.0001	0.5	0.5	0.7
M7	0.0198	0.0002	1.0	1.0	1.4

Table $P\left(Q \geq 23.4402 \middle| V_6\right)$

V matrix type 6

$P_T = 0.01$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0064	0.0036	36.0	36.0	36.4
M2	0.0421	0.0321	321.0	-321.0	-324.2
M3	0.0093	0.0007	7.0	7.0	7.1
M4	0.0099	0.0001	1.0.	1.0.	1.0
M5	0.0102	0.0002	2.0	-2.0	-2.0
M6	0.0100	0	0	0	0
M7	0.0100	0	0	0	0

Table $P\left(Q \geq 19.8676 \middle| V_7\right)$

V matrix type 7

$P_T = 0.10$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0742	0.0258	25.8	25.8	86.0
M2	0.2114	0.1114	111.4	-111.4	-371.3
M3	0.1000	0	0	0	0
M4	0.1008	0.0008	0.8	-0.8	-2.7
M5	0.1009	0.0009	0.9	-0.9	-3.0
M6	0.1000	0	0	0	0
M7	0.0996	0.0004	0.4	0.4	1.3

Table $P\left(Q \geq 22.1693 \middle| V_7\right)$

V matrix type 7

$P_T = 0.05$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \lvert P_T - P_{Mi} \rvert$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn} \hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn} \hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn} \hat{\delta}_{\%VMi})}$
M1	0.0298	0.0202	40.4	40.4	92.7
M2	0.1123	0.0623	124.6	-124.6	-285.8
M3	0.0484	0.0016	3.2	3.2	7.3
M4	0.0501	0.0001	0.2	-0.2	-0.5
M5	0.0505	0.0005	1.0	-1.0	-2.3
M6	0.0497	0.0003	0.6	0.6	1.4
M7	0.0494	0.0006	1.2	1.2	2.8

Table $P\left(Q \geq 24.9734 \middle| V_7\right)$

V matrix type 7

$P_T = 0.02$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \|P_T - P_{Mi}\|$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn}\hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn}\hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn}\hat{\delta}_{\%VMi})}$
M1	0.0087	0.0113	56.5	56.5	80.7
M2	0.0464	0.0264	132.0	-132.0	-188.6
M3	0.0184	0.0016	8.0	8.0	11.4
M4	0.0198	0.0002	1.0	1.0	1.4
M5	0.0204	0.0004	2.0	-2.0	-2.9
M6	0.0198	0.0002	1.0	1.0	1.4
M7	0.0197	0.0003	1.5	1.5	2.1

Table $P\left(Q \geq 26.9706 \middle| V_7\right)$

V matrix type 7

$P_T = 0.01$

Method	P_{Mi}	$\hat{\delta}_{VMi} = \|P_T - P_{Mi}\|$	$\hat{\delta}_{\%VMi}$	$\operatorname{sgn}\hat{\delta}_{\%VMi}$	$\frac{\operatorname{sgn}\hat{\delta}_{\%VMi}}{\sigma(\operatorname{sgn}\hat{\delta}_{\%VMi})}$
M1	0.0034	0.0066	66.0	66.0	66.7
M2	0.0231	0.0131	131.0	-131.0	-132.3
M3	0.0088	0.0012	12.0	12.0	12.1
M4	0.0098	0.0002	2.0	2.0	2.0
M5	0.0104	0.0004	4.0	-4.0	-4.0
M6	0.0099	0.0001	1.0	1.0	1.0
M7	0.0098	0.0002	2.0	2.0	2.0

Appendix C

Box plots of distributions

This appendix contains 12 box plots of the distributions of the estimated percentage signed delta of methods 3 – 7. For each variance matrix dimension (10, 20 and 25) four plots are presented, one for each quantile probability (90%, 95%, 98% and 99%). There are generally 15 values of estimated percentage signed delta (as there are 15 sub-/types of variance matrices). Figures D1-D4 contain box plots of variance dimension 10 at the four different quantile probabilities, figures D5-D8 contain box plots of variance dimension 20 and finally figures D9-D12 contain box plots of variance dimension 25. However for plots D5-D12 (dimension 20 and 25) for method 6 there are only 11 values estimated percentage signed delta, since in these cases the algorithm for generating the approximated value for this method did not converge. It is important to observe that for different variance matrix dimensions and for different quantiles there are different scales in the box plots.

Variance matrix dimension 10

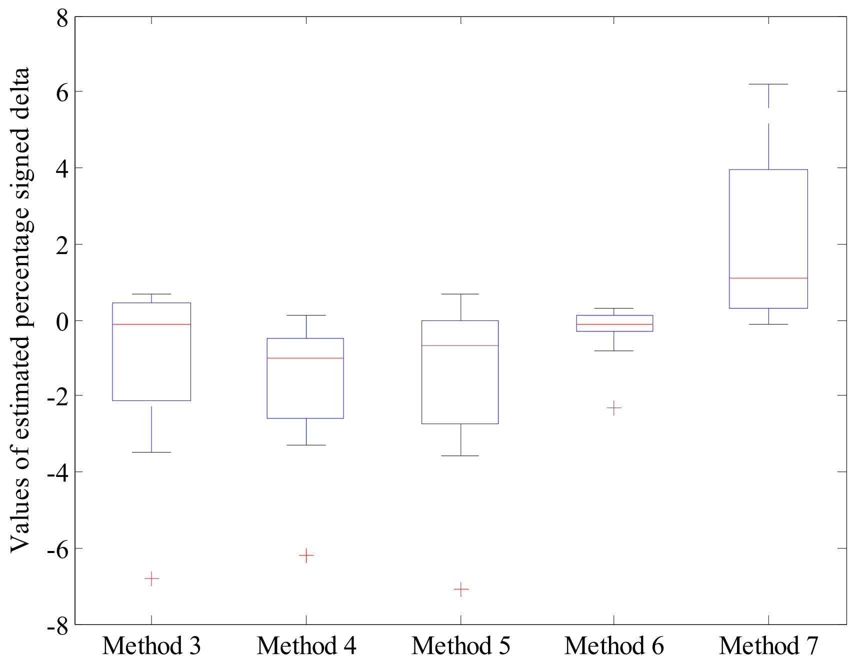

Figure D1 Distributions for variance matrices of size 10 and 90% quantile probability

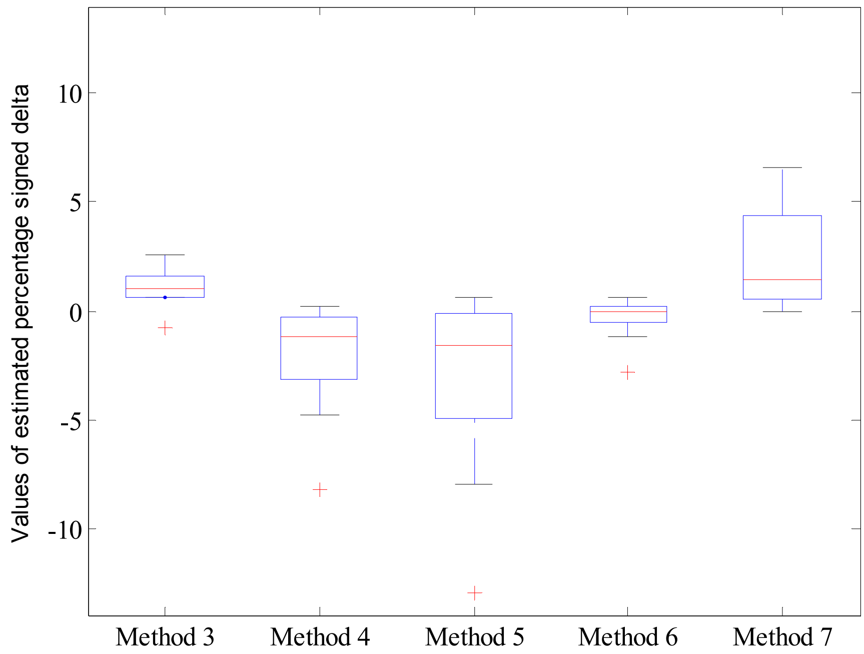

Figure D2 Distributions for variance matrices of size 10 and 95% quantile probability

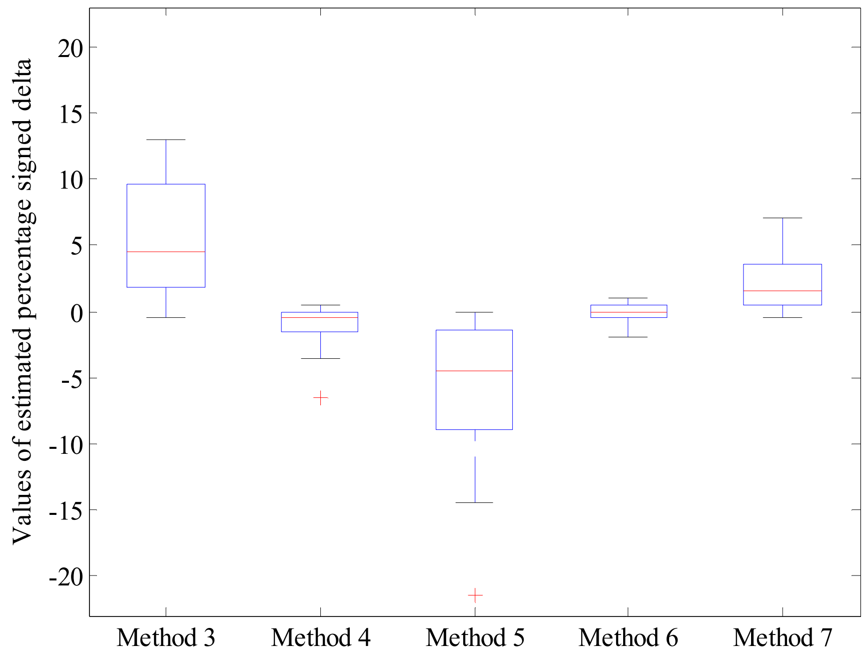

Figure D3 Distributions for variance matrices of size 10 and 98% quantile probability

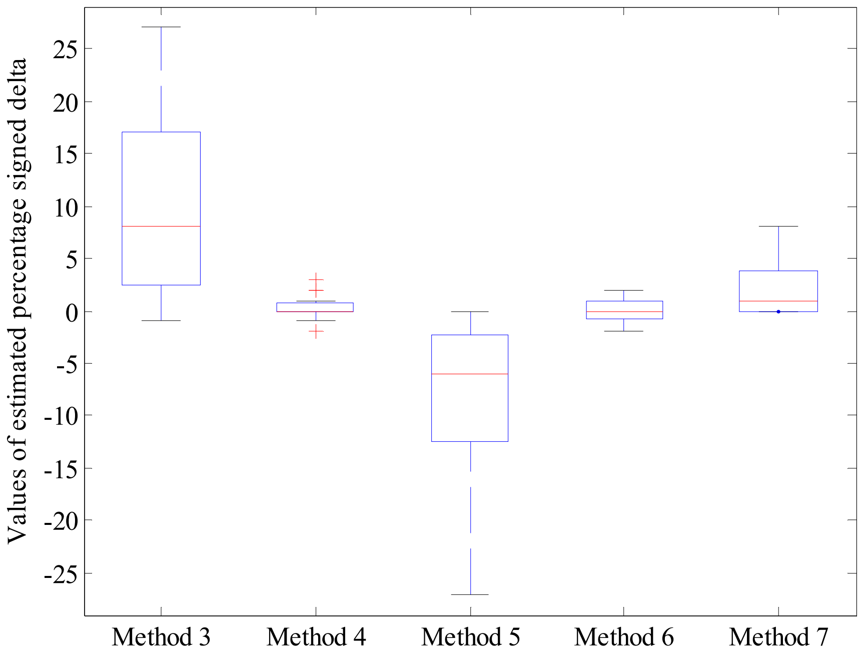

Figure D4 Distributions for variance matrices of size 10 and 99% quantile probability

Variance matrix dimension 20

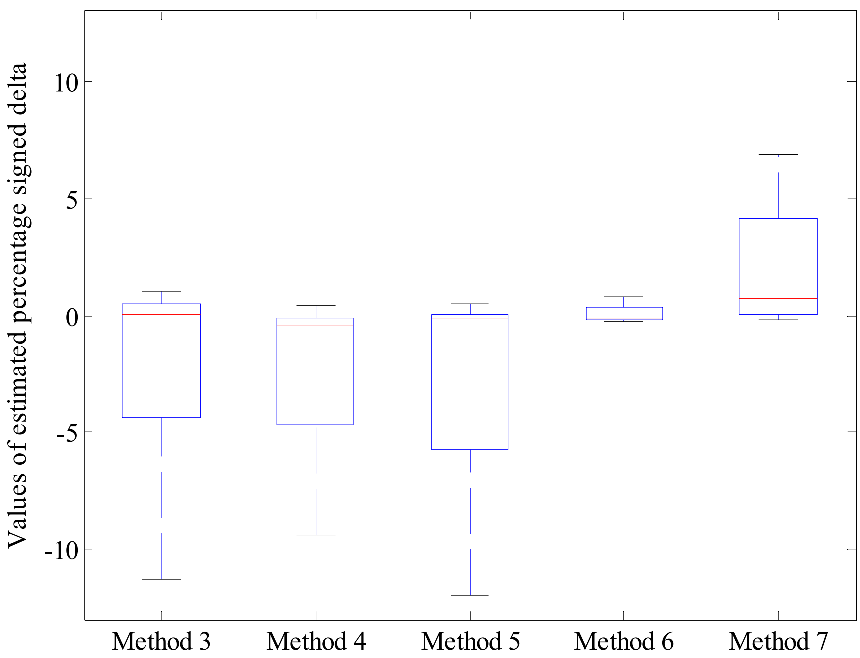

Figure D5 Distributions for variance matrices of size 20 and 90% quantile probability

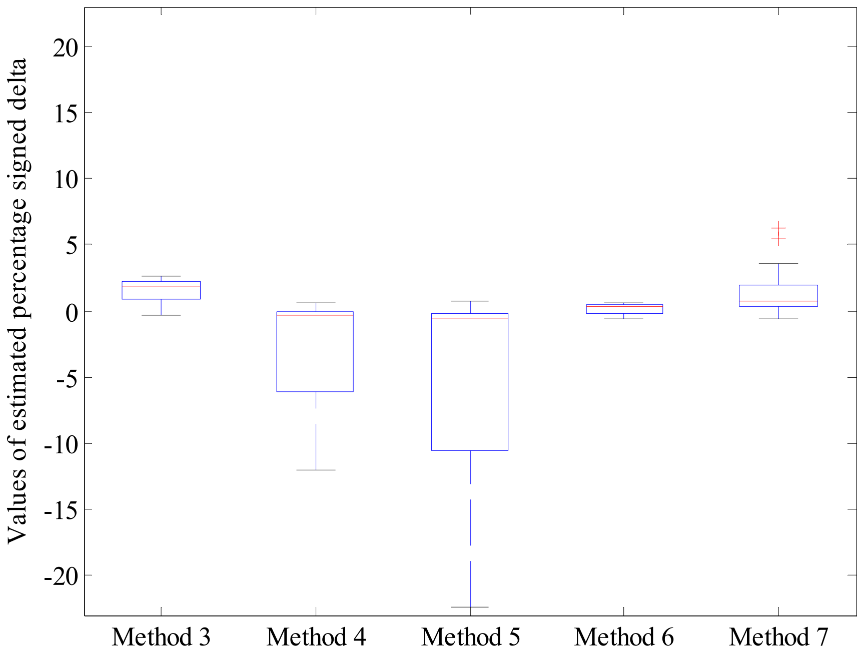

Figure D6 Distributions for variance matrices of size 20 and 95% quantile probability

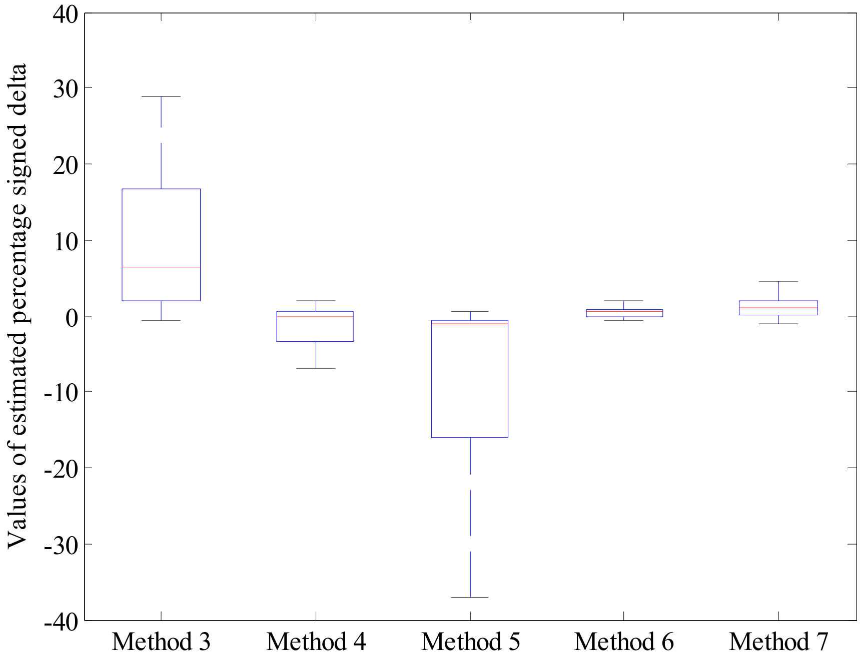

Figure D7 Distributions for variance matrices of size 20 and 98% quantile probability

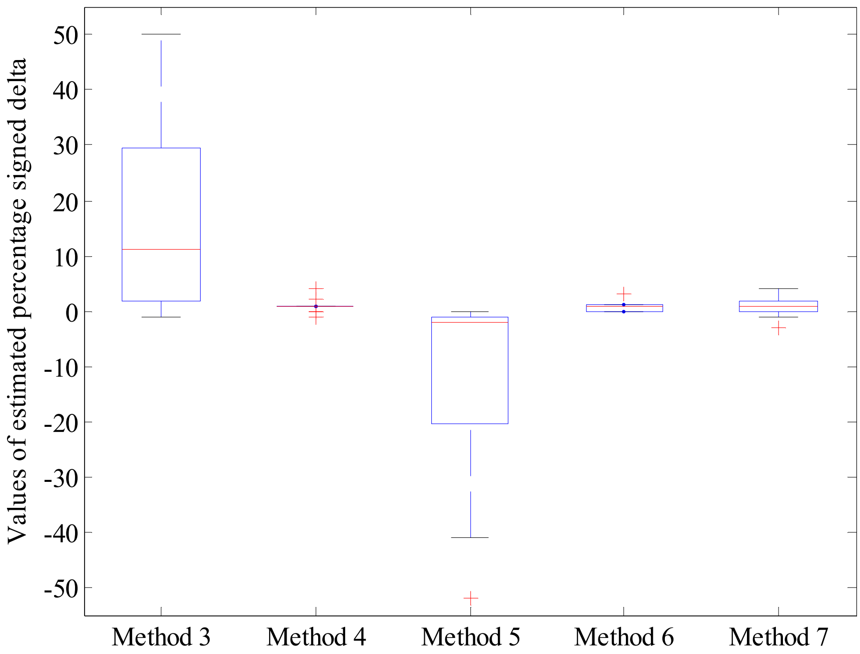

Figure D8 Distributions for variance matrices of size 20 and 99% quantile probability

Variance matrix dimension 25

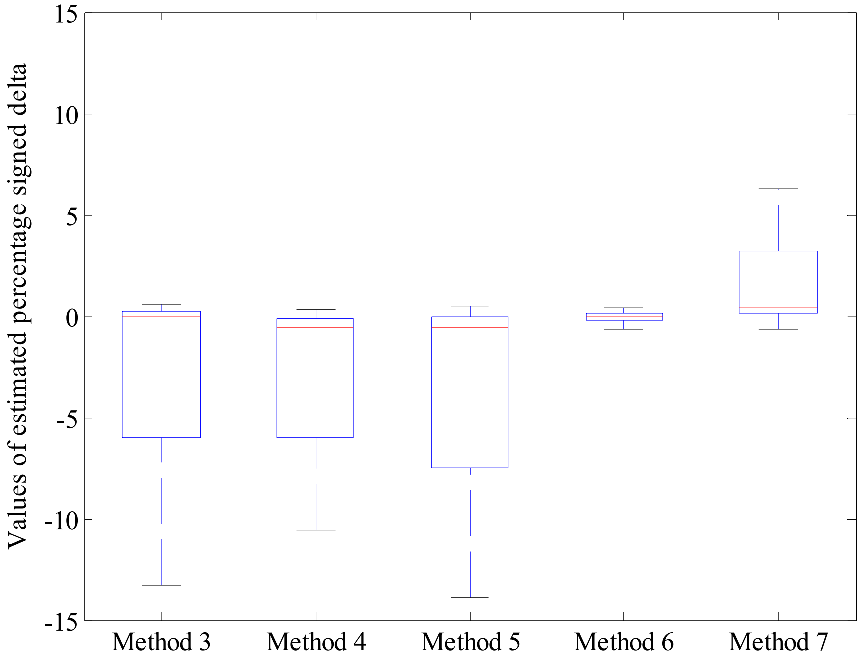

Figure D9 Distributions for variance matrices of size 25 and 90% quantile probability

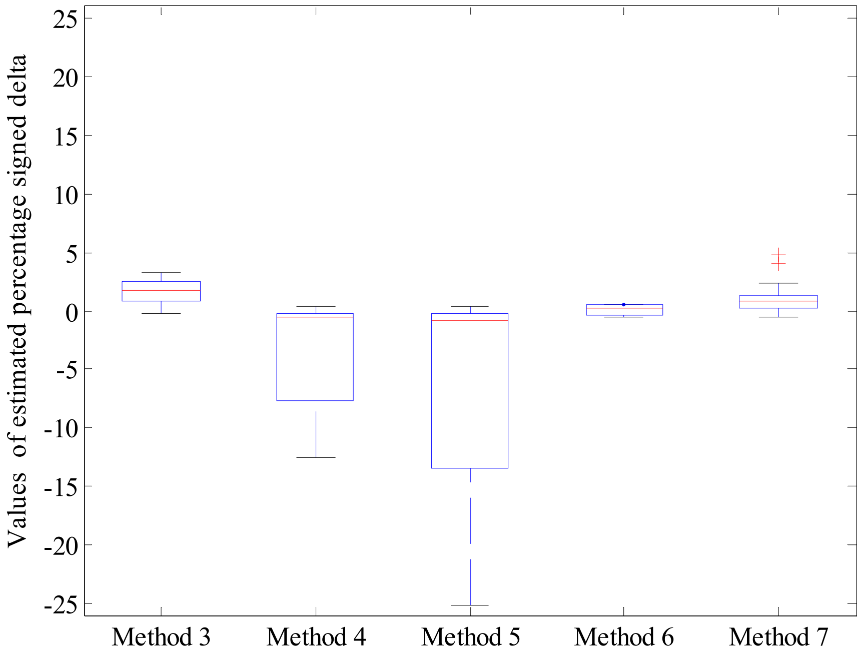

Figure D10 Distributions for variance matrices of size 25 and 95% quantile probability

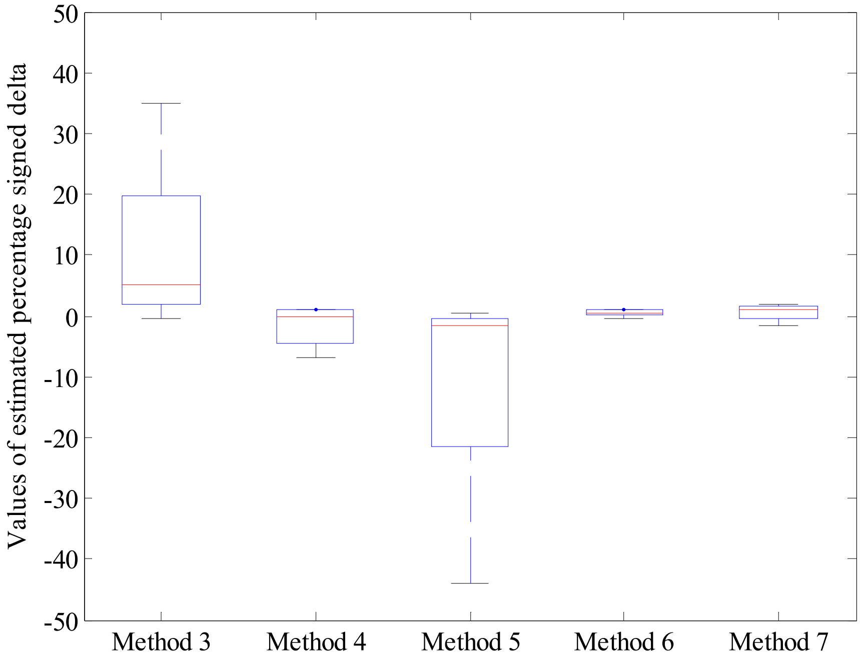

Figure D11 Distributions for variance matrices of size 25 and 98% quantile probability

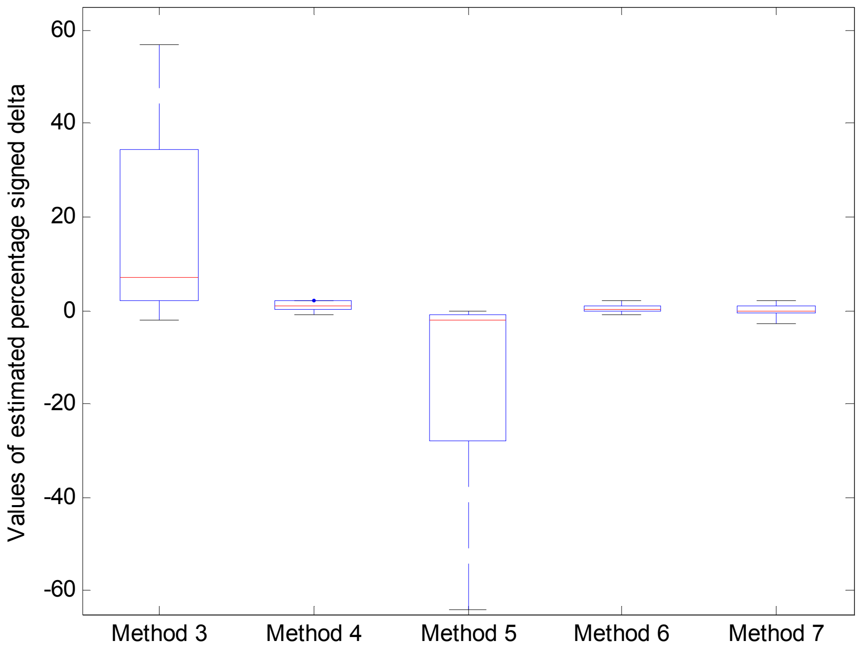

Figure D12 Distributions for variance matrices of size 25 and 99% quantile probability

Appendix D

Matlab programs

Reference variance matrices

```
% Type 1
% Input n, S2,r
% n=dimension of variance matrix
% S2=sigmatwo
% r=correlationcoefficient
% V=variance matrix
V=S2*((1-r)*eye(n)+r*ones(n))


% Type 2
% input n1, S21, r1, n2, S22, r2
% n1,n2=dimension of variance matrix
% S21,S22=sigmatwo
% r1,r2=correlation coefficient
V1=S21*((1-r1)*eye(n1)+r1*ones(n1));
V2=S22*((1-r2)*eye(n2)+r2*ones(n2));
V=[V1 zeros(n1,n2);zeros(n2,n1) V2]


% Type 3
% input n, a, b, vnn, r
% n=dimension of variance matrix
% a,b=constants particular to subtype (a,b,c)of type 3
v=a*rand(n-2,1)+b
v11=1;
v=[v11;v;vnn];
v=-sort(-v);
s=sqrt(v);
q=s*s'
V=r*s*s'+(1-r)*diag(v)


% Type 4
% input n
% n=dimension of variance matrix
v=0.75*rand(n-2,1)+0.25
v11=1;
vnn=0.25;
v=[v11;v;vnn];
v=-sort(-v);
s=sqrt(v);
q=s*s';

% case 4a
r=0;
V1=diag(v)

% case 4b
r=0.07*rand(n)
for i=1:n
    r(i,i)=1;
  for  j=2:n
      r(i,j)=r(j,i);
  end
```

```
end
std=diag(s);
V2=std*r*std


% case 4c
r=0.1*rand(n)
for i=1:n
    r(i,i)=1;
  for  j=2:n
      r(i,j)=r(j,i);
  end
end
std=diag(s);
V3=std*r*std


% Type 5
% input n
v=ones(n,1);

% case 5a
r1=0.07*rand(n)
V1=r1-diag(diag(r1))+diag(v);
for i=1:n
    for j=1:i
        V1(i,j)=V1(j,i);
    end
end
V1

% case 5b
r=0.1*rand(n)
V=r-diag(diag(r))+diag(v);
for i=1:n
    for j=1:i
        V(i,j)=V(j,i);
    end
end
V


% Type 6
% input n
v=0.75*rand(n-2,1)+0.25
v11=1;
vnn=0.25;
v=[v11;v;vnn];
v=-sort(-v);
s=sqrt(v);
q=s*s';

r=0.05*rand(n)+0.15
for i=1:n
    r(i,i)=1;
  for  j=1:i
      r(i,j)=r(j,i);
  end
end
std=diag(s);
V=std*r*std


% Type 7
```

```
% input n
v=0.75*rand(n-2,1)+0.25
v11=1;
vnn=0.25;
v=[v11;v;vnn];
v=-sort(-v);
s=sqrt(v);
q=s*s';

r=0.3*rand(n)
% valjer undre halva kvadraten
for i=1:n
    r(i,i)=1;
  for  j=1:i
      r(i,j)=r(j,i);
  end
end
std=diag(s);
V=std*r*std
```

Calculate the eigenvalues from matrix C

```
% Input=V
% G=eigen vectors as columns
% D=matrice with eigenvalues in the diagonal
[G,D]=eig(V)
H=G*sqrt(D)
B=eye(n)-ones(n)/n;
% control diff=G*D*G'-V
% create a C matrix
C=H'*B*H;
% lambda=eigen values of matrix C
lambda=eig(C)
```

Simulation of the distribution Q

```
% Simulation
% n=dimension of variance matrix
% M=number of replications in the simulation
% lambda=eigen values of matrix C
b=randn(M,n);
y=b.*b*lambda

% Program to calculate the different quantile values a, b, c and d
% M=number of replications in the simulation
result=[];
phatt=[0.90 0.95 0.98 0.99];
x=sort(y);
for i=1:4
phatt1=phatt(i);
critlimit(i)=x(M*phatt1);
end
result=[result;critlimit];
result

a=critlimit(1)
b=critlimit(2)
c=critlimit(3)
d=critlimit(4)

phatt1=sum(y>=a)/M
```

```
phatt2=sum(y>=b)/M
phatt3=sum(y>=c)/M
phatt4=sum(y>=d)/M
```

Approximating methods for calculating the upper tail probabilities of the null hypothesis distribution of the test statistic (2.25)

Method 1

```
% Crude method 1
% Input V, n
% V=variance matrix
% n=dimension of variance matrix
sigma2tot=(1/n)*sum(diag(V))
sigma2s=1/(n*(n-1))*(sum(sum(V))-sum(diag(V)))
sigma2r=sigma2tot-sigma2s

% Input the quantile value A=(a,b,c,d)
Y=A/sigma2r;
% phatt= upper tail probability
% phatt=P(Q>=Y)~1-P(Q<=Y)=1-F(Y)
phatt=1-chi2cdf(Y,n-1)
```

Method 2

```
% Crude method 2
% Input V, n
% V=variance matrix
% n=dimension of variance matrix
sigma2tot=(1/n)*sum(diag(V))

% Input the quantile value A=(a,b,c,d)
Y=A/sigma2tot;
% phatt= upper tail probability
% phatt=P(Q>=Y)~1-P(Q<=Y)=1-F(Y)
phatt=1-chi2cdf(Y,n-1)
```

Method 3

```
% Two moment approximation, Grad and Solomon
% Input V, lambda
% V=variance matrix
% lambda=eigen values of matrix C
% c=constant
% u=number of degrees of freedom
c=sum(lambda.^2)/sum(lambda)
u=(sum(lambda))^2/sum(lambda.^2)

% Input the quantile value A=(a,b,c,d)
Y=A/c;
% Z=standardised Gaussian variable
Z=(((Y/u)^(1/3))-1+2/(9*u))/(sqrt(2/(9*u)))
% phatt= upper tail probability
% phatt=P(Q>=Z)~1-P(Q<=Z)=1-F(Z)
phatt=1-normcdf(Z)
```

```
% Input the quantile value A=(a,b,c,d)
Y=A/c;
% phatt= upper tail probability
% phatt=P(Q>=Y)~1-P(Q<=Y)=1-F(Y)
phatt2=1-chi2cdf(Y,u)
```

Method 4

```
% Three moment approximation, Imhof
% Input V, lambda
% V=variance matrix
% lambda=eigen values of matrix C
% u=number of degrees of freedom
u=(sum(lambda.^2))^3/(sum(lambda.^3))^2

% Input the quantile value A=(a,b,c,d)
% c=Chi2(u)-value
c=sqrt(2*u)*((A-sum(lambda))/(sqrt(2*sum(lambda.^2))))+u
% phatt= upper tail probability
% phatt=P(Q>=c)~1-P(Q<=c)=1-F(c)
phatt=1-chi2cdf(c,u)
```

Method 5

```
% Gaussian approximation, Jensen and Solomon
% Input V, lambda
% V=variance matrix
% lambda=eigen values of matrix C
t1=sum(lambda);
t2=sum(lambda.^2);
t3=sum(lambda.^3);
h=1-(2*t1*t3)/(3*(t2^2));

% Input the quantile value A=(a,b,c,d)
% Z=standardized Gaussian variable
Z=(t1*((A/t1)^h-1-(t2*h*(h-1))/(t1^2)))/(h*sqrt(2*t2))
% phatt= upper tail probability
% phatt=P(Q>=Z)=1-P(Q<=Z)=1-F(Z)
phatt=1-normcdf(Z)
```

Method 6

```
%(1) PROGRAM NEWTON RAPHSON'S METHOD -"DIRECT COMPUTATION" WHEN GAMMA %
     ARGUMENT IS NOT TOO BIG

% program calculate
% Code for the functions f,g and their derivatives
f=(gamma(2*r+p/2)*gamma(p/2))/(gamma(r+p/2))^2-constf;
f1=(gamma(2*(r+e)+p/2)*gamma(p/2))/(gamma(r+e+p/2))^2-constf;
fr=(f1-f)/e;
f2=(gamma(2*r+(p+e)/2)*gamma((p+e)/2))/(gamma(r+(p+e)/2))^2-constf;
fp=(f2-f)/e;
g=(gamma(3*r+p/2)*(gamma(p/2))^2)/(gamma(r+p/2))^3-constg;
g1=(gamma(3*(r+e)+p/2)*(gamma(p/2))^2)/(gamma(r+e+p/2))^3-constg;
gr=(g1-g)/e;
g2=(gamma(3*r+(p+e)/2)*(gamma((p+e)/2))^2)/(gamma(r+(p+e)/2))^3-constg;
gp=(g2-g)/e;
```

```
% program method6march10.m
% input lambda n
% lambda=eigen values of matrix C
% n= dimension of variance matrix
% p=number of degrees of freedom
% r=constant
my1Q=sum(lambda);
my2Q=2*sum(lambda.^2)+(sum(lambda))^2;
my3Q=8*sum(lambda.^3)+6*(sum(lambda))*(sum(lambda.^2))+(sum(lambda))^3;
constf=my2Q/my1Q^2;
constg=my3Q/my1Q^3;

% initial values for p and r
p=(sum(lambda))^2/(sum(lambda.^2));
r=1;
[p r];
% start the iteration with these values of p and r
i=1;
d=0.01;
delta=0.01;
e=10^(-5);
calculate;
A=[fr fp;gr gp];
result=[i p r d delta f g];


for i=1:n
    calculate
    b=[-f;-g];
    A=[fr fp;gr gp];
    C=inv(A)*b;
    rold=r;
    pold=p;
    r=rold+C(1);
    p=pold+C(2);
    d=max(abs(rold-r)/abs(rold),abs(pold-p)/abs(pold));
    delta=max(abs(f),abs(g));
    result=[result;i p r d delta f g];
end
result
constf
constg


%(2) PROGRAM NEWTON RAPHSON'S METHOD WITH STIRLING'S EXPANSION

% program beraknanya
% Stirling approximation, "long version"
% Code for the functions f,g and their derivatives
f=(2*r+p/2-1/2)*log(2*r+p/2)+1/(12*(2*r+p/2))-
1/(360*(2*r+p/2)^3)+1/(1260*(2*r+p/2)^5)-
1/(1680*(2*r+p/2)^7)+1/(1188*(2*r+p/2)^9)+1/2*(p-
1)*log(p/2)+1/(12*(p/2))-1/(360*(p/2)^3)+1/(1260*(p/2)^5)-
1/(1680*(p/2)^7)+1/(1188*(p/2)^9)-2*((r+p/2-
1/2)*log(r+p/2)+1/(12*(r+p/2))-1/(360*(r+p/2)^3)+1/(1260*(r+p/2)^5)-
1/(1680*(r+p/2)^7)+1/(1188*(r+p/2)^9))-log(constf);

f1=(2*(r+e)+p/2-1/2)*log(2*(r+e)+p/2)+1/(12*(2*(r+e)+p/2))-
1/(360*(2*(r+e)+p/2)^3)+1/(1260*(2*(r+e)+p/2)^5)-
1/(1680*(2*(r+e)+p/2)^7)+1/(1188*(2*(r+e)+p/2)^9)+1/2*(p-
1)*log(p/2)+1/(12*(p/2))-1/(360*(p/2)^3)+1/(1260*(p/2)^5)-
1/(1680*(p/2)^7)+1/(1188*(p/2)^9)-2*((r+e+p/2-
1/2)*log(r+e+p/2)+1/(12*(r+e+p/2))-
```

```
1/(360*(r+e+p/2)^3)+1/(1260*(r+e+p/2)^5)-
1/(1680*(r+e+p/2)^7)+1/(1188*(r+e+p/2)^9))-log(constf);

fr=(f1-f)/e;

f2=(2*r+(p+e)/2-1/2)*log(2*r+(p+e)/2)+1/(12*(2*r+(p+e)/2))-
1/(360*(2*r+(p+e)/2)^3)+1/(1260*(2*r+(p+e)/2)^5)-
1/(1680*(2*r+(p+e)/2)^7)+1/(1188*(2*r+(p+e)/2)^9)+1/2*(p+e-
1)*log((p+e)/2)+1/(12*((p+e)/2))-
1/(360*((p+e)/2)^3)+1/(1260*((p+e)/2)^5)-
1/(1680*((p+e)/2)^7)+1/(1188*((p+e)/2)^9)-2*((r+(p+e)/2-
1/2)*log(r+(p+e)/2)+1/(12*(r+(p+e)/2))-
1/(360*(r+(p+e)/2)^3)+1/(1260*(r+(p+e)/2)^5)-
1/(1680*(r+(p+e)/2)^7)+1/(1188*(r+(p+e)/2)^9))-log(constf);

fp=(f2-f)/e;

g=(3*r+p/2-1/2)*log(3*r+p/2)+1/(12*(3*r+p/2))-
1/(360*(3*r+p/2)^3)+1/(1260*(3*r+p/2)^5)-
1/(1680*(3*r+p/2)^7)+1/(1188*(3*r+p/2)^9)+2*(1/2*(p-
1)*log(p/2)+1/(12*(p/2))-1/(360*(p/2)^3)+1/(1260*(p/2)^5)-
1/(1680*(p/2)^7)+1/(1188*(p/2)^9))-3*((r+p/2-
1/2)*log(r+p/2)+1/(12*(r+p/2))-1/(360*(r+p/2)^3)+1/(1260*(r+p/2)^5)-
1/(1680*(r+p/2)^7)+1/(1188*(r+p/2)^9))-log(constg);

g1=(3*(r+e)+p/2-1/2)*log(3*(r+e)+p/2)+1/(12*(3*(r+e)+p/2))-
1/(360*(3*(r+e)+p/2)^3)+1/(1260*(3*(r+e)+p/2)^5)-
1/(1680*(3*(r+e)+p/2)^7)+1/(1188*(3*(r+e)+p/2)^9)+2*(1/2*(p-
1)*log(p/2)+1/(12*(p/2))-1/(360*(p/2)^3)+1/(1260*(p/2)^5)-
1/(1680*(p/2)^7)+1/(1188*(p/2)^9))-3*((r+e+p/2-
1/2)*log(r+e+p/2)+1/(12*(r+e+p/2))-
1/(360*(r+e+p/2)^3)+1/(1260*(r+e+p/2)^5)-
1/(1680*(r+e+p/2)^7)+1/(1188*(r+e+p/2)^9))-log(constg);

gr=(g1-g)/e;

g2=(3*r+(p+e)/2-1/2)*log(3*r+(p+e)/2)+1/(12*(3*r+(p+e)/2))-
1/(360*(3*r+(p+e)/2)^3)+1/(1260*(3*r+(p+e)/2)^5)-
1/(1680*(3*r+(p+e)/2)^7)+1/(1188*(3*r+(p+e)/2)^9)+2*(1/2*(p+e-
1)*log((p+e)/2)+1/(12*((p+e)/2))-
1/(360*((p+e)/2)^3)+1/(1260*((p+e)/2)^5)-
1/(1680*((p+e)/2)^7)+1/(1188*((p+e)/2)^9))-3*((r+(p+e)/2-
1/2)*log(r+(p+e)/2)+1/(12*(r+(p+e)/2))-
1/(360*(r+(p+e)/2)^3)+1/(1260*(r+(p+e)/2)^5)-
1/(1680*(r+(p+e)/2)^7)+1/(1188*(r+(p+e)/2)^9))-log(constg);

gp=(g2-g)/e;


% program method6march16.m
% input lambda n
% lambda=eigen values of matrix C
% n= dimension of variance matrix
% p=number of degrees of freedom
% r=constant
my1Q=sum(lambda);
my2Q=2*sum(lambda.^2)+(sum(lambda))^2;
my3Q=8*sum(lambda.^3)+6*(sum(lambda))*(sum(lambda.^2))+(sum(lambda))^3;
constf=my2Q/my1Q^2;
constg=my3Q/my1Q^3;

% initial values for p and r
p=(sum(lambda))^2/(sum(lambda.^2));
r=1;
```

```
[p r];
% start the iteration with these values of p and r
i=1;
d=0.01;
delta=0.01;
e=10^(-5);
beraknanya
A=[fr fp;gr gp];
result=[i p r d delta f g];

for i=1:20
    beraknanya
    b=[-f;-g];
    A=[fr fp;gr gp];
    C=inv(A)*b;
    rold=r;
    pold=p;
    r=rold+C(1);
    p=pold+C(2);
    d=max(abs(rold-r)/abs(rold),abs(pold-p)/abs(pold));
    delta=max(abs(f),abs(g));
    result=[result;i p r d delta f g];
 end
result
constf
constg

%END PROGRAM FOR BOTH (1) AND (2)
% input p r cv
% p=number of degrees of freedom
% r=constant
% cv=quantile value (a,b,c,d)
% P= upper tail probability
%K=constant before chi2=distri.
K=(my2Q*(2^r)*gamma(r+p/2))/(my1Q*(2^(2*r))*gamma(2*r+p/2))
% K=K2, K2=(my1Q*gamma(p/2))/((2^r)*gamma(r+p/2))
P=1-chi2cdf((cv/K)^(1/r),p)
```

Method 7

```
% Method 7 step 1, Saddle Point Approximation, Field
% "metod7step1" for finding the solution t0
% input m lambda
% m=number of lambdavalues
% lambda=eigen values of matrix C

iter=0;
eps=1.;
tmin=0.;
tmax=.5/max(lambda);
t0=.5*(tmin+tmax);
result=[iter eps tmin tmax t0];
while abs(eps)>0.000001
    iter=iter+1;
    t0=.5*(tmin+tmax);
    eps=-A;
    for i=1:m
        eps=eps+lambda(i)/(1-2*lambda(i)*t0);
    end
    if eps>0 tmax=t0;
    else tmin=t0;
    end
```

```
    result=[result; iter eps tmin tmax t0];
end
result
t0


% Method 7 step 2, Saddle Point Approximation, Field
% Input lambda m A t0
% lambda=eigen values of matrix C
% m=number of lambdavalues
% A=(a,b,c,d) the quantile value
f=0;
for i=1:m
    f=f+log(1-2*lambda(i)*t0);
end
f=-0.5*f;
fprim=0;
for i=1:m
    fprim=fprim+lambda(i)/(1-2*lambda(i)*t0);
end
fprim;
fbis=0;
for i=1:m
    fbis=fbis+(lambda(i))^2/(1-2*lambda(i)*t0)^2;
end
fbis=2*fbis
f0=f-t0*A

% Method 7 step 3, Saddle Point Approximation, Field
% P= upper tail probability
% P=P(Q>=y0)~1-P(Q<= y0)=1-F(y0)
P=1-normcdf(sqrt(-2*f0))+(exp(f0)/sqrt(2*pi))*(1/(t0*sqrt(fbis))-
1/sqrt(-2*f0))
```