

Mathematical Statistics Stockholm University

# Combining leading indicators and a flash estimate

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#### Abstract

A Flash Estimator (FE), using monthly production data to obtain early estimators on quarterly values of Manufacturing is combined with Leading Indicators (LI), both monthly and quarterly. The leading information is extracted from the Business Tendency Survey using Kalman Filters. The result is called a Leading Flash Estimator (LFE). LFE proves to be more timely than a conventional FE and more accurate then the LI.

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#### Preface

This paper is part of a master thesis in Mathematical Statistics at Stockholm University. The work was done at Statistics Sweden<sup>1</sup> where the results, if any satisfactory, are to be used.

<sup>&</sup>lt;sup>1</sup>Statistics Sweden is a central government authority for official statistics and other government statistics and in this capacity also has the responsibility for coordinating and supporting the Swedish system for official statistics.

## Introduction

All over the world statisticians are working on their production routines so as to make the data more timely, without too much of accuracy being lost in the process. In the Swedish quarterly Production Accounts, Manufacturing  $(Y_Q)$  is an important variable that reflects the business cycle, maybe even more distinctly than the much larger aggregate, GDP. The data on this variable are published 70 days after the quarter has expired.

Leading indicators are industrial and economic statistics from which an indication of the value or direction of another variable might be obtained. In Öller & Tallbom (1996) [10] Quarterly Leading Indicators (QLI) were presented, which accurately forecast the preceding quarter in real time (co-incident) 30 days after the quarter has expired. A forward-looking indicator provides a first estimate of the current quarter (in real time), which started 30 days ago. The QLI have been published regularly since 1994, first by the National Institute of Economic Research, Sweden, and since 2003 by Statistics Sweden.

The Index of Industrial Production (IIP) is a monthly series closely related to  $Y_Q$ . It has been published regularly since 1913, nowadays with a delay of 45 days. In Dahllöf & Öller (2003) [3], Monthly Leading Indicators (MLI) are constructed for this series. The MLI gives us a coincident indication 30 days after the month ended and a forward-looking indication in the end of the month. By combining QLI for quarterly data with the monthly IIPand MLI we want to construct a Leading Flash Estimate (LFE) of Swedish  $Y_Q$ . This would precede the QLI coincident estimate by one month and is expected to be more accurate than the forward-looking QLI for that quarter, published two months earlier. In other words, we combine the early information in quarterly and monthly leading indicators with the early outcome registered in related monthly data. Conventional Flash Estimates (FE) use only outcome data, for a European model, see Mitchell & Weale (2001) [9].

Conventional Flash Estimators are just intended to speed up the data production process<sup>1</sup>. The estimates are compared to the preliminary value and if the discrepancy is considered moderate the FE is introduced. In the present case Leading Indicators are also available as early estimates of the forthcoming preliminary figures. For the *LFE* to be contributing something new, it is not enough for them to be earlier than Flash Estimatates, they must also be more accurate than the Leading Indicators. We have found only one slightly similar study where a bivariate monthly variabel contains interpolated values for GDP and monthly inflation data, see Salazar & Weale (1999) [14]. They found that monthly data improve the "nowcast" of the current quarter, but add nothing to the forecast of the next quarter.

Both the quarterly and monthly leading indicators use Business Tendency Survey data for early signals, which are then combined with autoregression of the statistical manufacturing variables in a Kalman filter, see Öller & Tallbom (1996) [10] and Rahiala & Teräsvirta (1993) [13]. The indicators are exponentially smoothed and include a turning point warning mechanism, which has worked well during the 10 years the quarterly indicators have been in use.

In Section 2 we present the model. We then describe the data in Section 3. Section 4 contains the results. Finally, section 5 gives some remarks and conclusions.

<sup>&</sup>lt;sup>1</sup>The Eurostat Handbook on Quartely National Accounts defines a flash estimate as: The earliest picture of the economy according to national accounts concepts, which is produced and published as soon as possible after the end of the quarter, using a more incomplete set of information than that used for traditional quarterly accounts.

## The Model

Consider the linear regression model:

$$y_t = c + \boldsymbol{\alpha}' \boldsymbol{y}_{t-1} + \boldsymbol{\beta}' \boldsymbol{x}_t + \varepsilon_t \qquad t = 1, 2, \dots, T,$$
(2.1)

where  $\boldsymbol{\alpha}$  is an  $m \times 1$  column vector of autoregressive (AR) coefficients and  $\boldsymbol{y}_{t-1}$  is also  $m \times 1$  vector containing lags from 1 to m of  $y_t$ . The regression coefficient vector  $\boldsymbol{\beta}$  and the regressor  $\boldsymbol{x}_t$  are both  $n \times 1$ . The error term  $\varepsilon_t$  is assumed to be independently and identically distributed normal variables with zero mean and variance  $\sigma^2$  ( $\varepsilon_t \sim i.i.dN(0, \sigma^2)$ ).

Let t denote quarter t and  $\hat{\boldsymbol{\alpha}}$  and  $\hat{\boldsymbol{\beta}}$  the estimates for  $\boldsymbol{\alpha}$  and  $\boldsymbol{\beta}$ , respectively, obtained from (2.1) using Ordinary Least Squares method (*OLS*). If  $\boldsymbol{x}_{T+1}$  is known the estimate of  $y_{T+1}$  is given by

$$\hat{y}_{T+1} = \hat{c} + \hat{\boldsymbol{\alpha}} \boldsymbol{y}_T + \hat{\boldsymbol{\beta}} \boldsymbol{x}_{T+1}$$
(2.2)

Let  $y_t$  be Manufacturing Variabel  $(Y_Q)$  and use QLI and outcome values of IIP as explanatory variables  $x_t$  in (2.1) to estimate the parameters. Now use MLI instead of unknown values of IIP in (2.2) to get a LFE, also called a *nowcast* of  $Y_{Q_{T+1}}$ .

Figure 2.1 shows how the available information depends on where the nowcaster stands in real time. At the end of a quarter (point  $P_0$ ) the information consists of quarterly and monthly values. On the quarterly frequency:

the outcome  $Y_{Q_T}$  of the preceding quarter and the forward-looking QLI for  $Y_{Q_{T+1}}$  have been published. The latest monthly values are: *IIP* outcome from the first month of the quarter, the coincident MLI of the second month and a forward-looking value for the third month.

Moving one month ahead to point  $P_1$  the latest quarterly outcome is still  $Y_{Q_T}$ , but now the *QLI* produces a coincident value for  $Y_{Q_{T+1}}$ . Outcomes of *IIP* are available for months one and two. For month three *MLI* has generated a coincident value. This timing procedure is made operative by estimating (2.1) using data on  $Y_Q$ , *QLI* and *IIP* from the estimation period. Some observation at the end of the data set are saved for testing the model in real situation. Here *MLI* figures substitute for unknown monthly *IIP* outcomes.

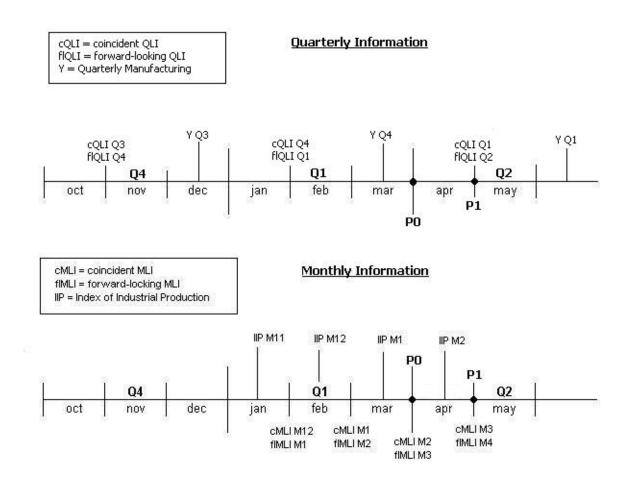


Figure 2.1: **Publishing times of quarterly and monthly statistics**, here specified for quarter one

### Data

#### **3.1** Quarterly Manufacturing $(Y_Q)$

Quarterly Manufacturing  $(Y_Q)$  is a Statistical Time Series (STS) in quarterly levels and constant prices, 1980:Q1–2003:Q2. Here data from 1990:Q1 onwards are used. Calendar and outlier effects are estimated in TRAMO and seasonal adjustment is performed on the TRAMO output in SEATS<sup>1</sup>.

#### 3.2 Quarterly Leading Indicator (QLI)

The model for QLI in Oller & Tallbom (1996) [10] uses data from 1970:Q1–1993:Q4, where 1970:Q1–1987:Q4 was used to estimate the model. The series was transformed into calendar corrected seasonal differences of logarithms; data were not seasonally adjusted. Here the coincident and the forward-looking QLI are transformed into levels, calendar and seasonally adjusted using TRAMO/SEATS, the default option. The data cover the period 1990:Q1 – 2003:Q2.

<sup>&</sup>lt;sup>1</sup>More about TRAMO/SEATS in appendix A

#### 3.3 Index of Industrial Production (IIP)

The monthly Index of Industrial Production (IIP) is published both as a monthly STS  $(IIP_M)$  and as a quarterly STS  $(IIP_Q)$ , calculated as a mean of the three months of a quarter. All IIP figures in this study are in year 2000 prices and are published in levels. Raw figures where sesonally adjusted with TRAMO/SEATS. The  $(IIP_M)$  is divided into three quartely STS, one for each month:  $IIP_{m1}$ ,  $IIP_{m2}$  and  $IIP_{m3}$ . Data are from the period 1990:M1– 2003:M6.

#### 3.4 Monthly Leading Indicator (MLI)

Dahllöf & Öller (2003) [3] estimated their Monthly Leading Indicators (MLI) on data from 1996:M1–2000:M5; observations 2000:M6–2003:M7 were saved for testing. The MLI figure is given in annual differences of logarithms, but here they are transformed into levels. Since these figures are not used in the regression model and because the series are so short they are not seasonally adjusted. The data cover the period from 2001:M1–2003:M6.

## Results

The Augumented Dickey Fuller<sup>1</sup> test (ADF) for unit roots, shows that all time series used have a unit root in log levels. All were stationarized by one difference, except  $Y_Q$  for which ADF did not reject the  $H_0$  of a unit root in the difference. Given the short time series and the weak power of the ADF test, we trust in earlier results, Öller & Tallbom (1996) [10] and more generally in the econometric literature, that most macroeconomic time series have a single unit root on frequency zero. Consequently, in all models the time series are in differences of log levels.

<sup>&</sup>lt;sup>1</sup>More about Augumented Dickey Fuller test in appendix B

Table 4.1 shows the four models for the estimation period 1990:Q2–2001:Q1. There are two models each for points  $P_0$  and  $P_1$ , in Figure 2.1. Models 2.0 and 2.1 are identical within the sample. The difference between 1.0 and 1.1 is that in the former we only have a forward-looking *QLI* (flQLI), but one month later in  $P_1$  the more accurate coincident *QLI* (cQLI) is available.

Model	Model specification
1.0	$\Delta Log\hat{Y}_{q_{T+1}} = \hat{\alpha}_2 \Delta LogY_{q_{T-1}} + \hat{\beta}_1 \Delta LogflQLI_{T+1} + \hat{\beta}_6 \Delta LogIIP_{m3_{T+1}}$
2.0	$\Delta Log \hat{Y}_{q_{T+1}} = \hat{\alpha}_2 \Delta Log Y_{q_{T-1}} + \hat{\beta}_3 \Delta Log IIP_{Q_{T+1}}$
1.1	$\Delta Log \hat{Y}_{q_{T+1}} = \hat{\alpha}_2 \Delta Log Y_{q_{T-1}} + \hat{\beta}_2 \Delta Log c Q L I_{T+1} + \hat{\beta}_6 \Delta Log I I P_{m3_{T+1}}$
2.1	$\Delta Log \hat{Y}_{q_{T+1}} = \hat{\alpha}_2 \Delta Log Y_{q_{T-1}} + \hat{\beta}_3 \Delta Log IIP_{Q_{T+1}}$

Table 4.1: Model specification, Model 1.0 and 2.0 for time  $P_0$  and Model 1.1 and 2.1 for time  $P_1$ , see Fig. 2.1

Table 4.2 presents the estimation results of the models in Table 4.1. As soon as  $IIP_Q$  was included in a model the QLI variables became insignificant. The reason seems to be that IIP iformation on the entire quarter outperforms the Quartely Leading Indicator. Note, however, that according to the diagnostics in Table 4.2 Models 1.0 and 1.1 fit data slightly better than Models 2.0 and 2.1. The reason to this could be that the third month of IIP includes information that is poorly covered by QLI, based on BTS data, in Christofferson et al.(1992) [2]<sup>2</sup> it is shown that forward-looking QLIindicates the expected situation in the beginning of the next quarter rather than the average of the whole quarter.

	Model 1.0	Model 2.0 & 2.1	Model 1.1
$\hat{\alpha}_2$ (p-value)	$0,268\ (0,038)$	$0,463\ (0,000)$	$0,305\ (0,006)$
$\hat{\beta}_1$ (p-value)	$0,233\ (0,040)$	_	—
$\hat{\beta}_2$ (p-value)	_	_	$0,233\ (0,025)$
$\hat{\beta}_3$ (p-value)	_	$0,568\ (0,000)$	_
$\hat{\beta}_6$ (p-value)	$0,807\ (0,000)$	_	$0,775\ (0,000)$
Log-likelihood	134,22	123,65	134,70
BIC	-6,125	-5,710	-6,147
AIC	-6,249	-5,793	-6,271
RMSE	0,0106	0,0124	0,0097
$R^2$	0,83	0,71	0,83
Jarque-Bera (p-value)	2,219(0,330)	$0,537 \ (0,764)$	$1,741 \ (0,419)$

Table 4.2: Results in the sample of OLS estimation of the models in Table 4.1

 $<sup>^2\</sup>mathrm{Frequency}$  domaine methods reveal that BTS respondents tend to focus on the first part of a quarter

In Table 4.3 the models of Table 4.1 are presented as they appear in a real nowcasting situation (2001:Q2–2003:Q2), where MLI data have to stand in for IIP figures, not known in points  $P_0$  and  $P_1$  respectively.

Model	Model specification						
1.0*	Model 1.0 with month three from forward-looking $MLI$						
2.0*	Model 2.0 with $IIP_{Q_{T+1}}$ as a mean of <i>IIP</i> outcome for month						
	one, coincident and forward-looking <i>MLI</i> for month two and						
	three respectively						
1.1*	Model 1.1 with month three from coincident <i>MLI</i>						
2.1*	Model 2.1 with $IIP_{Q_{T+1}}$ as a mean of <i>IIP</i> outcome for month						
	one and two and coincident $MLI$ for month three						

Table 4.3: Model specification for now casting models at time  $\mathrm{P}_0$  and  $\mathrm{P}_1,$  see Fig. 2.1

Table 4.4 shows the root mean square error (RMSE) and the Granger-Newbold test<sup>3</sup>(G-N) statistic in and out of sample. Within sample the relevant comparison is between the models of Table 4.2 and *QLI*. One wants to know if the new models of Table 4.2 are more accurate than the *QLI*? If the answer is "yes" then the next question is: does substituting leading monthly data for missing observations in a real nowcasting situation significantly impair accuracy? If the answer is "no" then the Leading Flash Estimators of Table 4.3 can be expected to improve on the *QLI* both in accuracy and in timeliness.

<sup>&</sup>lt;sup>3</sup>More about Granger Newbold test in appendix C

In	Out of sample				
Model	RMSE	G-N(p-value)	Model	RMSE	G-N(p-value)
Forward-looking QLI	0,0215		Model 1.0	0,0136	
v.s.		$8,9 \times 10^{-8} (t > 0)$	v.s.		0,4111
Model 1.0	0,0106		Model $1.0^*$	0,0099	
Forward-looking QLI	0,0215		Model 2.0	0,0105	
v.s.		$2, 2 \times 10^{-5} (t > 0)$	v.s.		0,9324
Model 2.0	0,0124		Model $2.0^*$	0,0107	
Coincident QLI	0,0172		Model 1.1	0,0095	
v.s.		$3, 4 \times 10^{-5} (t > 0)$	v.s.		0,5320
Model 1.1	0,0097		Model $1.1^*$	0,0120	
Coincident QLI	0,0172		Model 2.1	0,0105	
v.s.		$0,0158 \ (t > 0)$	v.s.		$0,\!6576$
Model 2.1	0,0124		Model $2.1^*$	0,0099	

Table 4.4: Models compared in and out of sample

The RMSE and the Granger-Newbold test confirm that in  $P_0$  Models 1.0 and 2.0 provide a significantly better fit to  $Y_Q$  than the forward-looking QLI. The same conclusion can be made about Models 1.1 and 2.1 vs. the coincident QLI. The same statistical comparison is done between the models using *IIP* outcome and the "asterisk models" with *MLI* "stand ins". There are no major differences in RMSE; in fact in two cases RMSE decreases using surrogate *MLI* figures. *G-N* finds no significant differences in accuracy. In other words, we have been able to find Leading Flash Estimators that improve on the present QLI both in accuracy and in timeliness. The performance of the models is shown graphically in Figure 4.1. If one had to chose between the two model specifications in Table 4.1, the larger models (1.0) and (1.1) would be selected as slightly better than (2.0) and (2.1). The choise of (1.0) instead of (2.0) in  $P_0$  is supported by records in Table 4.4. But for  $P_1$  the record is ambivalent: (1.1) is more accurate within, but less accurate outside the sample. Anyway, as Figure 4.1 shows, differences between model nowcasts are small. This is the reason to suggesting two more or less even candidates for monitoring Manufacturing.

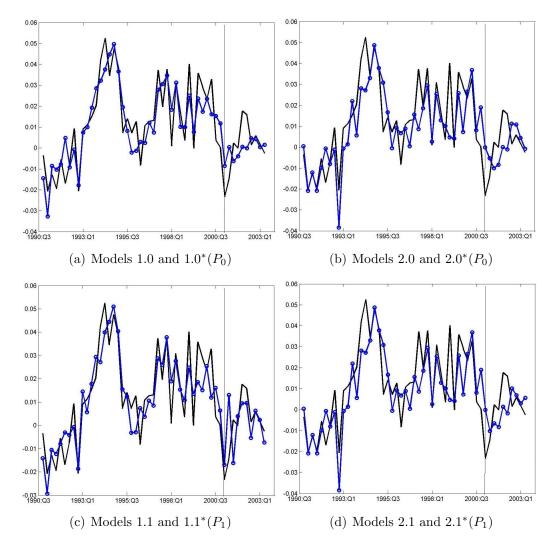


Figure 4.1: Model forecasts within and out of sample compared to outcome

## Conclusions

The results show that monthly observations, both real and survey-based leading indicator estimates can improve on both accuracy and timeliness of a reliable quarterly leading indicator of Manufacturing. Although this variable is an often cited and sensitive business cycle indicator, many people would prefer to see GDP in its place.

When attempting at a leading flash estimator of GDP, the models presented here could be both a building block and a blueprint for other variables included in GDP. Exports is an analogous case. Long time series of monthly exports are available to proxy for the slightly differently defined quarterly exports of the National Accounts. Business Tendency Survey data concerning quarterly exports have been recorded for decades and monthly analogues are available from 1996 on. The Exports share in Swedish GDP is close to one half. A third GDP component is Private Consumption, which also amounts to one half in the Expenditures Accounts of GDP. In this case the monthly proxy is Retail Trade. The leading information can in this case be obtained from the Consumer Survey.

Assuming that Leading Flash Estimators could be constructed for Exports and Consumption, too, a linear combination of all three could be quite close to GDP. All components would be important for analysts and forcasters in their own right, but the combination would be of even greater interest. Recall that the Leading Indicators contain a turning point alarm. This would be an additional asset, not provided by ordinary Flash Estimators.

## Appendix A TRAMO/SEATS

TRAMO and SEATS<sup>1</sup> are, in turn, two programs developed at the Bank of Spain, for time series analysis of data with a monthly or lower frequency of observations. When used for seasonal adjustment, TRAMO preadjusts the series to be adjusted by SEATS.

TRAMO (Time Series Regression with ARIMA<sup>2</sup> noise, Missing observations and Outliers) is a program for estimation and forecasting of regression models with errors that follow in general nonstationary ARIMA processes, when there may be missing observations in the series, as well as contamination by outliers and other deterministic effects. An important group of the latter is the Calendar effect, composed of the Trading day effect, Easter effect, Leap Year effect and hollidays effect.

If B denotes the lag operator, such that  $Bx_t = x_{t-1}$ , and f the number of observations per year, given the observations  $y = (y_{t_1}, y_{t_2} \dots y_{t_m})$  where  $0 < t_1 < \dots < t_m$ , TRAMO fits the general model

$$y_{t} = \sum_{i=1}^{n_{out}} \omega_{i} \lambda_{i}(B) d_{i}(t) + \sum_{i=1}^{n_{c}} \alpha_{i} cal_{i}(t) + \sum_{i=1}^{n_{reg}} \beta_{i} reg_{i}(t) + x_{t}$$
(A.1)

where  $d_i(t)$  is a dummy variable that indicates the position of the i:th outlier,  $\lambda_i(B)$  is a polynomial in B reflecting the outlier dynamic pattern,  $cal_i$ 

<sup>&</sup>lt;sup>1</sup>See Maravall (2002) [8], page 4–8.

 $<sup>^{2}</sup>$ An Auto Regressive Integrated Moving Average (ARIMA) model contains three different kinds of parameters: the p AR-parameters, the q MA-parameters and the variance of the error term.

denotes a calendar-type variable,  $reg_i$  a regression or intervention variable, and x is the ARIMA error. The parameter  $\omega_i$  is the instant i:th outlier effect,  $\alpha_i$  and  $\beta_i$  are the coefficients of the calendar and regression-intervention variables, respectively, and  $n_{out}$ ,  $n_c$  and  $n_{reg}$  denote the total number of variables entering each summation term in (A.1).

This can in compact notation be rewritten as

$$y_t = z_t' \gamma + x_t \tag{A.2}$$

where  $\gamma$  is th vector with the  $\omega$ ,  $\alpha$  and  $\beta$  coefficients, and  $z'_t$  denotes a matrix with columns the varibles

$$[cal_1(t),\ldots,cal_{n_c},\lambda_1(B)d_1(t),\ldots,\lambda_{n_{out}}(B)d_{n_{out}}(t),reg_1(t),\ldots,reg_{n_{reg}}(t)].$$

The first term of the addition in (A.2) represents the effects that should be removed in order to transform the observed series into a series that can be assumed to follow an ARIMA model; it contains thus the preadjustment component.

In compact form, the ARIMA model for  $x_t$  can be written as

$$\phi(B)\delta(B)x_t = \theta(B)a_t,\tag{A.3}$$

where  $a_t$  denotes the N(0, $\sigma_a^2$ ) white-noise innovation, and  $\phi(B)$ , $\delta(B)$  and  $\theta(B)$  are finite polynomials in B. The first one contains the stationary autoregressive (AR) roots,  $\delta(B)$  contains the nonstationary AR roots, and  $\theta(B)$  is an invertible moving average (MA) polynomial. Often they are assume the multiplicative form

$$\delta(B) = \nabla^d \nabla_f^{d_s}$$
  

$$\phi(B) = (1 + \phi_1 B + \ldots + \phi_p B^p)(1 + \Phi_1 B^f + \ldots + \Phi_{p_s} B^{p_s f})$$
  

$$\theta(B) = (1 + \theta_1 B + \ldots + \theta_q B^q)(1 + \Theta_1 B^f + \ldots + \Theta_{q_s} B^{q_s f})$$

where  $\nabla = 1 - B$  and  $\nabla_f = 1 - B^f$  are the regular and seasonal difference operators.

SEATS (Signal Extraction in ARIMA Time Series) decomposes the lineraized series into stochastic components, basicly by using the spectral density function,

$$x_t = \sum_i x_{it}$$

where i = p, c, s, u and the components are  $x_{pt} = trend - cycle$ ,  $x_{ct} = transitory$ ,  $x_{st} = seasonal$  and  $x_{ut} = irregular$ .

The trend-cycles captures the peak around zero in the series (pseudo) spectrum, the seasonal component captures the spectral peaks around the seasonal frequencies, the irregular component picks up white-noise variation, and the transitory component captures highly transitory variation different from white noise. The components are identified and given a ARIMA-identification. When estimating the components the rawseries are fore- and back-casted two years, so that a Wiener-Kolmogorov filter can be used.

## Appendix B

## Augumented Dickey Fuller (ADF) test

Consider a simple general autoregressive process AR(p) given by

$$Y_t = \mu + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \ldots + \phi_p Y_{t-p} + \varepsilon_t$$

If this is the process generating data but an AR(1) model is fitted, say

$$Y_t = \mu + \phi_1 Y_{t-1} + \nu_t$$

then

$$\nu_t = \phi_2 Y_{t-2} + \ldots + \phi_p Y_{t-p} + \varepsilon_t$$

and the autocorrelations of  $\nu_t$  and  $\nu_{t-k}$  for k > 1, will be nonzero, because of the presence of the lagged Y terms. Thus an indication of whether it is appropriate to fit an AR(1) model can be aided by considering the autocorrelations of the residual from the fitted models.

To illustrate how the DF<sup>1</sup> test can be extended to autoregressive processes of order greater than 1, consider the simple AR(2) process below.

$$Y_t = \mu + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \varepsilon_t$$

then notice that this is the same as:

$$Y_t = \mu + (\phi_1 + \phi_2)Y_{t-1} - \phi_2(Y_{t-1} - Y_{t-2}) + \varepsilon_t.$$

<sup>&</sup>lt;sup>1</sup>See Dickey & Fuller (1981) [4]

Subtracting  $Y_{t-1}$  from both sides gives:

$$\Delta Y_t = \mu + \beta Y_{t-1} - \alpha_1 \Delta Y_{t-1} + \varepsilon_t$$

where  $\beta = \phi_1 + \phi_2 - 1$  and  $\alpha_1 = -\phi_2$ 

This means that if the appropriate order of the AR process is 2 rather than 1, the term  $\Delta Y_{t-1}$  should be added to the regression model. A test of whether there is a unit root can be carried out in the same way as for the DF test with the test statistics provided by the  $\tilde{t}$  statistic of the estimated  $\hat{\beta}$  coefficient,  $H_0: \beta = 0$  (unit root) and  $H_1: \beta < 0$  (integrated of order zero). Notice that the critical values for  $\tilde{t}$  is not the same as for an ordinary tstatistic and depends on the sample size and whether you include a constant and/or a time trend<sup>2</sup>.

The same reasoning can be extended to a generic AR(p) process. Therefore to perform a Unit Root test on a AR(p) model the following regression should be estimated:

$$\Delta Y_t = \mu + \beta Y_{t-1} - \sum_{j=1}^p \alpha_j \Delta Y_{t-j} + \varepsilon_t$$

The Standard Dickey-Fuller model has been 'augmented' by  $\Delta Y_{t-j}$ . In this case the regression model and the  $\tilde{t}$  test are referred to as the Augmented Dickey Fuller (ADF) test.

<sup>&</sup>lt;sup>2</sup>See Dickey & Fuller (1981) [4]

## Appendix C The Granger-Newbold test

This is a test that compares the accuracy of two forecasts according to [5] p.279. Let  $\delta_1$  be the error of the first and  $\delta_2$  be the error in the second forecast, with the condition  $(\delta_{1,i}, \delta_{2,i})$  is independent of  $(\delta_{1,j}, \delta_{2,j})$  for  $i \neq j$ . Consider two new random variables  $\delta^+ = \delta_1 + \delta_2$  and  $\delta^- = \delta_1 - \delta_2$ . The expected value of the product:

$$E\left(\delta^{+}\delta^{-}\right) = E\left(\delta_{1}^{2} + \delta_{1}\delta_{2} - \delta_{1}\delta_{2} - \delta_{2}^{2}\right) = E\left(\delta_{1}^{2}\right) - E\left(\delta_{2}^{2}\right) = \sigma_{1}^{2} - \sigma_{2}^{2},$$

where  $\sigma_1^2$  and  $\sigma_2^2$  are equal if and only if the new variables  $\delta^+$  and  $\delta^-$  are uncorrelated so that

$$r = \frac{\sum_{i=T+1}^{M} \delta_i^+ \delta_i^-}{\sqrt{\sum_{i=T+1}^{M} (\delta_i^+)^2 \sum_{i=T+1}^{M} (\delta_i^-)^2}}$$

is zero. The corresponding test statistic for the hypothesis r = 0 of an unbiased estimate  $\hat{r}$  of r is

$$t = \frac{\hat{r}\sqrt{N-2}}{\sqrt{1-\hat{r}^2}} \quad \text{for } N = M - T,$$

which is distributed as Student's t with N-2 degrees of freedom.

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