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Monthly leading indicators using the leading information in the monthly Business Tendency Survey

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Abstract

A monthly Business Tendency Survey and a statistical time series are used to construct new early indicators for the total manufacturing in Sweden. As in Öller & Tallbom (1996) [9], by smoothing the forecasts, false turning point warnings are avoided. Introducing a relay that turns off the smoothing when there is a strong signal, the time shift that occurs in causal filter smoothing is eliminated.

Keywords: Business Tendency Survey, Exponential Smoothing, Kalman filter, Statistical Time Series, Smoothing Relay

Contents

Preface

This paper is part of a master thesis in Mathematical Statistics at Stockholm University. The paper is written at Statistics Sweden[1] where the results, if any satisfactory, are to be used.

[1]Statistics Sweden is a central government authority for official statistics and other government statistics and in this capacity also has the responsibility for coordinating and supporting the Swedish system for official statistics.

Chapter 1

Introduction

The purpose of this thesis is to create new early monthly indicators of the Swedish Volume of Production.

Since 1913 Statistics Sweden has published a monthly Statistical Time Series (STS) of the total manufacturing production volume of the National Accounts. Starting in the early 1950's the Swedish National Institute of Economic Research(NIER) has conducted a quarterly Business Tendency Survey (BTS), also of the total manufacturing production volume. In 1996 a corresponding monthly BTS of the same type was started. The STS is a more thorough investigation than the BTS and takes time to assemble. Thus the STS is published quite late as compared to the BTS. However, Öller (1990) [10] concludes that survey data alone result in poor forecasts of the time series of interest and suggests that combining STS and survey data may result in early warnings of turning points in the business cycle. Rahiala & Teräsvirta (1993) [6] developed a Kalman filter, using forecasted STS to regress the already known BTS and then adjust the STS forecast by adding the difference between the true BTS and the BTS value regressed with STS, multiplied by the *Kalman gain*. This was done to combine the STS and BTS for the quarterly production volume in Swedish and Finnish metal and engineering industries. Öller & Tallbom (1996) [9] applied this on quarterly data by using the same Kalman filter, but adding smoothing and a relay that turns it on/off in turning points. Here this filter is applied on a monthly BTS series and the STS, leading indicators on a monthly frequency are constructed, thus increasing the lead of the indicators. This type of analysis have not been done before on monthly data.

Christoffersson et al. (1992) [7] showed that there is a strong low (business cycle) frequency coherence in *quarterly* data between the STS and the BTS series Coincident Volume of Production (CVP), Forward-Looking Volume of Production (FLVP) and Total Orders (TO). This speaks in favour of making an experiment with monthly data that monitor the economy even more closely, as does the fact that the quarterly early indicators of Öller & Tallbom (1996) [9] have worked very well since their introduction in 1994. The main problem is that the monthly time series are so short.

Section 2 presents the approach of this study and the model. The analysis is made both in the frequency and in the time domains. The monthly data are introduced in Section 3, and the results in Section 4. Section 5 contains final remarks and conclusions.

Chapter 2

The model

2.1 The frequency domain

ARMA[1] models were estimated for the stationary BTS series and the stationarized STS series. The *Power Spectral Densities* ($P_j(\omega)$) were calculated using the ARMA parameter estimates, applying the formula:

$$P_j(\omega) = \frac{\hat{\sigma}_j^2 \left| 1 + \sum_{k=1}^{q} \hat{\theta}_{j,k} e^{-i2\pi\omega k} \right|^2}{\left| 1 - \sum_{k=1}^{p} \hat{\phi}_{j,k} e^{-i2\pi\omega k} \right|^2}, \quad j = 1, 2. \tag{2.1}$$

where the $\hat{\phi}_k$ and the $\hat{\theta}_k$ are maximum likelihood estimates of statistically adequate ARMA models for the series at hand and $\hat{\sigma}_j^2$ is the variance of the error.

The *coherence* between two stationary variables x_t and y_t can be estimated using the Fourier transforms, $F_X(\omega)$ and $F_Y(\omega)$:

$$Coh_{xy}(\omega) = \frac{E\left(|F_X(\omega)F_Y^*(\omega)|^2\right)}{\sqrt{S_X(\omega)S_Y(\omega)}} \tag{2.2}$$

where $S_X(\omega)$ is $E\left(|F_X(\omega)|^2\right)$, $S_Y(\omega)$ is $E\left(|F_Y(\omega)|^2\right)$ and $F_Y^*(\omega)$ is the complex conjugate of $F_Y(\omega)$. Here we used the Hanning window with the shortest width that still retains the main features of the coherence function.

[1]More about *ARMA models* in appendix A

2.2 The time domain

For natural reasons the STS is to be regarded as a more reliable indicator of production volume than the BTS. Hence STS is chosen as the reference variable to be forecasted, because the price of its reliability is that it is published later than the BTS. If the two types of time series have similarities for the long term variations (low frequencies), and act similarly in turning points, one approach to forecasting the STS is to use univariate inertia properties of the STS, transform the BTS to the STS scale and combine the two sources of predictive information. The forecast of the STS is then obtained as a weighted average of the rescaled BTS and a univariate forecast of the STS series. Both series may be so much contaminated by noise that no reliable signals for future values can be extracted. Abrupt changes, typical for both series, may wrongly be interpreted as turning points. By smoothing, the noise can be reduced, but a causal filter will then generate a time shift (late signals) on critical frequencies, as is shown in [11]. By using a relay that turns off the smoothing when there is a strong indication of an increase or decrease in the series we want to eliminate the time shift whenever there is a turning point[2]. We apply the state space[3] approach with a Kalman filter forecast of STS as introduced in [6] and modified in [9], and use it on monthly instead of quarterly data.

Following [9], let x_t be the logarithm of a monthly STS at time t, and let $\Delta_{12}x_t = x_t - x_{t-12}$. An autoregressive model of order p (AR(p)) for $\Delta_{12}x_t$,

$$\Delta_{12}x_t = \mu + \phi_1\Delta_{12}x_{t-1} + \phi_2\Delta_{12}x_{t-2} + \cdots + \phi_p\Delta_{12}x_{t-p} + \varepsilon_{1,t}, \qquad (2.3)$$

is estimated for $t = 12+1+p, 12+2+p, ..., T-1$, where $\varepsilon_{1,t}$ is i.i.d $(0,\sigma^2)$. The order of the model p is determined through ordinary univariate diagnostics (Ljung-Box test, AIC, BIC). AR parameterizations that whiten the time series are preferred to models containing MA parameters because the latter would make the model much more complicated. Model (2.3) can be stacked into a vector autoregressive form (VAR):

[2] More about turning points in Öller & Tallbom (1996) [9] and Koskinen & Öller (2003) [5].

[3] More about *state space* in appendix B

$$\begin{bmatrix} \Delta_{12}x_t \\ \Delta_{12}x_{t-1} \\ \cdot \\ \cdot \\ \cdot \\ \Delta_{12}x_{t-p+1} \\ 1 \end{bmatrix} = \begin{bmatrix} \phi_1 & \phi_2 & \cdot & \cdot & \phi_p & \mu \\ 1 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 1 & 0 & \cdot & \cdot & 0 \\ \cdot & \cdot & & & \cdot & \cdot \\ \cdot & \cdot & & & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 & 0 & 0 \\ 0 & \cdot & \cdot & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta_{12}x_{t-1} \\ \Delta_{12}x_{t-2} \\ \cdot \\ \cdot \\ \cdot \\ \Delta_{12}x_{t-p} \\ 1 \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ 0 \\ \cdot \\ \\ \cdot \\ \cdot \\ 0 \end{bmatrix}$$

and in matrix notation;

$$\mathbf{\Delta_{12}x_t} = \mathbf{A\Delta_{12}x_{t-1}} + \varepsilon_{\mathbf{1,t}} \qquad (2.4)$$
$$\text{for } t = 13+p, 14+p, ..., T-1.$$

which is called the *transition equation* for the STS series. The BTS series y_t is regressed on the analogous STS:

$$y_t = \gamma + \beta_1 \Delta_{12} x_t + \varepsilon_{2,t},$$

for $t = 12+1+p, 12+2+p, ..., T-1$. The scalar valued regression can be written in vector notation:

$$y_t = \mathbf{b\Delta_{12}x_t} + \varepsilon_{\mathbf{2,t}} \qquad (2.5)$$
$$\text{for } t = 13+p, 14+p, ..., T-1,$$

which is called the *measurement equation* for the BTS series and where $\varepsilon_{2,t}$ is i.i.d $(0, \sigma^2)$ and also independent of $\varepsilon_{1,t}$. Note that y_t is interpreted as an indirect observation of $\Delta_{12}x_t$. Note also that at time t we use only $y_t =$ Coincident Volume of Production, and att time $t+1$ we use regression with other BTS data to estimate a value of the Volume of Production. We forecast the STS for period T by using transition equation (2.4)

$$\mathbf{\Delta_{12}x}_{T|T-1} = \mathbf{A\Delta_{12}x}_{T-1}, \qquad (2.6)$$

where $\varepsilon_{1,T|T-1}$ is set to its expected value of zero. The BTS is forecasted for the same period by using the measurement equation (2.5), based on the AR prediction $\mathbf{\Delta_{12}x}_{T|T-1}$:

$$\hat{y}_{T|T-1} = \mathbf{b\Delta_{12}x}_{T|T-1}. \qquad (2.7)$$

Since we already know the outcome of the BTS, y_T, for period T, any discrepancy between the forecast $\hat{y}_{T|T-1}$ and the outcome y_T implies that the last observation deviates from the estimate derived from measurement equation (2.5). Now the Kalman[4] adjustment factor can be calculated and added to the VAR forecast in (2.6) to obtain the final *coincident forecast*:

$$\mathbf{\Delta_{12}x}^*_{T|T} = \mathbf{\Delta_{12}x}_{T|T-1} + \mathbf{k}_T \left[y_T - \hat{y}_{T|T-1} \right] \tag{2.8}$$

$$\mathbf{k}_T = \mathbf{V} \left(\mathbf{\Delta_{12}x}_{T|T-1} \right) \mathbf{b}' \mathbf{V}^{-1} \left(y_{T|T-1} \right).$$

Here $\mathbf{V} \left(\mathbf{\Delta_{12}x}_{T|T-1} \right)$ is the one-step-ahead forecast covariance-matrix of $\mathbf{\Delta_{12}x}_{T|T-1}$ and $\mathbf{V} \left(y_{T|T-1} \right)$ is the one-step-ahead forecast covariance-matrix of y, i.e. the Kalman gain vector $\mathbf{k}_T$ depends on the forecasting variances of the transition equation and of the measurement equation. In [4], ch. 4, it is shown that $\mathbf{\Delta_{12}x}^*_{T|T}$ is the *minimum mean square linear estimator* of $\mathbf{\Delta_{12}x}_{T|T}$.

The exponential smoothing algorithm:

$$\Delta_{12}\tilde{x}^*_t = \lambda \Delta_{12} x^*_t + (1 - \lambda) \Delta_{12} \tilde{x}_{t-1}, \tag{2.9}$$

where $\sim$ denotes "smoothed" and $0 < \lambda < 1$, with starting point:

$$\Delta_{12}\tilde{x}_{13} = \frac{\Delta_{12}x_{13} + \Delta_{12}x_{14}}{2}; \quad \Delta_{12}x_t = 0,\, t \leq 12$$

is used in the last stage. Smoothed and unsmoothed STS values are produced in parallel by the Kalman filter, where λ is a constant called "the forgetting factor".

Forecasting the STS one step further can be done using the forward looking BTS questions and by substituting the coincident forecast for the last observation of STS in the transition equation. Here we also incorporate the two BTS questions on coincident production and order stock in a vector $\mathbf{y}_{T|T}$. By linear regression we produce an estimated BTS value for $t = T + 1$:

$$\check{y}_{T+1|T} = h \left[y_{T+1|T}, \mathbf{y}_{T|T} \right] \tag{2.10}$$

Then the *forward looking* STS $\mathbf{\Delta_{12}x_{T+1|T}}$ is calculated by repeating the procedure in (2.8) for $T + 1 \mid T$.

[4]More about the Kalman filter in appendix B

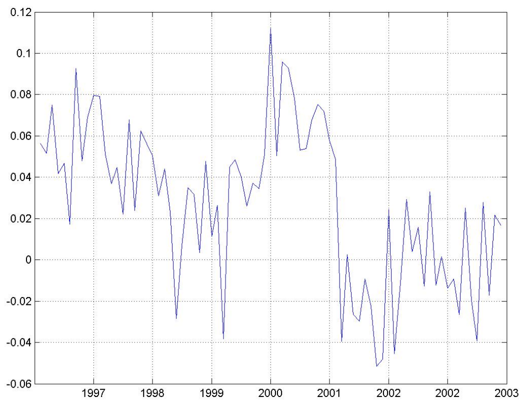
Chapter 3

The data

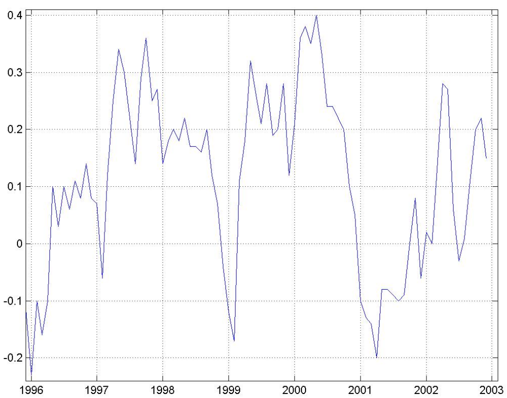
The Volume of Production Index (SNI 92, C+D) is a monthly time series which is not identical to the quarterly series used in [9], but very close. It has a strong seasonal component and the analysis has been done with seasonally differenced logarithmic STS data. This removes most of the seasonal variation, and eliminates unit roots on the zero and the seasonal frequencies. Seasonal differencing is a crude way of dealing with seasonality and unit roots, but given the short and noisy time series, simplicity was preferred to doubtful results of unit root tests. Because of the limited amount of BTS data we had to start the estimation period in 1996:1. The observations 2000:6 - 2003:7 were saved for testing. The testing period was chosen so that forecasts of the drop of the STS in 2001 could be evaluated. The data are working-day corrected, meaning that the Volume of Production is evaluated depending on how many working days there is in the month. The time series is shown in Figure(3.1(a)).

The early data used here to forecast the STS series are the following Swedish monthly BTS series: Coincident Volume of Production, (CVP),(Figure 3.1(b)), Total Orders, (TO), (Figure 3.1(d)) and Forward Looking Volume of Production (FLVP) (Figure 3.1(c)). The BTS series are measured as a distribution of answers across, "lower", "same" and "higher". Since the BTS answers are given on an ordinal scale a quantitative interpretation is difficult.

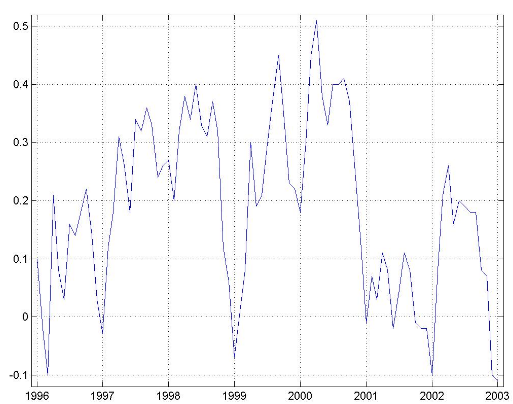
A visual inspection of Figure 1 (a-d) reveals that the series behave differently on all frequencies, lowering the hopes of achieving leading indicators for STS of a reliability comparable to the quarterly leading indicators. Note, however, that there is some similarities on the business cycle time interval (three to six years). Note also that the timing of turning points is promising.



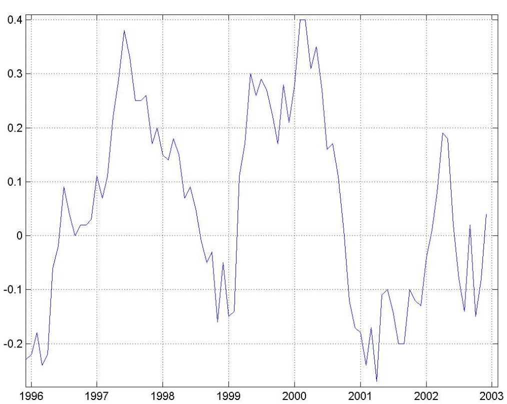
(a) The reference series $\Delta_{12}STS$



(b) Coincident Volume of Production (CVP)



(c) Forward-Looking Volume of Production (FLVP)



(d) Total Orders (TO)

Figure 3.1: The series used in the analysis

Chapter 4

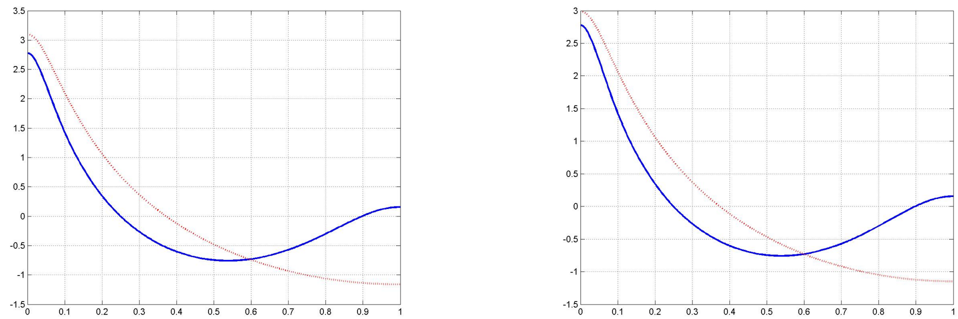
Results

Here we will present results from both the frequency domain and the time domain, with focus on the latter.

4.1 The frequency domain

Figures 4.1 and 4.2 show Power Spectral Density functions (PSD) of the variables involved and the coherence between the STS and two BTS variables. Table 1 presents the estimated parameter values that were inserted into (2.1) to obtain the PSD's. Actually, the complete model of the STS variable also contains a seasonal component, which we ignored in Figure 4.1. This is because in a complete model PSD, the unit root dips would completely dominate the figure, and we are just interested in the stationary dynamics. All PSD's have much power in the low (business cycle) frequencies, where the coherences are also high.

In Figure(4.2) the coherences between $\Delta_{12}STS$ and Coincident Volume of Production (CVP) and $\Delta_{12}STS$ and Forward Looking Volume of Production (FLVP) is measured. The local maxima around the annual frequency are either spurious, or reflections of remaining sasonality. In the former case they can be ignored. In the latter case the seasonality in the STS variable and the corresponding BTS variable can be expected to be positively correlated and is thus harmless in the measurement equation. Total Orders (TO) is a variable used in the regression in (2.10) for estimating BTS Volume of Production one step ahead and a coherence with $\Delta_{12}STS$ is not considered necessary.



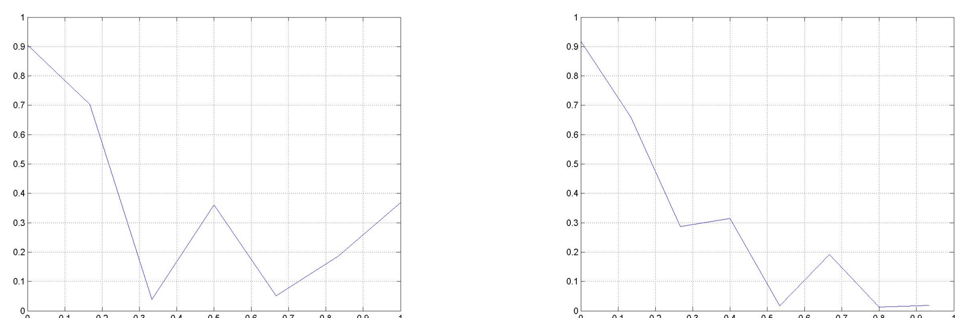
(a) $\Delta_{12}STS$(-) and Coincident Volume of Production($\cdots$)

(b) $\Delta_{12}STS$(-) and Forward-looking Volume of Production($\cdots$)

Figure 4.1: PSD's of $\Delta_{12}STS$ vs. the BTS series

Table 1. Models from which the PSDs are derived

STS Model: $x_t = 0.027{+}0.402x_{t-1}{+}0.305x_{t-2}{+}\hat{\varepsilon}_t$, Ljung-Box(12 Lags): $p < 0.0001$
CVP Model: $x_t = 0.132{+}0.787x_{t-1}{+}\hat{\varepsilon}_t$, Ljung-Box(12 Lags): $p < 0.0001$
FLVP Model: $x_t = 0.198{+}0.776x_{t-1} + \hat{\varepsilon}_t$, Ljung-Box(12 Lags): $p < 0.0016$



(a) $\Delta_{12}STS$ and Coincident Volume of Production

(b) $\Delta_{12}STS$ and Forward-looking Volume of Production

Figure 4.2: Coherences of $\Delta_{12}STS$ vs. the BTS series

Figures 4.1 and 4.2 above show that there are strong similarities between the monthly BTS variables and the Δ_{12}STS. These results indicate that we may be able to use the CVP and the FVLP series in the Kalman filter to produce useful monthly early indicators.

4.2 The time domain

In Table 2. the regression coefficients in the measurement equation (2.5) and on Total Orders (TO) in (2.10) do not quite reach statistical significance. Still, a Granger-Newbold test[1] on data outside the estimation period decisively rejected the hypothesis that the leading information in BTS does not improve forecasts generated by the transition equation, both in the coincident and in the forward-looking cases.

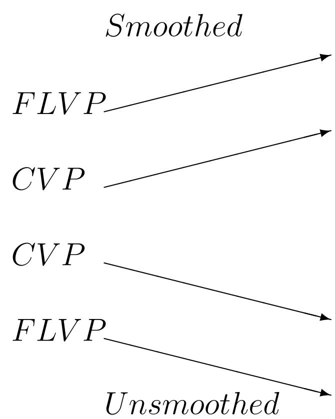
Table 2. The estimated Kalman model

	Eq(2.4)			**Eq(2.5)**		**Eq(2.10)**		
	μ	$\Delta_{12}x_{t-1}$	$\Delta_{12}x_{t-2}$	γ	$\Delta_{12}x_t$	CVP	FLVP	TO
Coeff	0.0274	0.4022	0.3053	0.1372	1.2044	0.4125	0.3048	0.2058
std	0.0111	0.1138	0.1164	0.0345	0.6897	0.1407	0.0718	0.1275
t-val	2.45	3.53	2.62	3.98	1.75	2.93	4.25	1.62

The coincident forecast is generated stepwise for the period 2000:6 - 2003:7 and the forward looking for 2000:7 - 2003:8. Both the coincident and the forward looking forecasts depend on the smoothing parameter, $0 < \lambda < 1$. When using the exponential smoothing algorithm in (2.9), a time shift occurs for higher frequencies, similar to the situation when a possible turning point is at hand. This is where the smoothing relay comes in. The relay turns off the smoothing only if both (coincident and forward-looking) smoothed forecasts indicate an increase or a decrease at the same time as both the unsmoothed forecasts indicate the opposite. Obviously this is because the smoothed forecast is stiffer in turning than the unsmoothed strong signal.

[1] See Appendix C

The smoothing relay



This eliminates time shifts whenever there is a downward turning point. The relay turns the smoothing on again when subsequently both the smoothed forecasts change signs and the absolute value of the smoothed coincident change is larger than the unsmoothed change, or if the two smoothed values start pointing in different directions. The larger λ is, the more noise gets through and the smaller λ the smoother is the signal, and the more the smoothing approaches a low pass filter, highlighting the interesting business cycle frecuesies. On the other hand the time shift increases, leading to late signals. The value $\lambda = 0.15$ was chosen as the smallest for which the relay works in the downturn in January 2001 and the upturn in April 2002.

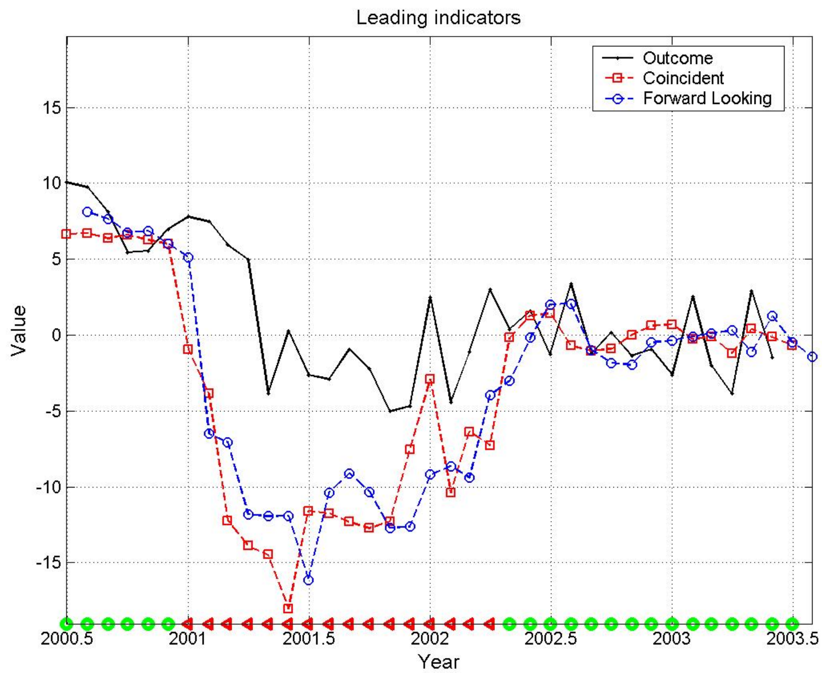


Figure 4.3: The STS (−) and the Coincident (□) and Forward-Looking (o) forecasts, smoothing off 2001:1 - 2002:4, otherwise on, relay active

In Figure 4.3 both indicators drop earlier and much deeper than the STS. Differently from quarterly indicators the monthly ones become misleading for the STS.

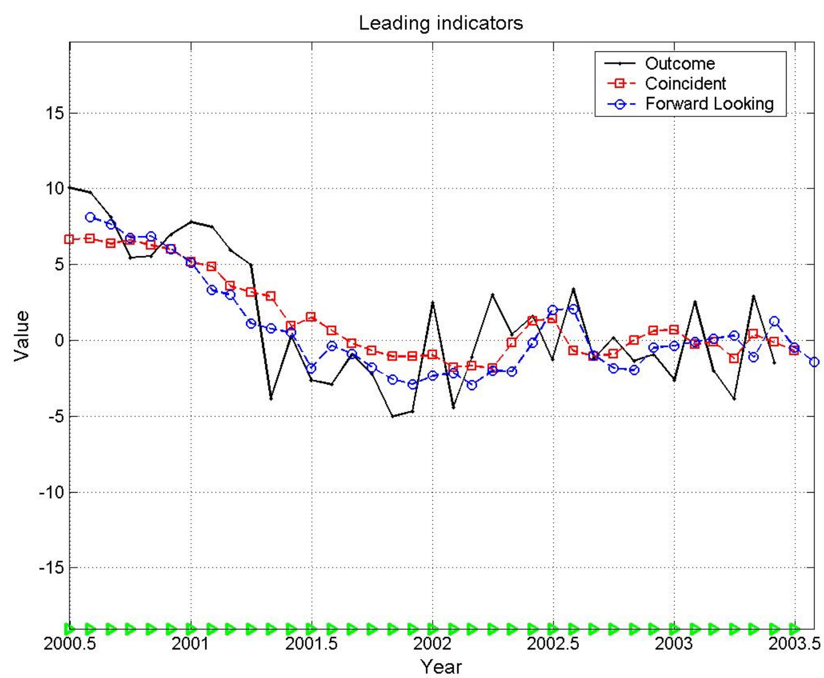


Figure 4.4: The STS (−) and the Coincident (□) and Forward-Looking (o) forecasts, smoothing on, relay inactive

Chapter 5

Conclusions

Given the circumstances with relatively short and noisy time series one could not hope for too much when trying to fit a model to such data. However, the Kalman filter in [6] is robust enough to extract leading signals from them. In Section 4.2, the results from the Granger-Newbold test show some evidence that monthly data contain the information required by leading indicators. The importance of smoothing is emphasized as in [5]. With no smoothing the indicators would have been misleading and no turning point signals would have been issued. The indicators work much better by preserving the smoothing in the contraction period (see Figure 4.4) and one just needs to register the turning point signals provided by the relay. Considering the small data set this does not need to be a general rule. The excessively deep dip may have resulted from an unusual pessimism in the business world at the time.

Acknowledgements

First I would like to thank Lars-Erik Öller for presenting the idea for this paper and for constantly guiding me. I would also like to thank Leif Persson for advise in the theoretical field. Finally I would like to thank Statistics Sweden for letting me use an office and a computer.

Appendix A

ARMA models

For a time series x_t, we can model that the level of its current observations depends on the level of its lagged observations. This can be represented by an AR model. The AR(p) (autoregressive of order p) can be written as:

$$x_t = \mu + \phi_1 x_{t-1} + \phi_2 x_{t-2} + \cdots + \phi_p x_{t-p} + \varepsilon_t \tag{A.1}$$

where $\varepsilon_t \sim WN(0, \sigma^2)$ and we keep this assumption through this appendix. One way of estimating the AR parameters is by letting $X_t = (x_t, x_{t-1}, ..., x_{t-n})'$ be a $(n \times 1)$ vector and

$$\mathbf{X}_{t-1} = \begin{pmatrix} x_{t-1} & x_{t-2} & ... & x_{t-p} \\ x_{t-2} & x_{t-3} & ... & x_{t-p-1} \\ \vdots & \ddots & & \vdots \\ x_{t-n-1} & x_{t-n-2} & ... & x_{t-n-p} \end{pmatrix},$$

and let $\Theta = (\phi_1, \phi_2, ..., \phi_p)'$ and $X_t = \mathbf{X}_{t-1}\Theta + \varepsilon_\mathbf{t}$ where $\varepsilon_\mathbf{t}$ is the $n \times 1$ vector of errors, then

$$(\mathbf{X}_{t-1}\mathbf{X}'_{t-1})^{-1}\mathbf{X}'_{t-1}X_t = \hat{\Theta}$$

In a second way of thinking, we can model that the observations of a random variable at time t are not only affected by the shock at time t, but also the shocks that have taken place before time t. This can represented by an MA model. The MA(q) (moving average of order q) can be written as

$$x_t = \gamma + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \cdots + \theta_q \varepsilon_{t-q} \tag{A.2}$$

If we combine these two models in (A.1) and (A.2), we get a general ARMA(p,q) model,

$$x_t = \psi + \phi_1 x_{t-1} + \cdots + \phi_p x_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q} \tag{A.3}$$

If $\hat{\rho_1}, ..., \hat{\rho_n}$ sample autocorrelation of an iid sequence $\varepsilon_1, ..., \varepsilon_n$ then the Ljung-Box statistic is defined,

$$Q_{LB} = n(n+2) \sum_{j=1}^{h} \frac{\hat{\rho}_j^2}{n-j}$$

Q_{LB} has a χ^2 distribution with h degrees of freedom. As a test for iid, we reject the hypothesis of iid at level α if $Q_{LB} > \chi^2_{1-\alpha}(h)$, where $\chi^2_{1-\alpha}(h)$ is the $1 - \alpha$ quantile of the chi-squared distribution with h degrees of freedom.

Appendix B

State space and the Kalman filter

Assume that at time $t-1$ we have the information set $K = \{x_{t-1}, x_{t-2}, ..., x_1\}$, the $m \times 1$ state vector x_t is not directly observable but its movements are assumed to be ruled by a well-defined process, the *transition equation*:

$$x_t = T_t x_{t-1} + R_t \varepsilon_{1,t}, \tag{B.1}$$

where T_t and R_t are fixed $m \times m$ respectively $m \times g$ matrices and $\varepsilon_{1,t}$ is a $g \times 1$ noise vector. The N observed variables are defined by a $N \times 1$ vector y_t. They are related to the state variables through the *measurement equation*:

$$y_t = Z_t x_t + S_t \varepsilon_{2,t}, \tag{B.2}$$

where Z_t and S_t are $N \times m$ respectively $N \times n$ matrices and $\varepsilon_{2,t}$ is a $n \times 1$ noise vector. The noise vectors $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ are mutually indipendent, have mean zero and covariance matrices Q_t and H_t,respectively. The equations (B.1) and (B.2) jointly constitute the *state space form.*

The estimate of x_t is adjusted by using the difference of the true value of y_t and the regressed value $\hat{y}_t$,

$$x_t^* = x_t + k_t \left[y_t - \hat{y}_t \right], \tag{B.3}$$

where k_t is called the *Kalman gain* and is defined as,

$$k_t = Q_t T_t H_t^{-1}.$$

Appendix C

The Granger-Newbold test

This is a test that compares the accuracy of two forecasts according to [2] p. 279. Let δ_1 be the error of the forecast (2.4) and δ_2 be the error in the corresponding Kalman filter (2.8) with the condition $(\delta_{1,i}, \delta_{2,i})$ is independent of $(\delta_{1,j}, \delta_{2,j})$ for $i \neq j$. Consider two new stochastic variables $\delta^+ = \delta_1 + \delta_2$ and $\delta^- = \delta_1 - \delta_2$. The expected value of the product:

$$E\left(\delta^+\delta^-\right) = E\left(\delta_1^2 + \delta_1\delta_2 - \delta_1\delta_2 - \delta_2^2\right) = E\left(\delta_1^2\right) - E\left(\delta_2^2\right) = \sigma_1^2 - \sigma_2^2,$$

where σ_1^2 and σ_2^2 are equal if and only if the new variables δ^+ and δ^- are uncorrelated so that

$$r = \frac{\sum_{i=T+1}^{M} \delta_i^+ \delta_i^-}{\sqrt{\sum_{i=T+1}^{M} \delta_i^+ \sum_{i=T+1}^{M} \delta_i^-}}$$

is zero. The corresponding test statistic for the hypothesis $r = 0$ of an unbiased estimate $\hat{r}$ of r is

$$t = \frac{\hat{r}\sqrt{N-2}}{\sqrt{1-\hat{r}^2}} \quad \text{for } N = M - T,$$

which is distributed as Student's t with $N - 2$ degrees of freedom.

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