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First passage percolation on $\mathbb{Z}^2$: a simulation study

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Abstract

First passage percolation on $\mathbb{Z}^2$ is a model for describing the spread of an infection on the sites of the square lattice. The infection is spread via nearest neighbor sites and the time dynamic is specified by random passage times attached to the edges. In this paper, the speed of the growth and the shape of the infected set is studied by aid of large-scale computer simulations, with focus on continuous passage time distributions. It is found that the most important quantity for determining the value of the time constant, which indicates the inverse asymptotic speed of the growth, is $\mathbf{E}[\min\{\tau_1, \ldots, \tau_4\}]$, where $\tau_1, \ldots, \tau_4$ are i.i.d. passage time variables. The relation is linear for a large class of passage time distributions. Furthermore, the directional time constants are seen to be increasing when moving from the axis towards the diagonal, so that the limiting shape is contained in a circle with radius defined by the speed along the axes. The shape comes closer to the circle for distributions with larger variability.

Keywords: First passage percolation, growth model, time constant, asymptotic shape, computer simulation.

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