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Routing on trees

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Abstract

We consider three different schemes for signal routing on a tree. The vertices of the tree represent transceivers that can transmit and receive signals, and are equipped with i.i.d. weights representing the strength of the transceivers. The edges of the tree are also equipped with i.i.d. weights, representing the costs for passing the edges. For each one of our schemes, we derive sharp conditions on the distributions of the vertex weights and the edge weights that determine when the root can transmit a signal over arbitrarily large distances.

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1 Introduction

Let $\mathcal{T}$ be a rooted infinite m-ary tree and assign i.i.d. weights $\{R_x\}$ to the vertices of $\mathcal{T}$ and i.i.d. weights $\{C_e\}$ to the edges. Assume that $\{R_x\}$ is independent of $\{C_e\}$. We think of the vertices as representing transceivers that can receive and transmit signals. The vertex weights represent the strength or range of the transceivers and the edge weights represent the cost or resistance when traversing the edges. We study three different schemes for signal routing in $\mathcal{T}$ and, for each of these schemes, we investigate when the root can transmit a signal over arbitrarily large distances. More specifically, write O for the set of vertices that are reached by a signal transmitted by the root, and say that a scheme can transmit indefinitely if $|O| = \infty$ with positive probability. Our main results are sharp conditions on the distributions of R and C that determine when the respective routing schemes can transmit indefinitely. Here and throughout the paper, R and C denote random variables with the laws of R_x and C_e, respectively.

Write $\Gamma_{x,y}$ for the path between the vertices x and y in $\mathcal{T}$, and write $y > x$ if y is located in the subtree below x in $\mathcal{T}$ (so that y is hence further away from the root than x). For each vertex x, let Λ_x be the set of all vertices y in the subtree below x for which the total cost of the path from x to y does not exceed the range of x, that is,

$$\Lambda_x = \Big\{ y > x : \sum_{e \in \Gamma_{x,y}} C_e \leq R_x \Big\}.$$

We say that the vertices in Λ_x are within the range of x. The schemes that we will consider are now defined as follows.

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