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Esbjörn Ohlsson

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Postal address:

Mathematical Statistics
Dept. of Mathematics
Stockholm University
SE-106 91 Stockholm
Sweden

Internet:

http://www.math.su.se/matstat

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Esbjörn Ohlsson*

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Abstract

In non-life insurance the provision for outstanding claims (the claims reserve) should include future loss adjustment expenses, i.e. administrative expenses to settle the claims, and therefore we have to estimate the expected *Unallocated Loss Adjustment Expenses* (ULAE) – expenses that are not attributable to individual claims, such as salaries at the claims handling department. The ULAE reserve has received little attention from European actuaries in the literature, supposedly because of the lack of detailed data for estimation and evaluation. Having good estimation procedures will, however, become even more important with the introduction of the Solvency II regulations, that require unbiased estimation of future cash flows for all expenses. We present a model for ULAE at the individual claim level that includes both fixed and variable costs. This model leads to an estimate of the ULAE reserve at the aggregate (line-of-business) level, as demonstrated in a numerical example from a Swedish non-life insurer.

KEY WORDS: Unallocated Loss Adjustment Cost, Claims Handling Expenses, Reserving, Paid-to-Paid method, Solvency II.

*Länsförsäkringar Alliance and Mathematical Statistics, Stockholm University, Sweden. E-mail: esbj@math.su.se.

1 Introduction

In non-life insurance the provision for outstanding claims (the claims reserve) should include future loss adjustment expenses, i.e. administrative expenses to settle the claims, for incurred claims, whether reported or not. Some of these expenses are attributable to individual claims and are often registered in the claims handling system. This part of the expenses is called *Allocated Loss Adjustment Expenses* (ALAE) and typically include external costs for lawyers or other expertise. For actuarial purposes, ALAE are thus contained in the claims figures and the provision for such costs is estimated together with the direct claim costs and require no special treatment. The rest of the adjustment costs are called *Unallocated Loss Adjustment Expenses* (ULAE) and typically consist of internal costs, mainly salaries at the claims handling department, together with their share of overhead costs, such as IT, management and premises costs. These costs are not only impossible to attribute to individual claims, but can neither be segmented at the granularity necessary to include them in claims triangles or other data used in traditional claims reserving. Therefore, the ULAE reserve (the provision for ULAE) has to be estimated separately and added to the provision for claim costs.

Note that the *Claims reserving manual* of the Institute of Actuaries (1997) uses the notation *direct* and *indirect expenses* while we have chosen the US terms ALAE and ULAE here.

Spalla (2001) notes that the distinction between ALAE and ULAE may be different at different companies, and reports that in the US there was a recommendation in 1998 on how to distinguish the two, with the objective of consistent reporting. Under the forthcoming European Solvency II regulations, the *Best Estimate* should contain all future expenses, without any explicit distinction between ALAE and ULAE. We follow this line and consider the ULAE reserve only as an operative means of covering expenses not already taken care of by the estimation of claim costs based on data from the claim systems. Hence, our distinction between ALAE and ULAE is just a question of how the data is organized, whith far-reaching implications on the methods for estimation. This implies that the split between ALAE and ULAE may differ between companies, but consistency between their reporting can nevertheless be achieved at the level of the total claims reserve.

The ULAE reserve has received little attention in the literature by European actuaries, an exception being Buchwalder, Merz, Bühlmann & Wütrich (2006). On the other hand, there are a number of papers from meetings of the CAS (Casualty Actuarial Society) in the US; besides the already quoted Spalla (2001) we like to mention Kittel (1981), Johnson (1989), Mango & Allen (1999) and Conger & Nolibos (2003). We refer the reader to these papers for further references.

The reason for the low interest from actuarial science on the subject is presumably the lack of data to perform statistical estimation, further discussed below. A consequence of the lack of data is also that methods can not be tested against experience. As a result, the proposed methods are based on assumptions and expert judgement that can not be verified empirically. Note also that ULAE reserves typically consist of just 5-10 % of the total reserve, which makes them much less important than the rest of the claims reserve, while still by no means negligible.

Notwithstanding these facts, we believe that it is valuable to base the assumptions and estimates on proper models, and this is even more so considering the requirements of Solvency II. The object of this paper is to present a relatively general framework for making assumptions and estimating the ULAE reserve, while keeping the estimate simple. This is done by introducing a micro-level (claim by claim) model, which is then estimated at the aggregated level.

2 Available data

The starting point is the data on historical ULAE payments, which should be available at least per accounting year. We will restrict our discussion to the latest accounting year, but the methods can trivially be extended to using data from several years or other periods. Last years observed ULAE is denoted L here. It should be possible to divide this amount into lines of business (LoB), with more or less accuracy, in which case the methods in this paper should be read as relating to a single LoB and L is the ULAE in that LoB.

The exact definition of L, e.g. in terms of which overhead expenses to include, is of course very important. However, this issue is beyond the scope of the present paper and we just assume that definition is already set, presumably by regulation, and that we have a useful observation of L available. In the words of Kittel (1981), the exact definition "does not matter" here, in spite of its importance.

A precise estimate of future ULAE would require more detailed data than just L. This could include separate costs for opening, maintaining and closing claims, all preferably split by claim size and types of claims. Such data are typically not available at a reasonable cost for the insurance companies. It is hard to think of how they could be, unless the expenses were registered on a case-by-case basis by the claim handlers, by e.g. noting the time used for handling each claim, with overhead allocated proportionally. But if such data was collected on a regular basis, the expenses would actually be *allocated* and could be treated as ALAE.

Johnson (1989) mentions the possibility of performing a time study for a temporary time period. Spalla (2001) discusses a study using data from the claims handling systems on time spent on different activities and notes that even with the use of such data "the project involved an investment of significant resources. The cost of such an investment goes beyond the benefit that would be derived by merely improving the accuracy of the estimation of ULAE liabilities." Indeed, the main justification for the mentioned study was to improve product pricing, not the ULAE reserve. With this in mind, we will assume here that no such study is available and we believe that this is the typical case. Then the lack of detailed data for ULAE follows more or less by definition, since expenses with detailed information at the claim level would be classified as ALAE, as argued above. We call this *the fundamental lack of data* for ULAE and this is the situation we discuss in the present paper.

In contrast to the lack of detailed data for expenses, we will typically have an actuarial database of insurances and claims, so that we can get detailed data on the number and cost of different types of claims. Let us first look at a traditional incremental claims triangle, in which C_{ij} denotes the observed paid claims for accident year i and development year j, $i = 1, 2, \ldots, m$; $j = 0, 1, \ldots, J$.

Table 2.1: Claims development triangle with $J = m$.

Accident year	*Development year* 0	1	2	$\cdots$	$J-1$	J
1	C_{10}	C_{11}	C_{12}	$\cdots$	$C_{1,J-1}$	$C_{1,J}$
2	C_{20}	C_{21}	C_{22}	$\cdots$	$C_{2,J-1}$	
3	C_{30}	C_{31}	C_{32}	$\cdots$		
$\vdots$	$\vdots$	$\vdots$	$\vdots$			
$m-1$	$C_{m-1,0}$	$C_{m-1,1}$				
m	$C_{m,0}$					

Of special interest here is the paid claims during the latest accounting year $C = \sum_{i+j=m} C_{ij}$. We further assume that we have an estimate of the outstanding claims, i.e. the sum of future C_{ij} in the lower part of the triangle, plus an eventual tail, computed by actuarial methods. Let us denote the reserve for reported but not settled claims (RBNS) by R and the reserve for incurred but not yet reported claims (IBNYR) by I. (We refrain from distinguishing between the quantity being predicted and the predictor here, since we are not investigating any stochastic properties of these estimates, but rather take them as given.) Note also that C, R and I should be given gross of reinsurance, undiscounted and without risk margin our any other extra margin that the company may hold for safety reasons, since these accounting measures are unlikely to influence the ULAE cost.

In many cases, R is just taken to be the sum of the case reserves (individual claims reserves), but it might be an estimate of the total cost for reported claims, as in the method by Schnieper (1991). As for I, if an estimate of IBNYR ("pure IBNR") is not available, we might approximate it by the difference between the actuary's best estimate of the total reserve and case reserves.

We also have the possibility to group the data by the opening and closing time of the claim. Following Mango & Allen (1999) the claims that are handled over the last accounting year are divided into four groups, here denoted $\mathscr{B}_1$–$\mathscr{B}_4$.

Table 2.2: Sectioning of claims

		End of the year	
		Closed	Open
Beginning	Not open	$\mathscr{B}_1$	$\mathscr{B}_3$
of the year	Open	$\mathscr{B}_2$	$\mathscr{B}_4$

Here $\mathscr{B}_1$ is the set of claims that were opened and closed during the year, irrespective of accident year, etc. The number of claims in group $\mathscr{B}_k$ is denoted A_k, $k = 1, 2, 3, 4$, so that we have

We will also assume that we have an estimate of the number A_I of unknown claims. This can be achieved by a straight-forward triangulation of data on

Table 2.3: Number of claims in the groups

		End of the year Closed	Open
Beginning	Not open	A_1	A_3
of the year	Open	A_2	A_4

the number of reported claims and will not be discussed further here.

We can also divide the paid claims during the year, C, into the same four sections, so that $C = C_1 + C_2 + C_3 + C_4$.

Table 2.4: Paid claims within the groups

		End of the year Closed	Open
Beginning	Not open	C_1	C_3
of the year	Open	C_2	C_4

3 The Paid-to-Paid method

Before presenting our model, we shall discuss the simple Paid-to-Paid (PTP) method. This method is based on the observed ratio of loss adjustment cost to amount paid, i.e. L/C. If this ratio is the same for the claims in the reserve $(R + I)$ as for those handled during the last accounting year, we get the simplest form of PTP estimate of the ULAE reserve U as

$$\hat{U} = \frac{L}{C}(R + I) \tag{3.1}$$

Mango & Allen (1999), as well as Institute of Actuaries (1997), note that the simple paid-to-paid estimate can be expected to be biased upwards:. While the ULAE itself increases with claim size, the percentage of ULAE should be a decreasing function of the claim size, and since claims that are closed quickly tend to be smaller on the average than those that are open longer, an upward bias will result.

As a remedy, it has been suggested to use some kind of "50/50 rule": approximately 50 % of the ULAE cost is paid at the opening of the claim and 50 % at the closing. Johnson (1989) describe the "classical" 50/50 rule in words as

$$\hat{U} = \frac{L}{C}(R/2 + I) = L\,\frac{R/2 + I}{C_1 + C_2 + C_3 + C_4} \tag{3.2}$$

and this is also the equation given in the *Claims reserving manual* by the Institute of Actuaries (1997, Section K4) in the UK; in fact, this is the only method mentioned for indirect expenses (ULAE) reserving there. This of course results in a lower ULAE reserve than the basic PTP method

$$\hat{U} = \frac{L}{C}(R + I) = L\,\frac{R + I}{C_1 + C_2 + C_3 + C_4}$$

The description of the 50/50 rule in Mango & Allen (1999), like Johnson (1989), is given without an explicit formula, but seems to imply

$$\hat{U} = L\,\frac{R/2 + I}{C_1 + \frac{1}{2}(C_2 + C_3)} \tag{3.3}$$

Indeed, this seems more logical than (3.2) if the 50/50 rule applies, under which C_4 generates no ULAE and C_2 and C_3 only half as much as C_1. Mathematically, (3.3) is not always lower than the basic PTP, as seen by considering the case $I = 0$, but it should result in a lower ULAE reserve in cases where I

is small relative to R and $C_1 > C_4$, as seen by setting $I = 0$ above. See also Section 4.1 for the method of Kittel (1981) which is quite close to (3.3). We conclude that the 50/50 rule in its two appearances is either not perfectly logical or not fail-proof to give a reduction of the bias.

As an alternative to methods having paid amounts as exposure, one might consider methods based on the number of claims. Conger & Nolibos (2003) mention an early method by R. E. Brian where the ULAE is split into different types of transactions: setting up the claim, maintaining the claim, etc. As discussed in the previous section, we do not assume such data to be available here.

A method based on claim counts that is simpler to apply was given by Johnson (1989) and is based on the assumption that the ULAE cost is given by a fixed amount each year the claim is open, with twice the cost during the first year when the claim is opened. In our experience, this idea of the ULAE being completely independent of the claims size is not quite realistic, but it seems reasonable that part of the cost is a fixed amount per claim as in these models. Therefore we will include a fixed, i.e. claim by claim, part of the ULAE in of our method below.

4 A micro-level model combining payments and claim numbers

Even if we do not have data on ULAE costs at the individual claim level, we think that it is informative to start by deriving a model at that level.

It seems natural to assume that the ULAE for a claim is the combination of a fixed cost, an ULAE per claim, and a cost proportional to the loss, an ULAE per unit of paid loss. This suggests a regression type model

$$u_i = \alpha + \beta c_i, \tag{4.1}$$

where u_i is the ULAE for claim i, and c_i is the paid claim cost for claim i, for the time period considered. This might be the entire time for handling the claim, but in our further specifications of the model below we will often refer to the latest accounting year, but also consider the time from the accounting date till the closing of the claim, i.e. the time during which we have a reserve for the claim.

Equation (4.1) of course just specifies the expected value structure. It could be made into a proper regression model by adding a stochastic error term, but since we do not have data at this level, it is not very meaningful to do so, cf. Buchwalder, Merz, Bühlmann & Wütrich (2006) who make a similar remark for the presentation of the "New York" method.

As mentioned above, several authors suggest methods based on the observation that there might be a cost for opening and another for closing the claim. In our model, this would imply that we should split the fixed part into two. So, we introduce s as the proportion of the fixed ULAE cost α that is incurred at the opening, $0 \leq s \leq 1$, so that $s\alpha$ is the fixed cost for opening a claim and $(1-s)\alpha$ the fixed cost for closing. There may of course also be a fixed claims maintenance cost between opening and closing, but presumably this is smaller than the others and in order to keep the model simple we will ignore this.

We will write $u_i = u_{1i} + u_{2i}$, where u_{1i} is the fixed part and u_{2i} is the variable part of the ULAE for claim i. In the time perspective of the latest accounting year, we then have

$$u_{1i} = \begin{cases} \alpha, & \text{if } i \epsilon \mathscr{B}_1 \text{ i.e. opened and closed during the period;} \\ (1-s)\alpha, & \text{if } i \epsilon \mathscr{B}_2 \text{ i.e. open from the beginning and then closed;} \\ s\alpha, & \text{if } i \epsilon \mathscr{B}_3 \text{ i.e. opened during the period, but not closed;} \\ 0, & \text{if } i \epsilon \mathscr{B}_4 \text{ i.e. open during the entire period;} \end{cases} \tag{4.2}$$

Aggregating this fixed cost model over all claims that have been open some time during the last year, i.e. over $\mathscr{B}_1 \cup \mathscr{B}_2 \cup \mathscr{B}_3 \cup \mathscr{B}_4$, we get

$$L_1 = \alpha(A_1 + (1-s)A_2 + sA_3), \tag{4.3}$$

with L_1 denoting the (unknown) fixed part of the ULAE for the last accounting year.

Note that if the entire ULAE cost would be fixed, then $L_1 = L$ and (4.3) gives us a (simplistic) cost per unit model. In order to estimate α we will have to make an assumption on the value of s. This might be done by expert judgement, consulting the claim handling department, but as default we will use the standard 50/50 rule, i.e. $s = 0.50$.

IF L_1 was known, we could now estimate α by equating L_1 to $\alpha(A_1 + sA_3 + (1-s)A_2)$, with the result

$$\hat{\alpha} = \frac{L_1}{A_1 + sA_3 + (1-s)A_2} \,. \tag{4.4}$$

If we were to use a pure fixed cost model, we should then apply this ULAE per normalized claim, to the the claims reserve. We have $A_3 + A_4$ open claims in the reserve and an estimated number A_I of unknown claims. The pure fixed ULAE reserve is then

$$U_1 = \hat{\alpha}((A_3 + A_4)(1-s) + A_I) = L_1 \frac{(A_3 + A_4)(1-s) + A_I}{A_1 + A_3\, s + A_2(1-s)} \,. \tag{4.5}$$

where $A_1 - A_4$ can be computed from the claim files and as default we set $s = 0.50$.

We now turn to the variable cost and go back to the individual claim level. As already mentioned, there is reason to believe that the cost per paid unit β is larger for small claims than for large ones. We do not always know which claims will become large and which will be small, so instead of splitting the claims by size, we recall the remark by Mango & Allen (1999) that claims that are closed quickly are on the average smaller than those that are open longer. Among $\mathscr{B}_1 - \mathscr{B}_4$, the first one consists of claims that are expected to be closed more rapidly, but this fact can not be used to construct an estimate since the proportions with with these claims are represented in the claims reserve is unknown. Instead we single out the claims payments for the current accident year, i.e. payments for claims incurred during the latest accounting year, and denote them by $\mathscr{B}_0$. The payments for these claims was denoted $C_{m,0}$ in the claims triangle in Table 2.1, but will here we write

$C_0 = C_{m,0}$ for short. These payments should include a lot of small claims that are quickly closed, even though of course we have a few large claims here, too. In any case, we expect more small claims among those paid for here than in the claims reserve, which does not contain any $\mathscr{B}_0$ type claims. Let r, $0 \leq r \leq 1$, denote how much less ULAE we expect per paid claim unit in the rest of the payments as opposed to $\mathscr{B}_0$. To get the variable part of the ULAE, we thus multiply c_i by β if the claim belongs to $\mathscr{B}_0$ and multiply it by $r\beta$ for all other claims.

The model for variable cost is then

$$u_{2i} = \begin{cases} \beta c_i, & \text{if } i \epsilon \mathscr{B}_0 \text{ i.e. is incurred during the period;} \\ r\beta c_i, & \text{else.} \end{cases} \tag{4.6}$$

By combining equations (4.2) and (4.6) we get our final model for individual ULAE $u_i = u_{1i} + u_{2i}$, being a generalization of (4.1). We now aggregate the variable cost model (4.6) over all claims that have been open some time during the last year. The result is

$$L_2 = \beta(C_0 + r(C - C_0)) \ , \tag{4.7}$$

with L_2 denoting the (unknown) variable part of the ULAE for the last accounting year. The payments C_0 and C are easy to compute and if we temporarily ignore that L_2 and r are unknown, we get the estimate

$$\hat{\beta} = \frac{L_2}{C_0 + r(C - C_0)} \tag{4.8}$$

Let us for a moment assume that the fixed cost is zero, so that $L_2 = L$. If we let $r = 1$ we are then back in the pure PTP method with $\hat{\beta} = L/C$. Looking for a default value of r less than 1, we again revert to something similar to a 50/50 rule, and choose $r = 0.5$, well aware that this is not really a 50-50 rule, but rather a "half the cost for later claims" rule.

The pure variable ULAE reserve U_2 is now found by applying $r\hat{\beta}$ to the entire claims reserve $R + I$

$$U_2 = \hat{\beta}(r(R + I)) = L_2 \frac{r(R + I)}{C_0 + r(C - C_0)} \tag{4.9}$$

As mentioned above, Johnsson (1989) states the 50/50 rule as the PTP ratio being applied to $(R/2+I)$. In analogue with this, it is tempting to use $(rR+I)$ in our case, recalling the default value $r = 0.5$. If the claims underlying the payments I are assumed to have a similar expected size as those in $\mathscr{B}_0$, it would be a good idea in our case, too, to thus use $(rR + I)$, but it is not obvious that this is a valid assumption.

Recall that we used (4.3) and (4.7) to find estimates $\hat{\alpha}$ and $\hat{\beta}$, in the pure fixed an pure variable model, respectively. In the full model, the corresponding equation would be

$$L = L_1 + L_2 = \alpha(A_1 + (1 - s)A_2 + sA_3) + \beta(C_0 + r(C - C_0))$$

Even with assumptions on s and r we can not estimate α and β from this model, having an unknown split of L into L_1 and L_2. We will solve this in a way that is similar to what is done in the "New york method" described by Buchwalder et al. (2006). That method does not use fixed and variable parts, but rather two variable parts for paid and incurred, respectively, but this leads to a similar problem which is resolved by splitting the ULAE into two parts by a factor. In our case, we split the observed ULAE L into a fixed part $L_1 = qL$ and a variable part $L_2 = (1-q)L$ for some q with $0 \leq q \leq 1$. As a default we take $q = 0.5$, invoking a 50/50 rule for the third time, noting that Buchwalder et al. do the same in their case, stating it to be the "usual choice" and the choice of the Swiss Solvency Test. We can now use (4.4) and (4.8) to get the estimates as before and then finally the total estimated ULAE is

$$U = U_1 + U_2 = L\left(q\frac{(A_3 + A_4)(1-s) + A_I}{A_1 + A_3\, s + A_2(1-s)} + (1-q)\frac{r(R+I)}{C_0\; + r(C - C_0)}\right). \tag{4.10}$$

Choosing $q = 1$ gives a pure fixed cost (cost per claim) model, and $q = 0$ gives a pure variable costs (cost per unit of paid claims) model. Our default value $q = 0.5$ then yields the mean of two such models.

4.1 Methods using incurred claims

There is a potential gain in modeling ULAE as proportional to reported claim cost, and not only to paid claims, in LoBs where claim handlers put in large effort in assessing the claims at inception, but most of the payments are made later on. This may be the case in, e.g.,property and liability insurance. Here most (large) claims are reported quite rapidly so that the IBNYR reserve is not very large and any method using reported claims would reduce the ULAE reserve substantially.

A simple model using incurred claims was suggested by Kittel (1981), who arrives at an equation close to (3.3), but with C_3 changed to the corresponding incurred amount, i.e. with case reserves for claims opened that remain open by the end of the year added to C_3. An extension of Kittel's model is given by Conger & Nolibos (2003).

The so called *New york* method is described by Buchwalder, Merz, Bühlmann & Wütrich (2006), who note that it is used in the Swiss Solvency Test. It assumes that 50 % (or some other percentage) of the ULAE is proportional to paid claim cost and the rest is proportional to incurred (reported) claim cost. A difficulty with the method is that it uses information on the pattern of *final* claim cost for incurred claims by reporting period, rather than the more easily available incurred claims pattern.

Reported claims could be incorporated in our micro-level model in (4.1) by introducing an extra term for incurred claims d_i reported during the period

$$u_i = \alpha + \beta c_i + \gamma d_i. \tag{4.11}$$

As in the New york method, one could set $\alpha = 0$ here, assuming no fixed cost, but it is still not the same method.

We could then proceed as above and estimate γ by equating a third part L_3 to γD, where D is the sum of d_i for all claims reported during the year, i.e. $\mathscr{B}_1 \cup \mathscr{B}_3$. Finally, we would apply the resulting factor to the IBNYR reserve I.

While this is an example of how our kind of modeling could easily be extended, we will not pursue this path further in this paper, partly because rather often the initial value of d_i is not very informative, being set by some template, and partly because we want to keep the model simple.

5 Numerical example

Here, we will look at an example from the Swedish insurance group Länsförsäkringar Alliance, for three different lines of business, viz. Private property, Motor TPL and Other motor. The federation consists of 23 local mutuals; for confidentiality reasons, will not reveal which company or companies the data is taken from. For the same reason, the amounts and claim counts presented here are multiplied by a undisclosed factor as are claims counts. This operation does not alter the conclusions, since all results remain the same, except a change to an unknown currency.

The annual ULAE expenses L is provided by the economics department(s) of the local mutual(s), were it is also divided into LoBs. Note that all methods mentioned above assume the ULAE reserve U to be proportional to L, the observed ULAE, i.e.

$$U = eL \tag{5.1}$$

for some $e > 0$. We will call e the expense reserving factor (ERF). In the PTP method, for example, we have $e = (R + I)/C$, while the ERF for our method can be read directly from (4.10). Since L is an identical input for all the methods, the estimation problem for the actuary is how to choose the method for computing the ERF.

The quantities that are computed from the claim system is presented in Table 5.1. We have three years data, which allows us to look at the stability of the estimates over the years.

Table 5.1: Claims data per LoB

Private Property

Year	A_1	A_2	A_3	A_4	A_I	$C - C_0$	C_0	R	I
2010	7 421	4 836	5 300	3 258	1 023	43 752	57 864	70 862	5 883
2011	7 976	5 379	5 113	3 180	1 060	54 797	60 737	71 522	6 582
2012	8 086	5 993	4 028	2 300	990	56 437	58 002	61 284	7 708

Other motor

Year	A_1	A_2	A_3	A_4	A_I	$C - C_0$	C_0	R	I
2010	14 943	5 035	4 628	798	1 420	17 325	82 650	13 386	13 715
2011	15 686	6 029	4 448	759	1 417	23 098	84 535	14 317	12 452
2012	15 233	5 808	4 559	702	1 300	21 543	84 106	13 698	13 611

Motor TPL

Year	A_1	A_2	A_3	A_4	A_I	$C - C_0$	C_0	R	I
2010	3 130	2 111	2 152	384	321	21 240	26 425	52 211	147 498
2011	3 102	2 455	1 945	389	256	27 019	25 723	56 808	176 567
2012	2 925	2 296	1 976	380	272	28 371	25 236	60 401	204 076

For the three LoBs in turn, we shall now use this data to compute the ERFs for different choices of the parameters q, s and r.

5.1 Private property

This LoB has a relatively short duration, so that most claims are finalized within a few years. After five years, 99 % of the final amount is paid out and the average payment duration is about one year. Claims in the reserve are on the average larger than claims paid during the first year, implying that $r < 1$ should be proper. As discussed above, we use $r = 0.5$ as the default choice.

In Table 5.2 we have computed the ERFs resulting from the above data. As a benchmark, we have added the ERF that would give the same ULAE percentage of the reserve as the reported average among Swedish non-life insurers. Note that this last factor, unlike the others, depends on L since it is backed out from a percentage. It is included here only for comparison and not as a candidate for estimation.

Table 5.2: ERFs for Private property

Model	PTP	Fixed	Variable	Default	Benchmark
Mix fixed/variable	$q = 0$	$q = 1$	$q = 0$	$q = 0.5$	
Open/close split		$s = 0.5$		$s = 0.5$	
First year loading	$r = 1$		$r = 0.5$	$r = 0.5$	
ERF 2010	0.76	0.42	0.48	0.45	0.34
ERF 2011	0.68	0.39	0.44	0.42	0.31
ERF 2012	0.60	0.32	0.40	0.36	0.25

For PTP the ERF of 0.76 suggests that an amount equal to 76 % of the loss adjustment cost for 2012 should be set off as ULAE reserve, and similar for the other ERFs.

It is notable that this LoB is not very sensitive to the balance q between the fixed and the variable cost models, so we can rather safely stay with the default choice $q = 0.5$. The sensitivity to the other two parameters is investigated in Table 5.3, where we compare the case with an opening cost, but no closing cost, i.,e. $s = 1$, to our default $s = 0.5$. The other extreme $s = 0$ would imply a fixed cost for closing and not for opening the claim, which seems rather unrealistic and is therefore not included in our comparison below. We also investigate the case with $r = 1$, i.e. equal ULAE per paid amount, as compared to the default with $r = 0.5$, i.e. double cost per currency unit for the first year's payments, which includes a high frequency of small claims. This gives us four combinations of s and r in Table 5.3.

Table 5.3: ERFs for Private property, sensitivity to s and r ($q = 0.5$)

Mixed model	Simple	Equal cost	No closing	Default
Open/close split	$s = 1$	$s = 0.5$	$s = 1$	$s = 0.5$
First year loading	$r = 1$	$r = 1$	$r = 0.5$	$r = 0.5$
ERF 2010	0.42	0.59	0.28	0.45
ERF 2011	0.38	0.53	0.26	0.42
ERF 2012	0.34	0.46	0.24	0.36

It is interesting that the simple model $s = 1$ and $r = 1$ gives very similar results to the default model. Comparing to the presumably overestimating PTP and the benchmark, which we guess might be a bit low, the other two choices might be a bit high and low, respectively, and we decide to stay with the default parameter values for Private property, i.e. $q = 0.5$, $s = 0.5$ and $r = 0.5$.

5.2 Other motor

This LoB has very short duration, so that about 99 % of the claims are finalized within the first two years. There are also few large claims. We do not really expect claims that are finalized the first accounting year to be very much smaller than the others and hence $r = 1$ seems a good candidate. For completeness, we nevertheless present the same tables as for Private property here.

Table 5.4: ERFs for Other motor

Model	PTP	Fixed	Variable	Default	Benchmark
Mix fixed/variable	$q = 0$	$q = 1$	$q = 0$	$q = 0.5$	
Open/close split		$s = 0.5$		$s = 0.5$	
First year loading	$r = 1$		$r = 0.5$	$r = 0.5$	
ERF 2010	0.27	0.21	0.15	0.18	0.22
ERF 2011	0.25	0.19	0.14	0.17	0.21
ERF 2012	0.26	0.19	0.14	0.17	0.20

Table 5.5: ERFs for Other motor, sensitivity to s and r ($q = 0.5$)

Mixed model	Simple	Equal cost	No closing	Default
Open/close split	$s = 1$	$s = 0.5$	$s = 1$	$s = 0.5$
First year loading	$r = 1$	$r = 1$	$r = 0.5$	$r = 0.5$
ERF 2010	0.17	0.24	0.11	0.18
ERF 2011	0.16	0.22	0.10	0.17
ERF 2012	0.16	0.23	0.10	0.17

For this LoB, the choice of method does not seem that dramatic, and actually the simple PTP might be considered, since we do not expect claims in the reserve to be that much larger than the ones paid first year. Considering the existence of zero claims makes us choose to include a fixed part anyway, but as argued above take $r = 1$ in this case. The resulting ERF is close to both PTP and the benchmark, which is a bit reinsuring, while the other choices seem a bit low. Hence we use the model with $q = 0.5$, $s = 0.5$ and $r = 1$ for Other motor.

5.3 Motor TPL

In Sweden, Motor TPL has no time limit for claiming income loss and therefore it is extremely long-tailed, with only a bit over 50 % paid after 10 years and an estimated 95 % after 30 years. Tables 5.6 contains the same information as we have seen for the other two LoBs.

Table 5.6: ERFs for Motor TPL

Model	PTP	Fixed	Variable	Default	Benchmark
Mix fixed/variable	$q = 0$	$q = 1$	$q = 0$	$q = 0.5$	
Open/close split		$s = 0.5$		$s = 0.5$	
First year loading	$r = 1$		$r = 0.5$	$r = 0.5$	
ERF 2010	4.19	0.30	2.70	1.50	1.05
ERF 2011	4.42	0.27	2.97	1.62	1.08
ERF 2012	4.93	0.29	3.35	1.82	1.24

As opposed to the other two LoBs, here we have a large difference between the pure fixed cost and the pure variable cost models, and hence the choice of q is crucial here. The pure fixed model seems to be heavily underestimating here, but the pure variable cost might be an alternative to the default method. We choose to stay with the default value, partly because it seems reasonable to have both fixed and variable costs, and partly because that brings us closer to the benchmark, however insecure that value is.

Table 5.7: ERFs for Motor TPL, sensitivity to s and r ($q = 0.5$)

Mixed model	Simple	Equal cost	No closing	Default
Open/close split	$s = 1$	$s = 0.5$	$s = 1$	$s = 0.5$
First year loading	$r = 1$	$r = 1$	$r = 0.5$	$r = 0.5$
ERF 2010	2.13	2.25	1.38	1.50
ERF 2011	2.24	2.35	1.51	1.62
ERF 2012	2.49	2.61	1.70	1.82

Now we turn to Table 5.7. There is not much sensitivity to the choice of s here and we might just as well do with $s = 0.5$. As for r, the choice is more important. With some very large losses in the reserve, we consider $r = 1$ not to be a good choice, but of course many $r < 1$ could be reasonable. Without any other guidance, we keep the default $r = 0.5$. All in all, we stay with the default model for Motor TPL, but this LoB illustrates the problem of choosing an ULAE model, considering the fundamental lack of evaluation data to guide our choice.

6 Discussion and conclusions

Given the definition of ULAE as the loss adjustment expenses that are not registered claim by claim, we have argued that there is normally a *fundamental lack of data* to estimate parameters and evaluate methods for ULAE reserving. In spite of this fact, we have tried to demonstrate that a model at the individual claims level can provide insight and guidance to the estimation at the aggregate level.

We have presented such a model that includes both fixed and variable costs at the claim level, leading to an estimator that only requires data at the aggregate level. With the fundamental lack of data, some parameters in the model have to be given arbitrary values, at the best based on expert judgement. In our model this is the case for q, s and r. Under these circumstances, an option would be that the supervisory authorities provided guidance or even stipulated default values for these parameters; the latter is the case for the "New york" model used in the Swiss Solvency Test.

In our numerical example, we tried to find a reasonable estimate of the expense reserving factor (ERF), which is to be multiplied by last years adjustment expenses L in order to get the ULAE reserve in currency units. The result is a *smaller* reserve than the PTP method would give – a desired result since the PTP is generally known to be biased upwards. By construction, our method with the default value $q = 0.5$ gives *larger* values than a pure fixed cost model, which is also desired for similar reasons. It is also seen to give larger values than the benchmark from Swedish non-life companies, which is not very surprising for not that large local companies. Note that this benchmark should be taken with a grain of salt, since we neither know the methods used, nor the accuracy of the ULAE reserves in these companies.

There are of course other candidates as micro-level models than the exact one used above, but to introduce a much more detailed model does not seem motivated in this situation. This is even more so, since the result in the end will depend heavily on the exact definition and computation of L, e.g. in terms of overhead expenses, which can result in quite different values in various companies.

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