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# The winner takes it all

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**Abstract**

We study competing first passage percolation on graphs generated by the configuration model. At time 0, vertex 1 and vertex 2 are infected with the type 1 and the type 2 infection, respectively, and an uninfected vertex then becomes type 1 (2) infected at rate $\lambda_1$ ($\lambda_2$) times the number of edges connecting it to a type 1 (2) infected neighbor. Our main result is that, if the degree distribution is a power-law with exponent $\tau \in (2, 3)$, then, as the number of vertices tends to infinity, one of the infection types will almost surely occupy all but a finite number of vertices. Furthermore, which one of the infections wins is random and both infections have a positive probability of winning regardless of the values of $\lambda_1$ and $\lambda_2$. The picture is similar with multiple starting points for the infections.

*Keywords:* Random graphs, configuration model, first passage percolation, competing growth, coexistence, continuous-time branching process.

MSC 2010 classification: 60K35, 05C80, 90B15.

## 1 Introduction

Consider a graph generated by the configuration model with random i.i.d. degrees, that is, given a finite number $n$ of vertices, each vertex is independently assigned a random number of half-edges according to a given probability distribution and the half-edges are then paired randomly to form edges (see below for more details). Independently assign two exponentially distributed passage times $X_1(e)$ and $X_2(e)$ to each edge $e$ in the graph, where $X_1(e)$ has parameter $\lambda_1$ and $X_2(e)$ parameter $\lambda_2$, and let two infections controlled by these passage times compete for space on the graph. More precisely, at time 0, vertex 1 is infected with the type 1 infection, vertex 2 is infected with the type 2 infection and all other vertices are uninfected. The infections then spread via nearest neighbors in the graph in that the time that it takes for the type 1 (2) infection to traverse an edge $e$ and invade the vertex at the other end is given by $X_1(e)$ ($X_2(e)$). Furthermore, once a vertex becomes type 1 (2) infected, it stays type 1 (2) infected forever and it also becomes immune to the type 2 (1) infection. Note that, since the vertices are exchangeable in the configuration model, the process is equivalent in distribution to the process obtained by infecting two randomly chosen vertices at time 0.

We shall impose a condition on the degree distribution that guarantees that the underlying graph has a giant component that comprises almost all vertices. According to the above

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