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On the expected time a branching process has K individuals alive

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Abstract

Consider a homogeneous time-continuous branching process where individuals have constant birth rate δ , and life length distribution Q having mean $E(Q) = 1$. Let $X(u)$ denote the number of individuals alive at time u , and assume that $X(0) = 1$. Let K be a positive integer and define $A_K := \int_0^\infty 1_{\{X(u)=K\}} du$, the accumulated time that the branching process has exactly K individuals alive. In this paper we prove that $E(A_K) = \delta^{K-1} / (k(1 \vee \delta)^K)$, irrespective of the life length distribution Q , subject to the normalizing condition $E(Q) = 1$.