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Simulation based claims reserving in general insurance

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Abstract

The aim of the present paper is to describe a simulation based "Best Estimate" reserving method for general insurance for which it is possible to obtain a coherent Solvency Capital Requirement (SCR) and Risk Margin (RM) under the Solvency II directive. This method is based on claim-level data and the idea that one can combine policy information with the claims database in order to make more efficient use of data. An important component in this reserving method is the modelling of the underlying time lines of all policies together with making the distinction between policies being Incurred But Not Reported (IBNR), Reported But Not Settled (RBNS) and Settled (S). The reserving method is described in terms of a simple algorithm.

Keywords: claim-level data, best estimate reserve, reserve risk, stochastic cash flows, Monte Carlo simulation, Solvency II

1 Introduction

Many traditional claims reserving methods are based on triangle schemes and various ways of projecting historically observed information forward in time, e.g. chain-ladder and related link ratio methods, the Bornhuetter-Ferguson method, the separation method. Apart from imposing various more or less realistic assumptions, the process of creating the needed triangles themselves is not un-problematic. When creating triangles much of the information contained in the data is lost due to aggregation. This critique has has already been acknowledged by e.g. England & Verall [4] and others. Moreover, there is increasing demand for the possibility to quantify the inherent uncertainty in the reserves and the corresponding reserve estimates, due to stricter supervisory regulations such as Solvency II and the Swiss Solvency Test. There are efforts made which deal with at least some of these matters using both analytical as well as simulation based techniques (e.g. bootstrap), see Merz and Wüthrich [11] and Björkwall, Ohlsson and Hössjer [3] and England & Verall [4]. Regarding the analytical approaches, these are in most situations only feasible for simple and "pure" methods, e.g. chain-ladder, see e.g. [7], under various assumptions. Also, bootstraping triangle methods may give rise to spurious residuals. For a practical evaluation of some methods used by the industry we refer to [5].

The authors have recently become aware of the work of Arjas, [2], and Norberg, [9],[10], and Haastrup & Arjas, [6], from the late 80's and early 90's where the modelling is done based on the idea to describe the time line of a claim using Marked Poisson Processes. This work has, to the authors knowledge, unfortunately had a limited impact on the actuarial community until rather recently, see Zhao et al [14], Zhao & Zhou [15], Antonio & Plat [1]. Other recent contributions on a more aggregated level is made by Verall et al. [13] and Miranda et al. [8] which are based on claim counts and claim amounts. We will refer to these papers and their ideas as "claim-level reserving methods".

The focus of the present paper is to introduce a general reserving method based on Monte Carlo simulation, i.e. a model framework, and we have therefore chosen to set most implementational issues aside. The underlying idea is to combine the entire policy database with the claims database making it possible to construct a simulated claims database where all relevant claims characteristics (from the undertaking's perspective) are known. Given that the claims characteristics are chosen in a suitable way, and are known, the cash flows of payments will then be induced from the simulated claims database. An important remark is that even though an undertaking may have few observed development years or accident years (or an unreliable historic database)

- the actually observed accident years may still consist of a large number of claims,

- there may still be a large number of policies.

This information is to a large extent destroyed by aggregating data into triangles. The authors believe that the main benefits with working within the proposed model framework are that

- available data is used effectively,
- it is possible to calculate "best estimate" claims provisions and a coherent solvency capital requirement without making exogenous distributional assumptions,
- to obtain stochastic cash flows that are built up constructively.

It is also important to note that since the cash flows are built up constructively it is possible to stress different claims characteristics and analyse how a particular stress will affect the resulting cash flow. For more on other potential uses of the model we refer to the closing discussion in Section 6.

The disposition of the remainder of the paper is as follows: In Section 2 the method is described in more detail, necessary notation is introduced and "best estimate" reserving is discussed. Section 3 is devoted to how the model can be incorporated into the Solvency II framework and general concerns regarding risk measurement are treated. In particular we describe how one can construct a consistent framework for calculating best estimate claims provisions, risk margin and solvency capital requirement. Section 4 briefly describes how the introduced framework relates to other claim-level reserving methods. In Section 5 we give some comments on how one can use the techniques introduced in Section 2 to price premia and treat premium risk. The paper ends with a closing discussion, Section 6, concerning potential extensions of the model together with some concluding remarks.

2 The simulation method

2.1 Simulating a claims database

As outlined above, even though an undertaking does not have many observed accident years there may still be many reported claims per accident year and the policy database might be large. This is information which will be destroyed if aggregated into a triangle. Therefore, we suggest that by simulating new claims or to add at present un-observed characteristics to partially observed claims, it is possible to constructively, and easily, build up a complete claims database from which cash flows may be obtained. Below we describe how this can be done using Monte Carlo simulation.

If we start at the policy level, all active policies in the database can from the undertakings perspective be categorised according to

(a) never having experienced any claims,

(b) having experienced claims which now are settled (S),

(c) having experienced claims that are reported but not settled (RBNS).

Note that the policies of class (a) and (b) may have claims that are yet unreported and thus they might belong to the, from the undertakings perspective, unobservable class "Incurred But Not Reported" (IBNR). Since we are interested in projecting the policy and claims database forward in time it is also necessary to include the possibility that a policy is terminated. A schematic representation of the possible transitions for a policy between the described states is given in Figure 1.

For each policy we will measure time relative to the time point of the last claim settlement or, if no claim has occurred, since the time when the policy was written up to calendar time t when the databases are observed. That is, in calendar time, the last claims settlement or the policy was written at time $t-\tau$. Introduce T_c, the time until a claim occurs relative to when either the policy was written or when the last claim was settled, and let T_o be the time until the claim becomes known to the insurer relative to T_c. Thus, a policy is

- IBNR if $T_c \leq \tau$ and $T_o + T_c > \tau$,
- RBNS if $T_c \leq \tau$ and $T_o + T_c \leq \tau$.

Moreover, in order to complete the partially known claims database we need to add extra information to the claims. Let T_s be the time it takes from T_o until the claim is settled. Further, we will distinguish between information of purely contractual nature, i.e. information which is known (read deterministic) for *every* policy, e.g. age and sex, which we will denote by $\tilde{X}(s) = \{\tilde{X}_1(s), \ldots, \tilde{X}_j(s)\}$, and $X = \{X_1, \ldots, X_k\}$ consisting of all additional information, from the undertakings perspective, except for T_c, T_o, T_s, affecting the claims payments and time until claims settlement. The reason for X not depending on time is that we assume that X can be described as a vector of vectors, e.g. one characteristic could be the time points when the severity of the claim is revised together with the corresponding changes in severity, which in this way can be generated at a single point in time. Or put more briefly, we will assume that X_i (as well as $\tilde{X}_j$) can be vector valued. Moreover, we associate X with a type space $\mathcal{T} = \mathcal{X}_1 \times \cdots \times \mathcal{X}_k$, where

$X_i \in \mathcal{X}_i$ for all $i = 1, \ldots, k$. For a figure depicting the time lines of policies being IBNR, RBNS and settled, see Figure 2.

We will now describe how we can use this information in order to model IBNR claims.

2.1.1 Modelling IBNR claims

The modelling of IBNR claims is done in a step-wise manner for each policy belonging to either of the categories a or b defined above. The steps of this process are as follows:

(i) Draw a random number T_c' from the distribution

$$\hat{F}^t_{T_c}(u|\tilde{X}(s); s \leq \tau) := \int_0^u \int_0^\tau \hat{f}^t_{T_c}(v|\tilde{X}(s))dsdv, \quad u \in \mathbb{R}_+.$$

The super-index t suggests that the empirical density/probability function is estimated or set subjectively at calendar time t.

(ii) If $T_c' > \tau$, the claim is not IBNR and we stop, but if $T_c' \leq \tau$, draw T_o' from the distribution

$$\hat{F}^t_{T_o}(u|T_o \leq \tau - T_c', T_c', \tilde{X}(s); s \leq \tau) := \int_0^u \int_0^\tau \hat{f}^t_{T_o}(v|T_o \leq \tau - T_c', T_c', \tilde{X}(s); s \leq \tau)dsdv, \quad u \leq \tau - T_c'.$$

(iii) Given the $\tilde{X}(s)$, $s \leq \tau$, T_c' and T_o', we generate the characteristics needed in order to settle the claim and hence describe the cash flow by drawing X' from the distribution

$$\hat{F}^t_X(x|T_c', T_o', \tilde{X}(s); s \leq \tau) := \hat{F}^t_X(x_1, \ldots, x_k|T_c', T_o', \tilde{X}(s); s \leq \tau)$$
$$= \int_{v_1 \leq x_1} \cdots \int_{v_k \leq x_k} \int_0^\tau \hat{f}^t_X(v_1, \ldots, v_k|T_c', T_o', \tilde{X}(s); s \leq \tau)dsdv_1 \cdots dv_k.$$

(iv) Given the information contained in X' we can finally draw the time until settlement T_s' from the distribution

$$\hat{F}_{T_s}(u|T_s > \tau - (T_c' + T_o'), X', \tilde{X}(s); s \leq \tau) := \int_{\tau-(T_c'+T_o')}^u \int_0^\tau \hat{f}^t_{T_s}(v|T_s > \tau - (T_c' + T_o'), X', \tilde{X}(s); s \leq \tau)dsdv,$$

where $u > \tau - (T_c' + T_o')$.

It is worth noting that we in (i) assume that the distribution of T_c is degenerate in the sense that it will have positive probability weight at infinity corresponding to that a claim never occurs.

A practical but nonetheless important remark regarding (iii) is to carefully consider how to generate the claims characteristics, i.e. to consider conditional dependencies between the chosen characteristics. More specifically, we might have three characteristics z_1, z_2, z_3 which due to their causal relation need to be generated in a specific order according to for example

$$\hat{f}_X(z_1, z_2, z_3) = \hat{f}_X(z_3|z_1, z_2)\hat{f}_X(z_2|z_1)\hat{f}_X(z_1).$$

Regarding the choice of estimated claims characteristics distributions, and the distributions of T_c, T_o and T_s, the simplest choice, if a sufficient amount of data is available, is to use empirical distributions. There are however risks with using empirical distributions if the underlying data is to sparse, where a fitted distribution might be more appropriate to use if data allows. There might also be situations where the characteristics which are desirable to model are unobservable or if a fitting process is unsuitable, where one is forced to make distributional assumptions. These distributional assumptions can, however, be seen as expert judgement. This situation also applies to the modelling of long tailed business at an early time point when only few claims are observed and where few or none are fully settled.

We will now continue with the modelling of RBNS claims.

2.1.2 Modelling RBNS claims

In essence the steps of modelling RBNS claims are the same as those made when we modelled IBNR claims. The only difference is that we know the values of T_c and T_o at (calendar time) t. Consequently, we can skip step (i) and (ii) in the algorithm described in the previous section. Moreover, at time t we may have observed some of the needed extra information X, say $Y' \subset X$, thus replacing the previous step (iii) by:

(iii′) Given that the information $Y' = \{X_i, i \in \mathcal{I}\}$ is known, we only need to model the remaining $Z' = \{X_i, i \notin \mathcal{I}\}$, which are drawn from the distribution

$$\hat{F}^t_{Z'}(z|Y', T'_c, T'_o, \tilde{X}(s); s \le \tau) := \int_{v_i \le z_i, i \notin \mathcal{I}} \int_0^\tau \hat{f}^t_X(v|Y', T'_c, T'_o, \tilde{X}(s); s \le \tau) ds dv.$$

Then, given that we have completed step (iii′), we can proceed according to step (iv) from the previous section.

By going through all policies classifying each of them as a potential IBNR claim or an active RBNS claim, we can hence generate a claims database where all necessary information needed in order to settle all claims is known. By repeating the above procedure a large number of times we will hence obtain a full predictive distribution of potential claims databases.

2.2 Reserving and best estimate claims provisions

In principle, once the (partially) simulated claims database is generated all necessary information for calculating the reserve is obtained. Since, given the information contained in the (partially) simulated claims database all characteristics needed to deduce the payments at each time point will be known, i.e. read out from the insurance contract. That is, the payments themselves are not stochastic but the claims' types are. In fact, it is always possible to construct a flow of payments in this way by increasing the type space. Hence, if we let $\hat{\mathcal{C}}^i_{x,t}$ denote the instantaneous payment resulting from a claim of type $x \in \mathcal{T}$ at t stemming from accident year i and let $\delta(t)$ denote the interest rate at time t, the *remaining cost* for accident year i evaluated at time t, henceforth denoted $\widehat{RC}^t_i$, can be expressed as

$$\widehat{RC}^t_i := \sum_{j=1}^{\hat{N}^{\text{ult}}_i} \int_t^\infty e^{-\int_t^v \delta(u)du} \hat{\mathcal{C}}^i_{x_j,v} dv. \tag{1}$$

Note that (1) only is a formal way of querying the (partially) simulated claims database for the discounted market valued payments. We have here assumed that all policies are re-indexed in such a manner that we only sum over those indices which belongs to a policy which is IBNR (according to the simulation procedure) or is RBNS. Moreover, $\hat{N}^{\text{ult}}_i$ is the number of claims that have occurred during accident year i, i.e. it is the number of policies that fulfil the condition $t - \tau + T'_c \in (i-1, i]$. It important to note that once the $\hat{\mathcal{C}}_{x_j,t}$'s are determined, i.e. the database is simulated, the integral in (1) is in itself deterministic. This yields that the the entire remaining cost is given by

$$\widehat{RC}^t := \sum_{i=0}^{I} \widehat{RC}^t_i. \tag{2}$$

Finally, the best estimate claims provisions is obtained by repeating the above described procedure a large number of times and taking the mean of the resulting empirical $\widehat{RC}^t$-distribution, i.e.

$$\hat{R}^t := \frac{1}{n} \sum_{j=1}^{n} \widehat{RC}^{t,j}, \tag{3}$$

where $\widehat{RC}^{t,j}$ is the jth realisation of the total remaining cost given all information available up to t.

Regarding the use of expert judgement, as mentioned earlier, the distributions needed for the simulation of the claims database can be chosen freely where each (set of) distributional assumptions can be seen as a "scenario" to which subjective probabilities can be attached. By doing so a reserve incorporating expert judgement will simply correspond to the probability weighted average taken over a number of empirical $\widehat{RC}^{t}$-distributions.

In the next section we describe how the method can be used in a partial internal model under the Solvency II directive and how the Solvency Capital Requirement (SCR) and Risk Margin (RM) can be calculated consistently in relation to the used reserving method. This also implies the possible use of the method under the Swiss solvency test (SST), where it can be used to obtain the standard deviation needed for the reserve risk calculation.

3 Calculating the capital requirement under Solvency II

The focus of the Solvency Capital Requirement (SCR) for reserve risk under Solvency II is on a one-year time horizon under run-off. That is, the risk meant to be captured is the one of mis-specifying the reserve at the end of year 0 in such a way that a substantial loss is made at the end of year 1 when the reserves are re-calculated for the same accident years. The standard formula under the Solvency II directive suggests that this loss is modelled to follow a log-normal distribution with mean 1 and variance σ^2, pre-specified or undertaking specific, which is shifted by -1. Thus, the "reserve loss-ratio" distribution is a shifted log-normal distribution centred at 0. More formally, the SCR corresponds to the 99.5%-percentile of this distribution, or equivalently: the 99.5% value at risk. This assumption is reasonable if the underlying reserving method is multiplicative (e.g. development factor models) and the uncertainty in the multiplicative factors are believed to be normal on a log-scale. In the present paper we instead argue that one could use the above described simulation method in order to obtain an empirical "reserve loss-ratio" distribution from which one can read off the, under the model, correct 99.5% percentile and use this as risk measure.

3.1 SCR calculation, risk measures and reserve loss-ratio's

In order to calculate the SCR we need to iterate the method one year forward in time. This requires that we need to obtain an estimate of the value of the reserve one year from now, $\hat{\hat{R}}^{t+1}$, as well as an estimate of the payments in the time interval between the two reserve assessments, henceforth denoted $\widehat{CF}_{(t,t+1]}$. Moreover, these two quantities have to be calculated for *each* of the realisations used in the calculation of the current reserve $\hat{R}^t$ from (3). Hence, we need to calculate $\hat{\hat{R}}^{t+1,j}$ and $\widehat{CF}^j_{(t,t+1]}$ for all $j = 1, \ldots, n$ from (3). Once this is done, the *reserve* loss-ratios

$$\widehat{LR}_j := \frac{\hat{\hat{R}}^{t+1,j} + \widehat{CF}^j_{(t,t+1]} - \hat{R}^t}{\hat{R}^t}. \tag{4}$$

can be computed and the capital requirement corresponds to the empirical 99.5%-percentile of this reserve loss-ratio distribution. In order for this process to be fully defined some clarifications are needed. To avoid cumbersome notation the dependence on the different realisations, manifested via super index j, will henceforth be omitted. The payments $\widehat{CF}_{(t,t+1]}$ are given by summing over the claims from all accident years and integrating over $(t, t+1]$:

$$\widehat{CF}_{(t,t+1]} := \sum_{i=0}^{I} \sum_{j=1}^{\hat{N}_i^{\text{ult}}} \int_t^{t+1} \hat{\mathcal{C}}^i_{x_j,v} dv. \tag{5}$$

The only quantity remaining to determine now is $\hat{\hat{R}}^{t+1}$ and this is done as follows: Given that we have added previously unknown characteristics to partially known claims (RBNS) and that we have added previously unknown claims and all of their characteristics (IBNR) we pick out the part of this information which, according to the model, *should* have been observed up to the end of year $t+1$. Thus, some of the RBNS claims may have been settled in the time interval $(t, t+1]$, and some of the IBNR claims may have become RBNS or settled. By using this additional information all empirical distributions (here in a wide sense) are re-estimated. In this way we obtain distributions $\hat{\hat{F}}^i_\bullet$ which are used to simulate a new claims database from the end of year $t+1$ and forward according to the algorithm laid out in section 2.1. This gives us that the remaining cost one year later, based on the updated data base, is given by

$$\widehat{\widehat{RC}}^{t+1} := \sum_{i=0}^{t} \widehat{\widehat{RC}}_i^{t+1-i}, \tag{6}$$

where the $\widehat{\widehat{RC}}_i^{t+1-i}$'s are defined analogously as (1)and the reserve $\hat{\hat{R}}^{t+1}$ is calculated analogously as in (3).

In this way we have obtained a way of calculating the SCR which is consistent with the reserving method and where the reserving method itself is very flexible. The above mentioned SCR calculation could also be used to provide an estimate of the standard deviation for the reserve risk needed in the Swiss Solvency Test. An important remark is that given the empirically obtained reserve loss-ratio distribution one can apply different risk measures and evaluate their effects, e.g. conditional value at risk ("expected shortfall").

An important remark is that the above described procedure for obtaining the SCR could be applied to other simulation based reserving methods as well.

3.2 Risk margin and best estimate claims provisions

Under the Solvency II directive the technical provisions as a whole consists of best estimate claims provision, see Subsection 2.2, and a Risk Margin (RM). The RM is thought to capture the cost which an undertaking is obliged to carry in order to be able to settle a business under complete run-off without any risk mitigating effects. According to the Solvency II directive the RM is supposed to capture the cost of capital of the discounted sum of all future SCR's when all assets are supposed to be put in bonds. The task of capturing "all future SCR's" is however not trivial if one wants to make a fair attempt. If we recollect how the SCR is calculated, the "new" information imputed into the simulated claims database corresponding to the information obtained in one year's time is used to re-fit/re-calibrate the empirical distributions that are used to model the claims characteristics. For the risk margin calculation, one needs to iterate this procedure forward in time while keeping track of the order with which the simulations are carried out whilst calculating the SCR's consecutively. A nice way of visualising this procedure is as a tree where each branching point corresponds to the state of the simulated database at a new future development year and the branches corresponds to new iterations of the updating procedure stemming from this particular state of the simulated database used in order to simulate future development years. It is not hard to see that this updating procedure in general will be very computer intensive, but unless one tries to simulate businesses with very heavy-tailed reporting lags or late, from a development year perspective, revisions of claims payments, one will often only need to carry out this iteration procedure for a few development years in order for the bulk of the claims to become known. Since even if the claims settlement period is long, e.g. annuities, most of the claims are often known relatively quickly, from a run-off perspective, and the remaining settlement period will quite often be of low risk nature even if the volume of payments might be large.

We will now briefly relate the above described method of claim-level reserving to other claim-level reserving methods.

4 Some comments on the relation to other claim-level reserving methods

One alternative to the above described procedure of a full claims database is to directly attack the problem of claim counts and claim amounts, see Verall et al. [13] and Miranda et al. [8]. In both papers it is assumed that the number of claims follows a mixed Poisson distribution and the claim amounts follows a mixed distribution together with a payment pattern which is independent between policies (here thought of as a lump payment with a discrete delay). That is, in terms of our model they model $N_i^{\text{ult,IBNR}}$ and $\hat{\mathcal{C}}^i_{x_j,t}$ directly.

The claim-level reserving methods stemming from the work of Arjas and Norberg, see e.g. [2],[9],[10] and [1], the use of marked Poisson processes implies an assumption of an underlying Poisson process governing the overall intensity with which a claim occurs and that there is a mark consisting of reporting delay and development of the claim. From an implementational perspective, they still follow a similar approach as Miranda et al. by modelling $N_i^{\text{ult},IBNR}$ directly, as being Poisson, and then adding reporting delay and additional claim development characteristics. The information about claim development is however more explicit than in Miranda et al.

Regardless of which model that is used, the simulation procedures for obtaining the SCR and risk margin outlined in Section 3 still applies.

5 Pricing of premia and premium risk

The method described in Section 2 can also be used in order to price premia. Since the method for obtaining the remaining cost for a policy which has not yet experienced a claim merely corresponds to the prospective reserve and the fair premium is as always merely the premium which is needed in order to balance this reserve. This is essentially the classical technique used in life-insurance, but based on a simulation approach.

Turning to premium risk, we will use the definition from Ohlsson & Lauzeningks [12]. In [12] the premium risk corresponds to that the expected earned premium for next calendar year proves insufficient to cover the costs which have arisen during the next new risk year. Two important differences between premium risk and reserve risk are that in order to capture the premium risk we need to model claims which have not yet occurred and policies to be written/terminated. Thus, within the above described model framework one needs to separate between

(I) existing policies which can experience a claim or which can be terminated,

(II) new policies which are written and which can experience a claim (or be terminated).

Regardless of which of (I) and (II) that is modelled, the underlying principle is similar to the IBNR-step in the algorithm described in Section 2.1.1, but that for (II) one also needs to model the number of newly written policies together with their fundamental characteristics such as age, sex, etc. Given that this is done both categories, (I) and (II), can be treated analogously: Assign a premium (obtained in some manner) in a suitable way to each policy. As long as the policy is not reported as a claim there will be a premium income, and otherwise a (possible) payment to the policy holder. By repeating this procedure for all policies similarly to how it was carried out for the remaining cost in Section 2.2 and Section 3.1 we obtain the premium risk.

6 Discussion and concluding remarks

The above description of the model/framework is deliberately made rather algorithmic in order not to focus too much on various details, both technical, and regarding the granularity of the model. This was done in order to keep the main ideas as clear as possible. Even if the present model seems to be too cumbersome to implement, if the available data for a particular implementation is too sparse or if the current business is run satisfactory using development factor methods, the present model still offers an opportunity to consider the underlying dynamics of the insurance in a systematic way. Another potential application of the model, which has not been discussed earlier, is as a liability "engine" which could be incorporated into an Asset Liability Management (ALM) model. Yet another potential use of the above described procedure is that it is natural to use when modelling correlations between e.g. different insurance policies related to the same underlying policy holder. It is also worth to stress that as a bi-product of generating a simulated claims database it is possible to use the attained simulated information to evaluate other reserving methods.

The level of granularity thought necessary for an intended application of the model will vary substantially from insurance to insurance. For more on these issues we refer the reader to [1], [8], [14] and [15].

An alternative approach could be to make use of the schematic representation laid out in Figure 1 and treat the reserving problem using standard (semi)

Markov process techniques, e.g. Thieles equation for obtaining the prospective reserve and Hattendorff's theorem for obtaining the reserve uncertainty.

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A Figures

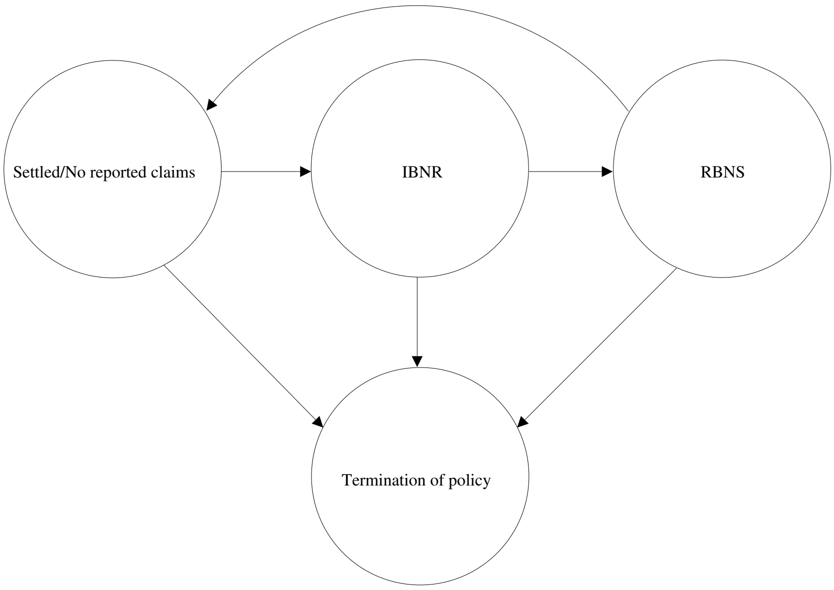

Figure 1: Schematic representation of possible transitions for a policy.

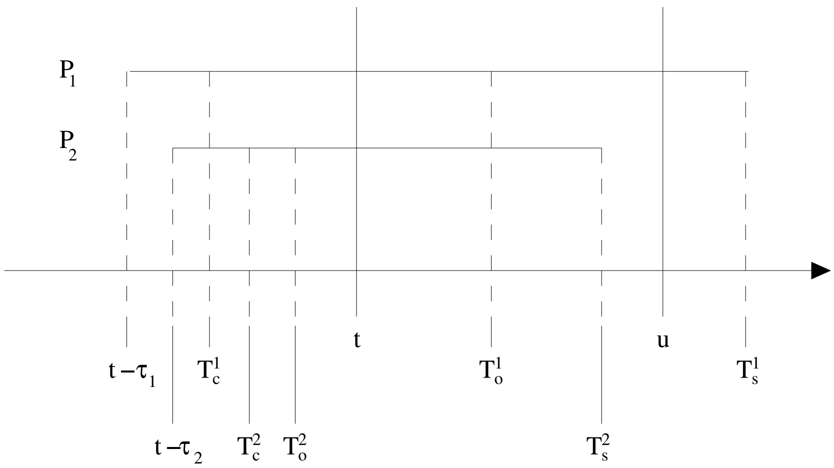

Figure 2: Policy 1 (P_1) is not identified as a claim by time t (IBNR), but is RBNS by time u. Policy 2 (P_2) is RBNS by time t and settled by time u.