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Argmax over Continuous Indices of Random Variables - An Approach Using Random Fields

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Abstract

In commuting research, it is common to model choices as optimization over a discrete number of random variables. In this paper we extend this theory from the discrete to the continuous case, and consider the limiting distribution of the location of the best offer as the number of offers tends to infinity.

Given a set $\Omega \subset \mathbb{R}^d$ of possible offers we seek a distribution over Ω, the argmax measure of the best offer. It depends on Λ, the sampling distribution of offer locations, and a measure index μ, which assigns to each point $x \in \Omega$ a probability distribution of offers.

This problem is closely related to argmax theory of marked point processes, altough we consider deterministic sequences of points in space, to allow for greater generality. We first define a finite sample argmax measure and then give conditions under which it converges as the number of offers tends to infinity.

To this end, we introduce a max-field of best offers and use continuity properties of this field to calculate the argmax measure. We demonstrate the usefulness of the method by giving explicit formulas for the limiting argmax distribution for a large class of models, including exponential independent offers with a deterministic, additive disturbance term. Finally, we illustrate the theory by simulations.

Key words: Argmax distribution, commuting, extreme value theory, exponential offers, marked point processes, max field.

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