



# Quasi Equilibrium Approximations of the Fixation Index under Neutrality: The Finite and Infinite Island Models

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April 30, 2012

## Abstract

The fixation index  $F_{ST}$  and the coefficient of gene differentiation  $G_{ST}$  are analyzed for the finite island model under short time spans, ignoring mutations. Dividing the reproduction cycle into the three steps gamete formation, fertilization, and migration we develop a new approach for computing quasi equilibrium formulas for  $F_{ST}$  (and  $G_{ST}$ ). Our formulas generalize earlier ones and reveal that the equilibrium value of  $F_{ST}$  is influenced not only by the migration rate and local effective population size,  $N_e$ , but also the local census size  $N$ , particularly so when the migration rate is high. The order of migration and fertilization is found to have a smaller effect on  $F_{ST}$ . A major advantage compared to previous approaches is that stochastic allele frequency of migrants is easily accommodated, thereby avoiding  $F_{ST}$  to be underestimated for large migration rates.

**Key words:** Effective population size, fixation index, fertilization, gamete formation, island model, migration, quasi equilibrium, reproduction.

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