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Splitting trees stopped when the first clock rings and Vervaat's transformation

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Abstract

We consider a branching population where individuals have i.i.d. life lengths (not necessarily exponential) and constant birth rate. We let N_t denote the population size at time t. We further assume that all individuals, at birth time, are equipped with independent exponential clocks with parameter δ. We are interested in the genealogical tree stopped at the first time T when one of those clocks rings. This question has applications in epidemiology, in population genetics, in ecology and in queuing theory.

We show that conditional on $\{T < \infty\}$, the joint law of $(N_T, T, X^{(T)})$, where $X^{(T)}$ is the jumping contour process of the tree truncated at time T, is equal to that of $(M, -I_M, Y'_M)$ conditional on $\{M \neq 0\}$, where : $M + 1$ is the number of visits of 0, before some single independent exponential clock $\mathbf{e}$ with parameter δ rings, by some specified Lévy process Y without negative jumps reflected below its supremum; I_M is the infimum of the path Y_M defined as Y killed at its last 0 before $\mathbf{e}$; Y'_M is the Vervaat transform of Y_M.

This identity yields an explanation for the geometric distribution of N_T and has numerous other applications. In particular, conditional on $\{N_T = n\}$, and also on $\{N_T = n, T < a\}$, the ages and residual lifetimes of the n alive individuals at time T are i.i.d. and independent of n. We provide explicit formulae for this distribution and give a more general application to outbreaks of antibiotic-resistant bacteria in the hospital.