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# A weighted configuration model and inhomogeneous epidemics

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**Abstract**

A random graph model with prescribed degree distribution and degree dependent edge weights is introduced. Each vertex is independently equipped with a random number of half-edges and each half-edge is assigned an integer valued weight according to a distribution that is allowed to depend on the degree of its vertex. Half-edges with the same weight are then paired randomly to create edges. An expression for the threshold for the appearance of a giant component in the resulting graph is derived using results on multi-type branching processes. The same technique also gives an expression for the basic reproduction number for an epidemic on the graph where the probability that a certain edge is used for transmission is a function of the edge weight. It is demonstrated that, if vertices with large degree tend to have large (small) weights on their edges and if the transmission probability increases with the edge weight, then it is easier (harder) for the epidemic to take off compared to a randomized epidemic with the same degree and weight distribution. A recipe for calculating the probability of a large outbreak in the epidemic and the size of such an outbreak is also given. Finally, the model is fitted to three empirical weighted networks of importance for the spread of contagious diseases and it is shown that $R_0$ can be substantially over- or underestimated if the correlation between degree and weight is not taken into account.