Mathematical Statistics
Stockholm University
Research Report **2011:1**,
http://www.math.su.se/matstat

Scale-free percolation

Maria Deijfen * Remco van der Hofstad † Gerard Hooghiemstra ‡

Mars 2011

Abstract

We formulate and study a model for inhomogeneous long-range percolation on $\mathbb{Z}^d$. Each vertex $x \in \mathbb{Z}^d$ is assigned a non-negative weight W_x, where $(W_x)_{x\in\mathbb{Z}^d}$ are i.i.d. random variables. Conditionally on the weights, and given two parameters $\alpha, \lambda > 0$, the edges are independent and the probability that there is an edge between x and y is given by $p_{xy} = 1 - \exp\{-\lambda W_x W_y/|x-y|^\alpha\}$. The parameter λ is the percolation parameter, while α describes the long-range nature of the model. We focus on the degree distribution in the resulting graph, on whether there exists an infinite component and on graph distance between remote pairs of vertices.

First, we show that the tail behavior of the degree distribution is related to the tail behavior of the weight distribution. When the tail of the distribution of W_x is regularly varying with exponent $\tau - 1$, then the tail of the degree distribution is regularly varying with exponent $\gamma = \alpha(\tau - 1)/d$. The parameter γ turns out to be crucial for the behavior of the model. Conditions on the weight distribution and γ are formulated for the existence of a critical value $\lambda_c \in (0, \infty)$ such that the graph contains an infinite component when $\lambda > \lambda_c$ and no infinite component when $\lambda < \lambda_c$. Furthermore, a phase transition is established for the graph distances between vertices in the infinite component at the point $\gamma = 2$, that is, at the point where the degrees switch from having finite to infinite second moment.

The model can be viewed as an interpolation between long-range percolation and models for inhomogeneous random graphs, and we show that the behavior shares the interesting features of both these models.

Keywords: Random graphs, long-range percolation, percolation in random environment, degree distribution, phase transition, chemical distance, graph distance.

AMS 2000 Subject Classification: 60K35, 05C80.

1 Introduction

The field of percolation has been very active the last few decades with important progress on questions concerning for instance the appearance and uniqueness of an infinite component and

*Department of Mathematics, Stockholm University, 106 91 Stockholm, Sweden. Email: `mia@math.se`

†Department of Mathematics and Computer Science, Eindhoven University of Technology, P.O. Box 513, 5600 MB Eindhoven, The Netherlands. Email: `rhofstad@win.tue.nl`

‡DIAM, Delft University of Technology, Mekelweg 4, 2628CD Delft, The Netherlands. Email: `g.hooghiemstra@tudelft.nl`