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Percolation in invariant Poisson graphs with i.i.d. degrees

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Abstract

Let each point of a homogeneous Poisson process in $\mathbb{R}^d$ independently be equipped with a random number of stubs (half-edges) according to a given probability distribution μ on the positive integers. We consider translation-invariant schemes for perfectly matching the stubs to obtain a simple graph with degree distribution μ. Leaving aside degenerate cases, we prove that for any μ there exist schemes that give only finite components as well as schemes that give infinite components. For a particular matching scheme that is a natural extension of Gale-Shapley stable marriage, we give sufficient conditions on μ for the absence and presence of infinite components.

Keywords: Random graph, degree distribution, Poisson process, matching, percolation.

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1 Introduction

Let $\mathcal{P}$ be a homogeneous Poisson process with intensity 1 on $\mathbb{R}^d$. Furthermore, let μ be a probability measure on the strictly positive integers. We shall study translation-invariant simple random graphs whose vertices are the points of $\mathcal{P}$ and where the degrees of the vertices are i.i.d. with law μ. Deijfen [7] studied moment properties achievable for the edge lengths in such graphs. Here, we shall instead be interested in the percolation-theoretic question of whether the graph contains a component with infinitely many vertices.

We next formally describe the objects that we will work with. For any random point measure Λ we write $[\Lambda] := \{x \in \mathbb{R}^d : \Lambda(\{x\}) > 0\}$ for its

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