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Random networks with preferential growth and vertex death in continuous time

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Abstract

A dynamic model for a random network evolving in continuous time is defined where new vertices are born and existing vertices may die. The fitness of a vertex is defined as the total number of connections that the vertex has held, that is, its accumulated degree. A new vertex is then connected to an existing vertex with probability proportional to a function b of the fitness of the existing vertex, and a vertex dies at a rate given by a function d of its fitness. Using results from the theory of general branching processes, an expression for the asymptotic empirical fitness distribution $\{p_k\}$ is derived and analyzed for a number of specific choices of b and d. When $b(i) = i + \alpha$ and $d(i) = \beta$ – that is, linear preferential attachment for the newborn and random deaths – then $p_k \sim k^{-(2\alpha+1)}$. When $b(i) = i + 1$ and $d(i) = \beta(i + 1)$, with $\beta < 1$, then $p_k \sim (1 + \beta)^{-k}$, that is, if also the death rate is proportional to the fitness, then the power law distribution is lost. Furthermore, when $b(i) = i + 1$ and $d(i) = \beta(i + 1)^\gamma$, with $\beta, \gamma < 1$, then $\log p_k \sim -k^\gamma$ – a stretched exponential distribution.

Keywords: Branching process, random network, preferential attachment, degree distribution, power law distribution.

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1 Introduction

Empirical studies on real networks has revealed that many of them exhibit features that are not captured by the classical Erdös-Rényi graph. In particular, many networks tend to have a quite heavy tailed degree distribution, often described by a power law, that is, the fraction of vertices with degree k decays as $k^{-\tau}$ for some exponent τ. To capture this, a number of new graph

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