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Stable Poisson Graphs in One Dimension

Maria Deijfen * Alexander E. Holroyd †

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Abstract

Let each point of a homogeneous Poisson process on $\mathbb{R}$ independently be equipped with a random number of stubs (half-edges) according to a given probability distribution μ on the positive integers. We consider two natural schemes for perfectly matching the stubs to obtain a simple graph with degree distribution μ, both derived from Gale-Shapley stable marriage. We prove results on the existence of an infinite component and on the length of the edges, with focus on the case $\mu(\{2\}) = 1$.

Keywords: Poisson process, random graph, degree distribution, matching, percolation.

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1 Introduction

Let $\mathcal{P}$ be a homogeneous Poisson process with intensity 1 on $\mathbb{R}^d$ and μ a probability measure on the strictly positive integers. We shall study translation-invariant simple random graphs whose vertices are the points of $\mathcal{P}$ and where, conditional on $\mathcal{P}$, the degrees of the vertices are i.i.d. with law μ. Previously, Deijfen [2] has studied achievable moment properties for the edges, and Deijfen, Häggström and Holroyd [3] have studied the question of whether the graph contains a component with infinitely many vertices. In the latter work a particular matching scheme, called the stable multi-matching, was introduced, leading to a number of challenging open questions. Here we restrict to $d = 1$ and the focus is on the case $\mu(\{2\}) = 1$, one of the simplest cases for which the question of existence of an infinite component is non-trivial. For the stable multi-matching and a variant of it with prescribed random stub directions,

*Department of Mathematics, Stockholm University, 106 91 Stockholm. mia at math.su.se

†Microsoft Research, 1 Microsoft Way, Redmond, WA 98052, USA; holroyd at microsoft.com