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Epidemics on random intersection graphs

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Abstract

In this paper we consider a model for the spread of a stochastic SIR (Susceptible $\rightarrow$ Infectious $\rightarrow$ Removed) epidemic on a network of individuals described by a random intersection graph. The number of cliques a typical individual belongs to follows a mixed-Poisson distribution, as does the size of a typical clique. Infection can be transmitted between two individuals if and only if they belong to the same clique. An infinite-type branching process approximation (with type being given by the length of an individual's infectious period) for the early stages of an epidemic is developed and made fully rigorous by proving an associated limit theorem as the population size tends to infinity. This leads to a threshold parameter R_*, so that in a large population an epidemic with few initial infectives can give rise to a large outbreak if and only if $R_* > 1$. A functional equation for the survival probability of the approximating infinite-type branching is determined; if $R_* \leq 1$, this equation has no non-zero solution, whist, if $R_* > 1$, it is shown to have precisely one non-zero solution. A law of large numbers for the size of such a large outbreak is proved by exploiting a single-type branching process that approximates the susceptibility set of a typical individual.