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Epidemics and vaccination on weighted graphs

Maria Deijfen *

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Abstract

A Reed-Frost epidemic with inhomogeneous infection probabilities on a graph with prescribed degree distribution is studied. Each edge (u, v) in the graph is equipped with two weights $W_{(u,v)}$ and $W_{(v,u)}$ that represent the (subjective) strength of the connection and determine the probability that u infects v in case u is infected and vice versa. Expressions for the epidemic threshold are derived for i.i.d. weights and for weights that are functions of the degrees. For i.i.d. weights, a variation of the so called acquaintance vaccination strategy is considered where vertices are chosen randomly and neighbors of these vertices with maximal edge weights are vaccinated. This strategy is shown to outperform the strategy where the neighbors are chosen randomly in the sense that the basic reproduction number is smaller for a given vaccination coverage.

Keywords: Reed-Frost epidemic, weighted graph, degree distribution, epidemic threshold, vaccination.

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1 Introduction

The Reed-Frost model is one of the simplest stochastic epidemic models. It was formulated by Lowell Reed and Wade Frost in 1928 (in unpublished work) and describes the evolution of an infection in generations. Each infected individual in generation t ($t = 1, 2, \ldots$) independently infects each susceptible individual in the population with some probability p. The individuals that become infected by the individuals in generation t then constitute generation $t + 1$ and the individuals in generation t are removed from the epidemic process. See [3] for a description of the asymptotic (as the population size grows to infinity) behavior of the process.

In the original version, an infective individual infects each susceptible individual in the population with the same probability. Realistically however

*Department of Mathematics, Stockholm University, 106 91 Stockholm. mia at math.su.se