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Bootstrapping the separation method in claims reserving

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Abstract

The separation method was introduced by Verbeek (1972) in order to forecast numbers of excess claims and it was developed further by Taylor (1977) to be applicable to the average claim cost. The separation method differs from the chain-ladder in that when the chain-ladder only assumes claim proportionality between the development years, the separation method also separates the claim delay distribution from influences affecting the calendar years, e.g. inflation. Since the inflation contributes to the uncertainty in the estimate of the claims reserve it is important to consider its impact in the context of risk management, too.
In this paper we present a method for assessing the prediction error distribution of the separation method. To this end we introduce a parametric framework within the separation model which enables joint resampling of claim counts and claim amounts. As a result, the variability of Taylor's predicted reserves can be assessed by extending the parametric bootstrap techniques of Björkwall *et al.* (2008). The performance of the bootstrapped separation method and chain-ladder is compared for a real data set.

Keywords

Bootstrap; Chain-ladder; Development triangle; Inflation; Separation method; Stochastic claims reserving.

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1 Introduction

One issue for the reserving actuary is how to deal with inflation, which contributes to the uncertainty in the estimate of the claims reserve. Even though some proper reserving techniques are suggested in the literature, little has been said about how to approach this issue when it comes to finding the variability of the actuary's best estimate either analytically or by bootstrapping.

Due to external forces the average cost per claim will change from one calendar year to another. Typically this *claims inflation* is specific to each line of business and depends on the economic inflation, which usually can be tied to some relevant index, as well as on factors like legislation and attitudes to policy holder compensation. The latter result in so called *superimposed claims inflation*.

The chain-ladder method makes implicit allowance for claims inflation since it projects the inflation present in the past data into the future, see e.g. Taylor (2000). Consequently, it only works properly when the inflation rate is constant. When the economic inflation rate is non-constant, the past paid losses can be converted to current value by some relevant index before they are projected into the future by the chain-ladder, but still there is no allowance for superimposed claims inflation.

Another approach of dealing with claims inflation is to incorporate it into the model underlying the reserving method. In this way the past inflation rate can be estimated and the future inflation rate can be predicted within the model. Verbeek (1972) introduced such a method in the reinsurance context and Taylor (1977) developed it further to be applicable to the average claim cost in a general context. The reserving technique is called *the separation method*. However, the separation method has, unlike its famous relative, remained quite anonymous in the literature on stochastic claims reserving. For instance, the mean squared error of prediction for the chain-ladder was analytically calculated by Mack (1993) and revisited by Buchwalder *et al.* (2006) and Mack *et al.* (2006) and a full predictive distribution was obtained for the chain-ladder by bootstrapping in England & Verrall (1999), England (2002) and Pinheiro *et al.* (2003). Recently the variability of other reserving methods has been investigated as well, e.g. the Bornhuetter-Ferguson method by analytical approximation in Mack (2008) and the Munich chain-ladder, see Quarg & Mack (2004), by bootstrapping of two correlated quantities in Liu & Verrall (2008).

The object of this paper is to analyze the variability of the separation method. Since bootstrapping easily gives a full predictive distribution and can also be used in risk management with Dynamic Financial Analysis (DFA) we develop a bootstrap procedure for the separation method. For this purpose we use an extended version of the parametric bootstrap technique described in Björkwall *et al.* (2008). To this end, we introduce a parametric framework within the separation model, in which claim counts are Poisson distributed and claim amounts are gamma distributed *conditionally* on the ultimate claim counts. This enables joint resampling of claim counts and claim amounts.

Section 2 contains the definitions and the theory behind the separation method. In Section 3 the suggested bootstrap methodology is discussed and it is studied numerically for the well-known data set from Taylor & Ashe (1983) in Section 4. Finally, Section 5 contains a discussion.

2 The separation method

Assume that we have a triangle of incremental observations of paid claims $\{C_{ij}; i,j \in \nabla\}$, where ∇ denotes the upper, observational triangle $\nabla = \{i = 0,\dots,t; j = 0,\dots,t-i\}$. The suffixes i and j refer to the origin year and the development year, respectively, see Table 2.1. In addition, the suffix $k = i + j$ is used for the calendar years, i.e. the diagonals of ∇. The purpose is to predict the sum of the delayed claim amounts in the lower, unobserved future triangle $\{C_{ij}; i,j \in \Delta\}$, where $\Delta = \{i = 1,\dots,t; j = t-i+1,\dots,t\}$, see Table 2.2. We write $R = \sum_{\Delta} C_{ij}$ for this sum, which is the outstanding claims for which the insurance company must hold a reserve. Furthermore, assume that we have a triangle of the incremental observations of the numbers of claims $\{N_{ij}; i, j \in \nabla\}$ corresponding to the same portfolio as in Table 2.1, i.e. the observations in Table 2.3. The ultimate number of claims relating to period of origin year i is then

$$N_i = \sum_{j \in \nabla_i} N_{ij} + \sum_{j \in \Delta_i} N_{ij} \,, \tag{2.1}$$

where ∇_i and Δ_i denotes the rows corresponding to origin year i in the upper triangle ∇ and the lower triangle Δ, respectively.

The separation method is based on the assumption that N_i is considered as known. Since the number of claims is often finalized quite early even for long-tailed business, N_i may very well be estimated separately, e.g. by the chain-ladder if a triangle of claim counts

	Development year					
Accident year	0	1	2	$\cdots$	$t-1$	t
0	C_{00}	C_{01}	C_{02}	$\cdots$	$C_{0,t-1}$	$C_{0,t}$
1	C_{10}	C_{11}	C_{12}	$\cdots$	$C_{1,t-1}$	
2	C_{20}	C_{21}	C_{22}	$\cdots$		
$\vdots$	$\vdots$	$\vdots$	$\vdots$			
$t-1$	$C_{t-1,0}$	$C_{t-1,1}$				
t	$C_{t,0}$					

Table 2.1: *The triangle ∇ of observed incremental payments.*

	Development year					
Accident year	0	1	2	$\cdots$	$t-1$	t
0						
1						$C_{1,t}$
2					$C_{2,t-1}$	$C_{2,t}$
$\vdots$					$\vdots$	$\vdots$
$t-1$			$C_{t-1,2}$	$\cdots$	$C_{t-1,t-1}$	$C_{t-1,t}$
t		$C_{t,1}$	$C_{t,2}$	$\cdots$	$C_{t,t-1}$	$C_{t,t}$

Table 2.2: *The triangle Δ of unobserved future claim costs.*

is provided, and then be treated as known. Henceforth estimates $\hat{n}_{ij}$ of the expectations $n_{ij} = E(N_{ij})$ is obtained by the chain-ladder for all cells in both ∇ and Δ. The estimated ultimate number of claims relating to origin year i is then

$$\hat{N}_i = \sum_{j \in \nabla_i} N_{ij} + \sum_{j \in \Delta_i} \hat{n}_{ij} \, . \tag{2.2}$$

The chain-ladder method operates on cumulative claim counts

$$A_{ij} = \sum_{\ell=0}^{j} N_{i\ell} \tag{2.3}$$

rather than incremental claim counts N_{ij}. Let $\nu_{ij} = E(A_{ij})$. Development factors g_j are estimated for $j = 0, 1, \ldots, t-1$ by

$$\hat{g}_j = \frac{\sum_{i=0}^{t-j} A_{i,j+1}}{\sum_{i=0}^{t-j} A_{ij}} \tag{2.4}$$

yielding

$$\hat{\nu}_{ij} = A_{i,t-i} \, \hat{g}_{t-i} \, \hat{g}_{t-i+1} \ldots \hat{g}_{j-1} \tag{2.5}$$

and

$$\hat{n}_{i,j} = \hat{\nu}_{i,j} - \hat{\nu}_{i,j-1} \tag{2.6}$$

for Δ, while estimates of $\hat{\nu}_{ij}$ for ∇ are obtained by the process of backwards recursion described in England & Verrall (1999).

	Development year					
Accident year	0	1	2	$\cdots$	$t-1$	t
0	N_{00}	N_{01}	N_{02}	$\cdots$	$N_{0,t-1}$	$N_{0,t}$
1	N_{10}	N_{11}	N_{12}	$\cdots$	$N_{1,t-1}$	
2	N_{20}	N_{21}	N_{22}	$\cdots$		
$\vdots$	$\vdots$	$\vdots$	$\vdots$			
$t-1$	$N_{t-1,0}$	$N_{t-1,1}$				
t	$N_{t,0}$					

Table 2.3: *The triangle ∇ of observed incremental numbers of reported claims.*

While the chain-ladder only assumes claim proportionality between the development years, the separation method in Taylor (1977) separates the claim delay distribution from influences effecting the calendar years, e.g. inflation. In the separation model we first assume that the proportion of the average claim amount paid in development year j is constant over i; denote this proportion by r_j. If the claims are fully paid by year t we have the constraint

$$\sum_{j=0}^{t} r_j = 1. \tag{2.7}$$

We then make a further assumption that the claim amount is proportional to some index, say λ_k, that relates to the calendar year k during which the claims are paid. The expected claim cost for development year j and calendar year k is then proportional to $r_j\,\lambda_k$.

	Development year					
Accident year	0	1	2	$\cdots$	$t-1$	t
0	$r_0\,\lambda_0$	$r_1\,\lambda_1$	$r_2\,\lambda_2$	$\cdots$	$r_{t-1}\,\lambda_{t-1}$	$r_t\,\lambda_t$
1	$r_0\,\lambda_1$	$r_1\,\lambda_2$	$r_2\,\lambda_3$	$\cdots$	$r_{t-1}\,\lambda_t$	
2	$r_0\,\lambda_2$	$r_1\,\lambda_3$	$r_2\,\lambda_4$	$\cdots$		
$\vdots$	$\vdots$	$\vdots$	$\vdots$			
$t-1$	$r_0\,\lambda_{t-1}$	$r_1\,\lambda_t$				
t	$r_0\,\lambda_t$					

Table 2.4: *The triangle ∇ of expected paid claims.*

The separation model can be given the following formulation, which is at a bit more detailed level than the one given in Taylor (1977). Let C_{ijl} denote the amount paid during calendar year k for the l:th individual claim incurred in origin year i and assume that C_{ijl} are conditionally independent for all i, j and l given N_i. According to the discussion above we also assume that

$$E(C_{ijl}|N_i) = r_j\,\lambda_k. \tag{2.8}$$

Since the total amount paid during calendar year k for claims incurred in origin year i is

$$C_{ij} = \sum_{l=1}^{N_i} C_{ijl} \tag{2.9}$$

we obtain

$$E\left(\frac{C_{ij}}{N_i}\middle|N_i\right) = \frac{1}{N_i}\sum_{l=1}^{N_i} E\left(C_{ijl}|N_i\right) = \frac{1}{N_i}\sum_{l=1}^{N_i} r_j\,\lambda_k = r_j\,\lambda_k \tag{2.10}$$

for the conditional expectation of the average claim costs given the ultimate number of claims and this relation is the basic assumption of the separation method. The expectations in equation (2.10) now build up the triangle in Table 2.4.

If N_i is estimated separately by (2.2), it follows from (2.8) and (2.9) that

$$\begin{aligned} E\left(\frac{C_{ij}}{\hat{N}_i}\middle|\nabla N\right) &= \frac{E(E(C_{ij}|N_i,\nabla N)|\nabla N)}{\hat{N}_i} \\ &= r_j\,\lambda_k\,\frac{(\sum_{\nabla_i} N_{ij} + \sum_{\Delta_i} n_{ij})}{(\sum_{\nabla_i} N_{ij} + \sum_{\Delta_i} \hat{n}_{ij})} \\ &\approx r_j\,\lambda_k \end{aligned} \tag{2.11}$$

where in the last equality we used $\hat{n}_{ij} \approx n_{ij}$.

Estimates $\hat{r}_j$ and $\hat{\lambda}_k$ of the parameters in the triangle in Table 2.4 can now be obtained using the corresponding triangle ∇s of observed values

$$s_{ij} = \frac{C_{ij}}{\hat{N}_i}\,, \tag{2.12}$$

and the method of moments equations

$$s_{k0} + s_{k-1,1} + \ldots + s_{0k} = (\hat{r}_0 + \ldots + \hat{r}_k)\,\hat{\lambda}_k\,, \qquad k = 0,\ldots,t \tag{2.13}$$

for the diagonals of ∇ and

$$s_{0j} + s_{1j} + \ldots + s_{t-j,j} = (\hat{\lambda}_j + \ldots + \hat{\lambda}_t)\,\hat{r}_j\,, \qquad j = 0,\ldots,t \tag{2.14}$$

for the columns of ∇.

Taylor (1977) shows that the equations (2.13) - (2.14), with the side constraint (2.7), have a unique solution that can be obtained recursively, starting with $k = t$ in (2.13) to solve for $\hat{\lambda}_t$, then $j = t$ in (2.14) to solve for $\hat{r}_t$, $k = t-1$ in (2.13) to solve for $\hat{\lambda}_{t-1}$ and so on. This yields

$$\hat{\lambda}_k = \frac{\sum_{i=0}^{k} s_{i,k-i}}{1 - \sum_{j=k+1}^{t} \hat{r}_j}\,, \qquad k = 0,\ldots,t \tag{2.15}$$

$$\hat{r}_j = \frac{\sum_{i=0}^{t-j} s_{ij}}{\sum_{k=j}^{t} \hat{\lambda}_k}\,, \qquad j = 0, \ldots, t\,, \tag{2.16}$$

where $\sum_{j=k+1}^{t} \hat{r}_j$ is interpreted as zero when $k = t$.

Estimates $\hat{m}_{ij}$ of the expectations $m_{ij} = E(C_{ij})$ for cells in ∇ are now given by

$$\hat{m}_{ij} = \hat{N}_i\, \hat{r}_j\, \hat{\lambda}_k\,, \tag{2.17}$$

but in order to obtain the estimates of Δ it remains to predict λ_k for $t+1 \leq k \leq 2\,t$, which requires some inflation assumption.

If there is a trend in the inflation indexes $\hat{\lambda}_k$ for $k \leq t$ then smoothing and extrapolation could be used in order to forecast the future inflation. An alternative is to use an average of the past indexes. In any case, with an inflation assumption of, say, $K\%$, the forecasted λ_{k+1} can be obtained as $\hat{\lambda}_{k+1} = (1 + \frac{K}{100})\, \hat{\lambda}_k$ for $t \leq k \leq 2\,t - 1$. The cell expectations of ΔC_{ij} are then estimated by equation (2.17) and estimators of the outstanding claims are obtained by summing per accident year $\hat{R}_i = \sum_{j \in \Delta_i} \hat{m}_{ij}$. The estimator of the total reserve is $\hat{R} = \sum_{\Delta} \hat{m}_{ij}$.

The separation model described by Taylor (1977) is more general than the one discussed in this paper, since the original model do not presume that N_i is the number of claims; it could be some other exposure relating to origin year i as well. However, in this paper we stick to the number of claims.

3 A conditional parametric bootstrap approach

For the purpose of obtaining the predictive distribution of the claims reserve R estimated by the separation method the bootstrap technique described in Pinheiro *et al.* (2003) and, in particular, the parametric approach in Björkwall *et al.* (2008) is used. For the sampling process we model the paid claims C_{ij} conditionally on N_i in accordance with (2.11). We provide models for the assumption of stochastic N_i as well as for the case when N_i is considered as known. The former assumption demands that we develop the technique described in Björkwall *et al.* (2008) in order to handle ∇N as well as ∇C.

3.1 Stochastic Poisson distributed claim counts

Verbeek (1972) adopted a Poisson distribution for the claim counting variable, while the method described in Taylor (1977) is distribution-free. However, the assumption of independent and Poisson distributed claim counts

$$N_{ij} \in Po(n_{ij}) \tag{3.1}$$

yields a very reasonable model for the sampling process.

In addition we assume that the conditionally independent claims $C_{ijl}|N_i$ in (2.8) are gamma distributed. We use the notation

$$C_{ijl}|N_i \in \Gamma\left(\frac{1}{\phi}, r_j\,\lambda_k\,\phi\right)\,, \tag{3.2}$$

where $1/\phi$ is the so called index parameter and $r_j\,\lambda_k\,\phi$ is the scale, so that the expected value is $r_j\,\lambda_k$. Moreover, $\phi > 0$.

Recalling (2.9) and the independence of the C_{ijl} we find that

$$C_{ij}|N_i \in \Gamma\left(\frac{N_i}{\phi}, r_j\,\lambda_k\,\phi\right)\,, \tag{3.3}$$

which is consistent with (2.10) since

$$E(C_{ij}|N_i) = N_i\,r_j\,\lambda_k\,. \tag{3.4}$$

The variance of the amounts in (3.3) is

$$Var(C_{ij}|N_i) = \phi\,N_i\,(r_j\,\lambda_k)^2 = \phi\,\frac{E^2(C_{ij}|N_i)}{N_i}\,, \tag{3.5}$$

which corresponds to a weighted generalized linear model under the assumption of a logarithmic link function and a gamma distribution. We use a Pearson type estimate of ϕ, cf.

McCullagh & Nelder (1989),

$$\hat{\phi} = \frac{1}{|\nabla| - q} \sum_{\nabla} \hat{N}_i \, \frac{(C_{ij} - \hat{E}(C_{ij}|N_i))^2}{\hat{E}^2(C_{ij}|N_i)} = \frac{1}{|\nabla| - q} \sum_{\nabla} \hat{N}_i \, \frac{(C_{ij} - \hat{N}_i \, \hat{r}_j \, \hat{\lambda}_k)^2}{(\hat{N}_i \, \hat{r}_j \, \hat{\lambda}_k)^2}\,, \tag{3.6}$$

where $|\nabla| = (t+1)\,(t+2)/2$ is the number of observations in ∇C, the estimators $\hat{N}_i$, $\hat{\lambda}_j$ and $\hat{r}_j$ are obtained from (2.2), (2.15) and (2.16) and $q = 2\,t+1$ is the number of parameters that have to be estimated by the separation method, i.e. r_j for $j = 0, 1, \ldots, t-1$ and λ_k for $k = 0, 1, \ldots, t$.

Notice that (3.3) could be interpreted as follows; given N_i claims we allocate claim amounts independently over the development years j according to the proportions $r_0, \ldots, r_t$ before the inflation is considered. According to (3.2) we not only allocate claim amounts but individual claims as well. This interpretation is consistent with the assumptions discussed in Section 2.

We adopt the bootstrap technique described in Pinheiro *et al.* (2003) and, in particular, the parametric approach in Björkwall *et al.* (2008). The relation between the true outstanding claims R and its estimator $\hat{R}$ in the real world is, by the plug-in-principle, substituted in the bootstrap world by their bootstrap counterparts. Hence, the process error is included in R^{**}, i.e. the true outstanding claims in the bootstrap world, while the estimation error is included in $\hat{R}^{*}$, i.e. the estimated outstanding claims in the bootstrap world. Henceforth we use the index $*$ for random variables or plug-in estimators in the bootstrap world which correspond to observations or estimators in the real world, while the index $**$ is used for random variables in the bootstrap world when the counterparts in the real world are unobserved.

The parametric bootstrap approach in Björkwall *et al.* (2008) can now be implemented for the separation method using (3.1) and (3.3) in the following way. We draw N^{*}_{ij} and N^{**}_{ij} from

$$N^{*}_{ij} \in Po(\hat{n}_{ij}) \quad \text{and} \quad N^{**}_{ij} \in Po(\hat{n}_{ij}) \tag{3.7}$$

B times for all $i, j \in \nabla$ and $i, j \in \Delta$, respectively. We thereby get the B pseudo-triangles

∇N^* and ΔN^{**}. The ultimate number of claims per origin year in the bootstrap world is then given by

$$N_i^{**} = \sum_{j \in \nabla_i} N_{ij}^* + \sum_{j \in \Delta_i} N_{ij}^{**} \tag{3.8}$$

according to (2.1).

Once N_i^{**} is calculated, C_{ij}^* is sampled B times from

$$C_{ij}^* | N_i^{**} \in \Gamma\left(\frac{N_i^{**}}{\hat{\phi}}, \hat{r}_j\, \hat{\lambda}_k\, \hat{\phi}\right), \tag{3.9}$$

for all $i, j \in \nabla$ yielding the B pseudo-triangles ∇C^*. Here $\hat{\lambda}_k$ and $\hat{r}_j$ are obtained from (2.15) and (2.16).

The heuristic estimation process described in Section 2 is then repeated B times for each pair of pseudo-triangles. The claim counts are first forecasted by $\Delta \hat{n}^*$, obtained by the chain-ladder from ∇N^*, in order to estimate the ultimate number of claims per origin year

$$\hat{N}_i^* = \sum_{j \in \nabla_i} N_{ij}^* + \sum_{j \in \Delta_i} \hat{n}_{ij}^* \tag{3.10}$$

according to (2.2). The future payments are then forecasted by estimating $\Delta \hat{m}^*$ according to (2.12) - (2.17). Finally, estimators for the outstanding claims in the bootstrap world are obtained by $\hat{R}_i^* = \sum_{j \in \Delta_i} \hat{m}_{ij}^*$ and $\hat{R}^* = \sum_{\Delta} \hat{m}_{ij}^*$.

In order to generate a random outcome of the true outstanding claims in the bootstrap world, i.e. $R_i^{**} = \sum_{j \in \Delta_i} C_{ij}^{**}$ and $R^{**} = \sum_{\Delta} C_{ij}^{**}$, we sample once again from (3.9) for all $i, j \in \Delta$ to get B triangles ΔC^{**}.

The final step is to calculate the B prediction errors

$$\mathrm{pe}_i^{**} = \frac{R_i^{**} - \hat{R}_i^*}{\sqrt{\widehat{Var}(R_i^{**})}} \quad \text{and} \quad \mathrm{pe}^{**} = \frac{R^{**} - \hat{R}^*}{\sqrt{\widehat{Var}(R^{**})}}. \tag{3.11}$$

The predictive distributions of the outstanding claims R_i and R are then obtained by plotting

$$\tilde{R}_i^{**} = \hat{R}_i + \mathrm{pe}_i^{**} \sqrt{\widehat{Var}(R_i)} \quad \text{and} \quad \tilde{R}^{**} = \hat{R} + \mathrm{pe}^{**} \sqrt{\widehat{Var}(R)} \tag{3.12}$$

for each B.

The conditional independence of C_{ij} for all i and j given N_i (3.3) implies that

$$\begin{aligned} Var(R_i) &= E\left(Var\left(R_i|N_i\right)\right) + Var\left(E\left(R_i|N_i\right)\right) \\ &= E\left(\sum_{j\in\Delta_i} \phi N_i \left(r_j\lambda_k\right)^2\right) + Var\left(\sum_{j\in\Delta_i} N_i r_j \lambda_k\right) \\ &= \phi E\left(N_i\right) \sum_{j\in\Delta_i} \left(r_j\lambda_k\right)^2 + Var\left(N_i\right)\left(\sum_{j\in\Delta_i} r_j\lambda_k\right)^2 \\ &= \left(\phi \sum_{j\in\Delta_i} \left(r_j\lambda_k\right)^2 + \left(\sum_{j\in\Delta_i} r_j\lambda_k\right)^2\right)\left(\sum_{j\in\nabla_i\cup\Delta_i} n_{ij}\right) \end{aligned} \tag{3.13}$$

since

$$E(N_i) = Var(N_i) = \sum_{j\in\nabla_i\cup\Delta_i} n_{ij}\,. \tag{3.14}$$

By plugging in the estimates we find

$$\widehat{Var}\left(R_i\right) = \left(\hat{\phi} \sum_{j\in\Delta_i} \left(\hat{r}_j\hat{\lambda}_k\right)^2 + \left(\sum_{j\in\Delta_i} \hat{r}_j\hat{\lambda}_k\right)^2\right)\left(\sum_{j\in\nabla_i\cup\Delta_i} \hat{n}_{ij}\right) \tag{3.15}$$

and

$$\widehat{Var}\left(R\right) = \sum_i \left(\hat{\phi} \sum_{j\in\Delta_i} \left(\hat{r}_j\hat{\lambda}_k\right)^2 + \left(\sum_{j\in\Delta_i} \hat{r}_j\hat{\lambda}_k\right)^2\right)\left(\sum_{j\in\nabla_i\cup\Delta_i} \hat{n}_{ij}\right)\,. \tag{3.16}$$

Analogously, the variances appearing in (3.11) are

$$\widehat{Var}\left(R_i^{**}\right) = \left(\hat{\phi}^* \sum_{j\in\Delta_i} \left(\hat{r}_j^*\hat{\lambda}_k^*\right)^2 + \left(\sum_{j\in\Delta_i} \hat{r}_j^*\hat{\lambda}_k^*\right)^2\right)\left(\sum_{j\in\nabla_i\cup\Delta_i} \hat{n}_{ij}^*\right) \tag{3.17}$$

and

$$\widehat{Var}\left(R^{**}\right) = \sum_i \left(\hat{\phi}^* \sum_{j\in\Delta_i} \left(\hat{r}_j^*\hat{\lambda}_k^*\right)^2 + \left(\sum_{j\in\Delta_i} \hat{r}_j^*\hat{\lambda}_k^*\right)^2\right)\left(\sum_{j\in\nabla_i\cup\Delta_i} \hat{n}_{ij}^*\right) \tag{3.18}$$

where

$$\hat{\phi}^* = \frac{1}{|\nabla| - q} \sum_{\nabla} \hat{N}_i^* \frac{(C_{ij}^* - \hat{N}_i^*\,\hat{r}_j^*\,\hat{\lambda}_k^*)^2}{(\hat{N}_i^*\,\hat{r}_j^*\,\hat{\lambda}_k^*)^2}\,. \tag{3.19}$$

in accordance with (3.6).

It is remarked in Björkwall *et al.* (2008) that for many bootstrap procedures, resampling of standardized quantities often increases accuracy compared to using unstandardized quantities. Nevertheless, the unstandardized prediction errors

$$\mathrm{pe}_i^{**} = R_i^{**} - \hat{R}_i^* \qquad \text{and} \qquad \mathrm{pe}^{**} = R^{**} - \hat{R}^* \tag{3.20}$$

are useful, e.g. for the purpose of studying the estimation and the process errors, but also since they are always defined.

The predictive distributions of the outstanding claims R_i and R are then obtained by plotting the B quantities

$$\tilde{R}_i^{**} = \hat{R}_i + \mathrm{pe}_i^{**} \qquad \text{and} \qquad \tilde{R}^{**} = \hat{R} + \mathrm{pe}^{**}\,. \tag{3.21}$$

The parametric predictive bootstrap procedure is described in Figure 1 and according to Björkwall *et al.* (2008) we will refer to it as standardized or unstandardized depending on which prediction errors that are used.

3.2 Known claim counts

In Section 2 it was remarked that the separation model is based on the assumption that N_i is considered as known at the moment when the reserving is being done. This can often be a reasonable assumption since the numbers of claims are usually finalized quite early even for long-tailed business. In Section 3.1 N_i was a random variable; in order to get a view of how much uncertainty N_i contributes to the predictive distribution of the claims reserve we now consider the special case when N_i is treated as deterministic, in contrast to (3.1). Consequently, $\hat{N}_i \equiv N_i$ in all equations above.

Assumption (3.3) can still be used and $\hat{\phi}$ is estimated as in (3.6), but the sampling process changes. We do not have to generate pseudo-triangles of claim counts in the bootstrap

Stage 1 - Real world

Substage 1.1 - The triangle of claim counts ∇N

· Forecast the future expected values $\Delta\hat{n}$ and calculate the fitted values $\nabla\hat{n}$ by chain-ladder.
· Calculate the estimated ultimate claim count per origin year $\hat{N}_i$.

Substage 1.2 - The triangle of paid claims ∇C

· Use $\hat{N}_i$ from Substage 1.1 for the purpose of forecasting the future expected values $\Delta\hat{m}$ and calculating the fitted values $\nabla\hat{m}$ by the separation method.
· Calculate $\hat{\phi}$ for the sampling process.
· Calculate the outstanding claims $\hat{R}_i = \sum_{j\in\Delta_i} \hat{m}_{ij}$ and $\hat{R} = \sum_{\Delta} \hat{m}_{ij}$.

Stage 2 - Bootstrap world

Substage 2.1 - The estimated outstanding claims

Substage 2.1.1 - The pseudo-triangle of claim counts ∇N^*

· Sample from (3.7) for $i, j \in \nabla$ to obtain the pseudo-reality in ∇N^*.
· Forecast the future expected values $\Delta\hat{n}^*$ by chain-ladder.
· Calculate the estimated ultimate claim count per origin year $\hat{N}_i^*$.

Substage 2.1.2 - The pseudo-triangle of paid claims ∇C^*

· Sample from (3.7) for $i, j \in \Delta$ to obtain the pseudo-reality in ΔN^{**}.
· Calculate the ultimate claim count per origin year N_i^{**} using ∇N^* from Substage 2.1.1 and ΔN^{**}.
· Sample from (3.9) for $i, j \in \nabla$ to obtain the pseudo-reality in ∇C^* conditionally on ΔN_i^{**}.
· Use $\hat{N}_i^*$ from Substage 2.1.1. for the purpose of forecasting the future expected values $\Delta\hat{m}^*$ by the separation method.
· Calculate the estimated outstanding claims $\hat{R}_i^* = \sum_{j\in\Delta_i} \hat{m}_{ij}^*$ and $\hat{R}^* = \sum_{\Delta} \hat{m}_{ij}^*$.

Substage 2.2 - The true outstanding claims

· Sample from (3.9) for $i, j \in \Delta$ to obtain the pseudo-reality in ΔC^{**} conditionally on ΔN_i^{**}.
· Calculate the true outstanding claims $R_i^{**} = \sum_{j\in\Delta_i} C_{ij}^{**}$ and $R^{**} = \sum_{\Delta} C_{ij}^{**}$.
· Store either the standardized prediction errors in (3.11) or the unstandardized ones in (3.20).
· Return to the beginning of the bootstrap loop in Stage 2 and repeat B times.

Stage 3 - Analysis of the simulations

· Obtain the predictive distribution of R_i and R, the true outstanding claims in the real world, by plotting the B values in either (3.12) or (3.21).

Figure 1: *The procedure of the parametric predictive bootstrap for the separation method.*

world, i.e. ∇N^* and ΔN^{**}, since N_i is considered as known. Thus, we just draw C^*_{ij} from

$$C^*_{ij} \in \Gamma(\frac{N_i}{\hat{\phi}}, \hat{r}_j\, \hat{\lambda}_k\, \hat{\phi}) \tag{3.22}$$

B times for all $i, j \in \nabla$ yielding ∇C^*. The estimation process of the separation method is then repeated for each ∇C^* using N_i as the exposure in the bootstrap world as well. Finally, we sample once again B times from (3.22) for all $i, j \in \Delta$ to get ΔC^{**}.

The prediction errors and the predictive distributions are as earlier obtained by (3.11) and (3.12), respectively, but since $Var(N_i) = 0$, we obtain the estimators

$$\widehat{Var}(R_i) = \hat{\phi} N_i \sum_{\Delta_i} (\hat{r}_j \hat{\lambda}_k)^2 \tag{3.23}$$

and

$$\widehat{Var}(R) = \sum_i \hat{\phi} N_i \sum_{\Delta_i} (\hat{r}_j \hat{\lambda}_k)^2 \tag{3.24}$$

instead of (3.15) and (3.16).

Analogously, the estimators appearing in (3.11) are

$$\widehat{Var}(R^{**}_i) = \hat{\phi}^* N_i \sum_{\Delta_i} (\hat{r}^*_j \hat{\lambda}^*_k)^2 \tag{3.25}$$

and

$$\widehat{Var}(R^{**}) = \sum_i \hat{\phi}^* N_i \sum_{\Delta_i} (\hat{r}^*_j \hat{\lambda}^*_k)^2 \,, \tag{3.26}$$

where $\hat{\phi}^*$ is estimated by (3.19).

The unstandardized prediction errors in (3.20) can of course be used as well. The predictive distributions are then obtained by (3.21).

This simplified approach is summarized in Figure 2.

4 Numerical study

The purpose of the numerical study is to illustrate the parametric bootstrap procedure for the separation method and to compare it to the approach for the chain-ladder described in

Stage 1 - Real world

Substage 1.1 - The triangle of claim counts ∇N

· Forecast the future expected values $\Delta \hat{n}$ by chain-ladder.
· Calculate the estimated ultimate claim count per origin year $\hat{N}_i$.

Substage 1.2 - The triangle of paid claims ∇C

· Use $\hat{N}_i$ from Substage 1.1 for the purpose of forecasting the future expected values $\Delta \hat{m}$ and calculating the fitted values $\nabla \hat{m}$ by the separation method.
· Calculate $\hat{\phi}$ for the sampling process.
· Calculate the outstanding claims $\hat{R}_i = \sum_{j \in \Delta_i} \hat{m}_{ij}$ and $\hat{R} = \sum_{\Delta} \hat{m}_{ij}$.

Stage 2 - Bootstrap world

Substage 2.1 - The estimated outstanding claims

· Sample from (3.22) for $i, j \in \nabla$ to obtain the pseudo-reality in ∇C^*.
· Use $\hat{N}_i$ for the purpose of forecasting the future expected values $\Delta \hat{m}^*$ by the separation method.
· Calculate the estimated outstanding claims $\hat{R}_i^* = \sum_{j \in \Delta_i} \hat{m}_{ij}^*$ and $\hat{R}^* = \sum_{\Delta} \hat{m}_{ij}^*$.

Substage 2.2 - The true outstanding claims

· Sample from (3.22) for $i, j \in \Delta$ to obtain the pseudo-reality in ΔC^{**}.
· Calculate the true outstanding claims $R_i^{**} = \sum_{j \in \Delta_i} C_{ij}^{**}$ and $R^{**} = \sum_{\Delta} C_{ij}^{**}$.
· Store either the standardized prediction errors in (3.11) or the unstandardized ones in (3.20).
· Return to the beginning of the bootstrap loop in Stage 2 and repeat B times.

Stage 3 - Analysis of the simulations

· Obtain the predictive distribution of R_i and R, the true outstanding claims in the real world, by plotting the B values in either (3.12) or (3.21).

Figure 2: *The procedure of the simplified parametric predictive bootstrap for the separation method.*

Björkwall *et al.* (2008). From now on $B = 10\,000$ simulations are used for each prediction. The upper 95 percent limits are studied and the coefficients of variation, i.e. $\sqrt{Var(\tilde{R}_i^{**})}/\hat{R}_i$ and $\sqrt{Var(\tilde{R}^{**})}/\hat{R}$, are presented as well.

We use the well-known data from Taylor & Ashe (1983), who also provide observations of number of claims. The triangles of paid claims ∇C and claim counts ∇N are presented in Table 4.1 and Table 4.2, respectively.

4.1 The estimate of the claims reserve and the payment pattern

The assumption of the future inflation rate has great impact on the claims reserve estimated by the separation method. The future inflation rate can of course be modeled by more refined approaches, but this is beyond the scope of this paper and we just consider a constant or the mean rate observed so far. In Table 4.3 the estimators are shown under three different assumptions. The inflation rate $11,01\%$ corresponds to the mean inflation rate observed so far, while 5% and 15% are chosen just for comparison. The estimated claims reserves obtained by the chain ladder are presented as well.

Table 4.4 shows the expected cumulative payment proportions

$$\hat{c}_j = \frac{\sum_{l=0}^{j}\sum_{i=0}^{t}\hat{m}_{il}}{\sum_{l=0}^{t}\sum_{i=0}^{t}\hat{m}_{il}}\,. \tag{4.1}$$

Obviously, a higher future inflation rate tends to delay the payments.

	0	1	2	3	4	5	6	7	8	9
0	357 848	766 940	610 542	482 940	527 326	574 398	146 342	139 950	227 229	67 948
1	352 118	884 021	933 894	1 183 289	445 745	320 996	527 804	266 172	425 046	
2	290 507	1 001 799	926 219	1 016 654	750 816	146 923	495 992	280 405		
3	310 608	1 108 250	776 189	1 562 400	272 482	352 053	206 286			
4	443 160	693 190	991 983	769 488	504 851	470 639				
5	396 132	937 085	847 498	805 037	705 960					
6	440 832	847 631	1 131 398	1 063 269						
7	359 480	1 061 648	1 443 370							
8	376 686	986 608								
9	344 014									

Table 4.1: *Observations of paid claims ∇C from Taylor & Ashe (1983).*

	0	1	2	3	4	5	6	7	8	9
0	40	124	157	93	141	22	14	10	3	2
1	37	186	130	239	61	26	23	6	6	
2	35	158	243	153	48	26	14	5		
3	41	155	218	100	67	17	6			
4	30	187	166	120	55	13				
5	33	121	204	87	37					
6	32	115	146	103						
7	43	111	83							
8	17	92								
9	22									

Table 4.2: *Observations of claim counts ∇N from Taylor & Ashe (1983).*

4.2 Predictive bootstrap results for the chain-ladder

In order to compare the separation method to the chain-ladder we summarize the results of the parametric predictive bootstrap procedures described in Björkwall *et al.* (2008), where data is bootstrapped according to the plug-in-principle under the assumption of a gamma distribution; see the reference for details. Tables 4.5 - 4.6 show the results for the standardized as well as the unstandardized approach.

Year i	Future inflation rate 5.00%	Future inflation rate 11.01%	Future inflation rate 15.00%	Chain-ladder
1	84 339	89 163	92 371	94 634
2	473 893	506 151	527 909	469 511
3	720 846	794 132	845 099	709 638
4	1 144 208	1 288 308	1 391 323	984 889
5	1 497 489	1 722 883	1 888 356	1 419 459
6	2 095 131	2 448 039	2 713 372	2 177 641
7	2 793 640	3 269 931	3 634 088	3 920 301
8	3 636 785	4 314 184	4 841 171	4 278 972
9	4 990 729	6 043 441	6 879 216	4 625 811
Total	17 437 060	20 476 232	22 812 905	18 680 856

Table 4.3: *The estimated claims reserves under the chain-ladder, compared to the separation method with different inflation assumptions. The mean inflation rate observed so far is* $11,01\%$.

Development year j	Future inflation rate 5.00%	Future inflation rate 11.01%	Future inflation rate 15.00%	Chain-ladder
0	7.1	6.7	6.4	6.9
1	25.2	23.9	23.0	24.2
2	44.5	42.5	41.0	42.2
3	63.3	60.9	59.1	61.5
4	73.7	71.3	69.5	72.2
5	81.2	79.0	77.4	79.7
6	87.7	86.0	84.7	86.6
7	92.3	91.0	90.1	91.3
8	98.6	98.3	98.1	98.3
9	100.0	100.0	100.0	100.0

Table 4.4: *The expected cumulative payment proportion (in %) under the chain-ladder, compared to the separation method with different inflation assumptions. The mean inflation rate observed so far is* $11,01\%$.

Year i	Standardized Gamma	Unstandardized Gamma
1	219 178	168 756
2	861 781	756 634
3	1 169 041	1 062 783
4	1 519 540	1 409 034
5	2 127 947	1 975 222
6	3 358 037	3 038 732
7	6 253 164	5 562 133
8	7 386 412	6 284 020
9	9 247 043	7 148 120
Total	23 991 467	23 123 593

Table 4.5: *The 95 percentiles of the parametric predictive bootstrap procedures described in Björkwall* et al. *(2008) for the chain-ladder. We work under the assumption of a gamma distribution and the procedure is either standardized or unstandardized.*

4.3 The standardized predictive bootstrap for the separation method

The results for the procedure described in Section 3.1, when the standardized prediction errors are used, are presented in Table 4.7 for the three different assumptions of the future inflation rate. Two of these are mean inflation rates observed so far, either treated as a constant (11.01%) or as stochastic in the bootstrap world. According to the plug-in-principle the inflation rate should be treated as stochastic, i.e. recomputed from $\{\hat{\lambda}_k^*\}$ for each resample, but the former alternative is shown as well for comparison. Table 4.8 contains the coefficients of variation. Tables 4.7 - 4.8 also include the results obtained by the chain-ladder for comparison.

As we can see the results are strongly affected by the inflation assumption and the coefficients of variation are naturally higher when the mean inflation is treated as stochastic, in particular for the grand total. As expected the coefficients of variation of the latest origin year are lower for the separation method than for the chain-ladder, since the extreme sensitivity to outliers for the chain-ladder in the south corner of the upper triangle is removed for the separation method. Less expected is that the separation method has

Year i	Standardized Gamma	Unstandardized Gamma
1	65	50
2	41	38
3	32	31
4	28	27
5	26	25
6	27	25
7	29	27
8	35	32
9	47	38
Total	15	16

Table 4.6: *The coefficients of variation of the simulations (in %) of the parametric predictive bootstrap procedures described in Björkwall* et al. *(2008) for the chain-ladder. We work under the assumption of a gamma distribution and the procedure is either standardized or unstandardized.*

Year i	Inflation 5.00%	Inflation 11.01%	Inflation Mean	Inflation 15.00%	Chain Ladder Gamma
1	197 907	201 184	190 418	208 028	219 178
2	839 849	882 300	858 679	926 020	861 781
3	1 137 848	1 253 445	1 204 288	1 336 139	1 169 041
4	1 704 374	1 908 980	1 859 360	2 066 985	1 519 540
5	2 178 017	2 513 476	2 446 109	2 751 393	2 127 947
6	3 033 630	3 526 976	3 516 529	3 901 976	3 358 037
7	4 223 019	4 893 910	4 807 925	5 359 921	6 253 164
8	5 564 419	6 540 182	6 489 287	7 239 800	7 386 412
9	8 261 189	9 852 469	9 540 033	11 081 546	9 247 043
Total	23 412 570	27 442 696	27 659 095	30 692 578	23 991 467

Table 4.7: *The 95 percentiles of the standardized predictive bootstrap procedure under the chain-ladder, compared to the separation method with different inflation assumptions. Two of these are mean inflation rates observed so far, either treated as a constant (11.01 %) or as stochastic (Mean).*

Year i	Inflation 5.00%	Inflation 11.01%	Inflation Mean	Inflation 15.00%	Chain Ladder Gamma
1	63	61	57	60	65
2	38	38	37	38	41
3	30	29	30	30	32
4	26	25	27	25	28
5	24	24	27	24	26
6	24	23	28	23	27
7	26	26	31	25	29
8	28	27	32	26	35
9	33	32	37	31	47
Total	18	18	25	17	15

Table 4.8: *The coefficients of variation of the simulations (in %) of the standardized predictive bootstrap procedure under the chain-ladder, compared to the separation method with different inflation assumptions. Two of these are mean inflation rates observed so far, either treated as a constant (11.01 %) or as stochastic (Mean).*

lower coefficients of variation for years 1-3.

4.4 The unstandardized predictive bootstrap for the separation method

In order to study the estimation and the process error we also investigate the procedure described in Section 3.1 when the unstandardized prediction errors are used. The results are shown in Tables 4.9 - 4.10.

Year i	Inflation 5.00%	Inflation 11.01%	Inflation Mean	Inflation 15.00%	Chain Ladder Gamma
1	152 189	158 866	158 108	163 797	168 756
2	765 412	803 966	792 018	840 344	756 634
3	1 071 483	1 180 997	1 150 577	1 262 227	1 062 783
4	1 632 010	1 825 048	1 780 637	1 967 326	1 409 034
5	2 082 197	2 413 236	2 340 763	2 644 546	1 975 222
6	2 916 401	3 389 043	3 327 822	3 754 270	3 038 732
7	4 024 333	4 666 419	4 547 122	5 125 141	5 562 133
8	5 270 015	6 180 526	6 027 989	6 874 970	6 284 020
9	7 528 152	9 024 898	8 787 987	10 208 677	7 148 120
Total	22 281 683	26 091 962	26 145 893	29 117 165	23 123 593

Table 4.9: *The 95 percentiles of the unstandardized predictive bootstrap procedure under the chain-ladder, compared to the separation method with different inflation assumptions. Two of these are mean inflation rates observed so far, either treated as a constant (11.01 %) or as stochastic (Mean).*

As remarked in Björkwall *et al.* (2008) the percentiles of the unstandardized predictive bootstrap tend to be lower than for the standardized one. This was explained by the left skewness of the predictive distribution of the unstandardized bootstrap compared to the distribution obtained by the standardized bootstrap. According to Figure 3 this seems to hold for the separation method too. Figure 3 (c) - (d) show the predictive distributions of the total claims reserve under the assumption of a stochastic future inflation rate corresponding to the mean inflation rate observed so far. The predictive distribution obtained by the unstandardized bootstrap in (c) is skewed to the left compared to the one obtained by the standardized bootstrap in (d), which is slightly skewed to the right. This follows since the process component in Figure 3 (a) has smaller variability than the estimation component in Figure 3 (b), and the latter is skewed to the right. The left skewness is to a large extent removed for the standardized prediction errors (3.11), because of the denominator, but not for the unstandardized prediction errors (3.20).

Recomputing the future inflation rate from $\{\hat{\lambda}_k^*\}$ for each resample in the bootstrap world yields some rates which are unreasonably high. These rates affect the estimation component, which become more skewed to the right than for a constant future inflation rate. Consequently, the predictive distribution of the outstanding claims is more skewed to the left for the stochastic future inflation rate than for the constant. This explains why most of the percentiles in Tables 4.7 and 4.9 are lower for stochastic inflation.

4.5 Known claim counts

In Tables 4.11 - 4.12 we present the results of the simplified approach in Section 3.2 where we treat N_i as known. As expected the variability has decreased compared to the results in Tables 4.7 - 4.8, but the difference is notably small. This is consistent with the separation

Year i	Inflation 5.00%	Inflation 11.01%	Inflation Mean	Inflation 15.00%	Chain Ladder Gamma
1	49	48	50	48	50
2	36	36	38	35	38
3	29	29	33	29	31
4	25	25	32	25	27
5	24	24	34	24	25
6	24	23	36	23	25
7	26	26	39	25	27
8	27	26	43	26	32
9	33	32	52	31	38
Total	18	18	35	17	16

Table 4.10: *The coefficients of variation of the simulations (in %) of the unstandardized predictive bootstrap procedure under the chain-ladder, compared to the separation method with different inflation assumptions. Two of these are mean inflation rates observed so far, either treated as a constant (11.01 %) or as stochastic (Mean).*

method assumption that the numbers of claims usually are finalized early enough to be considered as known. This is interesting, since Table 4.2 reveals that the data here is actually an example when claim numbers are not finalized very fast. As expected, the difference is largest for the last origin year, i.e. where we predict the ultimate number of claims based on one single observation.

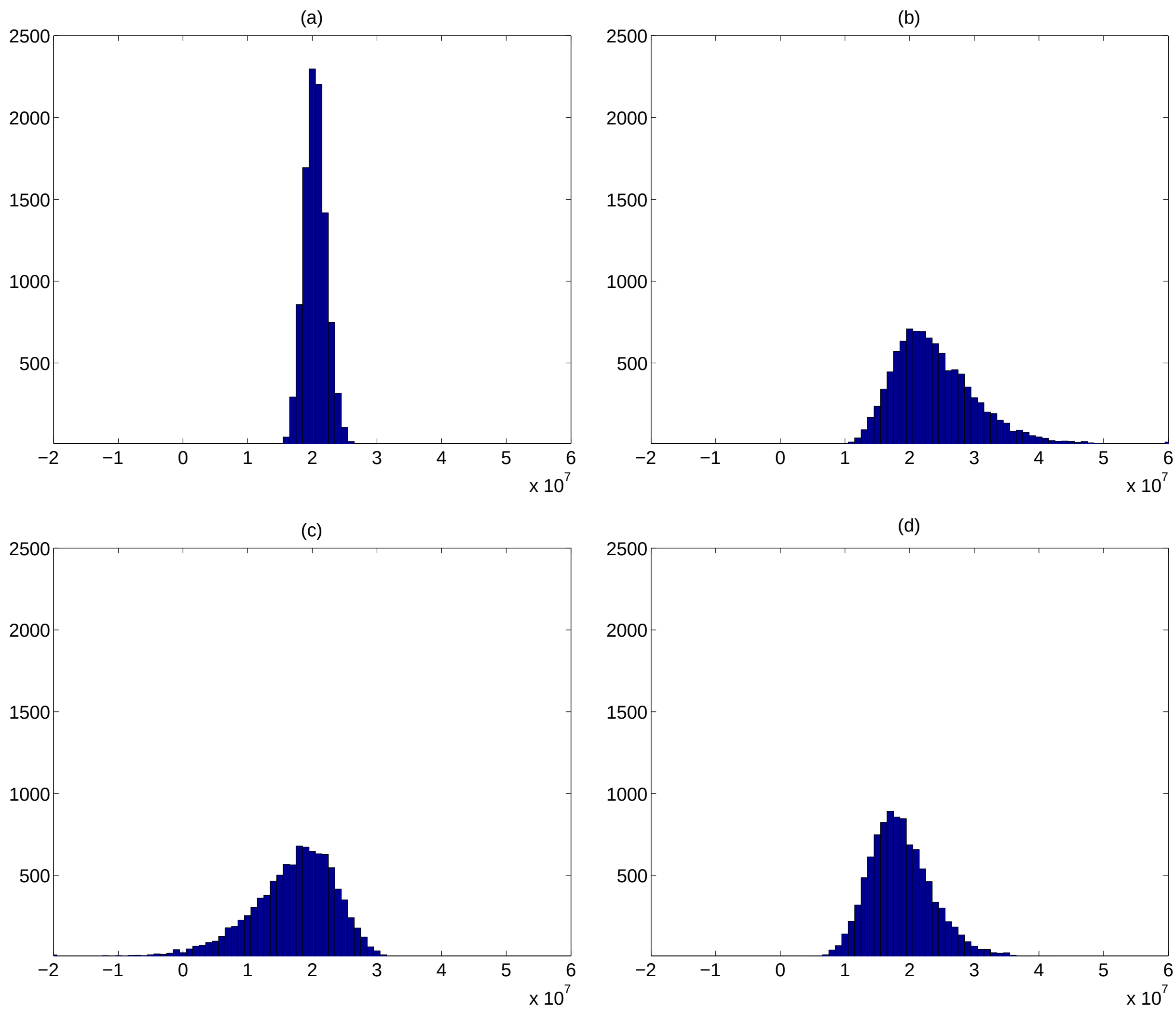

Figure 3: *Density charts of* R^{**} *(a),* $\hat{R}^{*}$ *(b) and* $\tilde{R}^{**}$ *for the unstandardized (c) and standardized (d) predictive bootstrap procedure under the assumption of a stochastic future inflation rate corresponding to the mean inflation rate observed so far.*

Year i	Inflation 5.00%	Inflation 11.01%	Inflation Mean	Inflation 15.00%	Chain Ladder Gamma
1	192 618	203 666	194 659	211 058	219 178
2	838 502	897 770	875 334	934 584	861 781
3	1 142 097	1 243 302	1 195 689	1 332 776	1 169 041
4	1 697 879	1 918 643	1 858 748	2 074 808	1 519 540
5	2 200 470	2 525 290	2 451 200	2 726 575	2 127 947
6	3 032 494	3 577 632	3 481 789	3 914 571	3 358 037
7	4 250 351	4 871 638	4 819 584	5 386 039	6 253 164
8	5 532 888	6 507 485	6 421 424	7 227 340	7 386 412
9	7 461 845	9 023 231	8 859 782	10 108 297	9 247 043
Total	23 398 840	27 417 470	27 783 761	30 304 007	23 991 467

Table 4.11: *The 95 percentiles of the simplified standardized predictive bootstrap procedure under the chain-ladder, compared to the separation method when N_i is considered as known. We work under different inflation assumptions. Two of these are mean inflation rates observed so far, either treated as a constant (11.01 %) or as stochastic (Mean).*

Year i	Inflation 5.00%	Inflation 11.01%	Inflation Mean	Inflation 15.00%	Chain Ladder Gamma
1	63	62	57	61	65
2	39	39	39	39	41
3	30	29	30	29	32
4	26	26	27	25	28
5	24	24	27	24	26
6	24	24	27	23	27
7	27	26	31	25	29
8	27	26	32	25	35
9	26	25	32	24	47
Total	17	17	24	17	15

Table 4.12: *The coefficients of variation of the simulations (in %) of the simplified standardized predictive bootstrap procedure under the chain-ladder, compared to the separation method when N_i is considered as known. We work under three different inflation assumptions. Two of these are mean inflation rates observed so far, either treated as a constant (11.01 %) or as stochastic (Mean).*

5 Conclusions

The separation method is a useful reserving technique for the purpose of modeling claims inflation, which contributes to the uncertainty of the claims reserve and therefore should be considered in risk management. This paper provides a parametric bootstrap procedure, which can be used to assess the uncertainty of the separation method. It is of course difficult to forecast the future inflation and in this paper simple assumptions have been used. We believe that the future inflation for real applications should be modeled by more refined approaches.

In one example we saw that whether we consider N_i as stochastic or known in the bootstrap procedure the results are still at the same level. Of course, the situation might be different in another example.

Furthermore, when we compare the percentiles obtained for the separation method with the ones for the chain-ladder in Tables 4.7 and 4.9 we can see that the result is more affected by the assumption of the future claims inflation rate than the choice between the chain ladder and the separation method. Since the separation method, under the assumption of a future inflation rate corresponding to the mean rate observed so far, indicates a higher risk than predicted by the chain-ladder the question of which method is preferable in a given situation immediately arises. Therefore, in a future paper, it would be interesting to compare the two methods in more situations than the one in Section 4 and in particular for long-tailed data.

The bootstrap approach for the separation method can also be used in a DFA context to simulate the reserve risk. However, as remarked by England & Verrall (2006), a DFA model usually includes an economic scenario generator (ESG), which simulates the future inflation, and it is important that the dependence between reserve risk and the inflation from the ESG is incorporated in the DFA model. Therefore, England & Verrall (2006) suggest that the data is adjusted to remove effects of the economic inflation before applying a reserving method, which use calendar year components to model superimposed claims inflation, is applied to forecast the future payments. Once the future payments has been simulated they are re-adjusted according to the inflation obtained from the ESG.

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