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Replacement of payoff function and hedging with a correlated asset in the case of a real power option

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Abstract

We study a special case of power derivative, a real option consisting of a strip of hourly power options, with spot prices S_t and the same strike price K. The payoff is a sum of payoffs, all of the type $\max(S_{T_i} - K, 0)$. As there is no forward market for specific hours, the fundamental question is to find a reasonable hedge using traded forward contracts, eg, on monthly deliveries. If we confine ourselves with stopping the dynamic hedging before delivery, the main result is a simple dynamic hedging strategy that reduces a significant part of the variance. The idea is to replace the payoff function with two parts that are analytically tractable and only depend on the traded asset. A benefit of the method is that the hedging strategy easily extends to more complex power derivatives and that only a few parameters need to be estimated. The simplified hedging strategy is compared with dynamic delta hedging strategies of Black76 type, using a correlated traded asset and local minimum variance hedging.

JEL classifications: C15; G13; G32; Q40

Keywords: Hedging; Power option; Black76; Swing option; Real option; Local minimum variance hedging

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1. Introduction

Energy companies are often faced with optionalities in their generation facilities as well as in retail contracts. Many of the energy companies now act on deregulated markets with a competitive pricing environment. Thus, hedging the inherent optionalities efficiently and practically is of great importance. The main purpose of the hedging is to reduce a great deal of the variance in the revenues of the company, but often some variance is allowed. Sometimes it is even the case that investors, rating institutes etc expect a certain fluctuation of the results when power prices move up and down. This leads to a situation where a company that is very efficient in the hedging might even be considered as odd in comparison with the rest of the market. Reasoning like above it is natural to look for hedging strategies that reduce a great part of the variance recognizing that it is not a stiff requirement to reduce all fluctuations. What we can hope for is that, when allowing some slack in the minimization of fluctuations, we can find simple strategies that are practically tractable. In this work we propose such a simplified hedging strategy for the case of a real power option, a swing option of a special type. The considered option is a strip of options on spot deliveries. The idea is to replace the original payoff with a new payoff which is much easier to work with. We should note here that hedging strategies have been derived for general swing options, see eg, Keppo [15] or Chapter 7 in Clewlow and Strickland [8]. The general case of a swing option is fairly complex, allowing the holder of the contract to specify the amount of energy that is bought each hour, at a fix price, in a certain period and within a set of restrictions. The set of restrictions is typically a minimum and a maximum amount of energy on each day and on the total. In our case the restrictions break down to the special case where the maximum of the total equals the sum of the maximum of the individual days and where the minimum constraint is dropped. The solution of the general case depends on complex decisions of stochastic nature, similar to pricing of American options. The path-dependency makes trees popular in the pricing and hedging as in Clewlow and Strickland [8]. Analytical tools typically relax some of the conditions and make assumptions on complete markets as in Keppo [15]. Analytical formulas for the simpler case of a strip of options, as we have, can be expressed in terms of a sum of individual Black-Scholes formulas. This is possible when the path-dependent structures are avoided or neglected. In Chapter 9 in the book by Eydeland and Wolyniec [11], there is an excellent introduction to these so-called spark-spread option formulas and hedging. Our analysis is comparable to this latter type of analysis, the important distinction being that we split the pricing formula in two analytically tractable parts, one Black-Scholes part and one 'burn analysis' part. We then make use of the assumption that hedging can be performed in terms of a correlated traded forward contract. We also show how the simplified hedging method can be used in more complex applications, which is a benefit of our work. Furthermore, we study, in some detail, a traditional hedging strategy of spark-spread type, using a correlated asset. We refer to Henderson and Hobson [13] that have developed such hedging strategies in the case of an option on a non-traded asset and hedging with a correlated traded asset. Original references for this type of hedging are Duffie and Richardson [9] and Schweizer [18]. General references treating pricing and hedging of swing options and power derivatives are Kluge [16] and Unger [19]. Several closely connected papers dealing with derivative pricing in power markets have also been written by Benth and co-authors [2], [3], [4].

We consider a power derivative market where forward contracts are traded continuously, each contract having a finite trading period. We adopt standard notation as in Audet et al [1] or Bjerksund et al [6]. Both references give a nice introduction to power derivatives. The smallest considered time period is hours, as we typically have in a power spot market. We work with continuous time when we set up the random processes but, whenever convenient, we refer to specific hours. Hopefully this is clear in each application. All cashflows related to the respective forward contracts are assumed to be paid/received when a maturity is reached, ie, there is no delay in payments. The forward prices $F(t, T)$ are defined as random variables on the probability space $(\Omega, \mathcal{F}_t, \mathbf{P})$ where the filtration $\mathcal{F}_t \subset \mathcal{F}$ is the filtration generated by the random variables and $\mathbf{P} : \mathcal{F} \to [0, 1]$ is a probability measure. $F(t, T)$ is the forward price, at time t, of the contract with a single delivery at time $T, t \leq T$. For the special case of the spot price we write $S_t = F(t, t)$. The forward price of contracts on multiple deliveries are denoted $F(t, \dot{T}_1, \dot{T}_2)$, $t \leq \dot{T}_1 \leq \dot{T}_2$, and defined by

$$F(t, \dot{T}_1, \dot{T}_2) = \sum_{T_i \in \mathcal{I}_{\dot{T}_1, \dot{T}_2}} w_t^i F(t, T_i), \quad w_t^i = \frac{e^{-r(T_i - t)}}{\sum_{T_j \in \mathcal{I}_{\dot{T}_1, \dot{T}_2}} e^{-r(T_j - t)}}, \tag{1}$$

where $\mathcal{I}_{\dot{T}_1,\dot{T}_2} = \left\{T_i : \dot{T}_1 \leq T_i \leq \dot{T}_2\right\}$, and r is a constant continuously compounded interest rate for a risk-free bank account. With the risk of being ambiguous, when we refer to a specific monthly delivery we write $F_t = F(t, \dot{T}_1, \dot{T}_2)$, thus dropping the indexation for the underlying time period. When speaking of hedging, we assume that dynamic hedging stops at time $\dot{T}$, $\dot{T} \leq \dot{T}_1 \leq \dot{T}_2$. Note that we have used a special dotted notation to single out the special fix times $\dot{T} \leq \dot{T}_1 \leq \dot{T}_2$. At all times, we assume that there is a price for all hours, T, in the forward curve specified by $F(t, T)$ for a finite horizon τ, $t \leq T \leq \tau$. This is certainly the case if we make sure that a forward curve, based on market data, is constructed each trading day, see, eg Fleten and Lemming [12]. A common way to construct prices for specific hours is to construct a forward curve with daily prices and then use weekly profile patterns to get hourly prices. The profile patterns are estimated based on historical observations of spot prices.

To draw relevant conclusions from the results in subsequent sections we need to give some basics on the price behavior. Later on we specify the stochastic behavior of the random variables explicitly but below we speak generally of price behavior. It is well known that spot prices have a spike behavior and seasonal components on daily, weekly as well as monthly basis, see Lucia and Schwartz [17]. Thus forward contracts with delivery during times of high consumption are, ceteris paribus, priced at higher levels than the corresponding yearly forward contracts introducing a seasonality effect between prices of forward contracts. The spike behavior, as well as the shape of the supply and demand curves of the spot market in general, possibly leads to a risk premium in the prices of the forward contracts. For a discussion on risk premia, see Benth et al [2]. We expect the risk premium to be positive, ie, the forward contracts are priced higher than their discounted expected payoffs. In this report we assume that the risk premium is small and therefore we set the premium to zero. We also expect that the volatility of a forward contract is increasing when the time to maturity decreases. The volatility of a forward contract is expected to be lower for a forward contract on a long period of delivery than for a contract on a short period of delivery. The correlation between two forward contracts is close to 1 when the time to maturity is long and decreasing when the time to maturity decreases, still expected to remain nonnegative. The correlation is also depending on the length of the delivery periods and on how close in time the underlying delivery periods are. The correlation is expected to be high if delivery periods are close in time. At least for forward contracts with a long time to maturity and with long underlying delivery periods, we expect that a lognormal distribution with zero drift, which can be assumed if the risk premium is small, give a good fit to observed market data. Our own analyses support the expectations on the price behavior as we describe above. Indeed many of the price models suggested in literature, eg, Eydeland and Wolyniec [11], Audet et al [1], Bjerksund et al [6], assume the characteristics above.

We move on to formulate the hedging problem we need to solve. Consider the contingent claim giving the following payoff at maturity

$$\Pi_{strip}(\dot{T}_2) = \sum_{T_i \in \mathcal{I}_{\dot{T}_1,\dot{T}_2}} e^{r(\dot{T}_2 - T_i)} \max\left(S_{T_i} - K, 0\right). \tag{2}$$

We assume that the hourly forward contracts $F(t,T)$ are not traded, except for $F(t,t) = S_t$ which are traded on a liquid spot market. However, based on forward curve calculations we observe prices $F(t,T)$. Forward contracts $F(t,\dot{T}_1,\dot{T}_2)$ are assumed to be traded on a liquid market where $\dot{T}_2 - \dot{T}_1$ make up underlying delivery periods of months. At time $\dot{T}$, $\dot{T} \leq \dot{T}_1 \leq \dot{T}_2$, the forward contracts $F(t,\dot{T}_1,\dot{T}_2)$ mature, ie, trading stops. Dynamic hedging is also assumed to stop at time $\dot{T}$. A static hedge, which is defined later, is maintained when dynamic hedging stops. The fundamental question is:

How can we best use the traded contracts to hedge the contingent claim on the correlated non-traded assets?

The rest of the document is organized as follows. Section 2 deals with the spark-spread setting and local minimum variance hedging which is very important as a comparison to our simplified hedging strategy. Section 3 gives the main results, the simplified hedging strategy. Then, for explicit stochastic processes, we perform simulations in Section 4. Especially, we are interested in the efficiency of the hedges. Section 5 gives extensions to more complex contingent claims and Section 6 concludes the results of the report.

2. Hedging strategy based on a strip of hourly Black76 options

Since there is no hedging strategy in terms of traded assets that can reduce all risks, we need to specify extra conditions on the optimal strategy. Here, we choose to minimize the variance locally. We derive such a minimum-variance method of hedging based on a couple of heuristic arguments. In Section 4 we evaluate the efficiency of the hedge and compare the hedge with the simplified hedge. Eydeland and Wolyniec [11], Pages 454-459, deal generally with hedging under global and local minimum variance settings. Only basic examples are considered but the general theory is outlined and discussed. When considering global minimum variance hedging, the problem is also closely connected to stochastic control theory, see Duffie [10], Pages 191-218, for an introduction. Henderson and Hobson [13], [14], develop hedging strategies for options on a single non-traded underlying in terms of hedging with a single correlated traded asset. The hedging strategy is derived using stochastic control theory and specific utility functions.

Let $C(s, T, K, F(s, T))$ be the price of a European style option with underlying hourly forward price $F(s, T)$, maturity T and strike price K. We are interested in the optimal fraction to invest in the traded asset $F_s = F(s, \dot{T}_1, \dot{T}_2)$. Below we define the meaning of optimal.

Definition 1. Given an $\mathcal{F}_s$-adapted control variable $u(s, \omega)$, we define the value process V_s^u by

$$dV_s^u = -u(s,\omega)dF_s + \sum_{T_i \in \mathcal{I}_{\dot{T}_1,\dot{T}_2}} dC(s, T_i, K, F(s, T_i)), \quad t \le s \le \dot{T}, \tag{3}$$

$$V_t = 0.$$

Note that the value process gives the development of a portfolio consisting of the underlying contingent claim as well as a hedging position.

Definition 2. A *local minimum variance* hedge is obtained as the solution to

$$\min_u Var\left(dV_s^u\right), \quad t \le s \le \dot{T}.$$

Note that the local minimum variance problem is defined in terms of finding the optimal hedge at all times of rebalancing of the portfolio. A *global minimum variance* problem is defined as $\min_{u(s,\cdot)} Var\left(V_{\dot{T}}^u\right)$, ie, a minimization of the total variance. For a discussion on the equivalence of these problems and further references, we refer to Eydeland and Wolyniec [11], Pages 458–459. Other utility functions than the variance could also be envoked and possibly lead to slightly different optimal strategies. However, as our aim is to get a comparison to the simplified hedging strategy given in Section 3, we are satisfied with the local minimum variance strategy.

Write $\Delta_s^i = \frac{\partial C(s,T_i,K,F(s,T_i))}{\partial F(s,T_i)}$. Assume the existence of a traded contract F_s with underlying delivery period $\mathcal{I}_{\dot{T}_1,\dot{T}_2}$. Using only first order terms, Expression (1) and Ito's formula, simplifying Equation (3) we get the approximate expression

$$dV_s^u = -u(s,\omega) \sum_{T_i \in \mathcal{I}_{\dot{T}_1,\dot{T}_2}} w_s^i dF(s,T_i) + \sum_{T_i \in \mathcal{I}_{\dot{T}_1,\dot{T}_2}} \Delta_s^i dF(s,T_i), \quad t \le s \le \dot{T}, \tag{4}$$

$$V_t = 0.$$

Note that Equation (4) is exact if all involved options are exactly replicable, ie, if the market is complete. We have assumed that there are traded hourly forward contracts, which is not the case and thus the market is not complete. However, we are of course free to make this assumption heuristically. Now, consider the following local minimum variance problem

$$\min_u Var\left(dV_s^u\right) = \min_u Var\left(\sum_{T_i \in \mathcal{I}_{\dot{T}_1,\dot{T}_2}} \left(-u(s,\omega)w_s^i + \Delta_s^i\right) dF(s,T_i)\right), \quad t \le s \le \dot{T}.$$

Based on the heuristic arguments above, differentiating and setting the expression equal to zero, we get the following proposition.

Proposition 2.1. *For the special case with a swing option of the type given by Equation (2) and hedging with the traded asset F_s, we have the locally optimal control variable*

$$u^*(s,\omega) = \frac{\sum_{i,j} w_s^i \Delta_s^j Cov_s^{i,j}}{\sum_{i,j} w_s^i w_s^j Cov_s^{i,j}},$$

where, conditioning on the information at time s,

$$Cov_s^{i,j} = Cov\left(dF(s,T_i), dF(s,T_j)\right).$$

We note that the proposed hedging strategy is self-financing and is well behaved. Exact conditions on well behaved trading strategies are given in, eg, Duffie [10]. We want to exclude strategies that are of the type 'double or nothing' which could result in infinite losses. Furthermore, we note that we are required to calculate the covariances for all involved quantities $dF(s, T_i)$, at all times s, which depend on the distributional properties that we assume for the forward curve, ie, for $F(s, T)$. In Section 4 we perform these calculations for a specific setup.

3. Simplified hedging strategy – Replacement of payoff function

The idea of the simplified hedging strategy is straightforward. Recall that dynamic hedging is assumed to stop at time $\dot{T}$, ie, before delivery. Let $N_{\mathcal{I}_{\dot{T}_1,\dot{T}_2}}$ denote # {hours in $\mathcal{I}_{\dot{T}_1,\dot{T}_2}$}. In the following, we multiply or divide with this number to get expressions in terms of the energy (MWh) or the power (MW). We make the following definition.

Definition 3. Define $\epsilon(F_{\dot{T}}, K)$ by

$$\epsilon(F_{\dot{T}}, K) = \frac{e^{-r(\dot{T}_2-\dot{T})}\Pi_{strip}(\dot{T}_2)}{N_{\mathcal{I}_{\dot{T}_1,\dot{T}_2}}} - \max\left(F_{\dot{T}} - K, 0\right).$$

Then, considering discounting, we can replace the original claim $e^{-r(\dot{T}_2-\dot{T})}\Pi_{strip}(\dot{T}_2)$ with the following equivalent claim:

$$\Pi_{simple}(\dot{T}) = N_{\mathcal{I}_{\dot{T}_1,\dot{T}_2}}\left(\max\left(F_{\dot{T}} - K, 0\right) + \epsilon(F_{\dot{T}}, K)\right). \tag{5}$$

We note that, at time $\dot{T}$, $\epsilon(F_{\dot{T}}, K)$ is certainly a random element.

Remark 1. A key is that we are able to estimate $\epsilon(F_{\dot{T}}, K)$ by observing historical payoff differences of the type $\left[\frac{e^{-r(\dot{T}_2-\dot{T})}\tilde{\Pi}_{strip}(\dot{T}_2)}{N_{\mathcal{I}_{\dot{T}_1,\dot{T}_2}}} - \max\left(\tilde{F}_{\dot{T}} - K, 0\right)\right]$, where $\tilde{}$ refers to observed quantities. Note that, in practice, when estimating $\epsilon(F_{\dot{T}}, K)$ we must give special consideration to seasonality and trends. We must find a method to filter out these effects.

We need to specify a class of approximating functions for the random elements $\epsilon(F_{\dot{T}}, K)$. We look for random elements of the form

$$\epsilon(F_{\dot{T}}, K) \in \mathcal{L}\left(\mu(F_{\dot{T}}, K), \sigma^2(F_{\dot{T}}, K)\right),$$

where $\mathcal{L}$ is the probability law, $\mu(F_{\dot{T}}, K)$ is the expected value function and $\sigma^2(F_{\dot{T}}, K)$ is the variance function. Our aim is to derive hedging strategies for the new claim. By construction, the new claim only depends on $F_{\dot{T}}$ and K, which makes it possible to derive hedging strategies directly in terms of F_t. Let E_t^* be the expected value with respect to the risk neutral martingale measure and conditional on the information at time t and let E_t be the expected value with respect to the real probability measure. We have the following proposition.

Proposition 3.1. *The contingent claim* $\Pi_{simple}(\dot{T})$ *can be priced by the following pricing formula*

$$\begin{aligned} C_{simple}(t) &= e^{-r(\dot{T}-t)}\mathbf{E}_t^*\left(\Pi_{simple}(\dot{T})\right) \\ &= N_{\mathcal{I}_{\dot{T}_1,\dot{T}_2}} e^{-r(\dot{T}-t)}\left(\mathbf{E}_t^*\left(\max\left(F_{\dot{T}}-K,0\right)\right)+\mathbf{E}_t^*\left(\epsilon(F_{\dot{T}},K)\right)\right) \\ &= N_{\mathcal{I}_{\dot{T}_1,\dot{T}_2}} e^{-r(\dot{T}-t)}\left(\mathbf{E}_t\left(\max\left(F_{\dot{T}}-K,0\right)\right)+\mathbf{E}_t\left(\epsilon(F_{\dot{T}},K)\right)\right)+\lambda, \end{aligned}$$

where λ *can be considered as the price that is paid to avoid risk.*

The pricing formula splits into two parts.

- *Black-Scholes part:* The first expected value above is easy. Assuming lognormality for $F_{\dot{T}}$ we have the ordinary Black76 formula.
- *Burn analysis part:* The second expected value corresponds to actual outcomes. Like the direct analysis of outcomes which is sometimes used in pricing of weather derivatives we may call this part 'burn analysis'.

Regarding the second expected value above, we are primarily interested in expected value functions $\mu(F_{\dot{T}}, K)$ which gives the possibility to calculate the expected value analytically. One way to get analytically tractable expressions for the expected value function is to estimate a suitable set of linear functions. This leads to closed-form solutions of Black-Scholes type, see Proposition 3.2.

It is straightforward to derive the hedging strategy. We calculate the hedging delta by

$$\Delta_s = \frac{\partial C_{simple}(s)}{\partial F_s}, \quad t \leq s \leq \dot{T}.$$

In Section 4 we perform explicit calculations. We estimate the expected value function and calculate the deltas.

3.1. Burn analysis part – Estimation of expected value function and pricing

Our aim is to estimate the expected value function of $\epsilon(F_{\dot{T}}, K)$, ie, $\mu(F_{\dot{T}}, K)$. From Remark 1, we recall that the estimation can be performed by observing differences of the type

$$\tilde{\epsilon}(F_{\dot{T}}, K) = \left[\frac{e^{-r(\dot{T}_2 - \dot{T})} \tilde{\Pi}_{strip}(\dot{T}_2)}{N_{\mathcal{I}_{\dot{T}_1, \dot{T}_2}}} - \max\left(\tilde{F}_{\dot{T}} - K, 0 \right) \right].$$

It turns out that the variance of the expression is considerably reduced, without any impact on the expected value, if we add the martingale increment

$$\Delta_{\dot{T}}^{BS} (e^{-r(\dot{T}_2 - \dot{T})} F_{\dot{T}_2} - F_{\dot{T}}),$$

where $F_{\dot{T}_2}$ is defined as the settlement price

$$F_{\dot{T}_2} = \frac{\sum_{T_i \in \mathcal{I}_{\dot{T}_1, \dot{T}_2}} S_{T_i} e^{r(\dot{T}_2 - T_i)}}{N_{\mathcal{I}_{\dot{T}_1, \dot{T}_2}}}$$

of the forward and

$$\Delta_{\dot{T}}^{BS} = \begin{cases} 0, & F_{\dot{T}} < K \\ -1, & F_{\dot{T}} \geq K \end{cases}. \tag{6}$$

The variance-reduction term is a natural candidate for a static hedge in delivery and thus it is not merely a theoretical construction. We have

$$\tilde{\epsilon}'(F_{\dot{T}}, K) = \left[\frac{e^{-r(\dot{T}_2 - \dot{T})} \tilde{\Pi}_{strip}(\dot{T}_2)}{N_{\mathcal{I}_{\dot{T}_1, \dot{T}_2}}} - \max\left(\tilde{F}_{\dot{T}} - K, 0 \right) \right] + \Delta_{\dot{T}}^{BS} (e^{-r(\dot{T}_2 - \dot{T})} \tilde{F}_{\dot{T}_2} - \tilde{F}_{\dot{T}}).$$

We can easily prove that $\tilde{\epsilon}'(F_{\dot{T}}, K) \geq 0$, which is nice when we estimate the expected value function. We make the observation that the expected value function $\mu(F_{\dot{T}}, K)$ declines to zero when $|F_{\dot{T}} - K|$ is large since all options then are 'in the money' or 'out of the money' respectively.

Let $x_j = F_{\dot{T}}(\omega_j)$ and $y_j = \epsilon'(F_{\dot{T}}, K, \omega_j)$, where we introduce ω_j to describe different observations. Denote the observed differences by (x_j, y_j), $1 \leq j \leq N$, where $N = \#$ observations. We estimate the simplest possible function, still being sufficiently rich. A set of n linear functions

$$y_{\text{est}}(x) = b_i + \left(\frac{b_{i+1} - b_i}{a_{i+1} - a_i} \right) (x - a_i) = b_i + k_i(x - a_i), \quad a_i \leq x \leq a_{i+1}, \quad 0 \leq i \leq n-1, \tag{7}$$

tied together in the $n+1$ points (a_i, b_i) are estimated. Let $\{a_i : 0 \le i \le n\}$, $a_0 = 0 < a_1 < \ldots < a_{n-1} < a_n = \infty$, be given. A natural estimate of an ordinate b_i is simply the average of the values in a suitable interval covering a_i. One way to describe such a method is to solve the following minimization problem for each ordinate b_i:

$$\min_{b_i} \sum_{j=1}^{N} I_{\{x_j \in (a_{i-1}, a_{i+1})\}} \left(b_i - y_j\right)^2 .$$

I is the indicator function. We assume $b_0 = 0, b_n = 0$, in order to make the expected value function meet the observed criteria when the options are deep 'in the money' or far 'out of the money'. We must also make sure that the shape of the resulting functions is 'monotone towards the ends'. As the variances around the expected values are dependent on the level of the respective expected value (higher expected value gives higher variance), we have a problem with heteroscedasticity. The observations should then be weighted accordingly in the estimation. However, it is not easy to do this since we do not have any good guess about the involved distributions in this context. Indeed, the estimates of the expected values are rough in all practical circumstances which lead to problems if we want to make statements about the variances around the expected values. To come around the problem, we should keep the heteroscedasticity in mind when we decide the number of linear functions to estimate. We have to compromise between the degree of heteroscedasticity and the degree of variance of the estimate.

Assuming that we have estimated the expected value function by the set of linear functions above, we are able to prove the following proposition.

Proposition 3.2. *Having estimated the linear functions given by Equation (7), and assuming $\lambda = 0$, we can calculate the expectations in Proposition 3.1. We have*

$$\begin{aligned} C_{simple}(t) &= e^{-r(\dot{T}-t)} \mathbf{E}_t^* \left(\Pi_{simple}(\dot{T}) \right) \\ &= N_{\mathcal{I}_{\dot{T}_1, \dot{T}_2}} e^{-r(\dot{T}-t)} \left(\mathbf{E}_t \left(\max\left(F_{\dot{T}} - K, 0 \right) \right) + \mathbf{E}_t \left(\epsilon(F_{\dot{T}}, K) \right) \right), \end{aligned}$$

where the first part is given by the Black76 formula, see Equation (9) below and the second part is given by

$$\begin{aligned} \mathbf{E}_t \left(\epsilon(F_{\dot{T}}, K) \right) &= F_t \sum_{i=0}^{n-1} k_i \left[\Phi\left(\frac{\ln \frac{F_t}{a_i} + \sigma^2(\dot{T}-t)/2}{\sigma\sqrt{\dot{T}-t}} \right) - \Phi\left(\frac{\ln \frac{F_t}{a_{i+1}} + \sigma^2(\dot{T}-t)/2}{\sigma\sqrt{\dot{T}-t}} \right) \right] \\ &+ \sum_{i=0}^{n-1} (b_i - k_i a_i) \left[\Phi\left(\frac{\ln \frac{a_{i+1}}{F_t} + \sigma^2(\dot{T}-t)/2}{\sigma\sqrt{\dot{T}-t}} \right) - \Phi\left(\frac{\ln \frac{a_i}{F_t} + \sigma^2(\dot{T}-t)/2}{\sigma\sqrt{\dot{T}-t}} \right) \right]. \end{aligned}$$

Φ is the standard normal cumulative distribution function.

Proof. See appendix.

Remark 2. In the appendix, we derive the formula by a straightforward calculation. It is possible to derive the formula in another way. We can set up a replicating portfolio consisting of long and short positions in European style options with different strikes. The number of units to buy and the strikes are easily realized by inspection of a plot of the payouts.

The practical implication of this remark is that we are able, in absence of transaction costs, to exactly replicate the expected payoff that we have at time $\dot{T}$ of the original contingent claim. This means that we are able to reduce all variance except the variance in delivery. However, using the static hedge in Equation (6) we are also able to reduce a great deal of the remaining variance in delivery.

4. Simulations

4.1. Description of the simulation model

In this section, we set up a specific stochastic model for the forward curve and then derive the necessary quantities in this setting. The forward price model should be sufficiently realistic for us being able to draw relevant conclusions, but it is not necessary to have a perfect model. We are interested in a setup in which we can analyse the two different hedging strategies, and particularly we are interested in the hedge efficiency. We recall that the locally optimal strategy is given by $u^*(s, \cdot)$ and that the simplified strategy is given by Δ_s. An analytically tractable and suitable three-factor model for the forward curve is given in Bjerksund et al [6] . Indeed, a principal components analysis reveals that 3 factors explain about 90 % of the variance. Parallel shift, twist and curvature are the dominant factors. Furthermore, in practice the magnitude of the risk premium is small. See the arguments in Eydeland and Wolyniec [11], Page 154. Hence we can, without loss of generality, use the present model to evaluate our results. The following price behavior of the forward curve is assumed under the risk neutral martingale measure:

$$\frac{dF(s,T)}{F(s,T)} = \frac{a}{T-s+b}dW_1^*(s) + \left(\frac{2ac}{T-s+b}\right)^{\frac{1}{2}} dW_2^*(s) + cdW_3^*(s), \quad t \leq s \leq \dot{T}_2. \tag{8}$$

We restrict a, b, and c to be positive constants and $dW_1^*(t)$, $dW_2^*(t)$, and $dW_3^*(t)$ are increments of three uncorrelated standard Brownian motions.

Recall that the forward contracts $F(t,T)$ are not traded but that we are able to observe the prices through forward curve calculations. Therefore we assume, heuristically, that we can calculate the option prices of the hourly options by Black76, see Black [7]:

$$C(t,T_i,K,F(t,T_i)) = e^{-r(T_i-t)}\left[F(t,T_i)\Phi(d_1) - K\Phi(d_2)\right], \tag{9}$$

where the interpretion of the time-dependent volatility is given below and

$$d_1 = \frac{\ln\left(\frac{F(t,T_i)}{K}\right) + \frac{\sigma^2(T_i-t)}{2}}{\sigma\sqrt{T_i-t}}, \quad d_2 = d_1 - \sigma\sqrt{T_i-t}.$$

The Black76 deltas are

$$\Delta_t^i = e^{-r(T_i-t)}\Phi(d_1).$$

The volatilities that are plugged into the Black76 model, see Bjerksund et al [6], shall be interpreted as the quantities

$$(T-t)\overline{\sigma}(t,T)^2 = \int_t^T \sigma^2(s,T)ds = \left[\frac{a^2}{T-s+b} - 2ac\ln(T-s+b) + c^2 s\right]_{s=t}^{s=T}. \tag{10}$$

The Black76 formula is valid for futures. When dealing with forward contracts we need to adjust the formula. The type of adjustment depends on how the forward is settled and when the cash settlements are received/paid. The volatility parameter for the forward contract, $F_t = F(t,\dot{T}_1,\dot{T}_2)$ must also be calculated. It turns out that a bit of calculation is required to derive an analytical expression for this volatility. We use an approximation derived by Bjerksund et al [6]. Here we simply recognize that we are able to derive the volatility parameter and that we denote this parameter with $\overline{\sigma}_{t,\dot{T}}^{\dot{T}_1,\dot{T}_2}$. The approximation formula is given in appendix. In practice we never have to estimate the volatility parameter in this way. Instead, we use the observed implied volatilities from traded European options, which is a nice practical feature.

By independency between the increments $dW_1^*(t)$, $dW_2^*(t)$, and $dW_3^*(t)$, and using $Cov(dW_k^*(t), dW_k^*(t)) = dt, k = 1, 2, 3$, we get

$$\begin{aligned} Cov\left(\frac{dF(t,T_i)}{F(t,T_i)}, \frac{dF(t,T_j)}{F(t,T_j)}\right) &= \Big[\frac{a^2}{(T_i - t + b)(T_j - t + b)} \\ &+ \frac{2ac}{\sqrt{T_i - t + b}\sqrt{T_j - t + b}} + c^2\Big] dt. \end{aligned}$$

Hence, conditioning on the information at time t, we have

$$Cov_t^{i,j} = Cov\left(dF(t,T_i), dF(t,T_j)\right) = Cov\left(\frac{dF(t,T_i)}{F(t,T_i)}, \frac{dF(t,T_j)}{F(t,T_j)}\right) F(t,T_i)F(t,T_j). \tag{11}$$

We are now able to calculate all required quantities in the simulation. We have expressions for the deltas, the covariances and for the forward curve process. We are also able to simulate outcomes of the payoffs and thereby we are able to estimate a proper function $\mu(F_{\dot{T}}, K)$ in this setting. Both trading strategies are therefore possible to analyse. We use the following definition of hedge efficiency to evaluate different hedging strategies.

Definition 4. Neglecting proper discounting, hedge efficiency is defined as

$$H = 1 - \sqrt{\frac{\text{Var (with hedge)}}{\text{Var (without hedge)}}}.$$

Sometimes the hedge efficiency is defined in terms of reduction of variance instead of reduction of volatility. We believe that the definition in terms of reduction of volatility is easier to interpret. Note that $H = 100$ % is a perfect hedge with a complete reduction of volatility. In our case however we always get a result less than 100 % since we stop dynamic hedging at time $\dot{T}$, $\dot{T} \leq \dot{T}_1 \leq \dot{T}_2$ and since the static hedge in delivery is not perfect. Eydeland and Wolyniec [11] experience a hedge efficiency, in terms of reduction of cash-flow volatility, of 50-60 % when the dynamic hedging stops before the delivery period. A hedge efficiency of around 80 % is experienced if so-called Balance of the Month contracts are utilized in the hedging. Given our theoretical model, we expect a fairly high hedge efficiency. A hedge efficiency above 50 % indicates that the strategies perform well.

4.2. Results of simulations – Hedge efficiency

A pre-simulation is performed to estimate the set of linear functions in the expected value function. Then, we perform the following simulation

1. Using the model given by Equation (8), simulate a sufficient number of price paths of the entire forward curve.

2. For each price path, calculate $u^*(s,\cdot)$ and $\Delta_s = \frac{\partial C_{simple}(s)}{\partial F_s}$.

3. For each price path, apply the static hedge given by Equation (6).

4. For each price path, sum up the respective results of the dynamic hedge, the static hedge and the strip of payoffs.

5. Calculate the respective hedge efficiencies.

Our simulation is based on the following setup:

- 31 daily forward prices are considered (we do not dig deeper into hourly prices), ie, a delivery period of one month. A monthly forward contract based on the same underlying period is used in the hedging. All prices are assumed to start at 30 EUR/MWh.
- The strike price is $K = 30$ EUR/MWh for all involved options.
- We assume $\dot{T}_2 = 1$ year, $t = 0$, $\dot{T} = \dot{T}_1$ and $\dot{T}_2 - \dot{T}_1 = 31/365$.
- We set $a = 9/80$, $b = 1/8$ and $c = 1/10$ in the model given by Equation (8). This implies a long-term volatility of 10 % and a short-term volatility of about 100 %.

Figure 1 illustrates a simulated plot of the payoff of the contingent claim of the strip of daily spot contracts. The payoff of the corresponding European option on the monthly forward is plotted in the same figure. We have applied the static hedge given by Equation (6). We see that the two payoffs coincide when $|F_{\dot{T}} - K|$ is large, as expected. The residual between the two payoffs, ie, $\epsilon(F_{\dot{T}}, K)$ is also plotted. The estimated set of linear functions is plotted together with $\epsilon(F_{\dot{T}}, K)$.

A simulated price path and the corresponding hedging strategies are displayed in Figure 2. Two forward prices are displayed. The deliveries are 30 days apart. We note that one of the forward prices are constant after the contract has been delivered. In

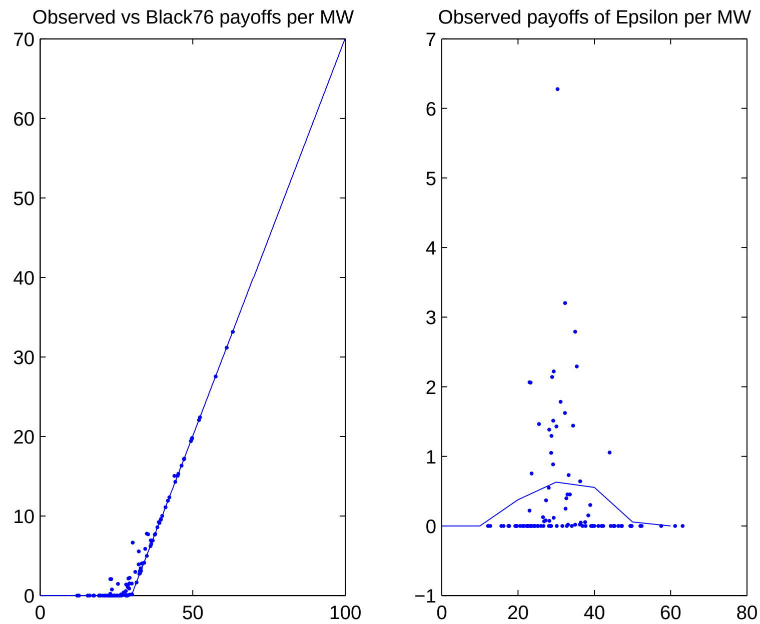

FIGURE 1: Simulated outcome of payoff and Epsilon element. Estimated set of linear functions are also displayed.

particular, we see that there is an increasing volatility when maturity approaches, as expected. We also see that the forward contracts are highly correlated, but not perfectly correlated. The correlation decreases when maturity approaches, as expected. We see that the hedges do not fully coincide with each other, but that they behave similarly.

A particular simulation of 100 paths resulted in the following hedge efficiencies:

Hedging strategy	Hedge efficiency
$u^*(s,\cdot)$	82 %
Δ_s	86 %

TABLE 1: Results of simulations – Hedge efficiencies

New simulations lead to similar results which indicate a stability of the results. It is surprisingly high hedge efficiencies, in both cases. In a real-world situation we would expect lower hedge efficiencies due to a number of reasons. The volatilities that are

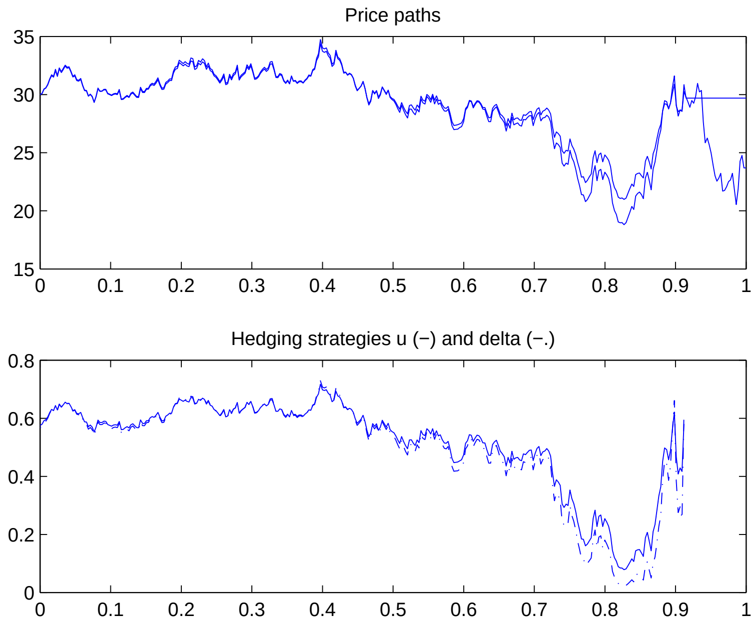

FIGURE 2: Simulated price paths and the resulting hedging stratgies.

fed to the hedging delta calculations would obviously not coincide perfectly with the observed volatilities. Forward prices are probably less correlated than we have in this specific model setup. Moreover, parameters are more difficult to estimate in a real-world situation. We have also neglected some of the characteristics of forward prices such as local trends. Anyhow, the simulated results give a very strong indication that both hedging strategies should work well which is the aim of this report. We note that it is not unreasonable that the simplified hedge performs better than the locally optimal. Indeed, different definitions of optimal stratgies may lead to different outcomes. In particular, the analytically attractive simplified hedging strategy seems to be a very good choice under all practical circumstances.

5. Extensions to more complex contingent claims

The strip of hourly payoffs with a fix strike price, dealt with in this report, is a contract that has been provided to customers in the Swedish retail market. Moreover, it is a good approximation to the case of a power plant where the price on the fuel is assumed to be stable and when there are no ramping effects, start-up costs etc. Even in many cases with restrictions of the type mentioned above, we are still able to use the proposed simplified hedging strategy. We give three examples of extensions here.

Example: Cross border contract. First, consider a cross border contract, ie, an inter-connector between two different price areas. The holder of this contract optimizes the direction and the amount of energy that flow in the inter-connector each hour based on the expectation of the price difference, ie, it is a typical example of a real option. Here we disregard operational characteristics such as planned or unplanned outage, transmission fees etc. For each hour, we are dealing with one spread option in each direction. In this case, we must extend our sample space to include random variables for forward and spot prices in both price areas, say $F_t^1, F_t^2, S_t^1, S_t^2$. We impose a dependency between the prices in the different price areas. We can replicate the previous ideas in this report in the new setting. We deal with an option payoff of the type

$$\Pi_{strip}(\dot{T}_2) = \sum_{T_i \in \mathcal{I}_{\dot{T}_1, \dot{T}_2}} e^{r(\dot{T}_2 - T_i)} \left(\max\left(S_{T_i}^1 - S_{T_i}^2, 0\right) + \max\left(S_{T_i}^2 - S_{T_i}^1, 0\right)\right),$$

and we replace the payoff above with the simplified payoff

$$\Pi_{simple}(\dot{T}) = \left(\max\left(F_{\dot{T}}^1 - F_{\dot{T}}^2, 0\right) + \max\left(F_{\dot{T}}^2 - F_{\dot{T}}^1, 0\right) + \epsilon(F_{\dot{T}}^1, F_{\dot{T}}^2)\right) N_{\mathcal{I}_{\dot{T}_1, \dot{T}_2}}.$$

Note that we have given $\epsilon(F_{\dot{T}}^1, F_{\dot{T}}^2)$ a somewhat new meaning here. Here $\epsilon(F_{\dot{T}}^1, F_{\dot{T}}^2)$ is the residual in the current context. See Berglund [5] for more details about this example.

Example: Spark spread. Second, consider a gas-fired power plant. We extend our sample space with the random variables $F_t^P, F_t^G, S_t^P, S_t^G$, where superscripts P and G stand for Power and Gas respectively. We also have to introduce the conversion rate ν, which tells us the relation between the amount of gas and the amount of power. Then, in a similar way as above we get

$$\Pi_{strip}(\dot{T}_2) = \sum_{T_i \in \mathcal{I}_{\dot{T}_1, \dot{T}_2}} e^{r(\dot{T}_2 - T_i)} \left(\max\left(S_{T_i}^P - \nu S_{T_i}^G, 0\right)\right),$$

and we replace the payoff above with the simplified payoff

$$\Pi_{simple}(\dot{T}) = \left(\max\left(F_{\dot{T}}^P - \nu F_{\dot{T}}^G, 0\right) + \epsilon(F_{\dot{T}}^P, F_{\dot{T}}^G)\right) N_{\mathcal{I}_{\dot{T}_1, \dot{T}_2}}.$$

Also in this example, $\epsilon(F^P_{\dot{T}}, F^G_{\dot{T}})$ shall be interpreted as the residual in the current context. In regions involved in emission trading we also need to expand the sample space with the price of CO_2. Corresponding correlations must be estimated.

Example: Power plant with ramping. Third, consider a power plant with ramping. Ramping means that there is a time lag when the power plant is started or shut off. Only after some time the plant can be run on its maximum effect, which is a very common feature of power plants. Here we briefly mention how our setup can be used to include ramping. If there is an optimal dispatching strategy that is $\mathcal{F}_s$-measurable we can estimate a function $\epsilon(F_{\dot{T}}, K)$ based on historical outcomes. The historical outcomes are calculated using the optimal dispatch strategy. This is a straightforward exercise. Then we can proceed in the same way as in the other cases.

6. Conclusions

Our concern is to find feasible hedging strategies of a contingent claim on a strip of spot deliveries. As the corresponding individual forward contracts are not traded our aim is to find strategies in terms of the traded monthly forward contracts instead. In particular, we propose a simple way to construct such a hedging strategy. The idea is to split the payoff on the contingent claim into two parts, one European option on the monthly forward and a burn analysis part on the residual. The burn analysis part is by construction only depending on the traded monthly forward. Hence, we are able to get explicit trading strategies directly in terms of the traded contracts. The simplified strategy is compared with an optimal hedge in the sense of a local minimum variance optimization. Simulation of a certain three-factor model of the forward curve gives the possibility to evaluate the hedge efficiency of the two strategies. Dynamic hedging is assumed before delivery and a static hedge is applied through delivery. Our results indicate that the simplified strategy performs well in comparison with the locally optimal hedge. We note that the simplified strategy is especially attractive in practice, since the estimation of the involved parameters is straightforward and implicit volatilities are often available from traded European style options. The local minimum variance optimal strategy would either require a specific parametric model or a large number of parameters to be estimated.

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Appendix A. Proofs and technicalities

Proof of Proposition 3.2.

Proof. Define the partial expectation of a random variable X with threshold v as

$$g(v) = \int_v^{\infty} x f(x) dx,$$

where $f(x)$ is the density. For a lognormal density $f(x) = logN\left(\ln F_t - \frac{\sigma^2}{2}(\dot{T} - t), \sigma^2(\dot{T} - t)\right)$ it can be shown that

$$g(v) = F_t \Phi\left(\frac{\ln \frac{F_t}{v} + \sigma^2(\dot{T} - t)/2}{\sigma\sqrt{\dot{T} - t}}\right).$$

It is also straightforward to calculate

$$\begin{aligned} \int_{v_1}^{v_2} f(x)dx &= \mathbf{P}\left(X \in (v_1, v_2)\right) = \mathbf{E}\left(I_{\{X \in (v_1, v_2)\}}\right) \\ &= \Phi\left(\frac{\ln \frac{v_2}{F_t} + \sigma^2(\dot{T} - t)/2}{\sigma\sqrt{\dot{T} - t}}\right) - \Phi\left(\frac{\ln \frac{v_1}{F_t} + \sigma^2(\dot{T} - t)/2}{\sigma\sqrt{\dot{T} - t}}\right). \end{aligned}$$

Assume that $F_{\dot{T}}|F_t \sim f(x)$. We have

$$\begin{aligned}
\mathbf{E}_t\left(\epsilon(F_{\dot{T}}, K)\right) &= \sum_{i=0}^{n-1} \mathbf{E}_t\left(I_{\{F_{\dot{T}}\in(a_i,a_{i+1})\}} y_{est}(F_{\dot{T}})\right) \\
&= \sum_{i=0}^{n-1} \mathbf{E}_t\left(I_{\{F_{\dot{T}}\in(a_i,a_{i+1})\}} (b_i + k_i(F_{\dot{T}} - a_i))\right) \\
&= \sum_{i=0}^{n-1} k_i \mathbf{E}_t\left(I_{\{F_{\dot{T}}\in(a_i,a_{i+1})\}} F_{\dot{T}}\right) + \sum_{i=0}^{n-1} \mathbf{E}_t\left(I_{\{F_{\dot{T}}\in(a_i,a_{i+1})\}} (b_i - k_i a_i)\right) \\
&= \sum_{i=0}^{n-1} k_i \left(\int_{a_i}^{\infty} x f(x)dx - \int_{a_{i+1}}^{\infty} x f(x)dx\right) \\
&+ \sum_{i=0}^{n-1} (b_i - k_i a_i) \mathbf{E}_t\left(I_{\{F_{\dot{T}}\in(a_i,a_{i+1})\}}\right) \\
&= F_t \sum_{i=0}^{n-1} k_i \left[\Phi\left(\frac{\ln\frac{F_t}{a_i} + \sigma^2(\dot{T}-t)/2}{\sigma\sqrt{\dot{T}-t}}\right) - \Phi\left(\frac{\ln\frac{F_t}{a_{i+1}} + \sigma^2(\dot{T}-t)/2}{\sigma\sqrt{\dot{T}-t}}\right)\right] \\
&+ \sum_{i=0}^{n-1} (b_i - k_i a_i) \left[\Phi\left(\frac{\ln\frac{a_{i+1}}{F_t} + \sigma^2(\dot{T}-t)/2}{\sigma\sqrt{\dot{T}-t}}\right) - \Phi\left(\frac{\ln\frac{a_i}{F_t} + \sigma^2(\dot{T}-t)/2}{\sigma\sqrt{\dot{T}-t}}\right)\right].
\end{aligned}$$

Deltas are easy to obtain from the expression above.

Bjerksund's approximation formula: We refer to Bjerksund et al [6]. Note that our formulae below have corrected some errors in Bjerksund's results. For the volatility parameter of the forward contract $F_t = F(t, \dot{T}_1, \dot{T}_2)$, we have

$$\begin{aligned}
(\dot{T} - t)\left(\overline{\sigma}_{t,\dot{T}}^{\dot{T}_1,\dot{T}_2}\right)^2 &= \left(\frac{a}{\dot{T}_2 - \dot{T}_1}\right)^2 \int_t^{\dot{T}} \left(\ln\frac{\dot{T}_2 - s + b}{\dot{T}_1 - s + b}\right)^2 ds + \\
&+ \frac{2ac}{\dot{T}_2 - \dot{T}_1} \int_t^{\dot{T}} \ln\frac{\dot{T}_2 - s + b}{\dot{T}_1 - s + b} ds + c^2 \int_t^{\dot{T}} ds. \qquad (12)
\end{aligned}$$

The dilogarithm function is defined by

$$\text{dilog}(x) = \int_1^x \frac{\ln(s)}{1-s} ds, \text{ where } x \geq 0.$$

The dilog function is available in Maple. In Matlab, it is possible to call the Maple function by the syntax mfun('dilog',x). Set

$$\alpha = \frac{1}{2}(\dot{T}_2 - \dot{T}_1), \quad X(s) = b + \frac{1}{2}(\dot{T}_2 + \dot{T}_1) - s.$$

We have the following expressions for the integrals in Equation (12)

$$\begin{aligned}\int_t^{\dot{T}} \left(\ln \frac{\dot{T}_2 - s + b}{\dot{T}_1 - s + b}\right)^2 ds &= [(x+\alpha)(\ln(x+\alpha))^2 - 2(x+\alpha)\ln(x+\alpha)\ln(x-\alpha) + \\ &+ 4\alpha\ln(2\alpha)\ln\left(\frac{x-\alpha}{2\alpha}\right) - 4\alpha \mathrm{dilog}\left(\frac{x+\alpha}{2\alpha}\right) + \\ &+ (x-\alpha)(\ln(x-\alpha))^2]_{X(\dot{T})}^{X(t)},\end{aligned}$$

$$\int_t^{\dot{T}} \ln \frac{\dot{T}_2 - s + b}{\dot{T}_1 - s + b} ds = [(x+\alpha)\ln(x+\alpha) - (x-\alpha)\ln(x-\alpha)]_{X(\dot{T})}^{X(t)}.$$

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