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Do 15-year-old girls and boys influence each other's norms?: Modeling peer interaction using Gibbs measures.

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Abstract

Adolescence is the time when boys and girls make new friends not only within the same gender but interests in heterosexual relationships increase. Adolescent girls and boys spend considerable time in school settings, where possible interactions between individuals occur. The sample in the study is from 1995 and includes 570 15-year-old girls and 602 boys of the same age in the middle-sized Swedish town Örebro. A norm questionnaire covering the adolescents' personal norms and perception of the norms of friends was administered to this cohort of adolescents. Cluster analysis is employed for grouping individuals on the basis of norm pattern similarity. Norm interaction among adolescent boys and girls is investigated using stochastic contact process models in the form of Gibbs measures, which describes how teenage norms continuously change over time partly because of adaptation to surroundings and partly because of peer interaction. Parameters in the models are estimated and significance tests are carried out using standard likelihood methods and Bayesian inference based on Markov Chain Monte Carlo simulations. The results show that there exists significant norm interaction between boys, between girls but not between boys and girls. This is in some cases opposed to what literature on the socialisation process and interests in heterosexual relationship among 15-year-old boys and girls shows.

Keywords: Norm breaking behaviour; Norm interaction; Adolescent; Gender; Gibbs measure; Bayesian inference; Markov Chain Monte Carlo

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1 Introduction

"Gender segregation is the most robust gender phenomenon of childhood" (Maccoby, 1998). However, the two sexes are not totally disconnected. Boys and girls are intensely aware of each other and with increasing age there are changes in the nature of this awareness but also in the interactions between them. The majority of children in Western societies spend most of their time in settings where they are grouped into same-age segregated groups. It is also in school settings the opportunities for children and adolescents to interact with peers of opposite sex appear. Abandoning childhood with its play activities within homogeneous gender groups with parents as main socialisations agents the young adolescents are facing some important transitions. One of the main transitions is that from parent-centred to peer-centred relationship. These changes in socialisation agents from parents to peers will gradually influence the adolescent's norms. According to Caspi (1993) the selection of and the influence from peers are complementary processes that produce the adolescent's social context. During a certain time there will be continuity in this process. Adolescents may change their norms and behaviours as a result of the discrepancies between them.

The other opening is that of gender segregation which begins to break down at about ages 11-13. The first encounters between sexes usually appear in mixed social groups of teenagers. Early adolescents respond to features and behaviours of other adolescents that are representing independence as, for example, aggression or delinquency but are less attracted to peers who present features that are associated with childhood e.g. obedience to authority. Bukowski, Sippola and Newcomb (2000) studied diversity in attraction across school contexts, age, sex, and the same- and other-sex peer domains. As they have shown, the results confirm the predicted increase in the apparent attractiveness of aggressive peers following entry into early adolescence. After the transitions to middle school higher aggression scores were observed for the boys to whom both girls and boys were attracted and for the girls to whom boys were attracted. This increased tolerance of aggression is specifically strong for girls who were attracted by aggressive boys. Girls saw aggression as an undesirable feature in other girls but were more tolerant of aggression in boys under the transition to middle school. However, according to Sippola, Bukowski, and Noll (1997) the contact with the other sex is rather limited during early adolescence. This period can be seen as the initial stage of this form of contact.

Heimer (1996) in her study on gender, interaction, and delinquency found that delinquency both by girls and boys occurs through a process of role-taking. In this process adolescents consider different perspectives on delinquency of significant others. Heimer argues that gender definitions – our beliefs about femininity and masculinity – are internalised into the self and then serve to regulate behaviour in the same way as do other types of attitudes and values. In this study we assume that adolescents create gender identity with regard to their norms. According to Berk (1991) gender identity describes how relatively masculine or feminine individuals view themselves to be with regard to their characteristics, capabilities, and behaviours. Research results, for instance, support findings that gender definitions or gender identities have different consequences for females' than for males' law violation. In fact, some recent theoretical work on delinquency argues that internalising definitions of gender discourage and cause the reduction of delinquency among girls, but might increase the risk of delinquency among boys. For instance, this could signify that girls' gender identity possesses characteristics that would restrain their own norm breaking behaviours but indirectly, e.g. through attractiveness, encourage norm breaking in boys as for example in the case of tolerance of aggression. And aggressive behaviour is related to delinquency.

According to Urberg, Luo, Pilgrim and Degirmencioglu (2003) the adolescent social context is viewed as a dynamic social context. In this context there is interdependence between influence on and selection of friends that produce both change and continuity for the adolescent. Urberg et al. (2003) proposed “A model of influence among friends”. This model consists of two stages. The first stage deals with friendship selection. How do adolescents choose friends who have a potential to be influential? There are two possibilities of how the friend selection work: a) a selection of friends who are rather similar to oneself, or b) a selection of friends who are rather different. In the second stage there is a greater opportunity for interaction between adolescents. It is at this stage when adolescents either do or do not conform to the behaviours of theirs friends. Urberg's et al. (2003) model is a reciprocal one, “with friendship acquisition and influence viewed as ongoing processes that continuously and reciprocally impact one another”. These processes need to be studied with the regard to changes in the adolescent as well as the changes in the adolescent's social context. With consideration to functional interaction these integrated developmental processes are irreducible and indivisible. “Functional interaction is a characteristic of processes at all levels of the individual-environment system, including the interaction among working mental, behavioral, and biological elements at all levels of the integrated individual as well as the individuals reciprocal, intentional interaction with the environment” (Magnusson, 2001). The model we use to study the interaction is designed to take care of the influence which comes from outside. This can for example be home milieu and its influence on adolescent norms. Adolescence is then an intensive period of interactions between groups of teenagers, between girls, between boys and between girls and boys. For example, recent research indicates that the adolescents' cognitive structures are normally more open to change in new developmental directions (Spear, 2000). This period is one of the most important of so-called transformation processes over an individual's lifetime. In the present study we assume that adolescents influence each other's norms under long time and develop norms that make them belong to the same norm clusters.

Normally, in psychology, ongoing interaction processes that develop over long time periods are studied longitudinally, i.e. data are collected at several evenly distributed time points. In the present study, data are available for only one single time point. However, the school environment, in particular the fact that pupils are organised in schools and classes, makes it possible to overcome this problem. It seems reasonable to assume that classes were composed completely arbitrarily, at least with respect to the pupils' norms at the time when they entered school. Since then, the pupils have had the opportunity to interact with each other, predominantly with their classmates and possibly also with friends in other classes within the same school, and thereby influence each other's norms. Hence, any tendency of norm similarity within classes and schools has to be the effect of that interaction. According to Magnusson (2001) the characteristics of the openness and high pace during adolescence imply that understanding of this developmental period requires a special type of analysis and methodological tools. So, how do one measure norm similarities and, consequently, the magnitude of interaction between adolescents?

In this study are used stochastic contact process models. These models describe how adolescents' norms continually change over time partly through adaptation to surroundings and partly through interaction with peers. Within these stochastic contact process models, the norm spreading process is constructed, where particular emphasis is placed on interaction within and between school classes. Distinction is also made with respect to gender in the sense that we consider interactions between boys, between girls and between boys and girls separately. The models presuppose that the pupils have had occasions to interact with each other and that this process has become stationary at the time when data were collected. The models are constructed as Gibbs measures, which are most commonly used for describing particle systems in statistical physics, e.g. the Ising model for magnetism (Georgii et al., 2001). In recent years, they have also been applied in other research disciplines for various problems like tree growth patterns (Särkkä & Tomppo, 1998), dynamics of stock markets (Maskawa, 2003) and image analysis (Schröder et al., 2000). A Gibbs measure describes how particles (pupils) assume certain states (norm clusters) with respect to an external field (society) and interaction with nearby particles (class- and schoolmates). These states can be perceived as I-states (Bergman, 2000), a concept used in similar psychological applications.

"At the end of the 20th century, then, a predominant perspective on gender development is a dual one focusing on individual differences. Its central themes are that children will differ in the degree to which they become sex-typed as a result of: (a) the strength of the socialisation pressures they have experienced; and (b) the nature and coherence of their gender schemas – their knowledge about the characteristics stereotypically associated with each sex, and about what the social expectations are for persons of their own sex" (Maccoby, 2000). Much of the theorising on gender today deals with the processes through which the social construction of gender appears. The analyses of gender include also questions about the relationships between sexes as an important complement to the questions about the differences between sex. One of the main points in this study is to take into consideration what significance sex has as an organising principle for the study of adolescents' norms and norm breakings. So, in this study, we focus first and foremost on the relationship between boys and girls norms. The differences are studied as a complement to the study of the relationships. With the support of the ongoing research the decision to categorise all pupils (both girls and boys) into a fixed numbers of norm clusters was taken. Chinapah (1998, 2000, 2004) studied the stability or change of norm breaking for two adolescent generations, one from year 1969 and the other from 1995. The same data information from 1995 is used also in this study. There a comparison of the typical individual norm profiles at two different points in time was performed. The analysis showed that three fairly stable cluster profiles appear in each of the generations as well as among both girls and boys: an over-conforming profile, a normal rejecting profile and a deviant profile. These clusters can be viewed as universals clusters.

In Sweden the length of compulsory school time has successively increased. Swedish adolescents stay in school classes composed of girls and boys of the same age. The school settings give the possibility for boys and girls to interact with each other. The forthcoming study investigates the interaction between same aged adolescents from three different perspectives. The possible interaction is studied through eight typical norm-breaking behaviours during adolescence (e.g. drinking, smoking, cheating etc.). We assume that adolescents have had the possibility to interact with each other during the whole school time. Swedish children start compulsory school at seven years of age and finish at fifteen. School time is divided in three stages. Every such stage lasts three years. Elementary school 1-3 class, middle or secondary school 4-6 class and junior high school 7-9 class. Most of the children stay in the same class during these three stages.

The overall aim in the present study is to study norm interaction and to what degree same-age teenagers influence each other's norms. We raised three questions: Is there a significant interaction between teenagers? Is there any difference in interaction within respectively between classes in the same school? Is there any difference in norm interaction between boys and boys, between girls and girls and between boys and girls?

To fulfil this overall objective a reliable and useful model for the cluster distribution with particular emphasis on interaction between individuals allowing for differences in short-range interaction (within classes) and long-range interaction (between classes within schools) and also differences with respect to sex has to be found.

2 Methods

2.1 Data and cluster analysis

In November 1995, a study was carried out in the medium-sized Swedish town of Örebro, where all pupils in the 8th grade (on average 15 years old) were asked to give their opinions about typical norm-breaking behaviour among adolescents. At this time, there were 53 classes in 13 schools. They were administered a questionnaire, which they were asked to fill in under supervision during one lecture hour. To increase participation, a follow-up was done two weeks later to catch some of those pupils that were absent for some reason at the first occasion. This had the result that as many as 1172 pupils out of a total of 1244 (94 %) completed the questionnaire.

The questionnaire included questions about the following norm-breaking behaviours:

1. pilfering
2. smoking hashish
3. truancy
4. staying out late without parents' permission
5. cheating on exams

6. drinking until intoxication
7. loitering about every evening
8. disobeying parents.

For each behaviour the pupils were asked about their A) own values, B) own intentions, C) own behaviour, D) friends' perceived values and E) friends' perceived intentions. Answers were given on a scale with seven categories ranging from *completely unacceptable* to *completely acceptable*, except for own behaviour where five categories were used ranging from *never* to *frequently*. Also, information about school, class and sex was recorded for each pupil.

In order to simplify the information contained in the data, cluster analyses were made on each of the variables A–E above to categorise all pupils into a fixed number of *norm clusters*. The purpose here was strictly descriptive, so no statistical inference was made about optimality of cluster solution. Several solutions with varying number of clusters were considered, but no significant improvement for solutions with more than five clusters was observed, so the number of clusters was fixed to five.

Individuals with more than two missing values for a particular variable were excluded from the analysis for that variable. Similarity of individuals were computed using the pseudo-distance[1]

$$d_{ij} = \sum_{k \in I_{ij}} |x_{ik} - x_{jk}|$$

where x_{ik} is the opinion of behaviour k of individual i and I_{ij} is the index set containing the indices where both individuals i and j have given an opinion. For each variable, clusters were determined using the *K-means method*, introduced by MacQueen (1967).

2.2 Model I: Gibbs measure

For each variable, the data can be represented as

$$\boldsymbol{y} = (y_1, y_2, \ldots, y_n)$$

where y_i denotes the cluster of individual i and n denotes the total number of individuals in the analysis. In addition, we have information about school, class and sex for each individual.

An alternative representation, which will be useful in the model formulations, is $N_{ijk\ell}$ which denotes the total number of individuals of sex i (where $i = 1$ for boys and $i = 2$ for girls) in cluster j, school k and class ℓ. Also, we let $N_{ijk} = \sum_\ell N_{ijk\ell}$ be the number of individuals of sex i in cluster j and school k and $N_{ij} = \sum_k N_{ijk}$ be the number of individuals of sex i in cluster j. The main objective is to find a reliable and useful model for the cluster distribution with particular emphasis on interaction between individuals allowing for

[1]It is not a proper distance since it does not satisfy the triangular inequality $d_{ij} \leq d_{ik} + d_{kj}$.

differences in short-range interaction (within classes) and long-range interaction (between classes within schools) and also differences with respect to sex.

The first model we are considering is constructed as a *Gibbs measure* where the probability p of the vector $\boldsymbol{y}$ can be written

$$p(\boldsymbol{y}) \sim \left(\prod_{j=1}^{5} q_{1j}^{N_{1j}} q_{2j}^{N_{2j}} \right) \exp(\theta_1 X_1 + \theta_2 X_2 + \theta_3 X_3 + \gamma_1 Y_1 + \gamma_2 Y_2 + \gamma_3 Y_3) \tag{1}$$

where

X_1 = the number of pairs of boys in the same class and cluster

X_2 = the number of pairs of girls in the same class and cluster

X_3 = the number of pairs of one boy and one girl in the same class and cluster

Y_1 = the number of pairs of boys in the same school and cluster and in different classes

Y_2 = the number of pairs of girls in the same school and cluster and in different classes

Y_3 = the number of pairs of one boy and one girl in the same school and cluster and in different classes

Note that all numbers above can be determined from available data.

The parameters in the model can be classified as global cluster distribution parameters (q_{1j} and q_{2j} for $j = 1, 2, \ldots, 5$), short-range interaction parameters ($\theta_1, \theta_2, \theta_3$) and long-range interaction parameters ($\gamma_1, \gamma_2, \gamma_3$). The parameters q_{1j} and q_{2j} can easily be interpreted as the probabilities of boys and girls belonging to cluster j independently of class and school. Note that we allow different distributions for boys and for girls.

The interaction parameters are a bit more elusive. For example, if boys do influence each other's norms strongly within classes, then it is more likely that X_1 assumes a large value, which, incidently, means that θ_1 also would have to assume a large positive value in order to increase $p(\boldsymbol{y})$. Hence, the interaction parameters indicate the magnitudes of different kinds of interaction between individuals.

Another illustrative way to look at the interaction parameters is the following. Assume that we want to determine the probability that a particular boy belongs to cluster j. First of all, q_{1j} gives us the overall probability, but for every other boy in his class belonging to cluster j it increases with factor e^{θ_1} and for every girl with factor e^{θ_3}. These exponentials of parameters are usually called the *odds ratios*.

Ideally, we would now use the exact expression for $p(\boldsymbol{y})$ as the likelihood function and obtain maximum likelihood estimates of the parameters by optimising this function. However, a common problem with Gibbs measures is that exact calculations are usually unfeasible. Since expression (1) is only proportional, we would have to calculate the proportionality constant for any set of parameter values, something which would require an enormous amount of calculations.

One way to avoid this is to consider the *pseudo-likelihood function*

$$p^*(\boldsymbol{y}) = \prod_{i=1}^{n} p(y_i|\boldsymbol{y}_{i-}) \tag{2}$$

where $\boldsymbol{y}_{i-}$ denotes the data for all individuals except individual i. The conditional probability $p(y_i|\boldsymbol{y}_{i-})$ gives the probability that individual i belongs to cluster y_i given that we know the clusters of all other individuals in the population.

This means that we calculate the cluster distribution for every individual knowing which clusters all the other individuals belong to. It can be shown that pseudo-likelihood functions approximate proper likelihood functions well under fairly weak conditions and it is therefore a common approach for models of clustered data (Aerts et al., 2002).

Every factor in (2) can be expressed as

$$p(y_i = c|\boldsymbol{y}_{i-}) = \frac{q_{1c}e^{\theta_1(N_{1ck\ell}-1)+\theta_3 N_{2ck\ell}+\gamma_1(N_{1ck}-N_{1ck\ell})+\gamma_3(N_{2ck}-N_{2ck\ell})}}{\sum_{j=1}^{5} q_{1j}e^{\theta_1(N_{1jk\ell}-1_{\{j=c\}})+\theta_3 N_{2jk\ell}+\gamma_1(N_{1jk}-N_{1jk\ell})+\gamma_3(N_{2jk}-N_{2jk\ell})}} \tag{3}$$

if i is a boy and

$$p(y_i = c|\boldsymbol{y}_{i-}) = \frac{q_{2c}e^{\theta_2(N_{2ck\ell}-1)+\theta_3 N_{1ck\ell}+\gamma_2(N_{2ck}-N_{2ck\ell})+\gamma_3(N_{1ck}-N_{1ck\ell})}}{\sum_{j=1}^{5} q_{2j}e^{\theta_2(N_{2jk\ell}-1_{\{j=c\}})+\theta_3 N_{1jk\ell}+\gamma_2(N_{2jk}-N_{2jk\ell})+\gamma_3(N_{1jk}-N_{1jk\ell})}} \tag{4}$$

if i is a girl. Derivations of expression (3) and (4) can be found in Appendix A.

Maximum pseudo-likelihood estimates (MPLE) are now obtained by maximising the pseudo-loglikelihood $\ell(\boldsymbol{q}, \boldsymbol{\theta}, \boldsymbol{\gamma}) = \log p^*(\boldsymbol{y})$ with respect to the parameters in the model, where $\boldsymbol{q}$, $\boldsymbol{\theta}$ and $\boldsymbol{\gamma}$ denote the vectors of cluster distribution, short-range and long-range parameters, respectively. Confidence intervals are calculated and hypothesis tests are carried out using the fact that the estimates are approximately normal distributed.

Goodness-of-fit of the model is based on the *deviance*, which informally can be expressed as

$$D = 2(\ell(H_1) - \ell(H_0)) = 2(\ell(H_1) - \ell(\hat{\boldsymbol{q}}, \hat{\boldsymbol{\theta}}, \hat{\boldsymbol{\gamma}}))$$

based on the likelihood ratio test of the null hypthesis H_0 that the Gibbs measure (1) holds against the alternative hypothesis H_1 that the saturated model

$$p^{H_1}(\boldsymbol{y}) = \prod_i \prod_j \prod_k \prod_\ell q_{ijk\ell}^{N_{ijk\ell}}$$

with independent parameters for every class holds. If H_0 holds then D is approximately χ^2-distributed with $\nu = r_1 - r_0$ degrees of freedom, where $r_1 = 424$ and $r_0 = 14$ are the numbers of free parameters under the saturated model and the Gibbs measure, respectively.

2.3 Model II: Hierarchical Gibbs measure

It turns out, which we will see later, that the Gibbs measure in Section 2.2 does not quite fit data sufficiently well. One possible reason for this is that we have assumed that all individuals, independently of school and class, follow the same global cluster distribution described by the parameters q_{ij}. It is probably more realistic to assume that there could be differences in this distribution in different schools due to socio-economic differences in society. Some schools are located in areas dominated by private homes while some are located where most people live in rental apartments and it seems reasonable to expect some differences in norms of adolescents attending those schools.

One way to account for such differences is to let the cluster distribution parameters q_{ijk} also depend on school in addition to sex and cluster. Unfortunately, this makes it impossible to estimate long-range interaction between individuals between classes within schools since differences between schools is completely determined by the extended cluster distribution parameters. However, as we will see later, there does not exist any strong evidence that such interactions play any significant role.

The extended Gibbs measure can now be written

$$p(\boldsymbol{y}) \sim \left(\prod_{j=1}^{5} \prod_{k=1}^{13} q_{1jk}^{N_{1jk}} q_{2jk}^{N_{2jk}} \right) \exp(\theta_1 X_1 + \theta_2 X_2 + \theta_3 X_3) \tag{5}$$

The short-range interaction parameters and numbers of pairs within classes are the same as before.

The number of free parameters in the model increases significantly from 14 to 107, which undoubtedly would improve the fit but at the cost of decreased precision in parameter estimates. We will overcome this problem by introducing further restrictions on the parameters to reduce the degree of freedom of the model.

Let us assume that the cluster distribution parameters q_{ijk} follow some common distribution rather than be completely independent of each other. This means that we henceforth consider these parameters as random variables rather than deterministic unknown quantities, making the model *hierarchical* (Novick et al., 1973). In some sense, we may interpret this as introducing some kind of random socio-economic variation into the model.

The most suitable choice of distribution for q_{ijk} is the Dirichlet distribution defined by

$$p(q_{i1k}, q_{i2k}, \ldots, q_{i5k}) = \frac{\Gamma(\alpha_{i1} + \alpha_{i2} + \ldots + \alpha_{i5})}{\Gamma(\alpha_{i1})\Gamma(\alpha_{i2}) \cdots \Gamma(\alpha_{i5})} \prod_{j=1}^{5} q_{ijk}^{\alpha_{ij}-1}$$

for $i = 1, 2; k = 1, 2, \ldots, 13$, where α_{ij} are the parameters, usually called *hyperparameters*, in this distribution. Note that these new hyperparameters are dependent on sex and cluster but independent of school.

Hierarchical models are unfortunately unsuitable to analyse using classical likelihood methods. A more common approach is to use *Bayesian analysis* and obtain estimates using Markov Chain Monte Carlo-algorithms (Gelman et al., 2004).

The first step in all Bayesian methods is to define prior distributions for all parameters and hyperparameters in the model to be analysed. In the absence of any prior information about the parameters, these prior distributions should be chosen to be as vague as possible so that the resulting posterior distributions will be dominated by data. It turns out that the improper uniform prior distribution

$$p(\theta_1, \theta_2, \theta_3) \sim 1 \tag{6}$$

for the short-range interaction parameters works well, i.e. it produces a proper posterior distribution. However, more care is needed for the prior distribution of α_{ij} in order not to obtain an improper posterior distribution. One sufficiently vague choice that satisfies this condition is

$$p(\alpha_{i1}, \alpha_{i2}, \ldots, \alpha_{i5}) \sim (\alpha_{i1} + \alpha_{i2} + \ldots + \alpha_{i5})^{-11/2} \qquad i = 1, 2 \tag{7}$$

Derivation of this prior distribution using the approach of Dempster et al. (1983) can be found in Appendix B.

Using the hierarchical model (5) and the prior distributions (6) and (7), a Markov Chain with suitable transitions and the posterior distribution as stationary distribution is constructed using the Metropolis-Hastings algorithm (Metropolis et al., 1953; Hastings, 1970). For each variable, 20 000 steps of this Markov Chain is simulated, where the first 10 000 steps are used as a "burn-in" period to attain equilibrium and the last M =10 000 steps are used for the analysis.

Bayesian estimates are obtained by averaging over all simulated values as

$$\begin{aligned} \bar{q}_{ijk} &= \frac{1}{M} \sum_{r=1}^{M} q_{ijk}^{(r)} \\ \bar{\theta}_k &= \frac{1}{M} \sum_{r=1}^{M} \theta_k^{(r)} \end{aligned}$$

where $q_{ijk}^{(r)}$ and $\theta_k^{(r)}$ are the simulated values at step r in the chain. *Double tail probabilities* (DTP), which are the Bayesian equivalents of p-values, are defined as

$$\text{DTP} = 2\text{P}(\theta_k < 0)$$

which are estimated as twice the observed proportion of negative simulated values.

Goodness-of-fit of this model is also based on the deviance (Spiegelhalter et al., 2002), which in the Bayesian framework can be expressed as

$$D(\boldsymbol{q}, \boldsymbol{\theta}) = 2(\ell(H_1|\boldsymbol{q}, \boldsymbol{\theta}) - \ell(H_0|\boldsymbol{q}, \boldsymbol{\theta}))$$

Here, the deviance is random so we need to estimate the average

$$\bar{D} = \frac{1}{M} \sum_{i=1}^{M} D(\boldsymbol{q}^{(i)}, \boldsymbol{\theta}^{(i)})$$

and the effective degree of freedom as

$$\nu_D = \bar{D}(\boldsymbol{q}, \boldsymbol{\theta}) - D(\bar{\boldsymbol{q}}, \bar{\boldsymbol{\theta}})$$

Par.	Var.	MPLE	p-value	OR	CI
	A	0.11	< 0.001	1.12	1.06–1.17
	B	0.11	< 0.001	1.11	1.06–1.18
θ_1	C	0.13	< 0.001	1.13	1.07–1.20
	D	0.11	< 0.001	1.12	1.06–1.18
	E	0.09	0.003	1.10	1.03–1.17
	A	0.07	0.029	1.07	1.01–1.14
	B	0.07	0.038	1.07	1.00–1.14
θ_2	C	0.09	0.009	1.09	1.02–1.16
	D	0.15	< 0.001	1.16	1.10–1.23
	E	0.14	< 0.001	1.15	1.08–1.22
	A	0.04	0.047	1.04	1.00–1.08
	B	0.02	0.35	1.02	0.98–1.06
θ_3	C	0.01	0.62	1.01	0.97–1.05
	D	0.03	0.076	1.03	1.00–1.08
	E	0.05	0.010	1.06	1.01–1.10

Table 1: *Estimates of short-range interaction parameters in Model I.*

3 Results

3.1 Model I: Gibbs measure

For each of the five variables described in Section 2.1, maximum pseudo likelihood estimates (MPLE) of the interaction parameters $\theta_1, \theta_2, \theta_3, \gamma_1, \gamma_2, \gamma_3$ were calculated together with p-values for the tests of the null hypotheses that parameters are zero against two-sided alternative hypotheses. Also, the odds ratios (OR), i.e. the exponentials of the parameters, were calculated with 95 % confidence intervals (CI). The results for short-range interaction parameters are depicted in Table 1 and for long-range interaction parameters in Table 2.

The odds ratios are the factors with which the odds for a certain cluster changes for each class- or schoolmate belonging to that cluster. For instance, the odds that a boy will "choose" a certain cluster based on variable A increases with 12 % for every boy and 4 % for every girl in the same class belonging to that cluster.

Although the results are slightly inconsistent for different variables, they still indicate that there seems to be a significant interaction between boys and between girls within classes but not between boys and girls within classes or any interaction at all between classes. Also, goodness-of-fit was considered for all five variables through the deviances with degree of freedom and p-values for the test of the Gibbs measure against the saturated model, which are summarised in Table 3. Since the scaled deviances are significantly larger than one and the p-values are extremely small, we may conclude that the Gibbs measure does not appear to fit data sufficiently well.

Par.	Var.	MPLE	p-value	OR	CI
γ_1	A	0.01	0.50	1.01	0.99–1.03
	B	-0.01	0.64	0.99	0.97–1.02
	C	0.01	0.64	1.01	0.98–1.03
	D	0.02	0.041	1.02	1.00–1.04
	E	0.01	0.53	1.01	0.98–1.04
γ_2	A	0.00	0.98	1.00	0.97–1.03
	B	-0.02	0.15	0.98	0.95–1.01
	C	0.01	0.23	1.01	0.99–1.03
	D	-0.01	0.48	0.99	0.96–1.02
	E	-0.01	0.42	0.99	0.96–1.02
γ_3	A	0.01	0.17	1.01	0.99–1.03
	B	0.03	0.003	1.03	1.01–1.05
	C	0.01	0.26	1.01	0.99–1.03
	D	0.01	0.22	1.01	0.99–1.03
	E	0.01	0.21	1.01	0.99–1.03

Table 2: *Estimates of long-range interaction parameters in Model I.*

Variable	Deviance	d.f.	Scaled dev.	p-value
A	555	410	1.35	$2 \cdot 10^{-6}$
B	560	410	1.37	$1 \cdot 10^{-6}$
C	531	410	1.30	$5 \cdot 10^{-5}$
D	636	410	1.55	$5 \cdot 10^{-12}$
E	591	410	1.44	$1 \cdot 10^{-8}$

Table 3: *Deviances for Model I with p-values.*

3.2 Model II: Hierarchical Gibbs measure

Bayesian estimates (BE), double tail probabilities (DTP) and 95 % posterior intervals (PI) for the short-range interaction parameters in the Hierarchical Gibbs measure are depicted in Table 4. Note that the estimates are more consistent for the five variables and also, for θ_1 and θ_2, more strongly significant, which may be a consequence of a better fit, se below. This yields even stronger evidence that boys do seem to interact with each other within classes and girls too. Moreover, these strengths of these interactions seem to be very similar. However, we have not found any clear indication that boys interact with girls or vice versa.

Goodness-of-fit of this model was also considered for all five variables through the deviances with effective degree of freedom for the test of the Gibbs measure against the saturated model, which are summarised in Table 5. Here, we see that all but one scaled deviance are smaller than one indicating a much better fit of the model. The fact that the effective degree of freedom is much smaller than for the Gibbs measure, which implies that

Par.	Var.	BE	DTP	OR	PI
	A	0.17	< 0.001	1.18	1.14–1.22
	B	0.16	< 0.001	1.17	1.14–1.21
θ_1	C	0.17	< 0.001	1.19	1.15–1.23
	D	0.18	< 0.001	1.20	1.15–1.24
	E	0.16	< 0.001	1.17	1.13–1.22
	A	0.16	< 0.001	1.17	1.13–1.22
	B	0.15	< 0.001	1.16	1.12–1.21
θ_2	C	0.17	< 0.001	1.18	1.15–1.22
	D	0.18	< 0.001	1.19	1.15–1.24
	E	0.17	< 0.001	1.19	1.14–1.24
	A	0.02	0.25	1.02	0.99–1.05
	B	0.00	0.92	1.00	0.96–1.04
θ_3	C	-0.02	0.21	0.98	0.94–1.01
	D	0.02	0.19	1.02	0.99–1.06
	E	0.04	0.023	1.05	1.01–1.08

Table 4: *Estimates of short-range interaction parameters in Model II.*

Variable	Deviance	d.f.	Scaled dev.
A	211	240	0.88
B	196	237	0.83
C	213	222	0.96
D	284	253	1.07
E	242	268	0.90

Table 5: *Deviances for Model II.*

this model is closer to the saturated model, is a consequence of the higher heterogeneity of cluster distribution parameters.

4 Discussion

The present study investigates norm interaction and to what degree same-age teenagers affect each other's norms. Three questions were of interest: a) If there is a significant interaction between teenagers; b) If there is any difference in interaction within respectively between classes in the same school; c) If there is any difference in norm interaction between boys, between girls and between boys and girls. We use two different models to study interactions between adolescents. The first model is an ordinary Gibbs measure and the second is a hierarchical Gibbs measure. The models generate similar results, with some improvements of fit when the second model is used. Our findings indicate that both models account for both interactions between some adolescents and no interactions at all between others. As expected,we can conclude that there is a significant interaction between boys and there is also a significant interaction between girls with respect to the studied norm breaking behaviours. We can also conclude that there is no interaction on norms between 15 years old boys and girls with respect to the studied norm breaking behaviours. Concerning studying different norm dimensions there is no difference in power of interaction for boys and girls. We can also conclude that there is no interaction between different school classes.

It is important to address the fact that there were two major limitations in the available data that had to be overcome. The first was that we only had data about norms of the adolescents in the study at one single time point. To assess change over time, it would seem necessary to observe individuals at several time points, at least two. The second limitation was that we did not have any information about the adolescents' social network, which of course is of vital importance when considering interactions between individuals.

However, since we have information about which class in which school every pupil did attend at the time when data were collected, we made some assumptions so that this information could be used efficiently. Firstly, we assumed that all pupils had attended the same class since first grade and that pupils had been assigned to classes completely arbitrarily. This means that we assume that the norm distribution in each class in first grade represents the overall norm distribution in society. Any deviations from this assumption in the eighth grade, e.g. overrepresentation of some norm clusters, then has to be the effect of interactions within classes. In the first model, the Gibbs measure, we made the same assumption on the school level, but this did not fit data very well, perhaps due to socio-economic differences between schools. Another assumption was that the degree of interaction was identical for all pupils within classes, except that we allowed for gender differences. This is naturally a very crude model of the actual social network within each class. However, if significant norm interactions appear using this simpler model, the actual interactions could only be stronger. The disadvantage is that we risk missing weaker interactions. We can be quite certain that boys interact with boys and that girls interact with girls, but this model does not manage to capture any significant interaction between boys and girls. In any case, this last interaction is most likely much weaker than the two first if it actually exists.

Caspi (1993) maintains that adolescents make friendship with other adolescents because of the similarity between them. There is always a portion of differences that stands for interaction between friends. The present study shows that the boys are consequential for interaction on each other's norms and so are the girls. We believe that there has been interaction between these pupils and that this interaction has had consequences for the influence that developed over the years since they started school. These results are in line with Smith's and Paternoster's (1987) study on gender and delinquency that shows that peers are consequential for offending by both girls and boys. Normally, the literature on adolescents' relationships shows that the interaction across gender starts on the threshold to the adolescent period. The models we used in the present study to investigate norm interaction between boys and girls are supposed to account for a long time perspective. So, if this long-term interaction between boys and girls exists, there should eventually be an influence on each other's norms. However, there is a possibility that this influence has not yet had time to manifest itself among fifteen-year-olds. Over the last decades a shift in studying gender development has occurred, where focus on gender differences has been replaced by an increasing interest in similarities and interactions (Maccoby, 2000). If gender segregation is still very pronounced up to the threshold to adolescence, then the period of interaction across gender may be too short to produce any significant influence on norms at the age of fifteen. Our results point in the opposite direction.

The possible explanation of no significant interaction between boys and girls can also be seen from the more general point of view. According to Bukowski, Sippola and Newcomb (2000), norm breaking behaviour can stand for greater significance of interaction among peers, both boys and girls, in general. It is difficult to find the specific matching between adolescent boys and girls,which will be less pronounced. There might exist interaction between boys and girls beyond these typically norm breaking behaviours.

According to Heimer (1996), gender determines the way social control is transformed to self-control in delinquent situations. In this way, the societal definitions of femininity and masculinity are transformed to adolescents' self-control. Research on gender and delinquency suggests that gender is a key aspect of self-control especially during adolescence (Markus, Crane, Bernstein, and Siladi, 1982; Gecas, Thomas, and Weigert, 1973). Results in the Heimer study can be interpreted that delinquency of girls as well as of boys occurs through a process of role taking. In this process adolescents consider the perspective of significant others about definitions of femininity and masculinity. These definitions regulate behaviour, just as do attitudes about rules and other types of attitude and values. "Gender definitions, however, have very different consequences for females' than for males' law violation: internalising gender definitions reduces delinquency among girls but not among boys" (Heimer, 1996). The results in the present study may mirror the societal definitions of gender in both boys and girls with respect to norm violation. Many other elements of gender socialisation might contribute to this gender gap with respect to norm breaking behaviours. Does gender identity prevent any interaction between boys and girls or is it possible that our fifteen years olds still live disconnected from each other?

One well known theory is the theory about early pubertal maturation in female development (Stattin and Magnusson, 1990). It might be that girls in this study are early matured girls and that they interact in the most cases with older boys. According to Magnusson (1988) there is a clear association between age of biological maturation in girls and norm violation. Norm violations in Magnusson's study were related to how far the girls matured physically. The results show that the frequency of girls who showed norm breaking behaviours was higher among early maturing girls. In addition, these early-matured girls were also considerably more advanced with their contacts with older boys. It is known that girls' strong interactive skills support interaction in their close relationships but in this case with older boys. According to Gilligan (1982) females are more concerned with interpersonal relationships compared to males. Hence, females may be more susceptible to others' disapproval of delinquency.

The predominant perspective on gender development at the end of the 20th century was a dual one that focused on individual differences. In the present study, the starting point was on gender similarities in the norm space, i.e. adolescents were classified in norm clusters irrespectively of gender. The design of this study was governed by the quality of available data. The data from the Örebro study was originally not collected for studying interactions between individuals. As pointed out earlier, it would be useful to have data from more than one time point to study change over time, data from several cohorts (ages) to study possible interaction between younger and older boys and girls and data over every individual's social network.

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References

[1] Aerts, M., Molenberghs, G., Ryan, L. M. & Geys, H. (2002). *Topics in Modelling of Clustered Data.* Chapman & Hall/CRC, Boca Raton, FL, USA.

[2] Bergman, L. R. (2000). I-states as objects analysis (ISOA) - a way to generate sequences of categories for longitudinal (CFA) analysis. *Psychologische Beiträge, 42*, 337-346.

[3] Berk, L. E. (1991). *Child development.* Allyn & Bacon, Needham Heights, MA, USA.

[4] Bukowski, W. M., Sippola, L. K., & Newcomb, A. F. (2000). Variations in patterns of attraction to same- and other-sex peers during early adolescence. *Developmental Psychology, 36*, 147-154.

[5] Caspi, A. (1993). Why maladaptive behaviors persist: Sources of continuity and change across the life course. In D. C. Funder, R. D. Parke, C. Tomlinson-Keasey, & K. Widenen (Eds.), *Studying lives through time: Personality and development* (pp. 209-230). Washington, DC: APA.

[6] Chinapah, E., Carlson, M., Bergman, L. R. & Magnusson, D. (1998) Modelling the relationship between values, intentions and behavior: A simultaneous LISREL analysis over time. *Reports from the Department of Psychology, 854*, Stockholm University, Sweden.

[7] Chinapah, E. (2000). Are adolescent norms changing? An empirical study of 15-year-old adolescents in a medium-sized Swedish town in 1969 and 1995. Department of Psychology, Stockholm University, Stockholm, Sweden (only in Swedish).

[8] Chinapah, E. (2004). Gender trends in norms and norm related behaviors: A 25-year study of Swedish adolescents. *submitted to Young*

[9] Dempster, A. P., Selwyn, M. R. & Weeks, B. J. (1983). Combining historical and randomized controls for assessing trends in proportions. *Journal of the American Statistical Association, 78*, 221-227.

[10] Gecas, V., Thomas, D. L. & Weigert, A. J. (1973). Social Identities in Anglo and Latin Adolescents. *Social Forces, 51*, 477-84.

[11] Gelman, A., Carlin, J. B., Stern, H. S. & Rubin, D. B. (2004). *Bayesian Data Analysis*, Chapman & Hall/CRC, Boca Raton, FL, USA.

[12] Georgii, H.-O., Häggström, O. & Maes, C. (2001). The random geometry of equilibrium phases. *Phase Transitions and Critical Phenomena*, Volume 18 (ed. Domb, C. & Lebowitz, J. L.), Academic Press, London, 1-142.

[13] Gilligan, C. (1982). *In a Different Voice*. Harvard University Press, Cambridge, MA, USA.

[14] Hastings, W. K. (1970). Monte Carlo sampling methods using Markov chains and their application. *Biometrika, 57*, 97-109.

[15] Heimer, K. (1996). Gender, Interaction, and Delinquency: Testing a Theory of Differential Social Control. *Social Psychology Quarterly, 59*, 39-61.

[16] Maccoby, E. E. (2000). Perspectives on gender development. *International Journal of Behavioral Development, 24*, 398-406.

[17] MacQueen, J.B. (1967). Some Methods for Classification and Analysis of Multivariate Observations. *Proceedings of 5th Berkeley Symposium on Mathematical Statistics and Probability* (pp. 281-297). Berkeley, CA: University of California Press.

[18] Magnusson, D. (1988). *Individual development from an interactional perspective.* Erlbaum, Hillsdale, NJ, USA.

[19] Magnusson, D. (2001). The Holistic-Interactionistic Paradigm: Some Directions for Empirical Developmental Research. *European Psychologist, 6*, 153-162.

[20] Markus, H., Crane, M., Bernstein, S. & Siladi, M. (1982). Self-Schemas and Gender. *Journal of Personality and Social Psychology, 42*, 38-50.

[21] Maskawa, J. (2003). Multivariate Markov chain modelling for stock markets. *Physica A: Statistical Mechanics and its Applications, 324*, 317-322.

[22] Metropolis, N., Rosenbluth, A. W., Rosenbluth, M. N., Teller, A. H., & Teller, E. (1953). Equations of state calculations by fast computing machine. *Journal of Chemical Physics, 21*, 1087-1091.

[23] Novick, M. R., Lewis, C. & Jackson, P. H. (1973). The estimation of proportions in m groups. *Psychometrika, 38*, 19-46.

[24] Särkkä, A. & Tomppo, E. (1998). Modelling interactions between trees by means of field observations. *Forest Ecology and Management, 108*, 57-62.

[25] Schröder, M., Walessa, M., Rehrauer, H., Seidel, K. & Datcu, M. (2000). Gibbs random field models: a toolbox for spatial information extraction. *Computers & Geosciences, 26*, 423-432.

[26] Sippola, L., Bukowski, W. M., & Noll, R. B. (1997). Age differences in Children's and early adolescents' liking for same-sex and other-sex peers. *Merill-Palmer Quarterly, 43*, 547-561.

[27] Smith, D. A. & Paternoster, R. (1987). The Gender Gap in Theories of Deviance: Issues and Evidence. *Journal of Research in Crime and Delinquency, 24*, 140-67.

[28] Spear, L. P. (2000). Neurobiological changes in adolescence. *Current Directions in Psychological Sciences, 9*, 111-114.

[29] Spiegelhalter, D. J., Best, N. G., Carlin, B. P. & van der Linde, A. (2002). Bayesian measures of model complexity and fit. *Journal of the Royal Statistical Society: Series B, 64*, 583-640.

[30] Stattin, H. & Magnusson, D. (1990). Pubertal maturation in female development. In Magnusson, D. (Ed.) *Paths through life* (Vol. 2), Erlbaum, Hillsdale, NJ, USA.

[31] Urberg, K. A., Luo, Q., Pilgrim, C., & Degirmencioglu, S. M. (2003). A two-stage model of peer influence in adolescent substance use: individual and relationship-specific differences in susceptibility to influence. *Addictive Behaviors, 28*, 1243-1256.

Appendix A

Let us assume that individual i is a boy in school k and class ℓ. The conditional probability of y_i given that we know the clusters of all other individuals can be written

$$p(y_i|\boldsymbol{y}_{i-}) = \frac{p(\boldsymbol{y})}{p(\boldsymbol{y}_{i-})}$$

The enumerator is given by (1) and the denominator can similarly be expressed as

$$p(\boldsymbol{y}_{i-}) \sim \left(\prod_{j=1}^{5} q_{1j}^{(N_{1j}-1)} q_{2j}^{N_{2j}}\right) \exp(\theta_1\tilde{X}_1 + \theta_2\tilde{X}_2 + \theta_3\tilde{X}_3 + \gamma_1\tilde{Y}_1 + \gamma_2\tilde{Y}_2 + \gamma_3\tilde{Y}_3) \qquad (8)$$

where the variables $\tilde{X}$ and $\tilde{Y}$ are the corresponding number of pairs in the population excluding individual i.

Let us denote the cluster of individual i by c. Since i is a boy, it follows immediately that $\tilde{X}_2 = X_2$ and $\tilde{Y}_2 = Y_2$. Regarding the other variables, the numbers of pairs of individuals decrease when individual i is lifted out of the population. Since the number of male classmates of individual i is $N_{1ck\ell} - 1$ in cluster c and $N_{1jk\ell}$ in any other cluster $j \neq c$, we get that

$$\tilde{X}_1 = \begin{cases} X_1 - (N_{1ck\ell} - 1) & j = c \\ X_1 - N_{1jk\ell} & j \neq c \end{cases}$$

The number of female classmates can simply be expressed as $N_{2jk\ell}$ regardless of cluster, which yields that $\tilde{X}_3 = X_3 - N_{2jk\ell}$.

Similar considerations can be made when considering schoolmates. However, since we are disregarding classmates here, we get that

$$\begin{aligned} \tilde{Y}_1 &= Y_1 - (N_{1jk} - N_{1jk\ell}) \\ \tilde{Y}_3 &= Y_3 - (N_{2jk} - N_{2jk\ell}) \end{aligned}$$

Summarising this yields that

$$\begin{aligned} p(y_i = c|\boldsymbol{y}_{i-}) &= Cq_{1c}e^{\theta_1(X_1-\tilde{X}_1)+\theta_2(X_2-\tilde{X}_2)+\theta_3(X_3-\tilde{X}_3)+\gamma_1(Y_1-\tilde{Y}_1)+\gamma_2(Y_2-\tilde{Y}_2)+\gamma_3(Y_3-\tilde{Y}_3)} \\ &= Cq_{1c}e^{\theta_1(N_{1ck\ell}-1)+\theta_3 N_{2ck\ell}+\gamma_1(N_{1ck}-N_{1ck\ell})+\gamma_3(N_{2ck}-N_{2ck\ell})} \end{aligned}$$

and

$$p(y_i = j|\boldsymbol{y}_{i-}) = Cq_{1j}\exp(\theta_1 N_{1jk\ell} + \theta_3 N_{2jk\ell} + \gamma_1(N_{1jk} - N_{1jk\ell}) + \gamma_3(N_{2jk} - N_{2jk\ell}))$$

for $j \neq c$, where C is an unknown proportionality constant.

Since we only have to consider five different combinations ($j = 1, 2, 3, 4, 5$), we can easily calculate the proportionality constant C (or rather the inverse C^{-1}) as

$$\begin{aligned} C^{-1} &= \sum_{j=1}^{5} p(y_i = j|\boldsymbol{y}_{i-}) \\ &= \sum_{j=1}^{5} q_{1j}e^{\theta_1(N_{1jk\ell}-1_{\{j=c\}})+\theta_3 N_{2jk\ell}+\gamma_1(N_{1jk}-N_{1jk\ell})+\gamma_3(N_{2jk}-N_{2jk\ell})} \end{aligned}$$

which yields expression (3).

The derivation of expression (4) when individual i is a girl is completely analogous.

Appendix B

The naive choice of a vague prior distribution for the parameters $\alpha_{i1}, \alpha_{i2}, \ldots, \alpha_{i5}$ is the uniform distribution

$$p(\alpha_{i1}, \alpha_{i2}, \ldots, \alpha_{i5}) \sim 1$$

but this does not work since it produces an improper posterior distribution. A common approach for these kinds of problems is to find an alternative parametrisation for which the uniform distribution does work.

One such parametrisation is based on the expected values

$$\mathrm{E}[q_{ijk}] = \frac{\alpha_{ij}}{\alpha_{i1} + \alpha_{i2} + \ldots + \alpha_{i5}} \qquad j = 1, 2, \ldots, 5$$

and standard deviations

$$\mathrm{SD}[q_{ijk}] \approx (\alpha_{i1} + \alpha_{i2} + \ldots + \alpha_{i5})^{-1/2}$$

for the Dirichlet distribution. Since the expected values are linearly dependent, we get the alternative parametrisation as

$$g(\alpha_{i1}, \alpha_{i2}, \ldots, \alpha_{i5}) = \left(\frac{\alpha_{i1}}{\alpha_{i\cdot}}, \ldots, \frac{\alpha_{i4}}{\alpha_{i\cdot}}, \alpha_{i\cdot}^{-1/2}\right)$$

where $\alpha_{i\cdot} = \sum_{j=1}^{5} \alpha_{ij}$.

By choosing the uniform distribution for the new parametrisation

$$p(g(\alpha_{i1}, \alpha_{i2}, \ldots, \alpha_{i5})) \sim 1$$

we get that

$$p(\alpha_{i1}, \alpha_{i2}, \ldots, \alpha_{i5}) = |J| p(g(\alpha_{i1}, \alpha_{i2}, \ldots, \alpha_{i5})) \sim |J|$$

where $|J|$ is the determinant of the Jacobian J with elements

$$J_{uj} = \frac{\partial g_u}{\partial \alpha_{ij}} \qquad u = 1, 2, \ldots, 5; j = 1, 2, \ldots, 5$$

Here, we get the elements

$$\begin{aligned} J_{uj} &= \frac{\alpha_{i\cdot} - \alpha_{iu}}{\alpha_{i\cdot}^2} && j = u = 1, 2, 3, 4 \\ J_{uj} &= -\frac{\alpha_{iu}}{\alpha_{i\cdot}^2} && u = 1, 2, 3, 4; j = 1, 2, 3, 4, 5; j \neq u \\ J_{5j} &= -\frac{1}{2\alpha_{i\cdot}^{3/2}} \end{aligned}$$

which yields the determinant

$$
\begin{aligned}
|J| &= \begin{vmatrix}
\frac{\alpha_{i\cdot}-\alpha_{i1}}{\alpha_{i\cdot}^2} & -\frac{\alpha_{i1}}{\alpha_{i\cdot}^2} & -\frac{\alpha_{i1}}{\alpha_{i\cdot}^2} & -\frac{\alpha_{i1}}{\alpha_{i\cdot}^2} & -\frac{\alpha_{i1}}{\alpha_{i\cdot}^2} \\
-\frac{\alpha_{i2}}{\alpha_{i\cdot}^2} & \frac{\alpha_{i\cdot}-\alpha_{i2}}{\alpha_{i\cdot}^2} & -\frac{\alpha_{i2}}{\alpha_{i\cdot}^2} & -\frac{\alpha_{i2}}{\alpha_{i\cdot}^2} & -\frac{\alpha_{i2}}{\alpha_{i\cdot}^2} \\
-\frac{\alpha_{i3}}{\alpha_{i\cdot}^2} & -\frac{\alpha_{i3}}{\alpha_{i\cdot}^2} & \frac{\alpha_{i\cdot}-\alpha_{i3}}{\alpha_{i\cdot}^2} & -\frac{\alpha_{i3}}{\alpha_{i\cdot}^2} & -\frac{\alpha_{i3}}{\alpha_{i\cdot}^2} \\
-\frac{\alpha_{i4}}{\alpha_{i\cdot}^2} & -\frac{\alpha_{i4}}{\alpha_{i\cdot}^2} & -\frac{\alpha_{i4}}{\alpha_{i\cdot}^2} & \frac{\alpha_{i\cdot}-\alpha_{i1}}{\alpha_{i\cdot}^2} & -\frac{\alpha_{i4}}{\alpha_{i\cdot}^2} \\
-\frac{1}{2\alpha_{i\cdot}^{3/2}} & -\frac{1}{2\alpha_{i\cdot}^{3/2}} & -\frac{1}{2\alpha_{i\cdot}^{3/2}} & -\frac{1}{2\alpha_{i\cdot}^{3/2}} & -\frac{1}{2\alpha_{i\cdot}^{3/2}}
\end{vmatrix} \\
&= \frac{1}{2\alpha_{i\cdot}^{19/2}} \begin{vmatrix}
\alpha_{i1}-\alpha_{i\cdot} & \alpha_{i1} & \alpha_{i1} & \alpha_{i1} & \alpha_{i1} \\
\alpha_{i2} & \alpha_{i2}-\alpha_{i\cdot} & \alpha_{i2} & \alpha_{i2} & \alpha_{i2} \\
\alpha_{i3} & \alpha_{i3} & \alpha_{i3}-\alpha_{i\cdot} & \alpha_{i3} & \alpha_{i3} \\
\alpha_{i4} & \alpha_{i4} & \alpha_{i4} & \alpha_{i4}-\alpha_{i\cdot} & \alpha_{i4} \\
1 & 1 & 1 & 1 & 1
\end{vmatrix} \\
&= \frac{\alpha_{i\cdot}^4}{2\alpha_{i\cdot}^{19/2}} \sim (\alpha_{i1}+\alpha_{i2}+\ldots+\alpha_{i5})^{-11/2}
\end{aligned}
$$