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# Nonparametric Volatility Density Estimation

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## Abstract

Stochastic volatility modelling of financial processes has become popular and most models contain a stationary volatility process. For volatility density estimation Van Es et al.(2003) introduced a deconvolution procedure; in this thesis we instead propose another nonparametric method. It is a two-step procedure, where we first apply some nonparametric regression technique to generate the process estimates, based on which we then use the ordinary kernel density estimator. To find the method parameters, we also suggest automatic parameter selectors using theories from the Nadaraya-Watson estimator and continuous-time kernel density estimation. To evaluate performance of the proposed method in comparison with the deconvolution approach, we apply both methods on data simulated from Heston model and real data. For simulated data, we divide it into two sets; high frequency(hourly) and low frequency(daily). We find that the proposed method slightly outperforms the deconvolution approach in terms of mean integrated squared error(MISE) for high frequency data. However, for low frequency data, the deconvolution procedure obtains far less MISE than the proposed method. Unfortunately, their performances on the real data are hardly comparable. **Keywords:** Volatility Density Estimation, Deconvolution, Bandwidth Selection, Nadaraya- Watson Estimator, Continuous-time Kernel Estimation, Heston Model.

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