Stockholms universitet

Ruin Probabilities for Stochastic Flows of Financial Contracts

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Masteruppsats i matematisk statistik
Master Thesis in Mathematical Statistics

Masteruppsats 2013:1
Matematisk statistik
Juni 2013

www.math.su.se

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Master Thesis **2013:1**
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June 2013

Abstract

In this thesis, ruin probabilities connected with stochastic flows of financial contracts are studied. We show that the model can be embedded in the specific variant of Sparre Andersen model and the corresponding ruin probabilities connected with stochastic flows of financial contracts considered in the thesis can be interpreted as a ruin probability for the corresponding Sparre Andersen model. Cramér-Lundberg type bounds are obtained for ruin probabilities and explicit equations are given for computing of the adjustment parameter. Finally, results of numerical studies are presented to illustrate the theoretical results.

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ACKNOWLEDGEMENT

I would like to express my profound gratitude to my supervisors, Dmitrii Silvestrov whose strong knowledge in the Mathematical Statistics guided me step by step right from the onset of this paper to the end and Håkan Andersson whose strong background in Finance and Insurance gave me the practical views on grounds and the application of mathematics into these fields. The discussions with these two supervisors have given me an overview of how the application of mathematical statistics in the finance and insurance field is important.

Special thanks to Dr. Kennedy Quaigrain, my Dad for the motivation, encouragement and support throughout the whole project. I would also like to thank Isaac Awuku Achaempong and Fatemeh Aramian for their immerse help throughout this project.

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1 INTRODUCTION

Ruin theory is based on estimates of ruin probabilities for risk process, including insurance and also financial processes as well. The classical origin for the ruin theory results was obtained by F. Lundberg (1903, 1909) and H. Cramér (1926, 1930). There exists a huge literature in this area. We refer to the books by Rolski, Schmidli, Schmidt, and Teugels (1998) and Asmussen (2000), where one can find the detailed presentation of the results and extended bibliographies of works in this area.

A risk reserve process $\{R_t\}_{t\geq 0}$ or a surplus process (aggregate loss process) $\{S_t\}_{t\geq 0}$ for a financial institution (specifically of an insurance company) is a model defined for the evolution of the reserves (in terms of capital) with $u = R_0$ being the initial reserve or capital. These two are given as:

$$R_t = u + ct - \sum_{j=1}^{Y_t} U_j \quad \text{and} \quad S_t = \sum_{j=1}^{Y_t} U_j - ct,$$

where:

(a) Y_t, $t \geq 0$ is a Poisson process;

(b) $U_j, j = 1,2, \ldots$ are independent and identically distributed (i.i.d.) non-negative random variables;

(c) Process Y_t, $t \geq 0$ and random variables $U_j, j = 1,2, \ldots$ are independent

(d) u, c = premium ≥ 0.

The probability of ruin in infinite time horizon is the probability defined as,

$$\varphi(u) = \mathbb{P}(\inf_{t\geq 0} R_t < 0) = \mathbb{P}(\inf_{t\geq 0} R_t < 0 | R_0 = u).$$

The concept of ruin theory has been used widely in many financial areas and applied to different models in assessing the performance of portfolio. The classical Cramér- Lundberg approximation outlines some natural conditions asymptotic for ruin probabilities in the following form,

$$\varphi(u) \sim e^{-\gamma u} \quad \text{as} \quad u \to \infty,$$

where the constant $\gamma > 0$.

The Cramér-Lundberg model was extended by E. Sparre Andersen (1957) where he assumed the claim that inter-arrival times have arbitrary distribution functions instead of the specified Poisson distribution as it is assumed in the classical Cramér-Lundberg model. The concept and analysis of the Sparre Andersen model will be the main focus in analysing the model studied in this thesis.

2 DESCRIPTION OF MODEL

The model of the portfolio is in continuous time measured in years with today being $t = 0$. The outline of the portfolio is as follows.

Arrival of deals follows a Poisson Process $N(t)$ with an intensity λ.

The notations in the model are described as follows:
L_j is the size of the loan deal j, M_j is the time to maturity, D_j is the time at which default deal j happens, T_j is the effective time that client j remains in the system. The client j amortizes at $\frac{L_j}{M_j}$ and pays a risk premium $L_j r_j$ per time unit. Moreover, the rate at which a client j defaults on its obligated payment is α (that is the probability of default) which is constant.
Assumption of the Model:

- There is no collateral taken by the bank, which means in case of default all future cash flows between the client and the bank is removed.
- The initial capital or reserve the bank holds is constant given as v.
- The loan size process L_j, Poisson process N and the default arrival process D_j are mutually independent.
- $N = \{N(t)\}_{t \geq 0}$ is a counting process on $[0, \infty)$: $N(t)$ is the number of deals which occurred by time t.

The effective time that client j remains in the system, T_j follows a truncated exponential distribution given as:

$$\acute{T}_j = min(M_j, D_j) \ , \quad \text{where} \qquad D_j \sim Exp(\alpha_j)$$

In general, the risk reserve process $V(t)$ of the bank generated by all the deals that have arrived between 0 and t is

$$V(t) = v + \sum_{j=1}^{N(t)} \left\{ \left(\frac{L_j}{M_j} + L_j r_j \right) \acute{T}_j - L_j \right\}, \qquad t \geq 0 \tag{2.1}$$

Since $\{N(t)\}_{t\geq 0}$ is a homogeneous Poisson process with the intensity λ, the random variable $\{N(t)\} \sim Poi(\lambda t)$ with deals arriving randomly of time $0 < \sigma_1 < \sigma_2 \ldots$ If Z_j is defined as the time between the $(j-1)th$ and the jth arrival of deals (i.e the inter-arrival times of the homogenous Poisson process) then

$$\sigma_n = Z_1 + \cdots + Z_n \quad \text{for} \quad n = 1,2, \ldots$$

And $Z_1 = \sigma_1, Z_2 = \sigma_2 - \sigma_1, \ldots Z_j = \sigma_j - \sigma_{j-1}$ are i.i.d. exponentially distributed random variables with $\mathbb{E}Z_1 = {}^1\!/_{\lambda}$.

3 METHOD OF ANALYSIS

It is of a significant importance to study the performance of portfolios in risk management. There are many ways of going about this but in our case the ruin theory used by actuaries was considered. In using the concept of ruin theory described earlier, it can be seen that (2.1) is more or less related to that of the ruin theory of non-life insurance. Where $V(t)$ is the balance of the bank at a given time t, $(V(t))_{t\geq 0}$ and is the cash flow in the portfolio over time. The income received from the jth client is $\frac{L_j}{M_j} + L_j r_j$. Thus the total money the bank receives by time t is $\sum_{j=1}^{N(t)} \left(\frac{L_j}{M_j} + L_j r_j \right) T_j$; L_j is the capital the bank lends out to the jth client with $\sum_{j=1}^{N(t)} L_j$ being the total outflow of capital as of loan lending occurring in $[0, t]$. Obviously, the process decreases at the arrival points $0 < \sigma_1 < \sigma_2 \ldots$ reducing by size L_j at the arrival time σ_j of the jth client. When the lending size is sufficiently large, there is a possibility that $V(t)$ will take negative values. From ruin theory, it is known that when V falls below zero there is ruin. This then leads to the concept of ruin probability for the model.

The case of homogeneous loans (where the loan sizes, time to maturity, the premium and the default are assumed to be constants and identical for all clients) was applied in this computation for simplicity and is given as:

$V(t) = v + \sum_{j=1}^{N(t)} \left\{ \left(\frac{L}{M} + Lr \right) \min\left(M, D_j \right) - L \right\}$ with $D_j \sim Exp(\alpha)$ (3.1)

For the model (3.1) to be more convenient to work with mathematically, the model was modified a little more. Letting,

$$\widetilde{M} = \left(\frac{L}{M} + Lr \right) M = L + MLr$$

This is obtained from the fact that M is constant. This also indicates that the total money the bank receives is greater than what the bank lends out.

This condition let one avoid the trivial case, where ruin occurs with probability one.

Thus, we assume that the following condition holds:

A: *The total money received (total cash inflow) should be greater than the total money lent out (total cash outflow) i.e.* $\widetilde{M} = \left(\frac{L}{M} + Lr \right) M = L + MLr > L$*.*

Also, it is known that D_j is exponentially distributed and therefore it can be deduced that letting,

$F(x) = \mathbb{P}\left\{ \left(\frac{L}{M} + Lr \right) D_j > x \right\} = e^{-\alpha \frac{x}{\left(\frac{L}{M}+Lr\right)}} = e^{-x \frac{\alpha}{\left(\frac{L}{M}+Lr\right)}} = e^{-x\tilde{\mu}}$.

Hence,

$\tilde{\mu} = \frac{\alpha}{\left(\frac{L}{M}+Lr\right)}$ and $N(t) = \max\left(n : \sum_{j=1}^{n} \min\left(\widetilde{M}, D_j \right) \le t \right)$

The modified model with the parameters defined above is:

$\tilde{V}(n) = v + \sum_{j=1}^{n} \min\left(\widetilde{M}, D_j \right) - L,$ (3.2)

where

$$\begin{cases} T_j = \min\left(\widetilde{M}, D_j \right) \\ D_j \sim Exp(\widetilde{\mu}), iid \\ \widetilde{M} > L > 0 \end{cases}$$

The ruin probability is given as:

$$\varphi(v) = \mathbb{P}\left(\inf_{t\geq 0} \tilde{V}(t) < 0 \middle| \tilde{V}(0) = v\right)$$

$$= \mathbb{P}\left(\underbrace{\min_n(v + \sum_{j=1}^{n}\{\min(\widetilde{M}, D_j) - L\} < 0}_{\tau}\right) \quad (3.3)$$

The sample path of the risk process can be illustrated in the diagram below:

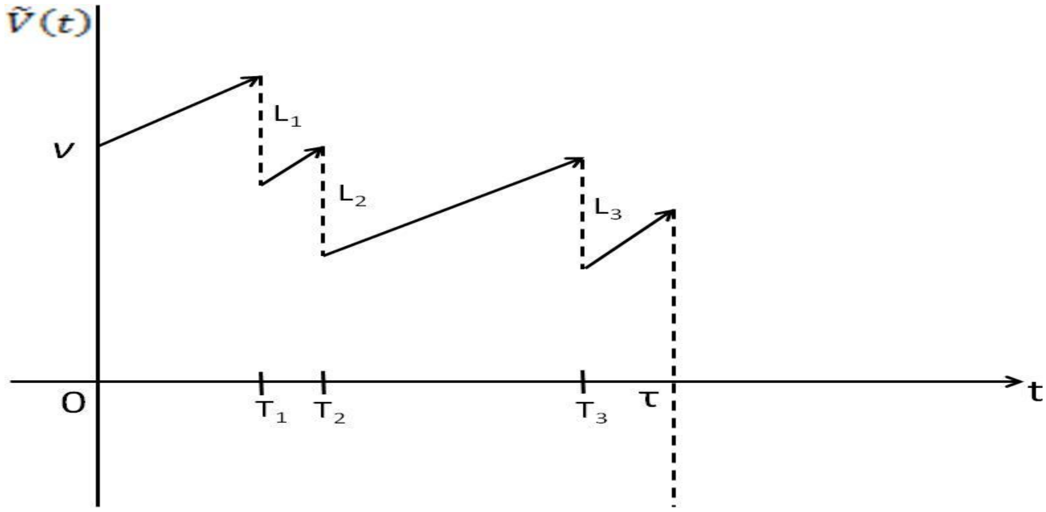

Figure 3.1: A trajectory illustration of the Risk Process

The ruin probability (3.3) will now be compared and analysed using the concept of Sparre Andersen Model.

3.1 Sparre Andersen Model

E. Sparre Andersen (1957) extended the classical Cramér- Lundberg model by allowing claim inter-arrival times to have arbitrary distribution functions. The Sparre Andersen model will be described, followed by the description of how the ruin probability of the model was determined and finally the application of Cramér-Lundberg type bound for the ruin probability of the model. The model is described as follows:

Letting the inter-claim times be $T_1, T_2, \ldots$, then T_n is the time between the $(n-k)th$ and the nth claim. T_1 is then the time between the zero point and the first claim and U_n is the nth claim amount. The following assumptions were made by him:

- The inter-claim times have a general distribution instead of an independent exponential distribution;
- The claim sizes are exponentially distributed;
- The risk premium per time unit is a positive constant;
- The inter-claim times and the claim sizes are independent random variables;
- The inter-claim times and claim sizes are both identically distributed;
- The means of the distribution of both the inter-claim times and claim sizes exist and are finite.

Hence, the risk reserve process of the Sparre Andersen model is given as:

$$W(t) = w + \beta t - U(t)\ ,\ t \geq 0, \tag{3.4}$$

where $w \geq 0$ is the initial risk reserve, β is the risk premium, $U(t) = 0$ if $T_1 > t$ and $U(t) = \sum_{k=1}^{\widetilde{N(t)}} U_k$.

Where $\widetilde{N}(t)$ is a renewal process and U_k independent and identically non-negative random variables with no probability mass at zero.

Mathematically, it is more convenient to consider the claim surplus process instead of the risk reserve process which is given as:

$$S(t) = \sum_{k=1}^{\widetilde{N(t)}} U_k - \beta t; \qquad t \geq 0 \tag{3.5}$$

3.2 Ruin Probability for the Sparre-Andersen Model

The main focus of the study is the ruin probability of this model. The ruin probability $\tilde{\varphi}(w)$ is thus given as:

$$\begin{aligned}\tilde{\varphi}(w) &= \mathbb{P}(\inf_{t\geq 0} W(t) < 0 | W(0) = w) \\ &= \mathbb{P}(\min_n (w + \sum_{k=1}^{n} \{\beta \mathrm{T_k} - U_k\} < 0)\end{aligned} \tag{3.6}$$

The ruin probability or function for the Sparre-Andersen model with exponential claim size distribution, say, $Exp(\delta)$ is given as:

$$\tilde{\varphi}(w) = \left(1 - \frac{\gamma}{\delta}\right) e^{-\gamma w} \tag{3.7}$$

for all $w \geq 0$, where γ, the adjustment coefficient is the unique positive root of the moment generating function given as:

$$\widehat{m}_Y(s) = \frac{\delta}{\delta - s}\hat{l}_T(\beta s) = 1 \qquad \text{with } \widehat{m}_U = \frac{\delta}{\delta - s}$$

where $Y = U - \beta T$ and $\widehat{m}_U(s)$ is the moment generating function of the claims size and $\hat{l}_T$ is the Laplace Stieltjes Transform.

For the existence of γ, the Cramér-Lundberg type bounds for the ruin probability of the Sparre Andersen Model can be computed.

3.3 Cramér-Lundberg type bounds

For the existence of the parameter γ, the lower and upper bounds of the ruin probability can be determined. These bounds are called the Cramér-Lundberg type bounds. The parameter γ is the unique positive root of the following equation,

$$\widehat{m}_Y(s) = \widehat{m}_U(s)\hat{l}_T(\beta s) = 1 \tag{3.8}$$

Indicating that 1 is a root of (3.8) when $s = 0$, that is, $\widehat{m}_Y(0) = 1$ and equation may have a second root γ which is unique and strictly positive when $s \neq 0$.

The Cramér-Lundberg type bounds for the ruin probability used in our studies, given in Rolski, Schmidli, Schmidt, Teugels (1998) (Theorem 6.5.4, Chapter 6; pages 255-256), takes the following form,

$$b_- e^{-\gamma w} \leq \tilde{\varphi}(w) \leq b_+ e^{-\gamma w}, \text{ for } w \geq 0, \tag{3.9}$$

where

$$b_- = inf_{x \in [0, x_0)} \frac{e^{\gamma x}\bar{F}_Y(x)}{\int_x^\infty e^{\gamma y} dF_Y(y)}, \quad b_+ = sup_{x \in [0, x_0)} \frac{e^{\gamma x}\bar{F}_Y(x)}{\int_x^\infty e^{\gamma y} dF_Y(y)} \tag{3.10}$$

With $x_0 = sup\{x: F_Y < 1\}$, $\bar{F}_Y(x)$ is the tail function, $dF_Y(y)$ is the density function and b_- and b_+ are constants.

Equation (3.10) can be given in terms of the claim size distribution F_U which is a bit weak but useful and simple to compute, it's given as:

$$b^*_- = inf_{x\in[0,x'_0)} \frac{e^{\gamma x}\bar{F}_U(x)}{\int_x^\infty e^{\gamma y} dF_U(y)} \quad , \qquad b^*_+ = sup_{x\in[0,x'_0)} \frac{e^{\gamma x}\bar{F}_U(x)}{\int_x^\infty e^{\gamma y} dF_U(y)}, \tag{3.11}$$

where γ is the solution to (3.8) and $x'_0 = sup\{x: F_U < 1\}$.

The constants b^*_-, b_-, b_+, b^*_+ given by (3.10) and (3.11) respectively, satisfy $0 \leq b^*_- \leq b_- \leq b_+ \leq b^*_+ \leq 1$

The ruin probability $\tilde{\varphi}(w)$ in the Sparre Andersen Model with general distributions of inter-claim times, say F_T and exponentially distributed claim sizes with the parameter δ, the adjustment coefficient γ is the solution to (3.8). Also, the constants $b^*_- = b^*_+ = \frac{\delta-\gamma}{\delta}$ and through this (3.7) is derived for the ruin probability.

4 MODEL ANALYSIS AND RESULTS

The financial contract under study is modelled in a way like the risk reserve where the rate at which loans (which are constant) arrives follow a Poisson process. The model constructed for studying the ruin probabilities of this financial contract is incorporated in the Sparre Andersen model which also considers the risk reserve process in continuous time. The loan (claim) counting process is also Poisson process which is governed by a sequence of independent and identically distributed inter-occurrence times with the claim being constant and are independent of the inter occurrence time. Furthermore, the Cramér-Lundberg bound for the ruin function used in the Sparre Andersen model with general distributions of inter-occurrence time and claim sizes will be used in the model understudy in creating the bounds for the ruin probability of the model.

4.1 Embedding of the model into Sparre-Andersen Model

Recalling from Chapter 3, the risk reserve and ruin probability given by (3.4) and (3.6) respectively is similar to (3.2) and (3.3) which are the model and ruin probability respectively understudy. Therefore our model can be embedded and analysed the same way as it was done for the Sparre Andersen model. The following comparison can be done by assigning the variables in the Sparre Andersen Model to that of the model understudy resulting in:

$$\begin{cases} \beta = 1 \\ \mathrm{T_k} = \min(M, D_j), k = j = 1,2 \ldots \quad \text{and} \quad w = v \\ U_k = L, \quad k = 1,2 \ldots \end{cases}$$

Giving:

$$\tilde{\varphi}(v) = \mathbb{P}\left(\min_n (v + \sum_{j=1}^{n} \{\min\,(M, D_j) - L\} < 0\right)$$

Hence the ruin probability for our model and the ruin probability after embedding into Sparre Andersen model are equal: $\varphi(v) = \tilde{\varphi}(v)$.

According to one of the assumptions made by Sparre Andersen, the means of the distribution of both the inter-claim times and claim sizes exist and are finite. So this assumption is checked by the model understudy in the following way:

The loan (claim sizes) is constant for the model understudy so the mean of that is:

$$\mathbb{E}L = L < \infty$$

The inter-claim times follow a truncated exponential. Therefore the mean is given as:

$$\begin{aligned} \mathbb{E}T &= \mathbb{E}\min(\widetilde{M}, D_j) = \mathbb{E}(\mathbb{E}(T|D) \\ &= \mathbb{E}(T|D > \widetilde{M})\mathbb{P}(D > \widetilde{M}) + \mathbb{E}(T|D < \widetilde{M})\mathbb{P}(D < \widetilde{M}) \\ &= \mathbb{E}(\widetilde{M})\mathbb{P}(D > \widetilde{M}) + \mathbb{E}(D|D < \widetilde{M})\mathbb{P}(D < \widetilde{M}) \\ &= \widetilde{M} * \mathbb{P}(D > \widetilde{M}) + \mathbb{E}(D|D < \widetilde{M})\mathbb{P}(D < \widetilde{M}) \\ &= \widetilde{M} * (1 - \mathbb{P}(D < \widetilde{M})) + \mathbb{E}(D|D < \widetilde{M})\mathbb{P}(D < \widetilde{M}) \end{aligned}$$

Since D is exponentially distributed with parameter $\tilde{\mu}$ we then have:

$F(x) = \mathbb{P}(D < \widetilde{M}) = 1 - e^{-\tilde{\mu}x}$, $dF(x) = \tilde{\mu}e^{-\tilde{\mu}x}dx$, $(D|D < \widetilde{M}) = x$

Hence

$$\widetilde{M} * \left(1 - \mathbb{P}\left(D < \widetilde{M}\right)\right) = \widetilde{M}(1 - \left(1 - e^{-\tilde{\mu}x}\right) = \widetilde{M}e^{-\tilde{\mu}\widetilde{M}}$$

$$\mathbb{E}\left(D \middle| D < \widetilde{M}\right)\mathbb{P}(D < \widetilde{M}) = \int_0^{\widetilde{M}} x\tilde{\mu}e^{-\tilde{\mu}x}dx$$

Using integration by part, i.e. $uv - \int vdu$; $u = x$, $dv = \tilde{\mu}e^{-\tilde{\mu}x}$, $v = -e^{-\tilde{\mu}x}$, $du = 1$

$$\int_0^{\widetilde{M}} x\tilde{\mu}e^{-\tilde{\mu}x}dx = \left[-xe^{-\tilde{\mu}x}\right]_0^{\widetilde{M}} + \int_0^{\widetilde{M}} e^{-\tilde{\mu}x}$$

$$= -\widetilde{M}e^{-\tilde{\mu}\widetilde{M}} + \left[-e^{-\tilde{\mu}x}\right]_0^{\widetilde{M}}$$

$$= -\widetilde{M}e^{-\tilde{\mu}\widetilde{M}} - \frac{1}{\tilde{\mu}}\left[-e^{-\tilde{\mu}\widetilde{M}} - 1\right]$$

$$= \frac{1}{\tilde{\mu}} - \left(\frac{1}{\tilde{\mu}} + \widetilde{M}\right)e^{-\tilde{\mu}\widetilde{M}}$$

Hence,

$$\mathbb{E}T = \widetilde{M}e^{-\widetilde{\mu}\widetilde{M}} + \frac{1}{\tilde{\mu}} - \left(\frac{1}{\tilde{\mu}} + \widetilde{M}\right)e^{-\tilde{\mu}\widetilde{M}} = \frac{1 - e^{-\tilde{\mu}\widetilde{M}}}{\tilde{\mu}} < \infty$$

The above result is also very useful and important because it tells something useful about the probability of ruin and this leads to the condition, which let one avoid a trivial case, where ruin occurs with probability equal one.

Thus, we assume that the following condition holds:

B: *The expectation or the mean of the distribution of inter-claim times should be greater than the loan size i.e.* $\mathbb{E}T = \frac{1 - e^{-\tilde{\mu}\widetilde{M}}}{\tilde{\mu}} > L$.

Embedment of our model into that of the Sparre Andersen Model and fulfilling the assumptions and conditions made by that model allows us to find the constants for the Cramér-Lundberg type bounds for the model and this is done in the next section.

4.2 Cramér-Lundberg type bounds for ruin probabilities

The Cramér-Lundberg type bound for the ruin probability of the initial capital v is given as,

$$b_- \, e^{-\gamma v} \leq \tilde{\varphi}(v) \leq b_+ e^{-\gamma v} \tag{4.1}$$

with b_- and b_+ given by (3.10) in section 3.3 and γ is the adjustment coefficient and also the unique positive root to the moment generating function of the model given as,

$$\widehat{m}_Y(s) = \widehat{m}_L(s)\hat{l}_T(s) = 1 \tag{4.2}$$

The equations (3.9) and (4.1) are of the same form and also (3.8) and (4.2) are similar.

Calculating for the moment generating function of the model leads to:

$$\widehat{m}_L(s) = \mathbb{E}(e^{sL}) = e^{sL}$$

This is obtained as a result of L being constant and

$$\begin{aligned}
\hat{l}_T(s) &= \mathbb{E}(e^{-sT}) = \mathbb{E}\left(e^{-s*min(\widetilde{M},D)}\right) \\
&= \int_0^M e^{-sx}\tilde{\mu}e^{-\tilde{\mu}x}dx + e^{-s\widetilde{M}} * e^{-\tilde{\mu}\widetilde{M}} \\
&= \left[-\frac{e^{-(s+\tilde{\mu})x}}{s+\tilde{\mu}}\tilde{\mu}\right]_0^{\widetilde{M}} + e^{-s\widetilde{M}} * e^{-\tilde{\mu}\widetilde{M}} \\
&= \frac{\tilde{\mu}}{s+\tilde{\mu}} - \frac{\tilde{\mu}}{s+\tilde{\mu}}e^{-(s+\tilde{\mu})\widetilde{M}} + e^{-(s+\tilde{\mu})\widetilde{M}}
\end{aligned}$$

Hence,

$$\widehat{m}_Y(s) = \mathbb{E}e^{sY} = \mathbb{E}\left(e^{s(L-T)}\right) = \left[\frac{\tilde{\mu}}{s+\tilde{\mu}}\left(1 - e^{-(s+\tilde{\mu})\widetilde{M}}\right) + e^{-(s+\tilde{\mu})\widetilde{M}}\right]e^{sL}$$

and equation (4.2) takes the following form

$$\widehat{m}_Y(s) \quad = \frac{\tilde{\mu}}{s+\tilde{\mu}}\left[1 - e^{-s\widetilde{M}}e^{-\tilde{\mu}\widetilde{M}}\right]e^{sL} + e^{-\tilde{\mu}\widetilde{M}}e^{-s(\widetilde{M}-L)} = 1 \tag{4.3}$$

When $s = 0$ then (4.3) has a root of 1:

$$\begin{aligned}
\widehat{m}_Y(0) &= \frac{\tilde{\mu}}{0+\tilde{\mu}}\left[1 - e^{-0*\widetilde{M}}e^{-\tilde{\mu}\widetilde{M}}\right]e^{0*L} + e^{-\tilde{\mu}\widetilde{M}}e^{-0*(\widetilde{M}-L)} \\
&= 1 - e^{-\tilde{\mu}\widetilde{M}} + e^{-\tilde{\mu}\widetilde{M}} \\
&= 1
\end{aligned}$$

Moreover, $\widehat{m}_Y(s) = \mathbb{E}e^{sY_j} = \int_{-\infty}^{\infty} e^{sx}F_Y(dx)$ is a convex function in s since e^{sx} is a convex function. Then, the convex function can be in any of these four forms illustrated below:

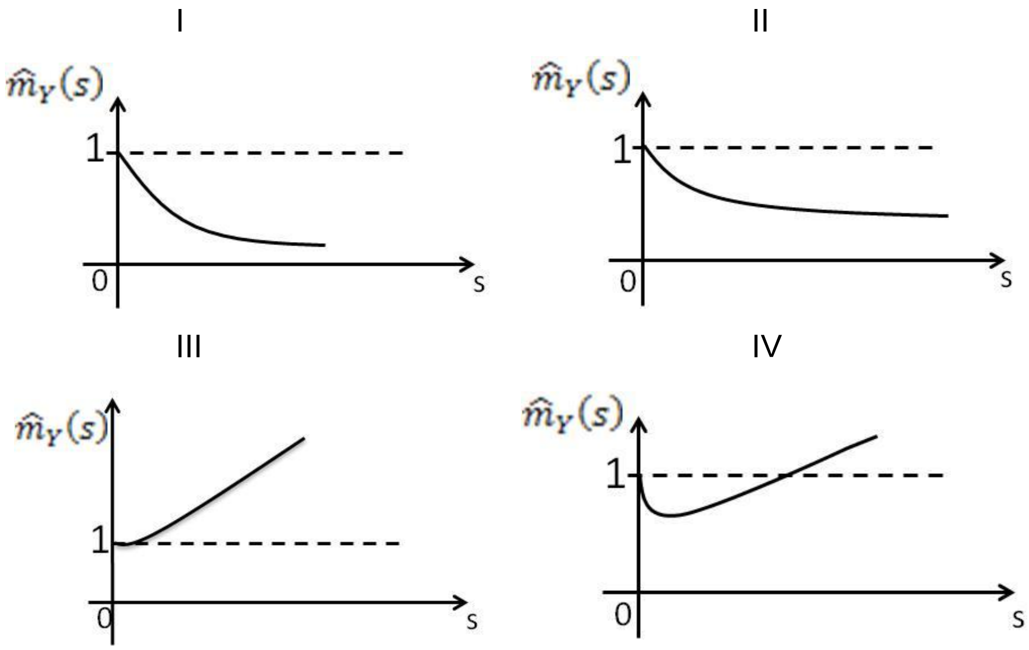

Figure 4.1: Illustration of the shapes of the convex function

Investigating the convexity of $\hat{\boldsymbol{m}}_{\boldsymbol{Y}}(\boldsymbol{s})$

$\hat{m}_Y(s)$ is further investigated to show which of the above convex function it takes.

With $Y_j = L - \min(\widetilde{M}, D_j)$, the distribution function of Y can be illustrated as:

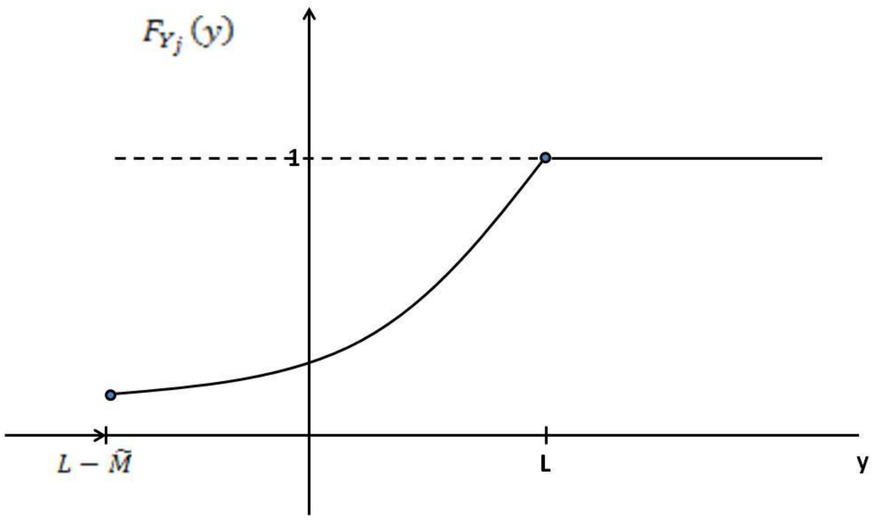

Figure 4.2: Illustration of the distribution function of Y

That is, $F_{Y_j}(y) = \mathbb{P}\{L - \min(\widetilde{M}, D_j) \le y\} = \mathbb{P}\{\min(\widetilde{M}, D_j) \ge L - y\}$

$$= \begin{cases} 1 & if \quad L - y < 0 \\ e^{-\widetilde{\mu}(L-y)} & if \quad 0 \le L - y \le \widetilde{M} \\ 0 & if \quad L - y > \widetilde{M} \end{cases}$$

$$= \begin{cases} 1 & if \quad L > 0 \\ e^{-\widetilde{\mu}(L-y)} & if \quad L - \widetilde{M} \le y \le L \\ 0 & if \quad y < L - \widetilde{M} \end{cases}$$

With $L - \widetilde{M} \le Y_j \le L$ being a bounded random variable

The first and second differential of $\widehat{m}_Y(s)$ given as:

$$\begin{cases} \widehat{m}'_Y(s) = \int_{-\infty}^{\infty} x e^{sx} F_Y(dx) \\ \widehat{m}''_Y(s) = \int_{-\infty}^{\infty} x^2 e^{sx} F_Y(dx) > 0 \end{cases}$$

Indicating that $\widehat{m}_Y(s)$ is a convex function and finding which type of the above diagrams it takes. Standard calculations reveal that with

$\widehat{m}_Y(s) = \frac{\widetilde{\mu}}{s+\widetilde{\mu}} e^{sL} + e^{-\widetilde{\mu}\widetilde{M}} e^{-s(\widetilde{M}-L)} \left(1 - \frac{\widetilde{\mu}}{s+\widetilde{\mu}}\right)$, then $\lim_{s\to\infty} \widehat{m}_Y(s) = +\infty$

As a result of this together with condition A and B holding, the convex function of $\widehat{m}_Y(s)$ will take the form:

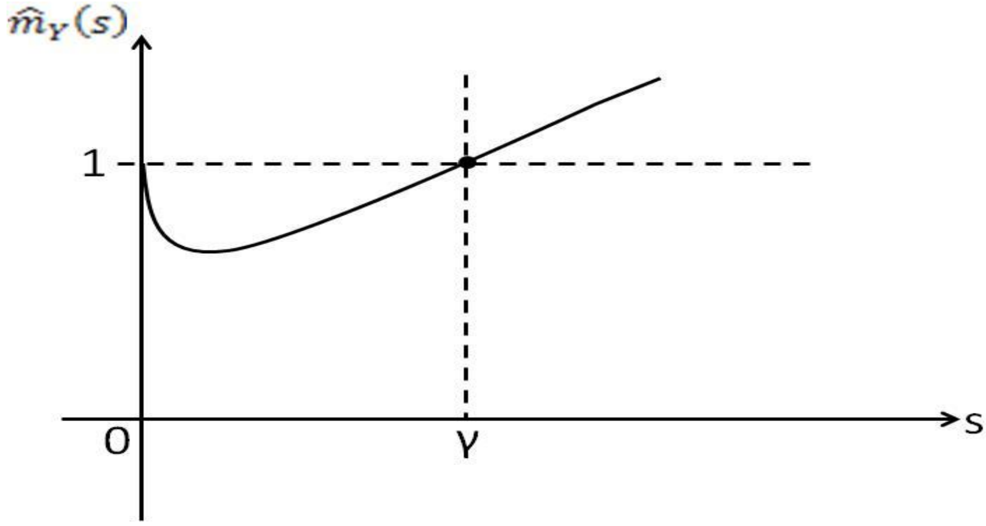

Figure 4.3: Illustration of the convex function of $\widehat{m}_Y(s)$

Thus, $\widehat{m}_Y(s)$ is a nonnegative continuous, convex function such that $\widehat{m}_Y(0) = 1$ and $\widehat{m}_Y(s) \to \infty$ as $s \to \infty$. If γ exist, then we move on to find the constants b^*_-, b_-, b_+, b^*_+ with their expressions given by

$$b_{-} = inf_{x\in[0,x_0)} \frac{e^{\gamma x}\bar{F}_Y(x)}{\int_x^{\infty} e^{\gamma y} dF_Y(y)} \;,\quad b_{+} = sup_{x\in[0,x_0)} \frac{e^{\gamma x}\bar{F}_Y(x)}{\int_x^{\infty} e^{\gamma y} dF_Y(y)}$$

$$b^*_{-} = inf_{x\in[0,x'_0)} \frac{e^{\gamma x}\bar{F}_L(x)}{\int_x^{\infty} e^{\gamma y} dF_L(y)} \;,\quad b^*_{+} = sup_{x\in[0,x'_0)} \frac{e^{\gamma x}\bar{F}_L(x)}{\int_x^{\infty} e^{\gamma y} dF_L(y)}$$

with $x_0 = sup\{x: F_Y < 1\}\ and\ x'_0 = sup\{x: F_L < 1\}$.

Calculating for the constants b^*_{-}, b^*_{+} is as follows:

Let

$F_L(x) = \begin{cases} 1\ if\ x \geq L \\ 0\ if\ x < L \end{cases}$, $\bar{F}_L(x) = 1 - F_L(x) = \begin{cases} 0\ if\ x > L \\ 1\ if\ x < L \end{cases}$ and $x'_0 = L$,

Then

$$b^*_{-} = inf_{0\leq x<L} \frac{e^{\gamma x}*1}{e^{\gamma L}*1} = \frac{1}{e^{\gamma L}} \quad \text{and} \quad b^*_{+} = sup_{0\leq x<L} \frac{e^{\gamma x}*1}{e^{\gamma L}*1} = 1$$

The computation of the bounds using these results of constants b^*_{-}, b^*_{+} is quite straight forward with known parameters. This is as a result of the loan sizes being constants but the result is useful hence it is stated in the following theorem.

Theorem 4.1: *Let assume condition* **B** *holds. Then the following lower and upper bounds hold for the ruin probability,*

$$\frac{1}{e^{\gamma L}} * e^{-\gamma v} \leq \tilde{\varphi}(v) \leq 1 * e^{-\gamma v}$$

where parameter γ is the unique positive root of equation (4.3).

We then move on to calculate for the constants b_{-}, b_{+}. While the calculation of that is a bit complicated, it can be done.

Let

$$x_0 = L, \quad F_Y(y) = \begin{cases} 1 & if\ \ L > 0 \\ e^{-\tilde{\mu}(L-y)} & if\ \ L - \widetilde{M} \leq y \leq L \\ 0 & if\ \ y < L - \widetilde{M} \end{cases} \text{ and}$$

$$\bar{F}_Y(y) = 1 - F_Y(y) = \begin{cases} 0 & if\ \ L > 0 \\ 1 - e^{-\tilde{\mu}(L-y)} & if\ \ L - \widetilde{M} < y \leq L \\ 1 & if\ \ y < L - \widetilde{M} \end{cases}$$

Then

$$b_- = inf_{x\in[0,L)} \underbrace{\frac{e^{\gamma x}\left[1-e^{-\tilde{\mu}(L-x)}\right]}{\int_x^L e^{\gamma y}\tilde{\mu}e^{-\tilde{\mu}L}e^{\tilde{\mu}y}dy}}_{*} \ , \ \ b_+ = sup_{x\in[0,L)} \underbrace{\frac{e^{\gamma x}\left[1-e^{-\tilde{\mu}(L-x)}\right]}{\int_x^L e^{\gamma y}\tilde{\mu}e^{-\tilde{\mu}L}e^{\tilde{\mu}y}dy}}_{*}$$

Computing *

$$\frac{e^{\gamma x}\left[1-e^{-\tilde{\mu}(L-x)}\right]}{\int_x^L e^{\gamma y}\tilde{\mu}e^{-\tilde{\mu}L}e^{\tilde{\mu}y}dy} = \frac{e^{\gamma x}\left[1-e^{-\tilde{\mu}(L-x)}\right]}{\tilde{\mu}e^{-\tilde{\mu}L}\left[\frac{e^{(\gamma+\tilde{\mu})y}}{\gamma+\tilde{\mu}}\right]_x^L}$$

$$= \frac{e^{\gamma x}\left[1-e^{-\tilde{\mu}L}e^{\tilde{\mu}x}\right]}{\frac{\tilde{\mu}}{\gamma+\tilde{\mu}}e^{-\tilde{\mu}L}\left[e^{(\gamma+\tilde{\mu})L}-e^{(\gamma+\tilde{\mu})x}\right]}$$

$$= \frac{e^{\gamma x}-e^{-\tilde{\mu}L}e^{(\gamma+\tilde{\mu})x}}{\frac{\tilde{\mu}}{\gamma+\tilde{\mu}}e^{\gamma L}-\frac{\tilde{\mu}}{\gamma+\tilde{\mu}}e^{-\tilde{\mu}L}e^{(\gamma+\tilde{\mu})x}}$$

Let

$$f(x) = \frac{e^{\gamma x}-e^{-\tilde{\mu}L}e^{(\gamma+\tilde{\mu})x}}{\frac{\tilde{\mu}}{\gamma+\tilde{\mu}}e^{\gamma L}-\frac{\tilde{\mu}}{\gamma+\tilde{\mu}}e^{-\tilde{\mu}L}e^{(\gamma+\tilde{\mu})x}}$$

and finding derivative using the quotient rule of differentiation result in:

$$f'(x) = \frac{\left[\gamma e^{\gamma x}-e^{-\tilde{\mu}L}(\gamma+\tilde{\mu})e^{(\gamma+\tilde{\mu})x}\right] * \left[\frac{\tilde{\mu}}{\gamma+\tilde{\mu}}e^{\gamma L}-\frac{\tilde{\mu}}{\gamma+\tilde{\mu}}e^{-\tilde{\mu}L}e^{(\gamma+\tilde{\mu})x}\right]}{\left[\frac{\tilde{\mu}}{\gamma+\tilde{\mu}}e^{\gamma L}-\frac{\tilde{\mu}}{\gamma+\tilde{\mu}}e^{-\tilde{\mu}L}e^{(\gamma+\tilde{\mu})x}\right]^2}$$

$$-\frac{\left[e^{\gamma x}-e^{-\tilde{\mu}L}e^{(\gamma+\tilde{\mu})x}\right] * \left[-\frac{\tilde{\mu}}{\gamma+\tilde{\mu}}e^{-\tilde{\mu}L}(\gamma+\tilde{\mu})e^{(\gamma+\tilde{\mu})x}\right]}{\left[\frac{\tilde{\mu}}{\gamma+\tilde{\mu}}e^{\gamma L}-\frac{\tilde{\mu}}{\gamma+\tilde{\mu}}e^{-\tilde{\mu}L}e^{(\gamma+\tilde{\mu})x}\right]^2}$$

Expanding $f'(x)$ can be a bit messy and we can lose track. So we will then tend to a different approach by simplifying $f(x)$ before finding the derivative.

This is done by adding and subtracting $e^{-\tilde{\mu}L}e^{(\gamma+\tilde{\mu})L}$ to $f(x)$ to the numerator which result in:

$$f(x) = \frac{e^{\gamma x}-e^{-\tilde{\mu}L}e^{(\gamma+\tilde{\mu})L}+e^{-\tilde{\mu}L}e^{(\gamma+\tilde{\mu})L}-e^{-\tilde{\mu}L}e^{(\gamma+\tilde{\mu})x}}{\frac{\tilde{\mu}}{\gamma+\tilde{\mu}}e^{-\tilde{\mu}L}\left[e^{(\gamma+\tilde{\mu})L}-e^{(\gamma+\tilde{\mu})x}\right]}$$

$$= \frac{e^{\gamma x} - e^{-\tilde{\mu}L} e^{(\gamma+\tilde{\mu})L}}{\frac{\tilde{\mu}}{\gamma+\tilde{\mu}} e^{-\tilde{\mu}L}\left[e^{(\gamma+\tilde{\mu})L} - e^{(\gamma+\tilde{\mu})x}\right]} + \frac{e^{-\tilde{\mu}L} e^{(\gamma+\tilde{\mu})L} - e^{-\tilde{\mu}L} e^{(\gamma+\tilde{\mu})x}}{\frac{\tilde{\mu}}{\gamma+\tilde{\mu}} e^{-\tilde{\mu}L}\left[e^{(\gamma+\tilde{\mu})L} - e^{(\gamma+\tilde{\mu})x}\right]}$$

$$= \frac{e^{\gamma x} - e^{-\tilde{\mu}L} * e^{\tilde{\mu}L} * e^{\gamma L}}{\frac{\tilde{\mu}}{\gamma+\tilde{\mu}} e^{-\tilde{\mu}L}\left[e^{(\gamma+\tilde{\mu})L} - e^{(\gamma+\tilde{\mu})x}\right]} + \frac{e^{-\tilde{\mu}L}\left[e^{(\gamma+\tilde{\mu})L} - e^{(\gamma+\tilde{\mu})x}\right]}{\frac{\tilde{\mu}}{\gamma+\tilde{\mu}} e^{-\tilde{\mu}L}\left[e^{(\gamma+\tilde{\mu})L} - e^{(\gamma+\tilde{\mu})x}\right]}$$

$$= \frac{e^{\gamma x} - e^{\gamma L}}{\frac{\tilde{\mu}}{\gamma+\tilde{\mu}} e^{-\tilde{\mu}L}\left[e^{(\gamma+\tilde{\mu})L} - e^{(\gamma+\tilde{\mu})x}\right]} + \frac{1}{\frac{\tilde{\mu}}{\gamma+\tilde{\mu}}}$$

$$= \frac{1}{\frac{\tilde{\mu}}{\gamma+\tilde{\mu}} e^{-\tilde{\mu}L}} * \frac{e^{\gamma x} - e^{\gamma L}}{e^{(\gamma+\tilde{\mu})L} - e^{(\gamma+\tilde{\mu})x}} + \frac{1}{\frac{\tilde{\mu}}{\gamma+\tilde{\mu}}}$$

$$f(x) = \underbrace{\frac{\gamma+\tilde{\mu}}{\tilde{\mu}} + \frac{\gamma+\tilde{\mu}}{\tilde{\mu}} e^{\tilde{\mu}L}}_{(A)} * \underbrace{\frac{e^{\gamma x} - e^{\gamma L}}{e^{(\gamma+\tilde{\mu})L} - e^{(\gamma+\tilde{\mu})x}}}_{(B)}$$

From the equation above, (A) can be determined since it does not depend on x, we then move on to take the derivative of (B).

Letting $g(x) = \frac{e^{\gamma x} - e^{\gamma L}}{e^{(\gamma+\tilde{\mu})L} - e^{(\gamma+\tilde{\mu})x}}$ and finding the derivative of this function as it what was done for $f'(x)$ will result in:

$$g'(x) = \frac{\gamma e^{\gamma x}\left[e^{(\gamma+\tilde{\mu})L} - e^{(\gamma+\tilde{\mu})x}\right] - \left[e^{\gamma x} - e^{\gamma L}\right]\left[-(\gamma+\tilde{\mu}) e^{(\gamma+\tilde{\mu})x}\right]}{\left[e^{(\gamma+\tilde{\mu})L} - e^{(\gamma+\tilde{\mu})x}\right]^2}$$

$$= \frac{\gamma e^{(\gamma+\tilde{\mu})L} e^{\gamma x} - \gamma e^{(2\gamma+\tilde{\mu})x} + (\gamma+\tilde{\mu}) e^{(2\gamma+\tilde{\mu})x} - (\gamma+\tilde{\mu}) e^{\gamma L} e^{(\gamma+\tilde{\mu})x}}{\left[e^{(\gamma+\tilde{\mu})L} - e^{(\gamma+\tilde{\mu})x}\right]^2}$$

This shows the function $f(x)$ is differentiable and the maximum and minimum of this function can be found. Hence we state the result in the theorem below.

Theorem 4.2: *Let assume condition* **B** *holds. Then the following lower and upper bounds hold for the ruin probability*

$$b_- e^{-\gamma v} \le \tilde{\varphi}(v) \le b_+ e^{-\gamma v}$$

where parameter γ *is the unique positive root of the equation* (4.3) *and* b_- *and* b_+ *are, respectively, the minimum and maximum values for the function* $f(x) = \frac{e^{\gamma x} - e^{-\tilde{\mu}L} e^{(\gamma+\tilde{\mu})x}}{\frac{\tilde{\mu}}{\gamma+\tilde{\mu}} e^{\gamma L} - \frac{\tilde{\mu}}{\gamma+\tilde{\mu}} e^{-\tilde{\mu}L} e^{(\gamma+\tilde{\mu})x}}$ *in the interval* [*0, L*].

We conclude from the above calculations that there exist explicit solutions in finding the values for the parameter $b^*{}_-, b_-, b_+, b^*{}_+$ for the Cramér-Lundberg bounds for our model with γ given which also has an explicit solution.

The values b_- and b_+ can be investigated numerically for given parameters, L, $\tilde{\mu}$ and γ.

5 NUMERICAL STUDIES

The functions derived for obtaining the approximate values of $\boldsymbol{\gamma}$ and $\boldsymbol{b_-}, \boldsymbol{b_+}$ in (4.4) and $\boldsymbol{f(x)}$ respectively was investigated numerically using Matlab. This is done by choosing some reasonable values for the constant variables in the model with the conditions A and B holding. The investigation of the convexity of $\boldsymbol{\hat{m}_Y(s)}$ in Figure 4.3 is done and the value of $\boldsymbol{\gamma}$ is obtained. This is then used to find the approximate values for the constants of the lower and upper bounds for the ruin probability which are $\boldsymbol{b_-}$ and $\boldsymbol{b_+}$ respectively.

Investigation of $\boldsymbol{\hat{m}_Y(s)}$ and numerical values for $\boldsymbol{\gamma}$

It is known from chapter 2 that, the model is considered in the homogenous state where the loan sizes, time to maturity, the premium and the default are assumed to be constants and identical for all clients. The effect of values of these parameters on the value of $\boldsymbol{\gamma}$ is investigated by keeping one value constants and varying the rest.

1. $\boldsymbol{L = 5000}$, $\boldsymbol{M = 2, Lr = 0.2 * L}$ and varying alpha ($\boldsymbol{\alpha}$)

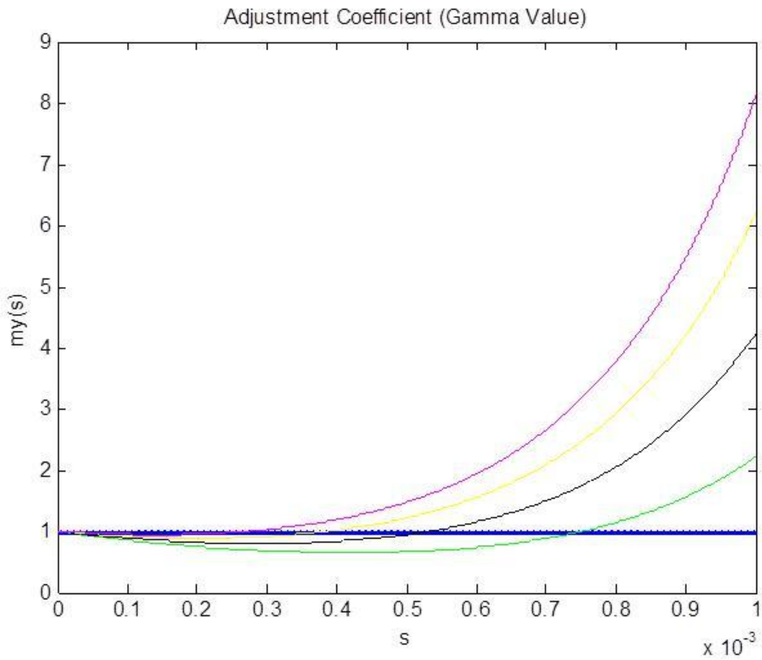

Figure 5.1: Effect of Default Probability on the adjustment coefficient (green is $\boldsymbol{\alpha = 0.2}$, black is $\boldsymbol{\alpha = 0.15}$, yellow is $\boldsymbol{\alpha = 0.1}$, and pink is $\boldsymbol{\alpha = 0.05}$)

Numerical Values for the approximate adjustment coefficient with different alphas.

Table 5.1

Default Probability $(\boldsymbol{\alpha})$	0.05	0.1	0.15	0.2
$\boldsymbol{\gamma}$	0.0007581	0.0005346	0.0003894	0.0002604

2. $\boldsymbol{L = 5000}$, $\boldsymbol{\alpha = 0.05}, \boldsymbol{Lr = 0.2 * L}$ and varying $\boldsymbol{M}$

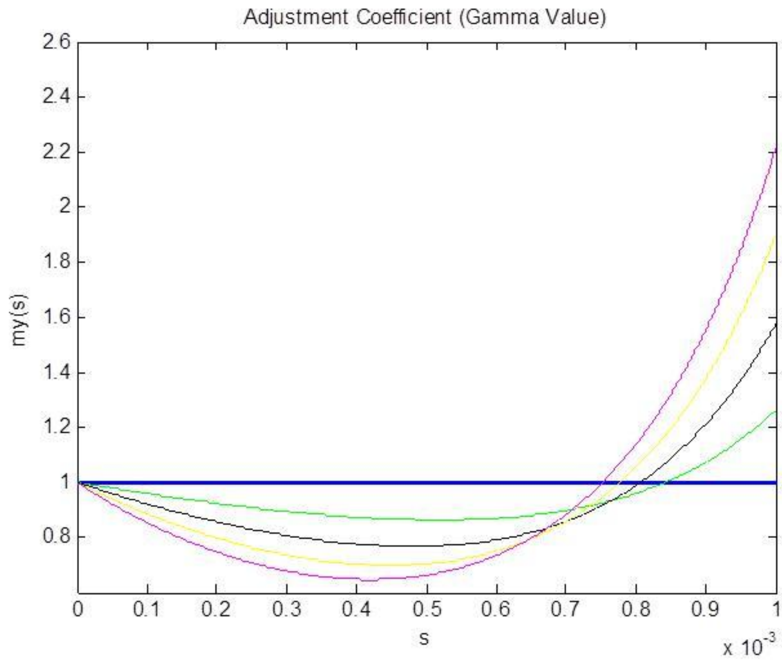

Figure 5.2: Effect of Time to Maturity on the adjustment coefficient (green is $M = 0.5$, black is $M =$, yellow is $M = 1.5$, and pink is $M = 2$)

Numerical Values for the approximate the adjustment coefficient with different time to maturity.

Table 5.2

Time to Maturity (M)	6months	1 year	1.5 years	2years
γ	0.0008502	0.0008111	0.0007834	0.0007558

3. $L = 5000$, $\alpha = 0.05$, $M = 2$ and varying Lr

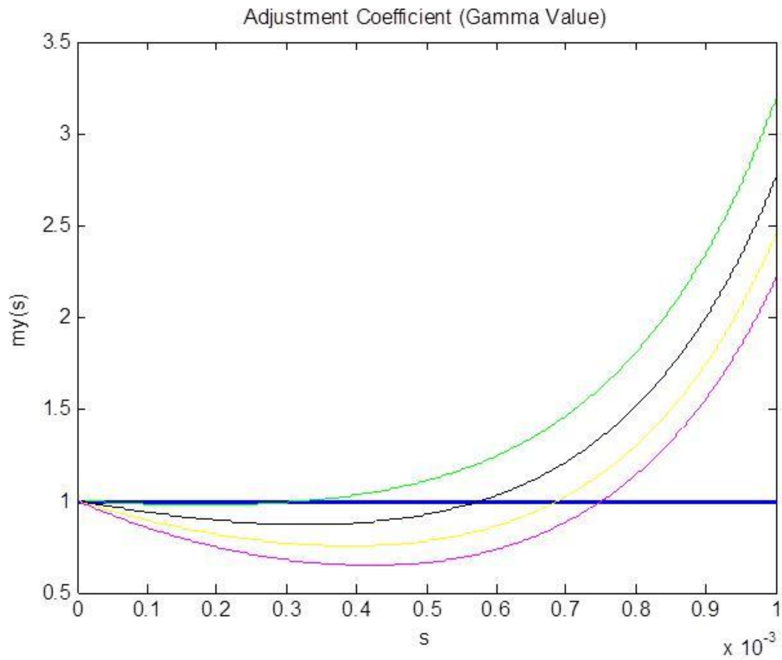

Figure 5.6: Effect of Premium on the adjustment coefficient (green is $Lr = 0.05 * L$, black is $Lr = 0.1 * L$, yellow is $Lr = 0.15 * L$, and pink is $Lr = 0.2 * L$)

Numerical Values for the approximate adjustment coefficient with different premium as.

Table 5.3

Premium (Lr)	0.05*L	0.1*L	0.15*L	0.2*L
γ	0.0003203	0.0005806	0.0006935	0.0007581

The constants b_-, b_+ for the Cramér-Lundberg type Bounds

We investigate $f(x)$ numerically by choosing some of the values of the adjustment coefficient γ obtained and using that to find their corresponding b_+ and b_- of the function which are the minimum (inf) and maximum (sup) of the function respectively. The value of the constant b_- and b_+ with different values of γ is as shown in the table below:

Table 5.4:

Adjustment Coefficient (γ)	**Upper bound constant** (b_-)	**Upper bound constant** $b_+)$
0.0002604	0	0.0003956
0.0003894	0	0.0006769
0.0005346	0	0.001296
0.0006935	0	0.002974
0.0007558	0	0.00451
0.0008502	-0.03	0.02185

After observing Figures 5.4-5.6 and Tables 5.1-5.4, the following results were obtained:

(1) It is proved numerically Figure 4.3 holds as it is presented in Figures 5.1-5.3 with conditions A and B holding.

(2) Increasing both the probability of default $\boldsymbol{\alpha}$ and the time to maturity $\boldsymbol{M}$ decreases the value of the adjustment coefficient $\boldsymbol{\gamma}$ but the adjustment coefficient decreases rapidly in the case of the probability of default than that of the time to maturity

(3) Increasing the premium $\boldsymbol{Lr}$ increases the value of the adjustment coefficient $\boldsymbol{\gamma}$.

(4) The adjustment coefficient is strongly affected by the premium and the default probability as compared to the time to maturity.

(5) The values of the upper bound constant are less than one satisfying $\boldsymbol{b}_+ \leq \mathbf{1}$ on the other hand the values for $\boldsymbol{b}_-$ are all zero except for when $\boldsymbol{\gamma} = \mathbf{0.0008502}$ which is negative. This indicates that values of $\boldsymbol{b}_-$ are not really useful. Hence only the constant upper bound of the function actually will be useful in determining the bound for the ruin probability.

(6) The upper bound constant increases with increasing value of the adjustment coefficient.

(7) The larger the value of $\boldsymbol{\gamma}$ the smaller the upper bound for $\boldsymbol{v}$ will be. That is larger values of $\boldsymbol{\gamma}$ imply smaller ruin probabilities.

6 SUMMARY AND CONCLUSION

In summary, the homogeneous portfolio of financial contracts for the lending aspect of a financial institution was used to investigate how the cash flow is affected by credit risk using ruin theory from insurance. The understudy model was analysed by embedding it in the Sparre Andersen Model with almost all the assumption made by that model holding. Theoretically, the distribution of the model was identified and the ruin probability of the model was calculated. The adjustment coefficient which was the second root of the moment generation function of the model was found and this was used to calculate the Cramér-Lundberg type bounds for the ruin probability. Numerical studies was further done to obtain an approximate value for the adjustment coefficient $\boldsymbol{\gamma}$ and the effect of changing the interest rate, time to maturity and probability of default with the Loan size fixed. The values obtained for the adjustment coefficient was further used to find the approximate numerical values for the constants $\boldsymbol{b}_-$ and $\boldsymbol{b}_+$ of the bounds for the ruin probability.

In conclusion, we need to appreciate the fact that the introduction of ruin probabilities for stochastic modelling techniques has aided the insurance and banking sector in quantifying market risks. It was found that there are explicit solutions in finding the adjustment coefficient $\boldsymbol{\gamma}$ and the constants $\boldsymbol{b}_-$ and $\boldsymbol{b}_+$ with all the assumptions made in place.

It has been established in this study that if a value of the adjustment coefficient can be found, then values of the constants for the bounds can be determined. The adjustment coefficient can be interpreted as a risk measure since it is a function of the parameters that determine or affect the ruin probability. Besides using the adjustment coefficient in determining the bounds of ruin probabilities which is also used as a risk measure for the credit risk exposure of financial contracts (loans), it can be used to determine initial capital/surplus the institution should hold and proper interest rate which is used to obtain the premium.

This project has examined the application of ruin probabilities of stochastic modelling techniques to credit risks facing the financial industry. This theoretical research is used to explore risks associated with financial institutions and approaches to minimize default using the specific, measurable, achievable and timely approach of ruin probabilities for stochastic modelling. This study is important in strengthening the early warning signs of credit default and is vital for controlling loans. It will be of an interest to further study other dynamic models instead of only in the case of homogeneous loans incorporating other elements that are practically applicable and also to implement the model and the bound found for the ruin probability using real data from different financial institutions with the aim to establishing case study realities that will have general and specific applications for financial institutions.

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APPENDIX: MATLAB CODES

```
%Moment Generating Function in calculating the value for the Adjustment
%Coefficient (Gamma)
function my=f(s,M,L,Lr,alpha)
M1=(L/M+Lr)*M; %M-tilda
mu=alpha/(L/M+Lr); %mu-tilda
my=((mu./(s+mu)).*(1-exp(-(s+mu)*M1))+exp(-(s+mu)*M1)).*exp(s*L);
end
%Function for calculating the upper and lower bounds for the constants b(+)
%and b(-) of the ruin probability.
function bvalue=f2(x,M,L,Lr,alpha,gamma)
mut=alpha/(L/M+Lr);%mu-tilda
bvalue=(exp(x.*gamma)-exp(-
mut*L)*exp(x.*(gamma+mut)))./((mut/(gamma+mut))*exp(-
mut*L)*(exp(gamma+mut)*L)-exp(x.*(gamma+mut)));
end
%Main Matlab Codes for the implementation of the two Functions
clear all
clc
close all
%Assigned values for the parameters (L, M, Lr, alpha) in the Moment
%Generating Function for Calculating the Adjustment Coefficient and
Upper&Lower bounds of the ruin probability. These
%Values Can be Varied.
M=2;         %Time to maturity
alpha=0.2;   %probability of default
L=5000;      %Loan size
Lr=0.2*L;    %premium
s=0:1e-8:1e-3; % This value be any value from zero to infinity
%% Calculating for the approximate value of the adjustment
%% Coefficent(Gamma)
```

```
my=f(s,M,L,Lr,alpha); %moment generating function
m0=1;% Value of the moment generating function when s=0
ml=exp(s.*L);
figure(1)
plot(s,my, 'b')
hold on
plot(s,m0, 'g')
xlabel('s')
ylabel('my(s)')
title('Gamma Value')
% %%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Calculating for the Upper and Lower Bound Constant for the ruin
%% probability
gamma=0.0002604; %Adjustment Coefficient
x=0:1e-3*L:L;% This value is from zero to the value of the size of Loans
bvalue=f2(x,M,L,Lr,alpha,gamma);
% Location of minimum point:
Low = find(bvalue == min(bvalue));
% Just in case there are multiple mins
Low = Low(1);
% Location of maximum point:
High = find(bvalue == max(bvalue));
% Just in case there are multiple maxes:
High = High(1);
%plot(x,bvalue)
% Plot and annotate the results:
figure
plot(x,bvalue,'b.', ...
x(Low),bvalue(Low),'rs', ...
x(High),bvalue(High),'g^')
grid on
hold on
```

```
text(x(Low),bvalue(Low),'Min')
text(x(High),bvalue(High),'Max')
hold off
```