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Argmax over Continuous Indices of Random Variables – An Approach Using Random Fields

Hannes Malmberg*

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Abstract

In commuting research, it has been shown that it is fruitful to model choices as optimization over a discrete number of random variables. In this essay we pass from the discrete to the continuous case, and consider the limiting distribution as the number of offers grow to infinity. The object we are looking for is an argmax measure, describing the probability distribution of the location of the best offer.

Mathematically, we have $\Omega \subseteq R^k$ and seek a probability distribution over Ω. The first argument of the argmax measure is Λ, a probability distribution over Ω, which describes the relative intensity of offers received from different parts of space. The second argument is a measure index $\mu : \Omega \to \mathcal{P}^R$ which associates every point in Ω with a distribution over R, and describes how the type of random offers varies over space.

To define an argmax measure, we introduce a concept called *point process argmax measure*, defined for deterministic point processes. The general argmax measure is defined as the limit of such processes for triangular arrays converging to the distribution Λ.

Introducing a theoretical concept called a max-field, we use continuity properties of this field to construct a method to calculate the argmax measure. The usefulness of the method is demonstrated when the offers are exponential with a deterministic additive disturbance term – in this case the argmax measure can be explicitly calculated.

In the end simulations are presented to illustrate the points proved. Moreover, it is shown that several research developments exist to extend the theory developed in the paper.

*Postal address: Mathematical Statistics, Stockholm University, SE-106 91, Sweden. E-mail:per.hannes.malmberg@googlemail.com. Supervisor: Ola Hössjer, Dmitrii Silvestrov.