Abstracts
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Invited Lectures

Actuarial Education in Sweden and Europe

Erik Alm

Hannover Re, Stockholm, Sweden
E-mail: erik.alm@hannover-re.com

An overview is given of the educational requirements for becoming a full member of an actuarial association in Europe, with special emphasis on recent changes in requirements. Some comments are given to the effect on corresponding changes in Sweden.

References

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[3] Svenska Aktuarieföreningen (2013). Kompetenskrav för olika medlemskap inom Svenska Aktuarieföreningen.

Some Mathematical Aspects of the New European Regime in Insurance, New Ideas Regarding Solvency and Occupational Pension through the Eyes of EIOPA

Gunnar Andersson

Folksam / KTH Royal Institute of Technology, Stockholm, Sweden
E-mail: gunnar.andersson@ActStrats.com

Supervision of the European insurance industry is carried out by national supervisory authorities and EIOPA, European Insurance and

Occupational Pensions Authority, located in Frankfurt. Several major changes are made in the legislative system relative the former system. One of the most important changes is that the solvency regime is about to change to a risk based system. This is not yet in place in the different national jurisdictions but is expected to be adopted fully within a few years. This will have a major impact in the insurance industry. Moreover, the mathematical basis for the new solvency regime is heavily discussed. This new system includes non-life insurance and life insurance as well as the pension industry. The risk based solvency system is built up on different modules. For each module risks are evaluated and the capital need for running the system is calculated. We will discuss certain features of the system. Furthermore, we will look at the system suggested to be adopted for occupational pensions. Since in many situations the liability side in the balance sheet is missing, EIOPA is introducing a so called holistic balance sheet, a tool to evaluate the solvency capital needed for IORP's.

Recent Results on the Ruin Time with Heavy Tail

Søren Asmussen

Aarhus University, Denmark
E-mail: asmus@imf.au.dk

Let $\{Z_n\}_{n\geq 0}$ be a random walk with a negative drift and i.i.d. increments with heavy-tailed distribution and let $M = \sup_{n\geq 0} Z_n$ be its supremum. Asmussen & Klüppelberg [2] considered the behavior of the random walk given that $M > x$, for x large, and obtained a conditional limit theorem, as $x \to \infty$, for the distribution of the quadruple that includes the time $\tau = \tau(x)$ to exceed level x, position Z_τ at this time, position $Z_{\tau-1}$ at the prior time, and the trajectory up to it (similar results were obtained for the Cramér-Lundberg insurance risk process). In particular, the conditional distribution of τ is (after normalization by the mean excess function) either Pareto or exponential. This contrast the light-tailed case, where a different normalization gives a normal limit (Segerdahl, 1955).

We obtain here several extensions of this result to various regenerative-type models and, in particular, to the case of a random walk with dependent increments. Particular attention is given to describ-

ing the limiting conditional behavior of τ. The class of models include Markov-modulated models as particular cases. We also study fluid models, the Björk-Grandell risk process, give examples where the order of τ is genuinely different from the random walk case, and discuss which growth rates are possible. Our proofs are purely probabilistic and are based on results and ideas from Asmussen, Schmidli & Schmidt [3], Foss & Zachary [5], and Foss, Konstantopoulos & Zachary [4], and provide a formal approach to the 'one big jump' heuristics.

References

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Robust Modelling and Management of Longevity Risk

Andrew Cairns
Heriot-Watt University, Edinburgh, U.K.
E-mail: A.J.G.Cairns@hw.ac.uk

In this talk I will focus on the importance of the choice of stochastic mortality model in relation to the intended risk management application. This will involve reviewing how modelling has developed in recent years and whether or not it is fit for purpose. I will argue that we need to be focusing on simpler, more robust models in order to deliver consistent and reliable results for the valuation and manage-

ment of longevity risk. Additionally, I will discuss how a focus on robustness can help determine what are the most appropriate hedging instruments for hedging longevity risk.

SAJ session: Prospects of Future Research in Insurance Mathematics

Boualem Djehiche

Royal Institute of Technology, Stockholm, Sweden
E-mail: boualem@math.kth.se

Paul Embrechts

ETH, Zürich, Switzerland
E-mail: embrechts@math.ethz.ch

Ragnar Norberg

Université Lyon 1, France
E-mail: ragnar.norberg@univ-lyon1.fr

In 2014 the Scandinavian Actuarial Journal will celebrate its 100 years Jubilee as a leading international journal in actuarial science. We will celebrate this event by organizing a session on prospects of future research in insurance mathematics. Boualem Djehiche will open the session by giving a short presentation of the journal and its founders and then Ragnar Norberg and Paul Embrechts each will present their own views on future research in insurance mathematics. The session will end by giving the audience the floor for questions and comments.

Counterparty Default Risk in Solvency II

Younes Elonq

Swedish Financial Supervisory Authority, Stockholm
E-mail: younes.elonq@fi.se

The treatment of counterparty default risk in the standard formula for the Solvency Capital Requirement shall reflect possible losses due to unexpected default, or deterioration in the credit standing, of the counterparties and debtors of insurance and reinsurance undertakings over the forthcoming twelve months. The counterparty default risk shall cover risk-mitigating contracts, such as reinsurance arrange-

ments, securitisations and derivatives, and receivables from intermediaries.

References

[1] Technical Specification on the Long Term Guarantee Assessment (Part I), Mars 2013.
[2] Ter Berg, Portfolio modeling of counterparty reinsurance default risk, Life & Pensions, April 2008.

Risk Aggregation

Paul Embrechts
ETH, Zurich, Switzerland
E-mail: paul.embrechts@math.ethz.ch

The quantitative regulation of banking and insurance is very much based on specific risk measures. Examples include Value-at-Risk (a quantile based measure) and Expected Shortfall (a conditional excess measure). Besides their statistical estimation, recent applications very much use the axiomatic theory of risk measures to investigate allocation and aggregation properties. In this talk I will present the necessary theory (going back to a question of Kolmogorov) on quantile based risk aggregation when only partial information on the underlying stochastic structure is known. Besides discussing some analytic results for sums of risk positions, I will also present a versatile, so-called Rearrangement Algorithm for the numerical calculation of best and worst bounds in a model uncertainty context. As an example we discuss the calculation of risk capital for operational risk within the Basel 3 framework of banking regulation.

Cramér Lecture

Anders Martin-Löf
Stockholm University, Sweden
E-mail: andersml@math.su.se

A short history of the development of the "Swedish school" in risk theory starting with Filip Lundberg and Harald Cramér is given. It is shown that a unified treatment of many problems can be given in the

framework of large deviation theory, which actually originated from the work of Fredrik Esscher in risk theory.

Building an Internal Model on Solid Ground

Ellinor Samuelsson

Swedish Financial Supervisory Authority, Stockholm
E-mail: ellinor.samuelsson@fi.se

According to Article 100 in Directive 2009/138/EC (the Solvency 2 Directive) insurance and reinsurance undertakings are allowed, after approval from the supervisory authority, to use an internal model for calculation of the capital requirement. An internal model has to fulfil a large number of legal requirements to get approved and in this talk I will make an exposition of the legal requirements and put them in relation to what is considered as good scientific modelling practice.

References

[1] Directive 2009/138/EC of the European Union.

An Explicit Expression for the Gerber-Shiu Function with an Application to Risk Measures

Hanspeter Schmidli

University of Cologne, Germany
E-mail: schmidli@math.uni-koeln.de

We consider a classical Cramér-Lundberg model. In recent years, several authors have studied the joint distribution of the time to ruin, deficit at ruin, and the surplus prior to ruin. A popular quantity is the discounted penalty at ruin, the so-called Gerber-Shiu function. We determine the Laplace transform of the Gerber-Shiu function without discounting and invert this formula. By change of measure, we are able to express the Gerber-Shiu function in terms of ruin probabilities also in the discounted case. Then we study the alternative risk measure of Eisenberg and S. (2009, 2011). If the surplus becomes negative, a capital injection has to be made. The risk measure is the expected value of the discounted capital injections. We express the measure as a Gerber-Shiu function, and study the asymptotic behaviour of the

value as the initial capital tends to infinity.

From Utility Optimization to Good Advice and Good Product Design

Mogens Steffensen
University of Copenhagen, Denmark
E-mail: mogens@math.ku.dk

We discuss three different problems, the structure of their solutions, and their relation to practical challenges concerning pension savings advice and product development. The three problems deal with the optimal consumption-investment plan for an individual or a household in the cases where a) the consumption-investment control in a stochastic framework is constrained to be deterministic, see [1], b) preferences are specified in terms of growth in smooth consumption rather than consumption itself, see [2], and c) risk aversion and elasticity of intertemporal substitution are separated in presence of uncertain lifetime and access to a life insurance market. The three problems and the structure of their solutions are quite different but they share the ability to shed light on important practical questions in personal finance and insurance and unveil appealing theoretical challenges.

References

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[2] Bruhn, K. and Steffensen, M. (2013). Optimal smooth consumption and annuity design. *J. Bank. Finance* (to appear).

Some Issues on the Extreme Value Index

Jef Teugels
Katholieke Universiteit Leuven, Belgium
E-mail: jef.teugels@wis.kuleuven.be

Under rather weak conditions the maximum of a sample from a distribution converges in law to an extreme value distribution. The class of such distributions is characterized by a single real-valued parameter

γ, the Extreme Value Index. We will deal with three kinds of results.

1. We first illustrate the general procedure with examples from geology and insurance.

2. Some examples seem to indicate that the extreme value index changes over time. We adapt methods from classical change point analysis to an extremal setting. Since we are not directly successful in applying the procedure to catastrophic losses, we investigate whether or not a trend analysis might be more appropriate. This part of the presentation refers to joint work with Goedele Dierickx.

3. The larger the value of the extreme value index the larger the extremes. We therefore investigate if the omission of some of the largest extremes results in safer statistical conclusions. This last part covers joint work with Hansjörg Albrecher.

From Ruin Theory to Solvency in Non-Life Insurance

Mario V. Wütrich
ETH, Zurich, Switzerland
E-mail: mario.wuethrich@math.ethz.ch

We start from ruin theory considerations in the classical Cramér-Lundberg process. These considerations will be modified step by step so that we arrive at a modern solvency assessment for non-life insurance companies, see [1]. These modifications include discussions about time horizons, risk measures, claims development processes, financial returns and valuation of insurance liabilities.

References

[1] Wüthrich, M.V., Merz, M. (2013). *Financial Modeling, Actuarial Valuation and Solvency in Insurance.* Springer.

Communications

A Simulation-Based ALM Model in Practical Use by a Norwegian Life Insurance Company

Kjersti Aas

Norwegian Computing Center, Oslo, Norway
E-mail: Kjersti.Aas@nr.no
Linda R. Neef
Norwegian Computing Center, Oslo, Norway
Dag Raabe
SpareBank 1 Forsikring, Oslo, Norway
Ingeborg D. Vårli
SpareBank 1 Forsikring, Oslo, Norway

A key aspect of the Solvency II regulatory framework is to compute the market value of the liabilities. In this paper we present an ALM model for computing this market value. The model, which is the result of a cooperation between the Norwegian Computing Center and the actuary and risk management departments of SpareBank 1 Forsikring, is able to produce an estimate of the liabilities for several different insurance products. In this paper the focus is, however, on one of these products; individual annuity insurance with a surrender option and an annual interest rate guarantee. In contrast to most of the existing literature, we consider a real-world portfolio consisting of 25,528 insurance policies. For this portfolio, we have computed the market value of the liabilities using two different approaches; the policy-by-policy and the aggregated approach. Moreover we have analysed the effect of different Solvency II related stress scenarios. Our experiments show that according to the Solvency II regulations, this product is unfortunately not profitable to the insurance company.

Optimal Design of Securitization in a Principal-Agent Relationship Based on Bayesian Inference for Moral Hazard

Ezatollah Abbasian
Bu Ali Sina University, Hamedan, Iran
E-mail: e.abbasian@gmail.com
Elham Farzangan
Bu Ali Sina University, Hamedan, Iran

The goal of this paper is to examine the optimal design of Mortgage

Backed Securities (MBS) in an agency relation framework. One difficulty along with securitization, in general, is moral hazard on the part of lender when it can sell loans in the secondary market. One way of mitigating these information asymmetries is to use signals about the lenders hidden action. In practice, lenders effort is often better characterized as a multidimensional variable. Thus, to infer hidden action, the observation of performance of pooled loans is modeled by infinite latent feature model. These hidden features that lead to those observations, namely defaults, are not always known. In this paper, inferring the hidden actions is done by using Nonparametric Bayesian approach. Inferred features, as informational signals, then are used for designing optimal incentive scheme to induce lender to choice the level of effort which investor wants to achieve.

Estimation of the Ruin Probability in Infinite Time for Heavy Right-Tailed Distributions

Rassoul Abdelaziz

National High School of Hydraulic, Blida, Algeria
E-mail: a.rassoul@ensh.dz

The aim of this work is to propose, an estimator of the ruin probability in infinite time in the case of heavy right-tailed claim amounts which have finite means but infinite variances for a large initial reserve. Our considerations are based on the extreme quantile approach. The asymptotic distributional properties of the proposed estimator is derived. A simulation study illustrates the main results and the comparison with parametric approach. Related references are [1 – 5].

References

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Modelling Adverse Selection in a Start-up Long-Term Care Insurance Market

Craig Adams

Heriot-Watt University, Edinburgh, U.K.
E-mail: cja3@hw.ac.uk

I will outline a model to measure the development of adverse selection costs in a start-up market of long-term care insurance. A major source of liability on long-term care insurance is due to Alzheimers disease and other forms of dementia, which can lead to long stays in nursing care facilities. Genetics can be a risk factor for Alzheimers disease and where underwriting cannot use the results of genetic tests, information asymmetry allows the potential for adverse selection. Additionally, since dementia is progressive, it might be noticed by the life prior to diagnosis. By incorporating a state for the initial signs of dementia, I will allow the possibility of adverse selection at this point. After setting premium rates, an insurer would analyse the cash flows in and out to adjust premium rates to new business. My model includes this response and in doing so, allows me to dynamically measure the adverse selection cost to the business.

Foreign-Currency Interest-Rate Swaps in Asset-Liability Management for Insurers

Jonas Alm

Chalmers University of Technology, Gothenburg, Sweden
E-mail: jonasa@chalmers.se

Filip Lindskog

KTH Royal Institute of Technology, Stockholm, Sweden

We consider an insurer with purely domestic business whose liabilities towards its policy holders have long durations. The relative shortage of domestic government bonds with long maturities makes the insurer's

net asset value sensitive to fluctuations in the zero rates used for liability valuation. Therefore, in order to increase the duration of the insurer's assets, it is common practice for insurers to take a position as the fixed-rate receiver in an interest-rate swap. We assume that this is not possible in the domestic currency but in a foreign currency supporting a larger market of interest-rate swaps. Monthly data over 16 years are used as the basis for investigating the risks to the future net asset value of the insurer from using foreign-currency interest-rate swaps as a proxy for domestic ones in asset-liability management. We find that although a suitable position in swaps may reduce the standard deviation of the future net asset value it may significantly increase the exposure to tail risk that has a substantial effect on the estimation of the solvency capital requirements.

Life Insurance Mathematics in the Master's Program in Actuarial Mathematics at Stockholm University

Gunnar Andersson

Folksam / KTH Royal Institute of Technology, Stockholm, Sweden
E-mail: gunnar.andersson@ActStrats.com

Life insurance mathematics in the master program at Stockholm University is separated in two courses. Both courses are describing life insurance mathematics from a Swedish perspective. The first course, Life Insurance Mathematics I, is dealing with basic life insurance mathematics with a fixed interest rate. The main aim is to learn how to calculate premiums and technical provisions for risk products as well as for pension products of different kinds. Life insurance is in Sweden presented using continuous technique. This course also deals with discounting of cash flows using the concept of commutation functions. One basic part in the first course is to model mortality, longevity and disability which are used in life and pension insurance mathematics. The second course, Life Insurance Mathematics II, uses the so called second order technique, with assumed parameters replaced by true observed values of the parameters, which then gives us the true evolution of the technical provision in, for instance, a pension product. The course also explains different types of allocation of bonuses in life- and pension-insurance in Sweden. The second course also includes a

presentation of semi-Markov modeling of life- and pension-insurance which is very practical.

Individual Loss Reserving with the Multivariate Skew Normal Framework

Katrien Antonio

KU Leuven, Belgium / University of Amsterdam, Holland
E-mail: katrien.antonio@kuleuven.be, K.Antonio@uva.nl

The evaluation of future cash flows and solvency capital recently gained importance in general insurance. The vast literature on stochastic loss reserving concentrates on data aggregated in run–off triangles. However, a triangle is a summary of an underlying data set with the development of individual claims. The talk considers a novel loss reserving model, designed for individual claims developing in discrete time. We model the occurrence of claims, as well as their reporting delay, the time to the first payment, and the cash flows in the development process. Our approach uses development factors similar to those of the well–known chain–ladder method. We suggest the Multivariate Skew Normal distribution as a multivariate distribution suitable for modeling these development factors. Empirical analysis using a real portfolio and out–of–sample prediction tests demonstrate the relevance of the model proposed. The talk concludes with an outlook to ongoing research in the domain of loss reserving with individual claims data.

References

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Liability-Driven Investment in Longevity Risk Management

Helena Aro

Aalto University, Helsinki, Finland
E-mail: helena.aro@aalto.fi

We study optimal investment from the point of view of an investor with longevity-linked liabilities. The relevant optimization problems are rarely analytically tractable, but we are able to show numerically that liability driven investment can significantly outperform common strategies that do not take the liabilities into account. In problems without liabilities the advantage disappears, which suggests that the superioity of the proposed strategies is indeed based on connections between liabilities and asset returns.

Actuarial Education in the UK

Andrew Cairns
Heriot-Watt University, Edinburgh, U.K.
E-mail: A.J.G.Cairns@hw.ac.uk

I will describe the various routes to qualification as an actuary in the UK. Qualification as an actuary with the Institute and Faculty of Actuaries is normally achieved by taking the profession's own examinations. However, students can make significant progress towards qualification through accredited university courses. I will contrast these approaches and discuss the pros and cons of each.

Optimal Reinsurance Under Coherent Comonotonic Risk Measures

M. L. Centeno
CEMAPRE-ISEG, Technical University of Lisbon, Portugal
E-mail: lcenteno@iseg.utl.pt
M. Guerra
CEMAPRE-ISEG, Technical University of Lisbon, Portugal
E-mail: mguera@iseg.utl.pt

This presentation deals with optimal reinsurance from the cedent point of view, assuming that the insurance company evaluates its net risk using a coherent comonotonic risk measure and that the reinsurance company prices the ceded risks using a risk adjusted premium calculation principle (which turns out to be also coherent and comonotonic). A more general problem has been studied by Jouini et al. (2008) with

respect to optimal risk sharing between two traders. We use their result to study the properties of the optimal reinsurance form and show that in some situations, the optimal treaty can lead to an increase on the finite time probability of ruin, when compared to no reinsurance at all.

References

[1] Jouini, E., Schachermayer, W., Touzi, N. (2008). Optimal risk sharing for law invariant monetary utility functions. *Math. Finance*, **18**, 2, 269–292.

Moments Analysis of a Markov-Modulated Risk Model with Stochastic Interest Rates

Guglielmo D'Amico

Università "G. d'Annunzio" of Chieti, Italy
E-mail: g.damico@unich.it

In this paper we determine explicitly the closed form of the moments of a Markov-Modulated Risk Model with Stochastic Interest Rate. The moments are derived by means of Laplace-Stieltjes transfoms. Equations and formulas are conveniently represented by using the 2-dimensional matrix formalism. This paper substantially extends the results of Kim and Kim [1], by allowing the possibility to work with a stochastic modulated interest rate and by considering a company having several business lines. A numerical example is provided to show possible applications of the model.

References

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A Real Data Application and Cost of Capital Estimation Based on Rating Migration Model

Guglielmo D'Amico

Università "G. d'Annunzio" of Chieti, Italy
E-mail: g.damico@unich.it

Giuseppe Di Biase

Università "G. d'Annunzio" of Chieti, Italy
E-mail: dibiase@unich.it

Jacques Janssen

Universitè Libre de Bruxelles, Belgium
E-mail jacques.janssen@skynet.be

Raimondo Manca

Sapienza Università di Roma, Italy
E-mail: raimondo.manca@uniroma1.it

This paper presents a duration dependent model which analyzes the credit rating evolution. It considers the backward recurrence process to face the problem of the time of permanence in the rating classes. In this way it is possible to manage the duration effects which represent one of the most important features in rating dynamics. A real data application using Standard & Poors historical database is provided. Furthermore the paper shows how it is possible to compute the cost of capital that an organization is required to pay for the capital used in financing its activities.

Grey Brownian Motion Local Time: Existence and Weak-Approximation

José Luís da Silva

University of Madeira, Portugal
E-mail: luis@uma.pt

We investigate the class of grey Brownian motions $B_{\alpha,\beta}$ ($0 < \alpha < 2$, $0 < \beta \leq 1$), which have a good potential for financial and insurance applications. It is a self-similar with stationary increments stochastic process and is completely determined knowing its expectation and second moment but is a non-Gaussian process. We show that grey Brownian motion admits different representations in terms of certain known processes, such as fractional Brownian motion, multivariate elliptical distribution or as a subordination. The weak convergence of the increments of $B_{\alpha,\beta}$ in t, w-variables is studied. Using the Berman criterium we show that $B_{\alpha,\beta}$ admits a λ-square integrable local time

$L^{B_{\alpha,\beta}}(\cdot, I)$ almost surely (λ Lebesgue measure). Moreover, we prove that this local time can be weak-approximated by the number of crossings $C^{B^{\varepsilon}_{\alpha,\beta}}(x, I)$, of level x, of the convolution approximation $B^{\varepsilon}_{\alpha,\beta}$ of grey Brownian motion.

References

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Large Deviations for a Damped Telegraph Process

Alessandro De Gregorio

Sapienza Università di Roma, Italy
E-mail: alessandro.degregorio@uniroma1.it

Claudio Macci

Università di Roma Tor Vergata, Italy
E-mail: macci@mat.uniroma2.it

The theory of large deviations gives an asymptotic computation of small probabilities on exponential scale; see e.g. [1] as a reference on this topic. Estimates based on large deviations play a crucial role in resolving a variety of questions in several fields, and in particular in risk theory. In this paper we consider a slight generalization of the damped telegraph process in [2]. We prove two results. We start with the large deviation principle for $\left\{\frac{D(t)}{t} : t \geq 0\right\}$. Moreover we prove an asymptotic estimate for the level crossing probabilities of $\{D(t) : t \geq 0\}$ (as the level goes to infinity) by combining the first result and an estimate in [3]. Finally we compare our result with the analogous well-known results for the standard telegraph process.

References

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totics for the supremum of a stochastic processes. *Ann. Appl. Probab.*, **13**, 2, 430–445.

Multi Period Risk Measurement for Long Term Guarantees

Pierre Devolder
Catholic University of Louvain, Belgium
E-mail: pierre.devolder@uclouvain.be

Valuation and solvency measurement of long term guarantees in the context of Solvency 2 have been for sure controversial elements, still on the agenda. The one year time horizon intrinsic to the Solvency 2 philosophy generates a short term view, potentially dangerous and inaccurate for pension or life insurance liabilities. The purpose of this presentation is to discuss various approaches in order to measure the financial and longevity risks of long term pension liabilities with guarantees. In particular, multi period risk measures based on the whole time horizon of the risks will be proposed and compared to the standard Solvency 2 metric.

Risk Analysis of Participating Survival-Indexed Annuities

Emilia Di Lorenzo
University of Napoli "Federico II", Napoli, Italy
E-mail: diloremi@unina.it
Albina Orlando
Istituto per le Applicazioni del Calcolo Mauro Picone, Napoli, Italy
E-mail: a.orlando@iac.cnr.it
Marilena Sibillo
University of Salerno, Fisciano (SA), Italy
E-mail: msibillo@unisa.it

The paper focuses on variable annuities characterized by a risk-profit sharing structure, where the installments are scaled by a demographic index and financial guarantees are involved in the embedded option linked to the financial profit participating quota. Within a risk framework, where systematic random fluctuations of capital return and of the lifetime trend interact, a performance analysis is developed. Con-

sistently to the firm evaluation scheme, profitability indexes are revised in the actuarial framework, in order to assess product profitability as well as prudential capital allocation.

Maximising Exponential Utility of Restricted Dividend Payments in a Brownian Risk Model

Julia Eisenberg

Vienna University of Technology, Austria
E-mail: jeisenbe@fam.tuwien.ac.at

We consider a diffusion approximation to the classical risk model. In this setting we study the following optimization problem:

$$\max_C \mathrm{E}[U(\int_0^\infty e^{-\delta t}\,\mathrm{d}C_t)],$$

where $U(x) = (1-\exp(-\gamma x))/\gamma$ denotes the utility function with risk aversion parameter γ, C denotes the accumulated dividend process, and δ a discounting factor. We only allow dividend strategies with restricted payments. The problem with unrestricted payments has been already considered before by Schachermayer et al. [1]. We will see that the introduction of a boundary for the dividend payments can simplify the considered problem. In some special cases we will be able to give a closed expression for the value function and for the optimal strategy.

References

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Modeling Stochasticity of Mortalities

Erland Ekheden

Stockholm University, Sweden
E-mail: erland@math.su.se

Ola Hössjer

Stockholm University, Sweden
E-mail: ola@math.su.se

It is apparent that mortality is not constant, but rather it evolves in time. The rate of change varies over ages and time periods. Forecasting of future mortality rates is important to life insurance companies in order to set proper reserves. A range of models have been proposed in order to project mortality. In this paper we start with an explorative data analysis with, amongst others, Swedish mortality data. Understanding the random structure of data is important for model selection and model validation. We look at variance and for evidence time and age dependencies. Even in quite large populations, mortality data displays a random behavior between years. A life insurers payments are governed by actual mortalities, not expected. Therefore we set up a model framework in order to predict not only expected values but also its variation in terms of a predicitve distribution. This makes it possible to calculate predicition intervals for future mortality rates.

Variations of PageRank and Model Considerations for Different Types of Data

Christopher Engström

Mälardalen University, Västerås, Sweden
E-mail: Christopher.engstrom@mdh.se

Sergei Silvestrov

Mälardalen University, Västerås, Sweden
E-mail: Sergei.silvestrov@mdh.se

We look at some alternative formulations of PageRank originally used by Google to rank home pages in a search query in order to rank, or classify other types of data with potential uses in for example text/data mining, finance, and insurance. We look at PageRank by considering a random walk on a graph where the data represents the vertices and the edges describe some relation between data. Different model considerations are made and looked at in order to highlight the potential use of PageRank outside of search engines as well as looking at potential model limitations.

Variable Annuities: Crossing a Bridge From Computational Finance to Quantitative Risk Management

Runhuan Feng

University of Illinois at Urbana-Champaign, USA
E-mail: rfeng@illinois.edu

Variable annuity guaranteed benefits are the insurance industry's answer to the increasingly fierce competition of individual products in the investment market. From the viewpoint of computational finance, these benefits are in fact variations of path-dependent financial derivatives. However, a series of unique challenges arise due to their long-term nature in contrast with the short-lived exchange-traded derivatives. Although the pricing is an important task, the quantification and assessment of long-term liabilities pose the most difficult but intriguing technical challenges. Over the past decade, the insurance industry in North America has gradually adopted new standards on the stochastic modeling of equity-linking insurance products. However, it has been reported in recent industrial surveys that in many cases the current market practice of risk assessment based on Monte Carlo simulations are found to be very time-consuming and cost-ineffective. This emerging problem naturally calls for new solutions from both practitioners and academics. Owing to the similarities between variable annuity guaranteed benefits and exotic options, we have been drawing various techniques known from the computational finance literature and developing new analytical solutions or algorithms for the computation of risk measures. Our numerical experiments have shown that these analytical methods are much more efficient and accurate than the Monte Carlo simulations. In this talk, we will make an introduction to the risk management of variable annuity guaranteed benefits, a brief summary of latest development as well as open questions for interested researchers.

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Discrete Time Non-Homogeneous Compound Renewal Processes for the G/G Risk Model Solution

Fulvio Gismondi

University Guglielmo Marconi, Roma, Italy
E-mail: fulvio.gismondi@parametrica.net

Jacques Janssen

Universitè Libre de Bruxelles, Belgium
E-mail: jacques.janssen@skynet.be

Raimondo Manca

Università di Roma La Sapienza, Roma Italy
E-mail: raimondo.manca@uniroma1.it

In this paper we define the discrete time non-homogeneous compound renewal process. This tool will be applied for the solution of the G/G risk model defined in Andersen (1967). The algorithm for solving this process will be also presented. In the last part of the paper the application of the G/G model will be applied to a disability insurance problem.

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IBNyR Claims Study by an Alternating Renewal Process Approach

Fulvio Gismondi

University Guglielmo Marconi, Roma, Italy
E-mail: fulvio.gismondi@parametrica.net

Jacques Janssen

Universit Libre de Bruxelles, Belgium
E-mail jacques.janssen@skynet.be

Raimondo Manca
Universit di Roma La Sapienza, Italy
E-mail raimondo.manca@uniroma1.it
Filippo Petroni
University of Cagliari, Italy
E-mail fpetroni@gmail.com

The problem of the evaluation of the IBNyR claims and of their properties is one of the most tortuous problem for the evaluation of claim reserving problems. In this paper we will evaluate all the probabilities properties of the IBNyR claim evolution by applying, for the first time in this topic, the alternating renewal process. An example will be also presented.

Discrete Time Homogeneous Semi-Markov Monte Carlo Reward Risk Process

Fulvio Gismondi
University Guglielmo Marconi, Roma, Italy
E-mail: fulvio.gismondi@parametrica.net
Jacques Janssen
Universit Libre de Bruxelles, Belgium
E-mail jacques.janssen@skynet.be
Raimondo Manca
Universit di Roma La Sapienza, Italy
E-mail raimondo.manca@uniroma1.it
Dmitrii Silvestrov
Stockholm University, Sweden
E-mail silvestrov@math.su.se

In this paper a semi-Markov process useful for following the time evolution of the aggregate claim amount in a homogeneous environment is defined. A mixture between the semi-Markov process and the Monte Carlo simulation model for the evaluation of the rewards paid by an insurance company for the claims permits the evaluation of the aggregate claim amount for each ensured of the company. Once the

semi-Markov process is solved a Monte Carlo model is activated. By means of this simulation process the values of claim rewards will be calculated. The states of process will be given by classes of money amount. In the reconstruction of trajectories by the Monte Carlo method the semi-Markov and the simulation algorithm will interact influencing the trajectory reconstruction.

Predicting Future Claims Among High Risk Policyholders Using Random Effects

Clara-Cecilie Günther
Norwegian Computing Center, Oslo, Norway
E-mail: Clara-Cecilie.Gunther@nr.no
Ingunn Fride Tvete
Norwegian Computing Center, Oslo, Norway
Kjersti Aas
Norwegian Computing Center, Oslo, Norway
E-mail: Kjersti.Aas@nr.no
Jørgen Andreas Hagen
Gjensidige Group, Oslo, Norway
Lars Kvifte
Gjensidige Group, Oslo, Norway
Ørnulf Borgan
University of Oslo, Norway

Insurance claims are often modelled by a standard Poisson model with fixed effects. With such a model, no individual adjustments are made to account for unobserved heterogenity between policyholders. A Poisson model with random effects makes it possible to detect policyholders with a high or low individual risk. The premium can then be adjusted accordingly. Others have applied such models without much focus on the model's prediction performance. As the usefulness of an insurance claims model typically is measured by its ability to predict future claims, we have chosen to focus on this aspect of the model. We model insurance claims with a Poisson random effects model and

compare its performance to the standard Poisson fixed effects model. We show that the random effects model both fits the data better and give better predictions for future claims for high risk policy holders than the standard model.

Claims Reserving Against Default Risk of Borrowers Based on Exchanges Options

Samuel Herrmann

Université de Bourgogne, France
E-mail: Samuel.Herrmann@u-bourgogne.fr

Geoffrey Nichil

Institut Élie Cartan Lorraine, Nancy, France
E-mail: geoffrey.nichil@gmail.com

Pierre Vallois

Institut Élie Cartan Lorraine, Nancy, France
E-mail: Pierre.Vallois@univ-lorraine.fr

We consider a mortgage insurer whose risk underwritten is to indemnify a bank for losses due to defaults of borrowers (failure to reimburse property loans). For the insurance company, the main goal is to determine, at time t_0, the amount of all future claims declared during the period of time $[t_0, t_0 + h]$. This amount is called loss provision and is denoted P_{t_0,t_0+h}. For a given borrower which has a default at time T, the value of the loss can be expressed as a function of T , the amount borrowed M and the term D of the loan including all possible recoveries. Moreover, we suppose that all couples (D, T) relating to losses generate a Poisson point process. Then P_{t_0,t_0+h} is defined as a random sum of individual amount of claims. Each loss will be interpreted as the payoff of an exchange option. Our model differs from Cramér – Lundberg's because the amount of claim is not independent from the reporting dates of loss. We then shall determine the mean and the variance of P_{t_0,t_0+h}. We will also calculate the quantile of order α of the loss provision under the assumption of normality. This assumption will be partly justified by the convergence, in law, of this provision to a Gaussian random variable. We shall conclude with a numerical application of our model.

Conditional Quantiles, Conditional Weighted Expected Shortfall and Application to Risk Capital Allocation

Karin Hirhager
Vienna University of Technology, Austria
E-mail: karin.hirhager@fam.tuwien.ac.at
Uwe Schmock
Vienna University of Technology, Austria
E-mail: schmock@fam.tuwien.ac.at
Jonas Hirz
Vienna University of Technology, Austria
E-mail: hirz@fam.tuwien.ac.at

Based on conditional quantiles we define the conditional expected shortfall via an explicitly given density on a general modelling setup with stochastic levels δ, involving conditional expectations based on σ-integrability. Then we give an overview of properties and some representations of conditional expected shortfall. Further, we point out the link to dynamic risk measures and prove a supermartingale property. In the next step we introduce contributions to conditional expected shortfall and list several properties. In particular, it is possible to derive the contribution of a subportfolio to the whole portfolio in order to be able to identify main risks. Finally, we define and analyse conditional distortion risk measures. Conditional weighted expected shortfall arises as a special case of conditional distortion risk measures.

Numerical Software and Tools for the Actuarial Community

David Humphris
Numerical Algorithms Group (NAG), U.K.
E-mail: david.humphris@anag.co.uk

Within the insurance industry there is a growing requirement from actuarial staff for high quality numerical and statistical routines to help meet todays ever changing and volatile requirements. This presentation will give a brief introduction to the numerical software used

by the Actuarial community; some of the problems that need to be considered, with an example of a typical problem that can occur using Excel. Then we will cover some of the areas that the NAG numerical Library can provide high quality and performant code to address many of the computational algorithmic requirements of the Actuarial community. NAG routines are used as the core building blocks in the development of broader applications. NAG's libraries provide a broad range of reliable and robust numerical and statistical routines in areas such as minimising and maximising functions (local and global optimisers), distribution functions, time series, PCA, copulas and RNGs to name but a few. Utilizing NAGs software in modelling critical business applications can help you more quickly build high quality and flexible solutions. By embedding NAG software within applications, analysts and software engineers are able to spend more time in other areas of their work where they are best utilized, and so ultimately improve productivity and time management.

When the Stress Factor Causes Small Changes in the Widow Pension?

Iurii Kartashov

Kyiv National Taras Shevchenko University, Ukraine
E-mail: kartashov.iurii@gmail.com

Vilaty Golomozy

Kyiv National Taras Shevchenko University, Ukraine

Mykola Kartashov

Kyiv National Taras Shevchenko University, Ukraine
E-mail: nkartashov@skif.com.ua

The model of joint life insurance with the stress factor is considered. The framework for maximal coupling of time-inhomogeneous Markov chains is given, the theorem on the stability of expectations of a function on a Markov moment is proven. Numerical examples are considered. This paper is a continuation of the paper [1], extending and enhanced it for the time-inhomogeneous Markov chains case. Following [1], coupling method is a key method of investigation. In the previous papers of the authors [2], [3], [4], one can find other stability results us-

ing coupling method for Markov chains including time-inhomogeneous case. However that works do not deal with the functionals on the stopping time. Results of this paper were applied to the calculation of the insurance premium for the joint life insurance is the case of stress-factor. Joint life insurance model is considered, particularly so-called widow's pension. The sense of it is in the following. Once one of two family members is dead another one receives life pension. Models of joint life insurance are considered, in particular, in [5]. As mentioned in [5](Chapter 9), considering life insurance for group of people mortalities probabilities are actually dependent, however the assumption of independence is very common and significantly simplifies all joint mortality rates calculation having only individual rates. It is proved statistically, that death rates of widows/widowers are significantly higher then ones whose spouse is alive. Mortality rates among marital statuses were firstly published in 1949 [6]. On average, recent statistics shows that at ages 40-60 death rates for widows/widowers are 45-65% higher [7]. In this paper we're investigate an impact of the stress-factor to the mortality, and so on the insurance premium size. In other words we're figuring out if that impact is essential, and if so, how to take it into account when calculating premium. Application of result, which gives estimate of stress factor impact on insurance premiums are given for simple geometrical model (which allows for exact numbers) and model based on Monte-Carlo simulations for life tables provided by Office for National Statistics, UK.

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Risk Measure Theory under Heavy-Tails and Conic Assumptions

Dimitrios G. Konstantinides
University of the Aegean, Samos, Greece
E-mail: konstant@aegean.gr
Christos E. Kountzakis
University of the Aegean, Samos, Greece
E-mail: chr.koun@aegean.gr

We attempt to establish a connection between the properties of the heavy-tailed distributions, the radial subsets of random variables and the coherent risk measures. An expected shortfall-like risk measure is introduced on radial subsets of heavy-tailed random variables. In the classical risk model, the solvency capital is calculated in certain classes of distributions. Existence and uniqueness of the solution in the optimization problem associated to the minimization of the risk over a set of financial positions in the same classes of heavy-tailed distributions is studied. An important aspect of this work is that the results of optimization hold on the $L^{1+\epsilon}$-spaces, for any $\epsilon > 0$, but they collapse on L^1, the canonical space for the law-invariant coherent risk measures.

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Minimization of a Function of a Quadratic Functional with Application to Optimal Portfolio Selection

Zinoviy Landsman

University of Haifa, Israel
E-mail: landsman@stat.haifa.ac.il

Udi Makov

University of Haifa, Israel

We present an explicit closed form solution of the problem of minimizing the combination of a linear functional and a function of quadratic functional, subject to a system of affine constraints. This is of interest for solving significant problems of financial economics which are related to optimal portfolio selection.The solution essentially generalizes the previous results of the authors concerning optimal portfolio selection with translation invariant and positive homogeneous risk measures. The important representatives of such risk measures are value-at-risk (VaR) and tail condition expectation (TCE). The classical mean-variance model, and the recently introduced and investigated tail mean-variance model, are special cases of the general setup considered here. The results are illustrated with the data of stocks from NASDAQ/Computers.

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Stable Variables in Non Life Insurance

Amel Laouar

Université des Sciences et de la Technologie Houari Boumediene, Bab-Ezzouar, Algeria
E-mail: amel.laouar@gmail.com

Rachid Sabre

Université des Sciences et de la Technologie Houari Boumediene, Bab-Ezzouar, Algeria

In this work, we are interested in modeling risk process of an Algerian insurance company and especially in estimating the distribution of car claims amounts. We assume that the claims amounts are infinite variance and we consider essentially the case of α-stable variables. Using various graphical and statistical tests we validate our hypotheses while estimating the four parameters of the corresponding distribution.

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Supervision of Insurance Undertakings and the Underlying Risks

Åsa Larson

Swedish Financial Supervisory Authority, Stockholm, Sweden
E-mail: asa.larson@fi.se

The communication presents current issues and activities at the Swedish Financial Supervisory Authority (Finansinspektionen) connected to supervision of insurance undertakings and the underlying risks. The Authorities' plans for a risk-based supervision under the up-coming Solvency 2 regulation will also be covered.

A Moment Test for Log-Normality

Char Leung

La Trobe University, Melbourne, Australia
E-mail: ltleung@students.latrobe.edu.au

This paper introduces a moment test for testing log-normality. Log-normal distribution has been found in different types of data such as reported loss in insurance claims and financial return of a portfolio. Given that the sample of insurance and financial data is usually large, commonly used tests such as Kolmogorov-Smirnov test and normality test with transformation involve sophisticated algorithm and computation such as computing the relative cumulative frequency, the use of Kolmogorov distribution or data transformation. Furthermore, these tests may not be applicable if only the descriptive statistics of the data are available. The proposed test has the test statistic follows chi-square distribution and is computed based on the empirical median, geometric mean and the two distribution parameters that can be easily computed by two maximum likelihood estimation equations or by the empirical arithmetic mean and standard deviation.

Asian Options, Jump Diffusion Processes on a Lattice and Vandermonde Matrices

Karl Lundengård

Mälardalen University, Västerås, Sweden
E-mail: karl.lundengard@mdh.se

Carolyne Adhiambo Oguto

University of Nairobi, Kenya
E-mail: coguto@uonbi.ac.ke

Sergei Silvestrov

Mälardalen University, Västerås, Sweden
E-mail: sergei.silvestrov@mdh.se

Jump diffusion processes can be used to model the value of options in a way that can be less biased than the Black-Scholes pricing model. One method of pricing options that involve these kinds of processes are lattice pricing algorithms. When describing the lattice pricing algorithm using matrices the well-known Vandermonde matrices appear. Applications of the inverse and other properties of these and related matrices will be presented within this context.

Long Term Disablility Model for Solvency II Purposes

Maryna Lundgren
SEB Trygg Liv, Stockholm, Sweden
E-mail: maryna.lundgren@seb.se

A Swedish long term disability insurance product with tree different waiting periods and three levels of insurance benefits is considered. A Markov chain cash flow model for best estimate and SCR calculations is realized in actuarial software IBM Algo Financial Modeller. En estimation of models parameters, models outputs, advantages and disadvantages of using a Markov chain model are discussed.

Reporting and Control of Risks in the Swedish Fund Industry

Robin Lundgren
Invest Systems, Stockholm, Sweden
E-mail: robin.lundgren@invest-systems.com

Swedish funds should report and control different types of risks to regulator, customers and internally to the board. To the regulator quantified market risk measures should be reported, also procedures on how the fund comply with limits should be described.This presentation will describe the Swedish fund industry from a risk perspective and how we can assist our customers with the help of our products in the Secura family.

Solvency Capital Requirement for Insurance Products via Dynamic Cash Flow Matching under Lattice Models

Alfred Ka Chun Ma
Chinese University of Hong Kong, China
E-mail: alfredkcma@cuhk.edu.hk
Yuen Ki Cheung
J.P. Morgan
E-mail: justina.yk.cheung@jpmorgan.com

We propose a framework based on cash flow matching for computing

the Solvency Capital Requirement under Solvency II. The time horizon of the insurance liabilities is typically longer than the maturities of bonds available in the market. With the assumption that a collection of bonds will be available for purchase in the future, we study the cash flow matching program under interest rate lattice models. The solution can be interpreted as the worst-case cost and the economic capital can be found accordingly.

A Stochastic Model for New Entrants in Pension Schemes

Alessandro Fiori Maccioni
CRENoS / University of Sassari, Italy
E-mail: alex.fiori.maccioni@gmail.com

We propose an original discrete-time Markov model for the estimation of new entrants in pension funds of professional categories, that highlights the interactions between demographic, economic and regulatory variables. The model considers the effects of trends in population, trends in education choices, appeal of the profession, and entrance regulations. We offer a numerical application to the pension fund of Italian Chartered Accountants, with a joint analysis of demographic and financial risks. The application reveal the effects of a main reform in the Italian university system on new entrants and on the sustainability of the fund. Demographic predictions are relevant for the financial self-sufficiency of PAYG pension schemes and for the long-term financial risk of fully-funded schemes. In PAYG schemes, pensions are funded by current employees' salary deductions and an "intergenerational pact" compels young generations to sustain the older age groups. In fully-funded pension schemes, contributors accumulate assets that will be sold at their retirement age to a younger generation of savers, thus influencing asset prices and returns. In both cases, it is important that in the long run there should be equilibrium between the number of pensioners and the number of workers.

Bonus-Malus Systems in Open and Close Portfolios

Rahim Mahmoudvand
Mellat Insuarance Company, Tehran, Iran

E-mail: R.mahmodvand@gmail.com

Samane Aziznasiri

Mellat Insuarance Company, Tehran, Iran
E-mail: S.aziznasiri@gmail.com

Bonus-malus system (BMS) is one of the types of experience ratemaking methods in automobile insurance in which the future premiums are adjusted according to the insured's claim history. Usually it is assumed the considered portfolio for designing BMS is closed in which the policyholders only have movement between specified levels in the portfolio without any assumption about their exit. In many applications, however, this assumption doesn't hold, which means a policyholder in every level maybe prefers to leave the portfolio. In this situation we have BMS in open portfolio. Certainly, ignoring this issue can result in unrealistic conclusions. This issue will need to be looked at more closely, in particular, in the deregulated markets, where each insurer is free to design its own BMS. The problem is open in general, although some results have been drawn over the last decade. This paper provides an overview of the importance of considering open potfolios in insurance markets and explores the challenges involved in designing and evaluating BMS in open portfolios.

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Extremal Behaviour, Weak Convergence and Argmax Theory for a Class of Non-Stationary Marked Point Processes

Hannes Malmberg

Stockholm University, Sweden

E-mail: hannes@math.su.se

We formulate a random utility model where we choose from n independent and identically distributed (i.i.d) random variables $\{X_i, U_i\}_{i=1}^n$, where X_i are objects characteristics and U_i is their associated utility. We study the distributional properties of the characteristics of our preferred choice. We show that this formulation allows us to use the connection between point processes and extreme value theory to analyze the asymptotic properties of choice characteristics. We derive analytic expressions of their asymptotic distribution for a range of distributional assumptions on the utilities U_i. In our discussion, we suggest an extension of our method to allow us to deal with an even larger class of distributions. We also show how our theoretical model can be used to analyze empirical patterns relating to commuting time distributions.

Utility and Ruin Probability Relationship through Premium Equivalence

Jonathan B. Mamplata

University of the Philippines Los Baños, Philippines
E-mail: jbmamplata@uplb.edu.ph

Jose Maria L. Escaner IV

University of the Philippines Diliman, Philippines
E-mail: jlescaner@gmail.com

Utility is a satisfaction received from consuming a commodity or service. Ruin probability is the probability that an insurers surplus will fall below zero for a certain period of time. Many researches focused on the pricing insurance premium using the concept of utility functions. Also some focused on finding the probability that the company or insurer will be insolvent at any given point in time via the ruin probability. This thesis focused mainly on the calculation of the insurance premium by setting a tolerance level for the ruin probability. After the premium was solved, the corresponding utility will be determined. It was assumed that the loss or claims amount follows an exponential distribution or geometric distribution. A closed form expression for

the finitetime ruin probability will be determined. This closed form expression will be used to generate algorithm that calculated the corresponding ruin probability. Computer program will be constructed for these calculations using Scilab.

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CVaR: Partial Hedging and Insurance Applications

Alexander Melnikov
University of Alberta, Canada
E-mail: melnikov@ualberta.ca

Hedging of options is one of the basic and comprehensive problems of mathematical finance which has very interesting insurance applications. The most visible developments in this area during the last decades were done by using the notion of partial or imperfect hedging. We formulate this problem as a possibility to create a terminal capital

which is close enough to given contingent claim in some probabilistic sense. Such understanding of the problem explains clearly why a reasonable statistical technique properly works here creating new types of hedging like quantile, efficient etc. Due to new developments in the theory of risk measures, hedging problem became a new insight. We investigate the partial hedging problem using the most applicable risk measure CVaR-Conditional Value at Risk. We develop the CVaR optimization technique along with the fundamental Neuman-Pearson lemma to solve hedging problem by minimizing CVaR under initial budget constraints. Explicit solutions will be derived in the framework of the Black-Scholes and regime-switching market models. Applications for pricing of equity-linked life insurance contracts and for valuation of regulatory capital requirements will be given. Besides that an efficient technique for CVaR estimation will be provided based on the so-called path-wise comparison theorem for strong solutions of stochastic differential equations. Theoretical findings will be numerically illustrated.

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Semi-Markov Models for Actuarial Technical Balances of Pension Funds

Marco Micocci

University of Cagliari, Italy

E-mail: micocci@unica.it

Filippo Petroni

University of Cagliari, Italy

E-mail: fpetroni@gmail.com

The paper faces the problem of actuarial valuations of future streams of salaries and contributions in actuarial technical balances of defined

benefit pension funds. Usually, in this kind of pension funds, future contributions paid by the participants are linked to their salaries trough a simple formula; for example they can be expressed as a determined percentage of the current salary. From this link arise the importance of a precise projection of the future levels of salaries to fully understand the actuarial sustainability of the fund and the adequateness of the technical reserves. Usually actuarial valuations of pension funds are performed by projections of the current level of salaries with the use of deterministic carrier lines that are calibrated by observing the past dynamics of the salaries vs the seniority of the participants eventually including the change of the working level of the active (i.e. from employee to manager). In this paper we model the future dynamic of salaries by applying a stochastic semi-Markov approach. The semi-Markov model describes the future changes of level of the participants and of their salaries. The model is applied to a large dataset of a real compulsory italian pension fund of the first pillar and the actuarial technical balance of the fund is calculated following a Monte Carlo approach. Results obtained with the semi-Markov model are compared with those coming from the use of traditional technics for the estimation of the carrier lines.

Continuous Chain Ladder

María Dolores Martínez Miranda
City University London, U.K.
E-mail: Maria.Miranda.1@city.ac.uk
Jens Perch Nielsen
City University London, U.K.
E-mail: Jens.Nielsen.1@city.ac.uk
Richard Verrall
City University London, U.K.
E-mail: R.J.Verrall@city.ac.uk

A novel approach to claims reserving is set out. Considering the data in continuous time, Martínez-Miranda, Nielsen, Sperlich and Verrall (2013) present the natural continuous version of the classical chain ladder method. One purpose of the paper is to establish that the

classical chain ladder estimator can be understood as a structured histogram. We argue that this interpretation of the classical chain ladder method is extremely relevant for its practical implementation. Our histogram interpretation of the classical chain ladder estimator has a number of immediate implications: first, the bandwidth selection problem of the histogram (whether chain ladder is based on monthly, quarterly or yearly data) is following the laws of bandwidth selection of non-parametric smoothing problems well known in the mathematical statistical literature. Secondly, it is well known from mathematical statistics that histograms are not efficient when estimating smooth curves. The most common and most immediate improvement to histogram methods is simply the kernel smoothing method. Thirdly, classical chain ladder is indeed a granular method, because the histogram works on continuous data. Then one could stay in the "good old chain ladder histogram world" and first derive two piece-wise functions from an under-smooth two-dimensional histogram. One in the underwriting year direction and other in the development year direction. And then smooth these one-dimensional functions. Unfortunately our study cannot recommend this. This is unfortunate, because this simple method of sieves would be intuitively appealing to many applied actuaries.

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Exponential Asymptotical Expansions for Ruin Probability in a Classical Risk Process with Non-Polynomial Perturbations

Ying Ni

Mälardalen University, Västerås, Sweden
E-mail: ying.ni@mdh.se

In this paper we investigate the asymptotical behaviour of ruin probability in a classical compound Poisson risk process associated with perturbations in the claim size distributions and/or other parameters of the risk process. The novelty of this study is that we consider

non-polynomial perturbations which includes the polynomial perturbations as particular cases. The aim of the study is to develop exponential asymptotical expansions for the ruin probability as the initial capital goes to infinity and the perturbation parameter goes to zero, simultaneously but in a balanced manner. Such asymptotical expansions are derived and presented using techniques of perturbed renewal equations with non-polynomial perturbations developed subsequently in [1] and [2 – 4]. The obtained asymptotical results cover two cases. In the first case, the loading rate of claims, roughly defined as the average claim amount per unit time divided by the premium rate, is equal to one in the limiting model, as typical in a *diffusion approximation* for ruin probability. For the second case the limiting loading rate of claims is less than one. The results from the second case generalizes the *Cramér-Lundberg approximation* to our perturbed model of risk process. The present results also generalize the corresponding part of results in [5], where similar problems with polynomial type of perturbations are investigated. Computer algorithm and programs for determining the coefficients in the exponential asymptotical expansions of ruin probability are developed and described in the paper. Numerical examples of risk processes with such type of perturbations are also given for illustrative purposes.

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sitions in Mathematics, **44**, Walter de Gruyter, Berlin.

The Geometric Chain-Ladder

Bent Nielsen
University of Oxford, U.K.
E-mail: bent.nielsen@nuffield.ox.ac.uk
Di Kuang
Hiscox Insurance Company, U.K.
E-mail: kuang.di@gmail.com
Jens Nielsen
City University, London, U.K.
E-mail: Jens.Nielsen.1@city.ac.uk

The log normal reserving model is considered. Explicit expressions for the maximum likelihood estimators are derived. These are expressed in terms of development factors which are geometric averages. The distribution of the estimators is derived. It is shown that the analysis is invariant to traditional measures for exposure.

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Ruin Theory Seen Differently: Adapting Premiums to a Prespecified Lundberg Bound

Ragnar Norberg
Université Lyon 1, France
E-mail: ragnar.norberg@univ-lyon1.fr

The traditional point of view in ruin theory is reversed: rather than studying the probability of ruin as a function of the initial reserve under fixed premium, the problem is to adjust the premium so as to obtain a given ruin probability (solvency requirement) for a fixed initial reserve (the financial capacity of the insurer). This programme works in quite general models with all sorts of dependence, the standing assumption being that increments of the claims process possess exponential moments so that variations of the Lundberg upper bound for the probability of ruin are in reach.

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Asymptotic Consistency and Inconsistency of the Chain Ladder

Michal Pešta

Charles University in Prague, Czech Republic
E-mail: pesta@karlin.mff.cuni.cz

The distribution-free chain ladder reserving method belongs to the most frequently used approaches in the general insurance. It is well known, see [1], that the estimators $\widehat{f}_j$ of the development factors are unbiased and mutually uncorrelated under some mild conditions on the mean structure and under the assumption of independence of the claims in different accident years. In [2], we deal with some asymptotic properties of $\widehat{f}_j$. Necessary and sufficient conditions for asymptotic consistency of the estimators of true development factors f_j are provided. A rate of convergence for the consistency is derived. Possible violation of these conditions and its consequences are discussed, and some practical recommendations are given. Numerical simulations and a real data example are provided as well.

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Asymptotics of Ruin Probabilities for Perturbed Discrete Time Risk Processes

Mikael Petersson

Stockholm University, Sweden
E-mail: mikpe@math.su.se

We consider the problem of approximating the infinite time horizon ruin probabilities for discrete time risk processes. The approach is

based on asymptotic results for non-linearly perturbed discrete time renewal equations obtained in [1-3]. Under some moment conditions on the claim distributions, the approximations take the form of asymptotic exponential expansions with respect to the perturbation parameter. We show explicitly how the coefficients of these expansions can be computed as functions of the coefficients of the expansions of local characteristics for perturbed risk processes. The results are also illustrated by computer simulations.

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Restructuration, Default and Bankruptcy for Lévy Insurance Risk Processes

Jean-François Renaud

Université du Québec à Montréal (UQAM), Canada

E-mail: renaud.jf@uqam.ca

We introduce an insurance ruin model where the possibility of restructuration is considered and in which classical ruin and bankruptcy are distinguished. In this model, the premium rate is increased as soon as the wealth process falls into the *red zone* and is brought back to its regular level when the process recovers. The analysis is mainly focused on the time the underlying surplus process spends in the red zone (analogous to the duration of the negative surplus). Building on results from [1] and [2], we identify the distribution of various func-

tionals related to occupation times of refracted spectrally negative Lévy processes. For example, these results are used to compute the probability of bankruptcy in this model with restructuration.

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Distortion Risk Measures, Ambiguity Aversion and Optimal Effort

Christian Y. Robert
Université Lyon 1, France
E-mail: christian.robert@univ-lyon1.fr

We consider the class of concave distortion risk measures to study how choice is influenced by the decision-maker's attitude to risk and provide comparative static results. We also assume ambiguity about the probability distribution of the risk and consider a framework à la Klibanoff, Marinacci, and Mukerji (2005) to study the value of information that resolves ambiguity. We show that this value increases with greater ambiguity, with greater ambiguity aversion, and in some cases with greater risk aversion. Finally we examine whether a more risk-averse and a more ambiguity-averse individual will invest in more effort to shift his initial risk distribution to a better target distribution.

A Survey of Actuarial Software

Oskar Schyberg
Mälardalen University, Västerås, Sweden
E-mail: oskar.schyberg@mdh.se
Anatoliy Malyarenko
Mälardalen University, Västerås, Sweden
E-mail: anatoliy.malyarenko@mdh.se
Dmitrii Silvestrov

Stockholm University, Sweden
E-mail: silvestrov@math.su.se

A survey of actuarial software is presented. The software is classified as free versus commercial, actuarial computer programs versus programs with actuarial components, and by groups: life-insurance, non-life-insurance, reinsurance, etc. The survey may be useful for researchers in the area of actuarial science, practitioners from insurance and other specialists interested in the current state-of-art information about modern actuarial software.

Penalized Least Squares Smoothing of Two-Dimensional Mortality Tables with Imposed Smoothness

Eliud Silva
Universidad Anáhuac, México, Mixico
E-mail: jsilvaurrutia@hotmail.com
Victor M Guerrero
Instituto Tecnológico Autónomo de México, Mixico

This paper presents a method useful to impose a percentage of smoothness when smoothing two-dimensional mortality tables via Penalized Least Squares. We can decide at the outset a desired percentage of smoothness to impose in the dimension of age, the dimension of year or both, in order to obtain comparable smoothed mortality trends, for different datasets. Since the smoothing method is mainly determined by two smoothing parameters, we employ some indices that relate those parameters to the percentages of smoothness. We define a smoothness index for the one-dimensional case and generalize it to the two-dimensional one. For illustrative purposes, we apply the proposed method to data from the Continuous Mortality Investigation Bureau of the UK. The numerical examples are useful to appreciate that different log-mortality patterns can be obtained when marginal smoothness changes, but the joint percentage of smoothness stays fixed. Thus, emphasizing the fact that, in practice, we should care about marginal as well as joint smoothness.

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Improved Asymptotics for Ruin Probabilities

Dmitrii Silvestrov
Stockholm University, Sweden
E-mail: silvestrov@math.su.se

This communication presents a survey of results on asymptotics of ruin probabilities and related functionals achieved by the author and his collaborators. These results are: exponential asymptotic expansions for ruin probabilities and related functionals [2–3], obtained by application of methods of asymptotic analysis for perturbed renewal

equations developed in [1 – 3]; necessary and sufficient conditions for convergence of ruin probabilities in the model of diffusion and stable approximations, obtained by application of results about necessary and sufficient conditions of weak convergence for distributions of the first-rare-event times for semi-Markov processes [4]; and explicit exponential rates of convergence in Cramér – Lundberg and diffusion approximations [8], obtained by application of results on coupling methods for regenerative processes [5–7].

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Default Probabilities for Markov Models Describing Credit Ratings Dynamics

Dmitrii Silvestrov

Stockholm University, Sweden
E-mail: silvestrov@math.su.se

Evelina Silvestrova

Mälardalen University, Västerås, Sweden
E-mail: evelina.silvestrova@mdh.se

Raimondo Manca

University of Rome "La Sapienza, Italy
E-mail: raimondo.manca@uniroma1.it

Credit ratings play nowadays a very important role in financial and risk management. Dynamics of credit ratings significantly influence investment flows at finance, insurance, energy, and other markets. At present, it becomes common to use Markov chains and related models to describe the dynamics of credit ratings as an indicator of the likelihood of rating default and other risk rating events. Stochastically ordered Markov models are used for stochastic modeling of credit rating dynamics. Default probabilities are studied for these models. Results of studies are partly presented in [1].

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Variability in the Level of Concentration in Indian Life Insurance Industry

Rajat Kumar Sinha

Insurance Regulatory and Development Authority (IRDA), India
E-mail: rksinha@irda.gov.in

The market structure of an industry can be classified into four brood stages viz. Monopoly, Oligopoly, Monopolistic Competition and Perfect Competition. The Indian life insurance industry remained in

monopoly till it was privatized in 1999. Since then, it is under the consistent process of transition from the stage of Monopoly (an extreme scenario) towards perfect completion (the other extreme scenario). The privatization in the industry has opened the doors for private insurers, which continue to enter into the market leading to increasing level of competitiveness in the industry. The paper attempts to analyze the level of market competition in the post-privatized era using two benchmark measures, viz. Concentration Ratio (CR) and Herfindahl-Hirschman Index (HHI). It reveals that the Indian life insurance industry is on the consistent path of moving from the stage of monopoly towards perfect competition and its current stage appears to be of a monopolistic competition. Further, the paper assesses the market competition in various states and union territories of India and reveals that there exists a wide variation in the concentration across various states. It concludes that many private insurers are underwriting their business in localized regions, which vary by companies. At the end, paper reveals that the state-wise composition of premium underwritten by insurers are highly correlated with the geographical distribution of the number of offices and number of agents. On carrying out a multiple linear regression analysis, both of these two variables are found to be statistically significant, which are influencing heavily in determining the business size of the insurers in the specific regions.

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Master's Programme in Actuarial Mathematics at Stockholm University

Joanna Tyrcha

Stockholm University, Sweden
E-mail: joanna@math.su.se

Ohlsson Esbjörn

Länsförsäkringar AB / Stockholm University, Sweden
E-mail: Esbjorn.Ohlsson@lansforsakringar.se

Gudrun Brattström

Stockholm University, Sweden
E-mail: gudrun@math.su.se

Actuarial education, as it is organized in e.g. the Core Syllabus of the Groupe Consultatif Actuariel Europen, consists of theoretical courses in mathematics and mathematical statistics, as well as more applied courses in insurance mathematics and finally some orientation in topics outside mathematics, viz. law, accounting and economics. The

Division of Mathematical Statistics at the Department of Mathematics, Stockholm University, is responsible for a Master's Programme of this kind. The programme meets the requirements of the Swedish Actuary Society, and hence those of the International Association of Actuaries and Group Consultatif Actuariel Européen. We will present some of the compulsory courses in the programme, and raise a couple of general issues in that context. The course Risk models and claims reserving in non-life insurance is part of the masters programme in actuarial mathematics at Stockholm University. Among other things it covers the collective model, re-insurance and claims reserving. The more practical aspects of these topics are taught using a version of the case method, where the case and the data come from an insurance company. In this way, students are prepared directly for their vocation as actuaries. The case studies comprise nearly half the course, and the students put a lot of effort into them. With examples from the course Non-life insurance pricing with GLMs, we will discuss the balance between theory and practice in the actuarial education from the point of view of the work at a non-life insurance company and in particular the requirements of an appointed actuary. Is it necessary to learn theory that is rarely or never applied at an insurance company? On the other hand: is it futile for universities to try to teach practical skills which are not of scientifically based? Or is there a common ground where the two may be combined?

Design of an Optimal Bonus-Malus System Using the Sichel Distribution as a Model of Claim Counts

George Tzougas

Athens University of Economics and Business, Greece

Nicholas Frangos

Athens University of Economics and Business, Greece
E-mail: nef@aueb.gr

Irini Dimitriyadis

Bahcesehir University, Turkey

This paper presents the design of an optimal Bonus-Malus System (BMS) using the Sichel distribution to model the claim frequency dis-

tribution. This system is proposed as an alternative to the optimal BMS obtained by the traditional Negative Binomial model (Lemaire, 1995). In fact the Sichel distribution has a thicker tail than the Negative Binomial distribution and it is considered as a plausible model for highly dispersed count data. We also consider the optimal BMS provided by the Poisson-Inverse Gaussian distribution, which is a special case of the Sichel distribution. In the above setup optimality is achieved by minimizing the insurers risk. Furthermore we develop a generalized BMS that takes into account both the a priori and a posteriori characteristics of each policyholder. For this purpose we consider to the generalized additive models for location, scale and shape (GAMLSS) in order to use all available information in the estimation of the claim frequency distribution. The GAMLSS were introduced by Rigby and Stasinopoulos (2001, 2005) and Akantziliotou et al. (2002) as a framework for fitting regression type models where the distribution of the response variable does not have to belong to the exponential family and includes highly skew and kurtotic continuous and discrete distribution. The GAMLSS allows all the parameters of the distribution of the response variable to be modelled as linear/non-linear or smooth functions of the explanatory variables. Within the framework of the GAMLSS we propose the Sichel GAMLSS for assessing claim frequency as an alternative to the Negative Binomial regression model used by Dionne and Vanasse (1989, 1992). We also consider the Negative Binomial Type I and the Poisson-Inverse Gaussian GAMLSS for assessing claim frequency. With the aim of constructing an optimal BMS by updating the posterior mean claim frequency, we adopt the parametric linear formulation of these models and we allow only their mean parameter to be modelled as a function of the significant a priori rating variables for the number of claims.

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A Compound Poisson Risk Model with Loss-Sensitive Premium

Lester Charles A. Umali

University of the Philippines Los Baños, Philippines
E-mail: umali.charles@yahoo.com.ph

Jose Maria L. Escaner IV

University of the Philippines Diliman, Philippines

We consider the surplus process of an insurance portfolio with a loss-sensitive feature wherein the premium varies depending on the portfolio's loss experience. Particularly, we give a compound Poisson risk model for such a loss-sensitive insurance, develop an integro-differential equation for the ruin probability and develop an upper bound for the ultimate probability of ruin.

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Value-at-Risk Aggregation When the Variance of the Sum is Known

Steven Vanduffel
Vrije Universiteit Brussel, Belgium
E-mail: steven.vanduffel@vub.ac.be
Carole Bernard
University of Waterloo, Canada
E-mail: c3bernar@uwaterloo.ca
Ludger Rüschendorf
University of Freiburg, Germany
E-mail: ruschen@stochastik.uni-freiburg.de

In the literature, Value-at-Risk bounds of sums of random variables when only the marginal distributions of the components are known have been extensively studied. In this paper, we study bounds when also the variance of the sum is known. We derive absolute bounds and discuss their sharpness. When the bounds are not sharp we propose an algorithm to obtain approximate sharp VaR-bounds. Numerical experiments show that the bounds obtained by the algorithm often correspond closely to the absolute bounds. In the second part of the paper, we specialize our results to the setting of Bernoulli distributed random variables. We show how our results for Bernoulli risks are particularly relevant to the case of credit risk.

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Corrected Phase-Type Approximations for Heavy-Tailed Risk Models in a Markovian Environment

Eleni Vatamidou

Eindhoven University of Technology, Holland
E-mail: e.vatamidou@tue.nl

Numerical evaluation of ruin probabilities in heavy-tailed risk models is a challenging problem, especially when there exist significant correlations between arrivals of claims. In this paper, we assume that claims arrive according to a Markovian Arrival Process, and we construct very accurate approximations of the ruin probabilities for every initial environment. Our approximations capture the tail behavior of the exact ruin probabilities and provide a small relative error. Motivated by statistical analysis, we assume that the claim sizes are a mixture of a phase-type and a heavy-tailed distribution. We consider the model that appears when all heavy-tailed claims are removed as a base model, and we evaluate the corresponding ruin probabilities. Using perturbation analysis, we derive our approximations as a sum of the ruin probabilities of the base model and a heavy-tailed component that depends on the perturbation parameter. We refer to our approximations as corrected phase-type approximations, and we use the classical Cramér-Lundberg model as test model to exhibit their performance.

Applications of Mixture of Priors in the Bayesian Credibility

Amin Hassan Zadeh

Shahid Beheshti University, Tehran, Iran
E-mail: am.hassanzadeh@sbu.ac.ir

Saman Ebrahimpour

Shahid Beheshti University, Tehran, Iran

In Bayesian credibility theory, modelling of loss data when it has more than one mode is challenging. While most actuaries' target model is a mixture of losses, we propose a mixure of priors. Using a mixure of priors has many advantages. First, an infinite mixture of natural conjugate is avoided. Secondly, the statistical inference is much more tractable and finally, in some situations, exact credibility occurs. This results are used to model real multi-modal data from an insurance company.

Fitting Grouped Insurance Data by PhaseType

Amin Hassan Zadeh

Shahid Beheshti University, Tehran, Iran
E-mail: am.hassanzadeh@sbu.ac.ir

Jafar Khezri Shahid

Shahid Beheshti University, Tehran, Iran

Phase type distributions are currently widely used in actuarial studies. In this paper, we model real heavy-tailed grouped loss data with a phase type distribution and the results are compared with some other distribution. The EM (Expectation Maximization) is used to estimate the representation of the distribution. The results are satisfactory.