# QUIVERS AND LIE SUPERALGEBRAS *) **)

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Indecomposable representations of quivers are in 1–1 correspondence with positive weight vectors of Kac-Moody algebras. The collection of indecomposable representations of the quiver is tame if the quiver corresponds to a Kac-Moody algebra of polynomial growth. What corresponds to positive roots of Lie algebras of polynomial growth different from Kac-Moody algebras? The classification problem for tame representations of quivers associated to Lie superalgebras is a natural step towards the answer to this question. As an aside we announce a classification of simple graded Lie superalgebras of polynomial growth.

*To Ernest Borisovich Vinberg*

## Introduction

This is a short announcement, the details will be given elsewhere.

**Classical results ([1] and refs. therein). A problem. Lie superalgebras as a step towards its solution.**

(A) I. Gelfand and V. Ponomarev showed that virtually all tame problems of finite dimensional linear algebra can be reduced to classification of *quadruples* of subspaces. This is one of numerous problems (ranging from perverse sheaves to quantum groups ([2]), to magneto-hydrodynamics ([3]), and so on, see [4]) that can be expressed in terms of representations of *quivers*, i.e. directed simple graphs (without edges-loops and multiple edges).

I. Gelfand and V. Ponomarev further demonstrated that unsolvable (*wild*) problems contain the classification problem for a pair of commuting linear operators as a subproblem and observed that wild problems can be classified, to an extent, in terms of representations of quivers.

(B) showed that only simply laced Dynkin diagrams (corresponding to Lie algebras $\mathfrak{sl}(n)$, $\mathfrak{o}(2n)$ and $\mathfrak{e}_i$) have finitely many indecomposable representations and

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established a 1–1 correspondence between the set of indecomposable representations of the Dynkin graphs and the set of positive roots of the corresponding Lie algebras. Nazarova obtained a result similar to that of Gabriel.

(C) J. Bernstein, I. Gelfand and V. Ponomarev found a way to construct any indecomposable representation of the simply laced quiver of the Dynkin diagram type from the simplest representations, thus clarifying Gabriel's result. J. Bernstein, I. Gelfand and V. Ponomarev also showed that classification of *quadruples* is connected with an extended Dynkin diagram but did not carry over their correspondence to such graphs.

(D) Dlab and Ringel extended the BGP correspondence to any Kac-Moody algebra of polynomial growth by replacing each Dynkin diagram with a *valued graph* and Kac [5] extended the BGP correspondence to any Kac-Moody algebra with Dynkin diagram, even to diagrams with edges-loops.

(E) Kac further showed [6] that tame problems of quiver representations are in 1-1 correspondence with the quivers equal to Dynkin diagrams of *affine* Kac-Moody algebras, i.e. the ones of polynomial growth.

(F) The above BGP correspondence is executed by *Coxeter functors*. These functors can be defined even in the absence of quivers [7].

Result (F) makes the following problem natural:

**Problem** *Several simple Lie algebras of polynomial growth have no Dynkin diagram (e.g., Lie algebras of vector fields, Lie algebras of matrices of complex size [8]). Is there anything like Coxeter functor corresponding to such algebras?*

**Our result**

We will demonstrate how Lie *super*algebras with Dynkin-Kac diagrams (these are graphs with vertices of three or four different types) constitute a natural intermediate stage in the general construction of Coxeter functors for Kac-Moody algebras and superalgebras with an arbitrary Cartan matrix, not necessarily corresponding to a Dynkin diagram, and describe via BGP method their tame indecomposable representations. We deduce that the superization of the classification problem for quadruples of subspaces is wild.

## 1 Quivers

A *directed graph* $Q$, i.e., a family of vertices $Q_0$ and ordered pairs (*arrows*) $Q_1 = \{(i,j) : i,j \in Q_0\}$ is sometimes called a *quiver*. Not all orientations are allowed: people who used arrows for hunting hardly ever kept their quivers untidy; neither do modern mathematicians: they do not let arrows be stuck in the quiver pell-mell: each vertex should be either a sink or a source for all arrows it belongs to. (This requirement, natural, perhaps, for hunters seems to be mathematically *ad hoc.*) Certain feebleness of mathematicians occasioned by excessive studies manifested itself in allowing the mathematicians' arrows degenerate into loops. Orientations of the whole graph that bring about loops composed of neighboring arrows are, however, forbidden.

A *representation* of the quiver is a collection of vector spaces $V_i$, $i \in Q_0$, and their homomorphisms $\varphi_{ij} : V_i \longrightarrow V_j$ for every $(i,j) \in Q_1$. The *sum* of representations $(U,\varphi)$ and $(V,\psi)$ of the same quiver $Q$ is the collection $(W,\theta)$ with $W_i = U_i \oplus V_i$ and $\theta = \begin{pmatrix} \varphi_{ij} & 0 \\ 0 & \psi_{ij} \end{pmatrix}$. A representation of $Q$ is called *indecomposable* if it can not be represented as a direct sum of nonzero representations.

Having in mind the study of quivers' representations we can confine ourselves to the connected quivers.

### 1.1 Gabriel's discovery. A trick of Dlab and Ringel.

Gabriel found out two amazing facts:

1. an indecomposable representation of a quiver does not depend on its orientation (we only consider admissible orientations).

2. Quivers with only finitely many indecomposable representations are the ones called Dynkin diagrams of types $A$, $D$, $E$, corresponding to finite dimensional Lie algebras $\mathfrak{sl}(n)$, $\mathfrak{o}(2n+1)$ and $\mathfrak{e}_6$, $\mathfrak{e}_7$, $\mathfrak{e}_8$. There is a 1–1 correspondence between the set of indecomposable representations of an $ADE$ quiver and the set of positive roots of the corresponding Lie algebra.

By a trick Dlab and Ringel extended this correspondence to any Dynkin diagram. They suggested a sophisticated cheating: to replace multiple edges and arrows with a rig — a pair of number — over the corresponding edge and call the simply laced (but rigged) graph obtained a *valued graph*.

At this stage we need precise formulations.

### 1.2 Valued graphs $\Longleftrightarrow$ Kac-Moody algebras

A *valued graph* $Q$ on the set $Q_0$ of its vertices is a function $d : Q_0 \times Q_0 \longrightarrow \mathbb{Z}_+$ such that (we write $d_{ij}$ instead of $d(i,j)$) (i) $d_{ij} \neq 0 \Longleftrightarrow d_{ji} \neq 0$; (ii) if $Q_0$ is infinite, then for every $i$ there is only finitely many $j$'s such that $d_{ij} \neq 0$.

An edge connects vertices $i$ and $j$ if and only if $d_{ij} \neq 0$; we rig the edge $(i,j)$ with a pair of numbers as follows: $d_{ij}$ over the $i$-th end of the edge and $d_{ij}$ over its $j$-th end.

Since a valued graph is completely recovered from its matrix $D = (d_{ij})$, we will not distinguish in what follows between the matrix $D$ and the quiver it determines.

A *Cartan matrix* is a matrix $A = (A_{ij})$ such that

(i) $A_{ij} \in -\mathbb{Z}_+$ for $i \neq j$ and $A_{ij} \neq 0 \Longleftrightarrow A_{ji} \neq 0$;

(ii) $A_{ii} = 2$ or, for a *generalized Cartan matrix*, $A_{ii}$ is an even integer $\leq 2$.

Additionally, if $A$ is an infinite matrix, then for every $i$ there is only finitely many $j$'s such that $A_{ij} \neq 0$. The 1–1 correspondence between valued graphs and Cartan matrices is as follows: $A_{ij} = -d_{ij}$; $\quad A_{ii} = 2 - 2d_{ii}$.

### 1.3 Tame and wild representations of quivers

For a quiver $Q$, the *dimension* of its representation $(V, \varphi)$ is a collection $(\dim V_1, \dots \dots, \dim V_n)$, where $n = \text{card} Q_0$.

Kac showed [5, 6] that if the number of parameters $\mu_\alpha$ of the set of indecomposable representations of dimension $\alpha = (\alpha_1, , \dots, \alpha_n)$ is $> 1$, then the problem of classification of the indecomposable representations of the quiver is wild. Contrarywise, if the number of parameters is $\leq 1$, it is tame. Kac gave an ingenuous proof of the fact that the number of parameters $\mu_\alpha$ is equal to

$$\mu_\alpha = 1 - (\alpha, \mu_\alpha),$$

where $(\cdot, \cdot)$ is the invariant bilinear form on the root system of the Lie algebra $\mathfrak{g}(A)$.

### 1.4 Coxeter transformations. A way to get any root from a simple one.

Let $Q$ be a simply laced graph, $E_Q$ the linear space over $\mathbb{Q}$, consisting of collections $x = (x_\alpha : \alpha \in Q_0)$. For each $\beta \in Q_0$ let $e_\beta$ be the basis vectors of $E_Q$. We say that $x > 0$ if $x \neq 0$ and $x_\alpha \geq 0$ for all $\alpha$. Let $B$ be the Cartan-Tits quadratic form on $E_Q$ given by the formula $B(x) = \sum_{\alpha \in Q_0} x_\alpha^2 - \sum_{l \in Q_1} x_{i(l)} x_{t(l)}$, where $i(l)$ and $t(l)$ are the initial and terminal points of the edge $l$. Let $< \cdot, \cdot >$ be the bilinear form associated with $B$. For each $\beta \in Q_0$ denote by $\sigma_\beta$ the linear transformation of $E_Q$ given by the formulas

$$(\sigma_\beta(x))_\gamma = \begin{cases} x_\gamma & \text{if } \beta = \gamma, \\ -x_\beta + \sum\limits_{l \in Q_1} x_\gamma(l) & \text{otherwise,} \end{cases}$$

where $x_\gamma(l)$ is the endpoint of $l$ different form $\beta$. Finally, let $W$ be the semigroup generated by the reflections $\sigma_\beta$. A miracle happens: $W$ is actually a group (called the *Weyl group*). Important properties of $W$: it preserves the integer lattice in $E_Q$ and $< \cdot, \cdot >$.

We say that $x \in E_Q$ is a *root* if $x = w e_\beta$ for some $\beta \in Q_0$ and $w \in W$. The basis vectors are called *simple roots*. Let $\alpha_1, \dots, \alpha_n$ be an enumeration of $Q_0$. The element

$$C = \sigma_{\alpha_n} \cdot \dots \cdot \sigma_{\alpha_1}$$

is called *Coxeter transformation*. Clearly, since $C$ depends on the enumeration of $Q_0$, there are $n!$ Coxeter transformations, generally.

Gabriel's theorem classifies indecomposable representations of Dynkin graphs of ADE type in terms of one discrete invariant: the vector of dimension. For more general graphs we need continuous invariants and these are the eigenvalues of the Coxeter transformations. Regretably, a review of (rather numerous) results on Coxeter transformations exists only in a pretty hidden form [4] and the results reviewed are also buried in the same depositions.

## 2 Lie superalgebras of polynomial growth

The classics considered various tame problems of linear algebra expressible in terms of quiver representations and associated them with positive roots of certain Kac–Moody algebras. These Lie algebras are qualified as simple Lie algebras of polynomial growth but not all such algebras. We recall what is known about the Lie superalgebras of the same type. In Sect. 3 we recall the definition of the analog of Weyl group for Lie algebras and Lie superalgebras without Cartan matrix.

### 2.1 Simple $\mathbb{Z}$-graded Lie algebras of polynomial growth

About 1966, V. Kac and B. Weisfeiler began the study of simple *filtered* Lie algebras of *polynomial growth*. Kac first considered the *$\mathbb{Z}$-graded* Lie algebras associated with the filtered ones and classified *simple $\mathbb{Z}$-graded* Lie algebras of *polynomial growth* under a technical assumption. It took more than 20 years to get rid of the assumption: see very technical papers by O. Mathieu, cf. [9] and refs. therein. Kac's list contains: finite dimensional Lie algebras and twisted loop algebras (these algebras possess Cartan matrices and, therefore, Weyl group). The remaining algebras from Kac's list are: Lie algebras of vector fields with polynomial of formal coefficients ($\mathfrak{vect}(m) = \mathfrak{der}\,\mathbb{C}[x]$ for $x = (x_1, ..., x_m)$ and its subalgebras of divergence-free, Hamiltonian and contact fields) and the Witt algebra $\mathfrak{witt} = \mathfrak{der}\,\mathbb{C}[t^{-1}, t]$. It seemed, they have no analog of the Weyl group.

Recent attempt [10] to classify *filtered* Lie algebras of polynomial growth provided us with a wealth of new algebras, some of them known ($\mathfrak{diff}(n)$, the Lie algebra of differential operators in $n$ indeterminates; $\mathfrak{gl}(\lambda)$, the algebra of matrices of complex size). Do *they* have an analog of Weyl group? Superization helps to answer, cf. [11].

### 2.2 Simple filtered Lie superalgebras of polynomial growth

Simple $\mathbb{Z}$-graded superalgebras are classified in several papers: [12] (finite dimensional ones); [13] and [14] (twisted loop algebras, without and with symmetrizable Cartan matrix, respectively); [15] (vectorial Lie superalgebras, i.e., homogeneous subalgebras of $\mathfrak{vect}(m|n)$) and [16] ("stringy", the analogs of $\mathfrak{witt}$).

For the list of examples of simple filtered superalgebras see [10].

## 3 System of simple roots. From the Weyl group to a skorpenser.

Superization naturally intermixes the classes considered in Subsect. 2.1: some vectorial superalgebras are finite dimensional; these are not the only simple finite dimensional Lie superalgebras without any Cartan matrix, there is also a *queer* series; the outcome of twisting of the loop algebra with values in a superalgebra without Cartan matrix can be an algebra with a Cartan matrix, though nonsymmetrizable one; the outcome of twisting of the loop algebra with values in a superalgebra *with*

Cartan matrix can be an algebra *without* any Cartan matrix; the Cartan matrix of a finite dimensional Lie superalgebra can have complex entries and, therefore, have no Dynkin-Kac graph, etc. Therefore, in supersetting the question of what is an analog of Coxeter transformation in the absence of Dynkin-Kac graph or even Cartan matrix is most natural.

Even if the Lie superalgebra has Cartan matrix, there are at least four different types of simple roots; we depict the vertices of Dynkin-Kac diagram corresponding to $\mathfrak{sl}(2)$, $\mathfrak{osp}(1|2)$, $\mathfrak{sl}(1|1)$ by white, black and grey circles, respectively, cf. [12]; it is natural to denote the node corresponding to $\mathfrak{sq}(2)$ by a square circle, cf. [17].

For the Lie superalgebras composed of these building blocks, the notion of the representation of the quiver, which, usually, is a valued graph, is naturally defined: with each vertex we associate a *super*space, the rest is routine, *provided* the vectors of dimension abide certain selection rules: the vectors of dimensions corresponding to simple roots of types $\mathfrak{sl}(2)$, $\mathfrak{osp}(1|2)$, $\mathfrak{sl}(1|1)$ and $\mathfrak{sq}(2)$ are: 1, $0|1$, $0|1$, and $1|1$, respectively.

I. Skornyakov, I. Penkov and V. Serganova introduced, first in presence of a symmetrizable Cartan matrix $A$, the notion of odd reflections, see [18, 19]. Thus, with one Lie superalgebra of the form $\mathfrak{g}(A)$ there were associated several competing analogs of the Weyl group, see [20]. (Actually, the Weyl group appears in various instances and superization of each of them brings about several versions, each with its own weak and strong points.)

In 1990 Vinberg said that to consider the fact that one can multiply reflections in disconnected (on the Dynkin diagram) simple roots a miracle; he said that he only saw neighboring systems of simple roots. But it is so tempting to consider the universal group formally generated by the reflections!

It is also important for applications to quivers' representations to be able to realize this universal group or its quotient *linearly* on weights, or at least, roots. Observe that in addition to the constructions of I. Skornyakov, I. Penkov and V. Serganova, for whose purposes the reflections in odd roots need not generate a group, there is an alternative approach by Manin [18] whose analog of the Weyl group is always a group. We suggest to call the collection of reflections *skorpenser* (after I. Skornyakov, I. Penkov and V. Serganova), if it is not a group. If the skorpenser *is* a group, it can be interpreted geometrically and identified. It turned out that only in rare cases, say for $\mathfrak{gl}(m|n)$, almost all superizations of the Weyl group considered above are identical and linearly act on the space of weights. Usually, even if a superization of the Weyl group preserves the root lattice it does not preserve the set of roots; or, if it does preserve the set of roots, it does not act on weights at all, or, at least, it does not act in the space of weights linearly.

Further studies brought Penkov and Serganova to a remarkable notion of an analog of the Weyl group for *ANY* Lie algebra, see [19], where this notion is considered in super setting. This notion — skorpenser — was further developed in [21] to match the infinite dimensional case. Therefore, we can apply it to the Lie (super)algebras of vector fields and $LU_{\mathfrak{g}}(\lambda)$ — the generalizations of $\mathfrak{g}(\lambda)$.

We started with the task to find out all the cases when it is possible to define a linear action of skorpenser on the space generated by roots and found several new cases.

## 4 Skorpenser – an analog of Weyl groups in super setting ([20])

Here are descriptions of the analogs of the Weyl group in presence of Cartan matrix $A = (a_{ij})$. For the definition of such an analog in the absence of Cartan matrix see [19, 21].

### 4.1 The universal skorpenser

Let $\mathfrak{g}$ be a simple Lie superalgebra, $\mathfrak{h}$ its maximal torus, $B$ a base, $\mathfrak{n}_B$ the nilpotent subalgebra generated by root vectors corresponding to $B$. A subalgebra of the form $\mathfrak{b}_B = \mathfrak{n}_B \oplus \mathfrak{h}$ is called a *Borel subalgebra*. Let $L(\lambda, B)$ be the finite dimensional irreducible $\mathfrak{g}$-module with $\mathfrak{b}$-highest weight $\lambda$; let $\Lambda = \Lambda_B$ be the set of $\mathfrak{b}$-highest weights of all the finite dimensional irreducible $\mathfrak{g}$-modules and $\Gamma_\lambda$ the set of weights of $L(\lambda, B)$.

Denote by $S_\alpha(\gamma)$ for any $\alpha \in R, \gamma \in \Gamma_\lambda$ the *$\alpha$-string through* $\gamma$, i.e., the set

$$\gamma - q\alpha, \dots, \gamma - \alpha, \gamma, \gamma + \alpha, \dots, \gamma + p\alpha$$

such that $\gamma - (q+1)\alpha, \gamma + (p+1)\alpha \notin \Gamma_\lambda$. The number $l_\alpha = p + q - 1$ is called the *length* of the $\alpha$-string.

Set $r_\alpha(\gamma) = \gamma + (p-q)\alpha$. Since the weight $r_\alpha(\gamma)$ is defined for any $\gamma \in \Gamma_\lambda$, there exists a map $r_\alpha : \Gamma_\lambda \mapsto \Gamma_\lambda$. This map will be called the *reflection with respect to $\alpha$*.

Let $F_R$ be the free group with generators $f_\alpha$ for every $\alpha \in R$. Then for any $\lambda \in \Lambda$ there is defined an $F_R$-action on $\Gamma_\lambda$ by the formula $f_\alpha(\gamma) = r_\alpha(\gamma)$.

Let $I_{\lambda,R}$ be the normal subgroup of $F_R$ singled out by the formula

$$I_{\lambda,R} = \{f \in F_R : f(\gamma) = \gamma \ \text{ for all } \ \gamma \in \Gamma_\lambda\}.$$

The group $UW_R = F_R/I_R$, where $I_R = \cap_{\lambda\in\Lambda} I_{\lambda,R}$, will be called the *universal skorpenser* of the root system $R$ (or of $\mathfrak{g}$ and we denote it by $UW_R$ or $UW_\mathfrak{g}$).

Denote by $r_\alpha$ the image of $f_\alpha$ under the natural projection. By construction, $UW_\mathfrak{g}$ acts on $\Gamma_\lambda$ for any $\lambda \in \Lambda$, in particular, it acts on $R$.

**Lemma** a) $r_\alpha^2 = 1$.

b) $r_{-\alpha} = r_\alpha$.

c) *The Weyl group $W_{\mathfrak{g}_{\bar 0}}$ of the Lie algebra $\mathfrak{g}_{\bar 0}$ is naturally embedded into $UW_\mathfrak{g}$.*

d) *Let $w \in W_{\mathfrak{g}_{\bar 0}}$. Then $r_{w(\alpha)} = w r_\alpha w^{-1}$.*

### 4.2 The linear skorpenser

Under the notations of the previous section, define the reflections $r_\alpha$ by the formulas

$$r_{\alpha_i}(\alpha_j) = \begin{cases} -\alpha_j & \text{for } i = j, \\ \alpha_j - a_{ij}\alpha_i & \text{for } i \neq j \text{ and } a_{ii} = 2, \\ \alpha_j - 2a_{ij}\alpha_i & \text{for } i \neq j \text{ and } a_{ii} = 1, \\ \alpha_j + \alpha_i & \text{for } i \neq j \text{ and } a_{ii} = 0\ ,\ a_{ji} \neq 0, \\ \alpha_j & \text{for } i \neq j \text{ and } a_{ii} = a_{ji} = 0. \end{cases}$$

Let $LW_R$ be the group generated by such reflections when $\alpha$ runs over simple roots of *all* bases of $\mathfrak{g}$. We will call $LW_R$ the *linear skorpenser* of $R$ (or of $\mathfrak{g}$).

## 5 Conclusion

It turned out that for the *distinguished* stringy Lie superalgebras of series $\mathfrak{k}^L$ and $\mathfrak{k}^M$ (for their definition see [16]) this skorpenser is a group that linearly acts on the root space and the geometric picture is identical to that for one of the Kac-Moody superalgebras with symmetrizable Cartan matrix provided we forget that the multiplicities of the roots are different. To compute the Coxeter transformation in the case of $\mathfrak{g}(A)$ with a symmetrizable $A$, even with complex entries, and the above distinguished stringy superalgebras is routine but tedious. It turns out that even for finite dimensional Lie superalgebras the Coxeter transformation does not vanish identically. The complete description of the continuous invariants is in progress.

Observe that from classification of simple Lie superalgebras of polynomial growth it is clear that not all problems of linear algebra depicted by a quadruple of subspaces are tame: the corresponding Dynkin-Kac diagram can not have nodes of arbitrary colors or be square ones to describe the Lie superalgera of polynomial growth.

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