of *simple Lie algebra* $\mathfrak{g}$ there are several deformations; the number of parameters in the last example is equal to the rank of $\mathfrak{g}$ and leads to generalizations of the **Lie algebras** of "*matrices of complex size*", cf. [5]. The Posson superalgebra on the vector **superspace** can be realized by vector fields D as

$$\{D \mid L_D(\alpha_1) = 0\},$$
$$\alpha_1 = dt - \sum(p_i dq_i - q_i dp_i) - \sum \varepsilon_j \theta_j d\theta_j.$$

Similarly, the *Buttin superalgebra* (with *Schouten bracket*, i.e., **antibracket**) is

$$\{D \mid L_D(\alpha_0) = 0\}, \quad \alpha = d\tau - \sum(\theta_i dq_i + q_i d\theta_i).$$

The *deformed Buttin superalgebra* is

$$\mathfrak{b}_{a,b}(n) = \{D \in \mathfrak{vect}(n|n+1) \mid L_D(\,{}^{a}_{q,\xi,\tau}\alpha_0^{a-bn}) = 0\}.$$

Instead of a, b, one can consider one parameter

$$\lambda = \frac{2a}{n(a-b)} \in P^1.$$

The *structure functions* (obstructions to flattening the corresponding *G-structure*) are computed in [3]. For infinite dimensional analogs see [4].

Bibliography

[1] D. Leites, I. Shchepochkina, preprint ESI-875; Theor. and Math. Physics, v. 126, no. 3, 339.

[2] Gomis J., Paris J., Samuel S., Phys. Rept. 259 (1995) 1.

[3] D. Leites, E. Poletaeva E. In: O. Kegel et al. (eds.) Proc. Intnat. algebraic conference, Novosibirsk, August 1989, Part 1, Contemporary Math. AMS, 603; id., In: L. Bokut' et al. (eds.) Second International Conference on Algebra (Barnaul, 1991) Contemp. Math., 184, Amer. Math. Soc., Providence, RI, 267; E. Poletaeva, Acta Appl. Math. 31 (1993), no. 2, 137; id., Math. Scand. 72 (1993), no. 2, 161, 191; id. In: C. Bartocci et al. (eds.) Differential Geometric Methods in Theoretical Physics Proc. DGM-XIX, 1990, Lecture Notes in Phys., 375, Springer, Berlin, 1991, 390.

[4] D. Leites, V. Serganova, G. Vinel. In: C. Bartocci et al. (eds.) Differential Geometric Methods in Theoretical Physics Proc. DGM-XIX, 1990, Springer, LN Phys. 375, 1991, 286.

[5] P. Grozman, D. Leites, In: E. Ramírez de Arellano, M. Shapiro, L. Tovar and N. Vasilevski (eds.) Proc. Internatnl. Symp. Complex Analysis and related topics, Mexico, 1996, Birkhauser Verlag, 1999, 73

Dimitry Leites

SUPERTIME — A **supermanifold** of values of a dynamical parameter. On manifolds, Time is usually 1-dimensional, a different example is *Kadomtsev–Petviashvili hierarchy* in which an infinite dimensional manifold is interpreted as Time.

On finite dimensional manifolds, Time is always 1-dimensional as follows from the *rectifyability* of vector field theorem studied at early courses of differential equations. Shander generalized the theorem on *rectifyability* of vector fields to nondegenerate fields on **supermanifolds** and gave the following characterization of such fields, in particular, the ones used in SUSY theories: the nondegenerate (at a point) vector field X can locally be reduced to the form $D_0 = \frac{\partial}{\partial x}$, where x is an even coordinate, if X is even, to the form $D_1 = \frac{\partial}{\partial \theta}$, where θ is an odd coordinate, if X is odd and $X^2 = 0$, or to the form $D = \frac{\partial}{\partial \theta} + \theta \frac{\partial}{\partial x}$, if X is odd and $X^2 \neq 0$.

Shander explained that for dynamical systems on **supermanifolds supertime** runs a $(1|1)$-dimensional **supermanifold** with parameters t, τ. Shander gave examples with **Poisson bracket** and **antibracket**, e.g., he showed that the most profound dynamics is given not by $D_0(f) = \{f, H\}$, but by

$$D(f) = \{f, H\},$$

where the parity of the Hamiltonian, H, should be opposite to that of the (anti)bracket $\{\cdot,\cdot\}$, indeed

$$D_0(f) = \frac{1}{2}\{f, \{H, H\}\}.$$

This explanation enables one to pick up odd parameters missed under the conventional crude approach, but no physical paper used this so far.

Bibliography

[1] V. Shander, Functional Anal. Appl. **14** (1980) 160.

Dimitry Leites

SUPERTRACE — A linear functional on a **Lie superalgebra** that vanishes on the superbracket and denoted by str (or just tr). For the definition of the "usual" **supertrace** of the *supermatrix*, not necessarily in the standard format, see [1,2]. There is also *queertrace*, qtr, defined on a "queer" superanalog of the matrix algebra $\mathfrak{q}(n)$ by the same characteristic property but since $\mathfrak{q}(n)$ is a subalgebra in $\mathfrak{gl}(n|n)$ we can compare str and qtr and see that they are totally different functions; in particular, one is even and the other one is odd, [3]. Both **supertrace** and *queertrace* have a contraction into the **Berezin integral** — the **supertrace** on the *Poisson–Lie superalgebra* $\mathfrak{po}(0|n)$ whose parity is equal to that of n.

These **supertraces**, being defined on finite dimensional algebras, can be integrated to groups, so they correspond to **superdeterminants**:

$$\det(\exp(X)) = e^{tr(X)}.$$

There are also superanalogs of trace on infinite dimensional **Lie superalgebras**, they do not necessarily correspond to **superdeterminants**. Examples: *stringy* superalgebras $\mathfrak{k}^L(1|4)$ and $\mathfrak{k}^M(1|5)$, cf. [4], *special Buttin superalgebras* $\mathfrak{sb}(n)$, and divergence free algebras $\mathfrak{svect}(1|n)$. The parity of these **supertraces** is equal to that of the number of odd indeterminates.

Bibliography

[1] D. Leites, Russian Math. Surveys, **35** (1980) 3.

[2] F. A. Berezin Introduction to superanalysis. D. Reidel Publishing Co., Dordrecht, 1987. xii+424 pp.

[3] J. N. Bernstein, D. Leites, C. r. Acad. Bulg. Sci., v. **35**, 285.
[4] P. Grozman, D. Leites, I. Shchepochkina, Acta Math. Vietnamica, **26**, (2001) 27; hep-th/9702120.
[5] D. Leites, I. Shchepochkina, Theor. Math. Physics, **126** 339.

Dimitry Leites

SUPERTRACE FORMULA — A linear form ϕ on a $\mathbb{Z}_2$-graded (super)algebra A satisfying $\phi([a,b]) = 0$ for all $a, b \in A$, where $[a, b]$ is the **supercommutator** (see entry). When $V = V^+ \oplus V^-$ is a $\mathbb{Z}_2$*-graded vector space*, the endomorphism algebra End(V) provides an example of a superalgebra for which there exists a canonical **supertrace** given by

$$Str(a) = \begin{cases} tr_+(a) - tr_-(a), & \text{if } |a|=0; \\ tr_+(a) + tr_-(a), & \text{if } |a|=1, \end{cases}$$

where $tr_\pm$ is the ordinary trace on End($V^\pm$), and $a \in End(V)$ is said to have degree $|a| = 0$ (resp. 1) if it maps $V^\pm$ into $V^\pm$ (resp. $V^\mp$). Observe that the **supertrace** of the odd matrix is given by the usual trace.

Bibliography

[1] D. Leites, Russian Math. Surveys, **35** (1984) 3.
[2] P. Deligne et al (eds.) Quantum fields and strings: a course for mathematicians. Vol. 1, 2. Material from the Special Year on Quantum *Field Theory* held at the Institute for Advanced Study, Princeton, NJ, 1996–1997. AMS, Providence, RI; Institute for Advanced Study (IAS), Princeton, NJ, 1999. Vol. 1: xxii+723 pp.; Vol. 2: pp. i–xxiv and 727–1501.
[3] Yu. Manin, Gauge field theory and complex geometry. Second edition. Grundlehren der Mathematischen Wissenschaften, 289. Springer-Verlag, Berlin, 1997.

Gert Roepstorff

SUPERTUBE — A stable supersymmetric configurations of cylindrical topology for D2-branes in *type IIA string theory* preserving a quarter of the supersymmetries of the flat *Minkowski spacetime* [1]. The supertubes are supported against collapse by an angular momentum, which is the result of a combined electric and magnetic field on the world volume. They can be obtained by starting from a sequence of *D0-branes* placed on a long string, like beads on a necklace. This structure then starts spinning around the longitudinal axis and blows up into a supertube [1,2].

The generalizations of the supertube in a T-dual *type IIB* picture, when the **T-duality** acts along a perpendicular direction and along the supertube were studied in [2] and [3]. The supertubes in the matrix model context were considered in [4]. A direct generalization of supertubes, when the string networks (or **web** [5]) indeed blow up into D3-branes, is called D3-supertube [6] or supersheet [7].

Bibliography

[1] D. Mateos, P. K. Townsend, Phys. Rev. Lett. **87** (2001) 011602;
[2] J.-H. Cho, P. Oh, Phys. Rev. **D64** (2001) 106010; D. Mateos, S. Ng, P. K. Townsend, J. High Energy Phys. **0203** (2002) 016.
[4] D. Bak, S.-W. Kim, Nucl. Phys. **B622** (2002) 95.
[5] O. Aharony, A. Hanany, B. Kol, J. High Energy Phys. **9801** (1998) 002; A. Sen, J. High Energy Phys. **9803** (1998) 005.
[6] D. K. Park, S. Tamaryan, H. J. W. Müller-Kirsten, Phys. Lett. **B551** (2003) 187; S. Tamaryan, D. K. Park, H. J. W. Müller-Kirsten, hep-th/0209239.
[7] E. Verlinde, M. Vonk, hep-th/0301028.

Steven Duplij

SUPERTWISTOR — A space $\mathcal{Y}_A = (\lambda_\alpha, \mu^\alpha, \chi)$, where $\lambda_\alpha, \mu^\alpha$ are real commuting **spinors** and χ is a real anticommuting **spinor**, $\alpha = 1, \ldots, 2^k$ and 2^k is equal to the dimension of an irreducible **spinor** representation of $SO(1, D-1)$ in $D = 3, 4, 6, 10 (mod\ 8)$ [1]. The generalized Penrose–Ferber correspondence between real **supertwistors** and real generalized **superspace** $(X^{\alpha\beta}, \Theta^\alpha)$ is [2]

$$\mu^\alpha = X^{\alpha\beta}\lambda_\beta - i\,\Theta^\alpha(\Theta^\beta\lambda_\beta),$$
$$\zeta = \Theta^\alpha\lambda_\alpha.$$

The generalization of the Cartan-Penrose representation is $P_{\alpha\beta} = \lambda_\alpha\lambda_\beta$ solving the BPS condition $\det P_{\alpha\beta} = 0$ [3].

The relativistic superparticle is a constrained dynamical system not all its dynamical variables are independent, and so by performing **supertwistor** transform one deals directly with independent physical degrees of freedom of the superparticle in a covariant way, which simplifies the quantization procedure and the analysis of the spectrum of quantum states of the model [1].

For **supertwistor** applications in various superparticle models [3–5].

Bibliography

[1] A. Ferber, Nucl. Phys. **B132** (1978) 55; T. Shirafuji, Progr. Theor. Phys. **70** (1983) 18.
[2] I. Bandos, J. Lukierski, Mod. Phys. Lett. **14** (1999) 1257; Lect. Notes Phys. **539** (2000) 195; I. Bandos, J. Lukierski; D. Sorokin, Phys. Rev. **D61** (2000) 045002.
[3] I. Bandos, J. Lukierski, Ch. Preitschopf, D. Sorokin, Phys. Rev. **D61** (2000) 065009; I. Bandos, hep-th/0208110.
[4] J. Lukierski, Int. J. Mod. Phys. **B16** (2002) 2039; W. Heidenreich, J. Lukierski, Mod. Phys. Lett. **A5** (1990) 439.
[5] E. Sezgin, I. Rudychev, hep-th/9711128; C. Fronsdal in Essays on Supersymmetry, ed. C. Fronsdal, D. Reidel Publ. Comp., 1986, p.164.

Steven Duplij

SUPERVECTOR BUNDLES, general ground rings formulation — A well-behaved theory of supervector bundles within the general **category** of **supermanifolds** is described in [3] and a more detailed treatment in [1].

Let $\mathfrak{X} = (X, \mathcal{A}, ev^X)$, $\mathfrak{Y} = (Y, \mathcal{B}, \mathrm{ev}^Y)$ be two **supermanifolds** based on a ground *graded Banach algebra* Λ satisfying appropriate (but quite loose) conditions, cf. [3]. The *product supermanifold* $\mathfrak{Z} = (Z, \mathcal{C}, ev^Z)$ is constructed by taking $Z = X \times Y$ as topological spaces while the *structure sheaf* $\mathcal{D}$ is the **sheaf** associated to