Nambu–Poisson cohomology [5]: the center of the *Leibniz algebra* $(\Omega^{n-1}(M), [\![\, , \,]\!])$ is the $C^\infty(M, \mathbf{R})$-module $\ker \#_{n-1}$ and thus the quotient space $\Omega^{n-1}(M)/\ker \#_{n-1}$ is a **Lie algebra**; then the *Nambu–Poisson cohomology* is the cohomology of this **Lie algebra** relative to a certain representation. If the structure is *regular*, $\Lambda^{n-1}(T^*M)\ker \#_{n-1} \to M$ is a *Lie algebroid* and the *Nambu–Poisson cohomology* is just the cohomology of this *Lie algebroid*.

The *characteristic foliation* of a the **Nambu–Poisson manifold** allows us to introduce the *foliated cohomology* which in the *regular* case coincides with the *Nambu–Poisson cohomology*. If $\mathcal{D}$ denote the *characteristic foliation* of M and $\Omega^k(M, \mathcal{D})$ the space of the k-forms α on M such that $\alpha(X_1, \dots, X_k) = 0$, for $X_1, \dots, X_k \in \#_{n-1}(\Omega^{n-1}(M))$, consider $\Omega^k(\mathcal{D})$ the $C^\infty(M, \mathbf{R})$-module $\Omega^k(M)/\Omega^k(M, \mathcal{D})$. Therefore, the exterior differential d induces a *cohomology operator* $\tilde{d} : \Omega^k(\mathcal{D}) \to \Omega^{k+1}(\mathcal{D})$ and the resultant cohomology $H^*(\mathcal{D})$ is called the *foliated cohomology*, and for *regular Nambu–Poisson structure* $H^*(\mathcal{D})$ is just the usual *foliated cohomology* of $(M, \mathcal{D})$.

Canonical Nambu–Poisson homology: related to a volume form ν there exists an isomorphism of $C^\infty(M, \mathbf{R})$-modules $\flat_\nu : \mathcal{V}^k(M) \to \Omega^{m-k}(M)$, defined by $\flat_\nu(P) = i(P)\nu$, and a homology operator $\delta_\nu = \flat_\nu^{-1} \circ d \circ \flat_\nu : \mathcal{V}^k(M) \to \mathcal{V}^{k-1}(M)$, which for a vector field X is the divergence of X with respect to ν, $div_\nu X$. For an *oriented Nambu–Poisson manifold*, consider $\mathcal{V}^k_t(M, \Lambda) = \{P \in \mathcal{V}^k(M)/i(\alpha)(P) = 0, \forall \alpha \in \Omega^1(M), \ \alpha \in \ker \#_1\}$. In the *regular* case, we have that $\mathcal{V}^k_t(M, \Lambda) = \#_{n-k}(\Omega^{n-k}(M))$. The homology of the **complex** $(\mathcal{V}^*_t(M, \Lambda), \delta_\nu)$ is the *canonical Nambu–Poisson homology* (nondepending on ν). **Duality**: let ν be a volume form on M and consider the mapping $\mathcal{M}^\nu_\Lambda$: $C^\infty(M, \mathbf{R}) \times \dots^{(n-1} \dots \times \ C^\infty(M, \mathbf{R}) \to C^\infty(M, \mathbf{R})$ defined by $\mathcal{M}^\nu_\Lambda(f_1, \dots, f_{n-1}) = div_\nu(X_{f_1 \dots f_{n-1}})$. Then $\mathcal{M}^\nu_\Lambda$ induces an $(n-1)$-vector on M which we also denote by $\mathcal{M}^\nu_\Lambda$. Moreover, the mapping

$$\mathcal{M}^\nu_\Lambda : \Omega^{n-1}(M) \to C^\infty(M, \mathbf{R}), \\ \alpha \mapsto i(\alpha)\mathcal{M}^\nu_\Lambda$$

defines a 1-*cocycle* in the *Leibniz cohomology complex* associated with the *Leibniz algebroid* $(\Lambda^{n-1}(T^*M), [\![\, , \,]\!], \#_{n-1})$ and its *cohomology class* $[\mathcal{M}^\nu_\Lambda]$ (the *modular class*) does not depend on the chosen volume form ($\mathcal{M}^\nu_\Lambda$ defines also a *Nambu–Poisson cohomology* class which is null if and only if the *modular class* is).

The vanishing of the *modular class* implies the existence of a **duality** between the *foliated cohomology* and the homology of the subcomplex $(\#_{n-k}(\Omega^{n-k}(M)), \delta_\nu)$ of the *canonical Nambu–Poisson homology* complex [5]. Thus, for *regular* structures with null *modular class* there is a duality between the *Nambu–Poisson cohomology* and the *canonical Nambu–Poisson homology*.

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NATURAL OPERATION — A linear operator acting in sections of various vector bundles naturally associated with the base (tensor bundles, spaces of jets, spaces of connections, etc.) and invariant with respect to certain groups of **diffeomorphisms**. Examples: the integral (the only known nonlocal operator), the exterior differential (unary operators), Poisson and *Schouten brackets* (binary operators). All physical laws are formulated in invariant terms of natural operations. Precise definitions:

Let M be a connected n-dimensional manifold over $\mathbb{R}$ and ρ a representation of $GL(n, \mathbb{R})$ in a finite dimensional space V. Denote by $T(\rho)$ or $T(V)$ the space of tensor fields of type ρ, i.e., the collection of the sections of the bundle over M with fiber V. On $T(\rho)$, the group Diff M of **diffeomorphisms** of M naturally acts: let J_A be the *Jacobi matrix* of A calculated in coordinates of points m and $(A^{-1}m$, then set

$$A(t(m)) = \rho(J_A)(t(A^{-1}m))$$

for $A \in \mathrm{Diff}\, M, m \in M, t \in T(V)$. Any operator $c : T(\rho_1) \longrightarrow T(\rho_2)$ is invariant if it commutes with the Diff M-action.

For irreducible fibers there is only one unary invariant operator, the exterior differential. On manifolds, there are no (nonscalar) natural operations in the spaces of sections of natural bundles depending on higher jets of **diffeomorphisms**, such as connections. On **supermanifolds** this is an open problem.

On manifolds, the order of invariant bilinear operators is ≤ 3, and apart from one (*Grozman operator* on the line) all operators of orders 2, 3 are compositions of 1st order operators. Amazingly, most of the *first order* bilinear operators define a **Lie superalgebra** structure [2] (most known are Poisson, Schouten and Nijenhuis brackets). On **supermanifolds**, no analog of this statement is known.

Classification of the skew-symmetic invariant operators on the line had lead Feigin and Fuchs [4] to a proof of the formula for *Shapovalov determinant* conjectured by Kac [5].

On the latest developments a review [6] where unary natural operations are invariant with respect to 11 of 15 exceptional **Lie superalgebras** of vector fields, some of them, conjecturally, pertaining to *Standard Models*, are listed. For history and more examples see [1]–[3].

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NESTED DEFECT — If a scalar field χ associated with a *topological defect*, such as a domain wall or a cosmic string [1–4], couples appropriately to another scalar field ϕ which has a vanishing vacuum expectation value, then by the *Witten mechanism* [5] it becomes possible for the system to stabilize by the formation of a ϕ scalar field condensate in the core of the **defect**. Thus the symmetry associated with the scalar field ϕ, which is respected in the vacuum outside of the **defect**, is broken within the **defect's** core where typically $\chi = 0$. If this symmetry is a gauged local symmetry, then the condensate (ϕ) can give rise to a current density within the host **defect** (χ) which, in turn, generates long-range **gauge fields**. If, instead of a local symmetry, the field ϕ is associated with a discrete symmetry, such as Z_2, which gets broken in the host **defect's** core, then *domain defects* (ϕ) can populate the core of the host **defect** (χ). For example, when the discrete symmetry is Z_2, the energetically degenerate configurations $\phi = \pm\phi_0$, where ϕ_0 is a constant, can minimize the energy of the ϕ field in the host defect core. During a phase transition in which the condensate forms within the host defect core, ϕ settles into energy minimizing states, assuming the value $+\phi_0$ in some spatial regions and a value $-\phi_0$ in others. These $\pm\phi_0$ domains, however, will be uncorrelated beyond a distance scale corresponding to the coherence length ξ, and two different adjacent domains must be separated by a region where $\phi = 0$, locating the *domain defect* (part of a domain wall) within the core of the host defect. These **nested defects** represent an interesting type of configuration where the host defect–a domain wall or a cosmic string– has a nontrivial internal structure, described by a population of other defects.

As an example, consider the case where $\chi = \frac{1}{\sqrt{2}} R\, e^{i\alpha}$ represents a cosmic string field interacting with a U(1) **gauge field** $A_\mu = \frac{1}{e}[P(\vec{x}, t) - 1]\partial_\mu \alpha$ and a real scalar field ϕ with a discrete Z_2 symmetry, with the potential of the system being given by

$$\begin{aligned} V &= \lambda\left(\chi^*\chi - \frac{1}{2}\eta^2\right)^2 + f\left(\chi^*\chi \right. \\ &\quad \left. - \frac{1}{2}\eta^2\right)\phi^2 + \frac{1}{2}m^2\phi^2 + \frac{1}{4}g\phi^4 \\ &= \frac{1}{4}\lambda(R^2 - \eta^2)^2 + \frac{1}{2}f(R^2 - \eta^2)\phi^2 \\ &\quad + \frac{1}{2}m^2\phi^2 + \frac{1}{4}g\phi^4 \end{aligned}$$

with λ, f, g, and m being real positive quantities. The stable vacuum state of the theory is given by $R = 0$, $\phi = 0$. However, in the string's core where $R \to 0$, if the model parameters assume a range where $\phi = 0$ is an unstable configuration, the energy can be minimized for $\phi = \pm\phi_0$, where

$$\phi_0 = [(f\eta^2 - m^2)/g]^{1/2},$$

which is positive provided that $f\eta^2 - m^2 > 0$. The adjacent $\pm\phi_0$ domains are separated by domain kinks and antikinks, with $\phi = 0$ in the kink and antikink cores.

Systems of **nested defects** can include *vortices* within walls [6], walls within walls [7–9], and walls within strings [7], and can arise from supersymmetric [8] and nonsupersymmetric *field theories*.

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NEUMANN COEFFICIENT — A matrix appearing in the *oscillator representations* of the *star product* in terms of a three string vertex in the operator formalism of the open string *field theory* [1].

The *star product* of two states $|A\rangle$ and $|B\rangle$ in the matter part of the **conformal field theory** is (see e.g. [2])

$$|A *^m B\rangle_3 =_1 \langle A|_2 \langle B|\, V_3\rangle,$$

where the three string vertex $|\, V_3\rangle$ is given by