$$\mathcal{L} = -\frac{1}{4g^2} G^a_{\mu\nu} G^a_{\mu\nu} + \frac{\theta}{32\pi^2} G^a_{\mu\nu} \tilde{G}^a_{\mu\nu} + \frac{i}{g^2} \lambda^{\alpha\alpha} \mathcal{D}_{\alpha\dot{\beta}} \bar{\lambda}^{\dot{\alpha}\beta}$$
$$= \frac{1}{4g^2} \int d^2\theta \, Tr \, W^2 + H.c.,$$

in the second and higher loops. The gauge coupling g^{-2} is complexified, the imaginary part is proportional to the θ angle,

$$\frac{1}{g^2} \to \frac{1}{g^2} - i\frac{\theta}{8\pi^2}.$$

Formally, the coefficient in front of $Tr\, W^2$ is predicted to holomorphically depend on the bare coupling g_0^{-2}, i.e. g^{-2} must be analytic in g_0^{-2}. The formal proof is valid for the Wilsonean action, which is renormalized only at one loop [1] but is violated for the action with the canonically normalized gauge term (the so-called canonic action). The anomaly can be seen as an infrared [1,2] or ultraviolet [3] effect. The *equivalence* between the infrared and ultraviolet derivations of the **holomorphic anomaly** can be established in the same way as for the chiral anomaly. This **holomorphic anomaly** is instrumental, e.g. in the derivation of the *NSVZ β function.*

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HOLONOMY GROUP — A group of a Riemannian or pseudo–*Riemannian manifold*, that is the **subgroup** of the orthogonal group defined by taking all possible parallel translations of vectors or **spinors** around closed paths, [1], Chapter 10. In *string theory compactifications*, the preservation of $N = 1$ **supersymmetry** requires [2] the existence of a spinor field which is covariantly constant. This restricts the **holonomy group** [3]. The possible **holonomy groups** of *Riemannian manifolds* have been classified by Berger [1], but note that this classification theorem applies to the simply connected case only. The nonsimply-connected case is not completely settled, but it is known [4] that there are precisely seven possible linear **holonomy groups** of compact locally irreducible four-dimensional *Riemannian manifolds*. In six dimensions, the most interesting **holonomy group** is $SU(3)$, regarded as a **subgroup** of $SO(6)$; this is the **holonomy group** of a **Calabi–Yau manifold**. In this particular case, the **holonomy group** is the same, whether the **Calabi–Yau manifold** is simply connected or not, but this is not true in other dimensions [for example, in eight dimensions].

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Brett McInnes

HOMOLOGICAL VECTOR FIELD — An odd field D such that

$$D^2 = \frac{1}{2}[D, D] = 0.$$

Examples: the exterior differential, so important in *cohomology theory*, translations in odd directions. For the normal forms of the simplest such fields see [1,2].

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HOPF ALGEBRA — A complex associative algebra A with additional linear operations: a *comultiplication* Δ: $A \to A \otimes A$, a *counit* $\varepsilon : A \to \mathbb{C}$, and an *antipode* S: $A \to A$, which satisfy certain conditions given below. The notion of a **Hopf algebra** first occurred in the work of H. Hopf [1] on algebraic topology.

In the theory of **Hopf algebras**, an associative algebra A is defined as a linear space A with linear operations such as a multiplication $m: A \otimes A \to A$ and a unit $\eta: \mathbb{C} \to A$ which satisfy the identities
(a) $m \circ (m \otimes \mathrm{id}) = m \circ (\mathrm{id} \otimes m)$ (associativity)
(b) $m \circ (\eta \otimes \mathrm{id}) = \mathrm{id} = m \circ (\mathrm{id} \otimes \eta)$

on $A \otimes A \otimes A$ and on $A \otimes A$, respectively, where id means the identity transformation on A. Now the conditions, which must be satisfied by a *comultiplication* Δ, a *counit* ε, and an *antipode* S of a **Hopf algebra** A, can be written as
(a) $(\Delta \otimes \mathrm{id}) \circ \Delta = (\mathrm{id} \otimes \Delta) \circ \Delta$ (coassociativity);
(b) $(\varepsilon \otimes \mathrm{id}) \circ \Delta = (\mathrm{id} \otimes \varepsilon) \circ \Delta = \mathrm{id}$;
(c) $m \circ (S \otimes \mathrm{id}) \circ \Delta = m \circ (\mathrm{id} \otimes S) \circ \Delta = \eta \circ \varepsilon$;
(d) the *comultiplication* Δ and the *counit* ε are algebra homomorphisms from A to $A \otimes A$ and from A to $\mathbb{C}$, respectively.

The theory of **Hopf algebras** is the base for the construction of *quantum groups*. A **Hopf algebra** is commutative if the underlying algebra structure is commutative. A **Hopf algebra** A is *cocommutative* if $\sigma \circ \Delta = \Delta$, where σ is the linear mapping on $A \otimes A$ such that $\sigma(a \otimes b) = b \otimes a$ for $a, b \in A$.

A super **Hopf algebra** is defined by means of the notion of Z_2-graded associative algebra. Note that a