Bibliography

[1] M. Bordemann, E. Meinrenken, and M. Schlichenmaier, Commun. Math. Phys. **165** (1994) 281.

[2] M. Schlichenmaier, in: Quantization, Coherent States and Poisson Structures, Proc. XIV'th Workshop on Geometric Methods in Physics (Białowieża, Poland, 9-15 July 1995) (A. Strasburger, S.T. Ali, J.-P. Antoine, J.-P. Gazeau, and A. Odzijewicz, eds.), Polish Scientific Publisher PWN, 1998, q-alg/9601016, p. 101.

[3] M. Schlichenmaier, in the Proceedings of the Conference Moshe Flato 1999, (eds. G. Dito, D. Sternheimer), Kluwer, math.QA/9910137.

[4] D. Borthwick, S. Klimek, A. Lesniewski, and M. Rinaldi, Commun. Math. Phys. **153** (1993), 49. [5] D. Borthwick, A. Uribe, Math. Res. Lett. **3** (1996), 845.

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BEREZIN, Felix Alexandrovich — (b. April 25, 1931 — perished in an accident on July 14, 1980). For a brief scientific biography and development of some of Berezin's ideas see [1]. Apart from being very nice, courageous and generous man, Berezin is the founder of **supersymmetry** and **supermanifold** theory. Several key notions in this field and in other fields bear his name (**Berezinian**, **Berezin integral**, Berezin quantization, Berezin transform) see [1,2]. Berezin made also a very important contribution to the quantization theory, in particular, to the theory of second quantization [3,4]. Results obtained by Berezin in representation theory significantly altered the field. In particular, he gave a description of Laplace operators on semisimple *Lie groups* later generalized to supergroups, he was the first to describe typical irreducible finite dimensional representations of unitary **Lie superalgebras** and first to use the *point functor* to describe invariant polynomials on **Lie superalgebras**, cf. [2]. Statistical physics is a direction of Berezin's research aborted by his untimely death, cf. [1].

Bibliography

[1] R. L. Dobrushin et al. (eds.) Contemporary Mathematical physics (F. A. Berezin memorial volume) AMS Translations, series 2, vv. **175**, **177**, 1996.

[2] F. A. Berezin Introduction to superanalysis. D. Reidel Publishing Co., Dordrecht, 1987.

[3] F. A. Berezin, Comm. Math. Phys., 40 (1975), 153–174; id., Math. USSR-Izv. 9 (1975), 341–379; id., Izv. Akad. Nauk SSSR Ser. Mat. 39 (1975), no. 2, 363–402, 472.

[4] F. Berezin, Methods of second quantization, Moscow 1976.

Dimitry Leites

BEREZIN INTEGRAL — A generalization of notion of integral for functions among arguments of which there are anticommuting ones.

Odd variables. An integral on *Grassmann algebra*, fermionic integral, Martin integral. The linear functional which assigns to the element

$$f(\xi) = f_0 + f_\mu \xi^\mu + \frac{1}{2}!f_{\mu\nu}\xi^\mu\xi^\nu + \dots + \frac{1}{m}!f_{\mu_1\dots\mu_m}\xi^\mu_m \dots \xi^\mu_1 \qquad (1)$$

of the *Grassmann algebra* with m fixed generators ξ^μ the number $f_{1\dots m}$, i.e. coefficient of the top order term. (Here we assume that the coefficients are skew-symmetric in indices, hence $\frac{1}{m!}f_{\mu_1\dots\mu_m}\xi^{\mu_m}\dots\xi^{\mu_1} = f_{1\dots m}\xi^m\dots\xi^1$.) Notation: $\int f(\xi)d\xi^1\dots d\xi^m$, $\int f(\xi)D\xi$ with $D\xi = D(\xi^1,\dots,\xi^m)$ for the 'element of measure' (notice that there is no measure theory behind this integral). The former notation is the most common in physical literature, but is very unfortunate because drastically contradicts to the formula for the change of variables (see below). The notation with $D\xi = D(\xi^1,\dots,\xi^m)$ introduced by J.N. Bernstein and D.A. Leites [1] corresponds to the classical notation for the Jacobian as Dx/Dy or similar and is best suited for changes of variables. The characteristic property of this functional is that the integral over the variables $\xi^1,\dots,\xi^m$ can be calculated as the repeated integral: $\int D\xi f(\xi) = \int D\xi^1 \dots \int D\xi^m f(\xi)$. In such formulas it is convenient to write the 'volume element' $D\xi$ to the left of a function and to introduce the parity as $\widetilde{D\xi} = m \bmod 2$ (hence $\widetilde{D\xi^\mu} = 1$ for a particular ξ^μ) and to postulate (super)commutativity. Such formulas rely on the (implicit) extension of the *Berezin integral* from a fixed *Grassmann algebra* with real coefficients to the case where the coefficients belong to some $\mathbb{Z}_2$*-graded space*, in particular, to another *Grassmann algebra*. The second characteristic property of this functional is that the integral is invariant under translations: if ξ_0^μ are some other *odd variables* ("constants"), then $\int f(\xi+\xi_0)D\xi = \int f(\xi)D\xi$. This can be symbolically rewritten as the property $D(\xi-\xi_0) = D\xi$. As a consequence of this invariance the integral is zero on every element of the *Grassmann algebra* which is a derivative and so has not the top order term. This makes it an analog of the usual Riemann integral of functions with compact support over the whole space mimicking the usual integration by parts where the boundary terms are thrown away. Canonical normalization is such that the integral of $\xi^m\dots\xi^1$ (reverse order) is 1. For a linear change of variables with the matrix $A = (A_\nu{}^\mu)$, we have $\int f(\xi A)D\xi = \det A \int f(\xi)D\xi$. This can be symbolically rewritten as $D(\xi A^{-1}) = \det A\, D\xi$. For example, just with one *odd variable* ξ, we have $\int f(5\xi)D\xi = 5 \int f(\xi)D\xi$, or $D(5\xi) = (1/5)D\xi$. Obviously, this is the opposite from what could be expected if one would try to consider $D\xi$ as a 'differential of ξ' or a product of such differentials for the multivariable case (assuming the natural behavior $d(5\xi) = 5d\xi$). That is why the