Mini-workshop in PDE: Theory, Applications and Numerics

Program

November 26, 2010
KTH, CSC,
Lindstedtvägen 3, room 4523, floor 5

10.00-10.45 Eskil Hansen, Lund: *High order splitting schemes for semigroups.*

Abstract: Splitting schemes constitute an effective class of time integrators and have been widely used for both ODEs and PDEs. The general error analysis of such methods have often been conducted in a fairly classical ODE setting, including Taylor expansions and nonstiff order conditions, which is not directly applicable in a PDE context.

In this talk we survey our recent work, where we prove that exponential splitting methods retain their nonstiff order when applied to abstract evolution equations which are evolved by semigroups. As a concrete application we consider parabolic PDEs and their dimension splittings.

We also present a systematic derivation of new splitting methods of orders three to fourteen based on complex coefficients. These results resolve the open question whether there exist splitting schemes with convergence rates greater then two in the context of semigroups.

This is joint work with Alexander Ostermann.

11.00-11.45 Ljudmila Bordag, Halmstad: *Nonlinear PDEs as models for financial markets with frictions.*

Abstract: Nowadays there exist many models which involve different kinds of frictions on the markets like transaction costs, feedback effects or illiquidity of different types. Most of them are developed as some improvements of the famous Black–Scholes equation. These models start from different assumptions on the underlying price process and market facilities. From the analytical point of view they end up with similar fully nonlinear parabolic differential equations which describe the price of derivatives on the market under the corresponding assumptions. Our main goal is to describe the Lie algebraic structure of the symmetry group admitted by these equations. In some special cases it is possible to solve reduced equations and to obtain invariant solutions explicitly. We

discuss the properties of these reductions and the corresponding invariant solutions. The analytical solutions obtained in this way can be used as benchmarks by development different numerical methods.

11.45-13.15 Lunch

13.15-14.00 Elisabeth Larsson, Uppsala: *Radial basis function approximation methods for solving PDE problems.*

Abstract: Radial basis function (RBF) based approximation methods are interesting in the context of PDE solvers due to their ease of implementation, their potentially spectral convergence rates, and their flexibility with respect to geometry. However, a persistent problem has been the severe ill-conditioning of the systems of equations that typically need to be solved. This ill-conditioning is partly related to the size of the system, but even more so to the shape parameter of the RBFs. As the shape parameter is decreased, the RBFs become increasingly flat, leading to a nearly linearly dependent basis. Even so, the nearly flat limit does in many cases provide the best approximation properties. Therefore, we have recently developed the RBF-QR method, which provides numerically stable evaluations for the small shape parameter range in up to three space dimension. With the conditioning obstacle removed, the focus can be turned to more general computational issues such as computational cost and memory requirements. Using a global RBF method results in a dense linear system with no established fast solution method. Here, a partition of unity approach with local RBF approximants is proposed. The locality reduces both memory usage and computational cost compared with the global method. We show that the RBF-QR algorithm is a key to success and provide numerical experiments showing spectral convergence with respect to the local problem size and algebraic convergence with respect to the partition size.

14:15–15.00 Peter Lindqvist, Trondheim: *Asymptotic Behaviour of Solutions of the Evolutionary p-Laplace Equation.*

Abstract: This is a much studied topic. I will use the celebrated Moser iteration on the difference $v(x,t) - u(x)$ to bound the supremum norm of this quantity. This yields a sharp decay estimate as t goes to infinity. Here $v(x,t)$ is a solution to the Evolutionary p-Laplace Equation, and $u(x)$ is the corresponding stationary solution having the same boundary values on the lateral boundary. – A somewhat surprising counterexample shows that the results in the much better known special case with $u = 0$ cannot be extended as if there were a superposition principle available.