PDE Seminars

You are invited to a PDE afternoon with three speakers.

Thursday, May 6, Lindstedsvägen 3, Room E33.

13.00-13.45 Wolfgang Reichel, Karlsruhe: *A-priori bounds and existence of solutions to nonlinear, subcritical higher-order elliptic Dirichlet problems.*
Abstract: We consider the $2m$-th order elliptic boundary value problem $Lu = f(x, u)$ on a bounded smooth domain $\Omega \subset \mathbb{R}^N$ with Dirichlet boundary conditions. For the nonlinearity we assume subcritical power-like behaviour at infinity.
We prove uniform a-priori bounds for every solution (the solutions are allowed to be sign-changing). The proof is done by a blow-up argument and a new half-space Liouville-type theorem. In combination with degree theory the a-priori bounds lead to existence of nontrivial solutions.

14.00-14.45 John Toland, Bath: *A Bernoulli free boundary-value problem for waves with rotation.*
Abstract: We present a new approach to the theory of two-dimensional steady surface waves on flows with vorticity. In it the Laplacian of the stream function is required to have the same distribution as a prescribed square-integrable function. In this approach, vorticity is not a prescribed function of the stream function. Instead, it emerges as an infinite-dimensional Lagrange multiplier in a variational characterisation of the problem.
Motivated by this variational principle, a notion of weak solution of Bernoulli free-boundary problems is introduced and non-trivial weak solutions are shown to arise from a minimax principle. At this level of generality there is no essential difference between the theory for irrotational waves and for waves with locally square-integrable vorticity.
To illustrate the minimax principle with a minimum of technical difficulties, the existence of non-trivial waves with a prescribed distribution of vorticity on the surface of a fluid confined beneath an elastic sheet is proved.
Much remains to be done.

15.15-16.00 Catherine Bandle, Basel: *The Fujita Exponent for the Cauchy Problem in the Hyperbolic Space*
Abstract: It is known that the heat kernel in the hyperbolic space has a different behavior for large times than the one in the Euclidean space. The main purpose is to study its effect on the positive solutions of Cauchy problems with power nonlinearities. Different types of solutions are introduced and compared. Existence and non existence results for local solutions are derived. Emphasis is put on their long time behavior, in particular on the discussion of Fujita's phenomenon. In order to have the same situation as for the Cauchy problem in the Euclidean space, namely finite time blow up for all solutions if the exponent is smaller than a critical value and existence of global solutions only for powers above the critical exponent, we have to introduce a weight depending exponentially on the time. In this respect the situation is similar to problems in bounded domains with Dirichlet boundary conditions. Important tools are estimates for the heat kernel in the hyperbolic space and comparison principles.

Welcome!