

Avd. Matematik
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Some general comments: If you cannot solve the full exercise, you may solve it in some special cases, to begin with. However, you should note which cases your solution covers.

If I'm not satisfied, you may get a chance to make improvements, and re-deliver the results.

Cooperation is encouraged! (However, be sure to be able to answer for your own set of delivered results.)

1. Show that $t^4 - at - 1$ is irreducible over \mathbb{Q} for each non-zero integer a .
Decide for which integers b the polynomial $t^4 - bt - 2$ is irreducible over \mathbb{Q} ; and factorise it when it isn't.
2. Show explicitly that $\mathbb{Q}(\sqrt{2}, \sqrt{3}, \sqrt{5})$ is a simple extension of \mathbb{Q} .
3. a) List all irreducible polynomials of degree ≤ 4 in $\mathbb{Z}_2[x]$.
b) Give a recursive definition of the sequence a_1, a_2, a_3, \dots , where a_n is the number of irreducible polynomials of degree n in $\mathbb{Z}_2[x]$.
Please note: I am not asking for more than just a recursive definition. It need not be elegant; nor should you worry about finding a closed expression for a_n .
4. Decide the cardinalities $|K|$ of the finite fields for which the polynomial $x^2 + 1$ is irreducible in $K[x]$.
If this is too hard, solve it for the finite prime fields, $K = \mathbb{Z}_p$.
5. Exercise 2.13. (Your first problem is to find out precisely what Stewart asks for ...) You may but do not have to follow Stewart's detailed advices on how to prove that indeed p is a polynomial in the s_i .
6. Exercise 3.11: Show that if K is a field of any other characteristic than 2, and $f(x) \in K[x]$ is of degree 2, then f splits in some field extension $K(\alpha)$, such where $\alpha^2 \in K$. (α in general depends on f , of course.)
Show that the same is true for some but not for all fields of characteristic 2. (If you wish, comment on Stewart's formulation of exercise 3.12.)
7. Exercise 4.13.
8. Show that if we consider the classical planar Euclid geometric operations, but also allow **putting marks** on rulers in the Archimedian manner, then for any algebraic number α which lives in a field which may be constructed from \mathbb{Q} by a finite number of extensions of degree 2 or 3, and for any given distance d between some initial points, we may construct a distance αd .
(Archimedes' marked ruler method is described in exercise 5.3. You may also have use of pondering exercise 5.7.)