Regulatory return targets

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AFA Insurance

- Owned by the labour market parties
  - The Confederation of Swedish Enterprise (Svenskt Näringsliv),
  - The Swedish Trade Union Confederation (LO), and
  - PTK.
- Colletively bargained insurances for sickness and work injuries.
- Liabilities $\approx$ 100 billion SEK
  - Most policies for work injury (4.5 million insured).
  - Sickness largest liability (top up social security)
  - Duration $\approx$ 8 years.
- Assets $\approx$ 220 billion SEK
  - 50% fixed income
  - 40% equity
  - 10% real estate

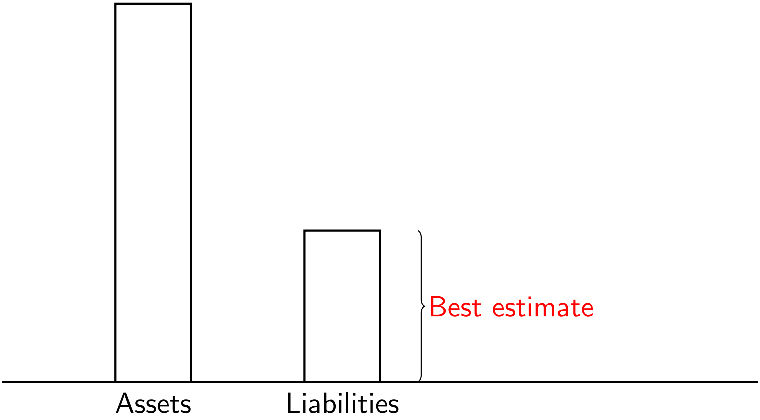
Disclaimer

Views expressed are my own, and not necessarily those of AFA Insurance!

Solvency II

- EU wide regulations coming into force next year.
- Quantitative and qualitative requirements, and reporting!
- In order to be solvent, assets must cover the sum of
  - *Best estimate* (BE) = "market value" of liabilities,
  - *Risk margin* (RM) = cost of capital for other (re)insurer to take over the liabilities, and
  - *Solvency Capital Requirement* (SCR) = 99.5%(!) yearly VaR of basic own funds, i.e. assets minus (BE + RM).
- SCR depends on both asset and liability composition.
- SCR can be calculated by a *standard formula* or an *internal model*.

Solvency II balance sheet



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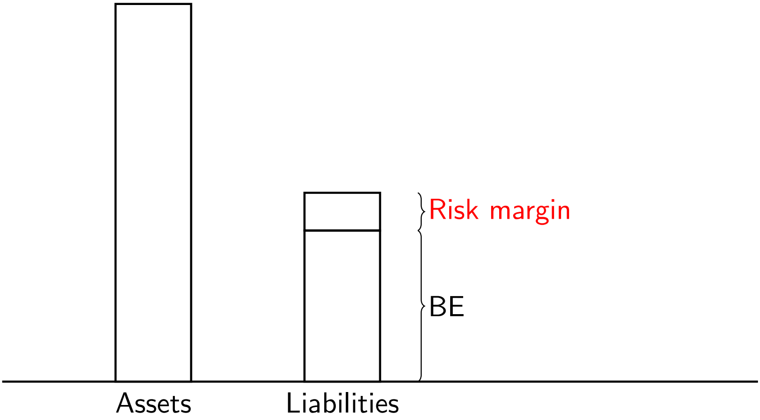

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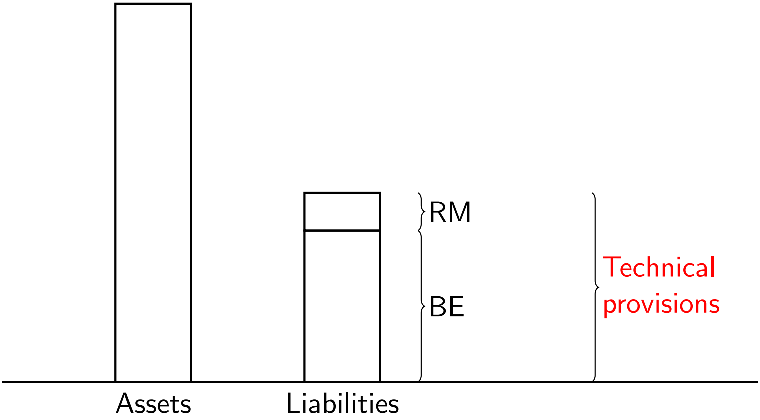

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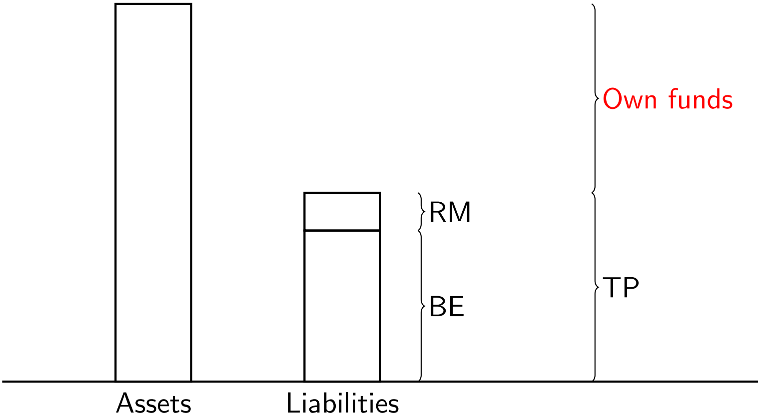

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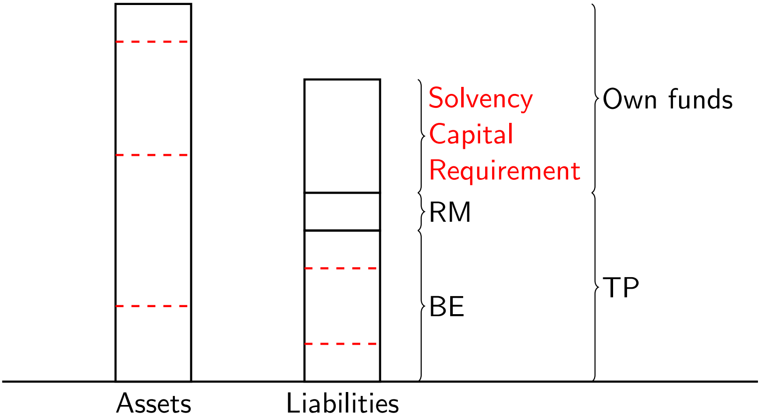

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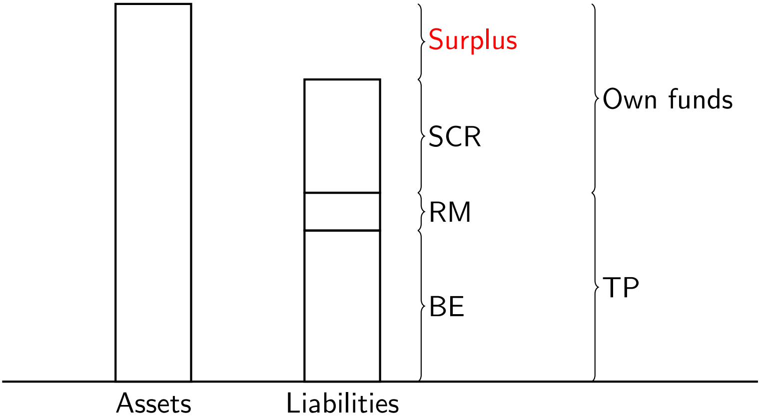

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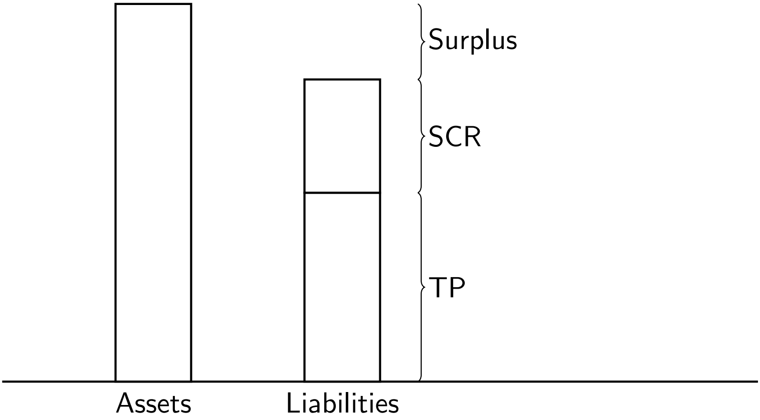

Standard model of SCR 1(3)

- The overall SCR is calculated from the SCR for different submodules, e.g. market risk, life risk, health risk, etc.

$$\mathrm{SCR} = \sqrt{\sum_{\alpha\beta} \rho_{\alpha\beta} \mathrm{SCR}_\alpha \mathrm{SCR}_\beta}$$

- The submodule SCR's are calculated by the same type of square root of correlated capital requirements for, e.g., equity risk, interest rate risk, foreign exchange risk, etc.

$$\mathrm{SCR}_\alpha = \sqrt{\sum_{ij} \rho_{ij} \mathrm{SCR}_i \mathrm{SCR}_j}$$

Standard model of SCR 2(3)

- Individual SCR's are typically expressed as $\text{SCR}_i = s_i x_i$ where s_i is the 99.5% VaR ("stress") for one SEK of exposure, and x_i is the exposure in SEK, to asset or liability of type i.
- Even though the structure with a square root of a quadratic expression looks like the expression for the quantile of an elliptical distribution such as the normal distribution, the *nesting* of such square root expressions is in general inconsistent with any multivariate distribution of risks (Filipović).

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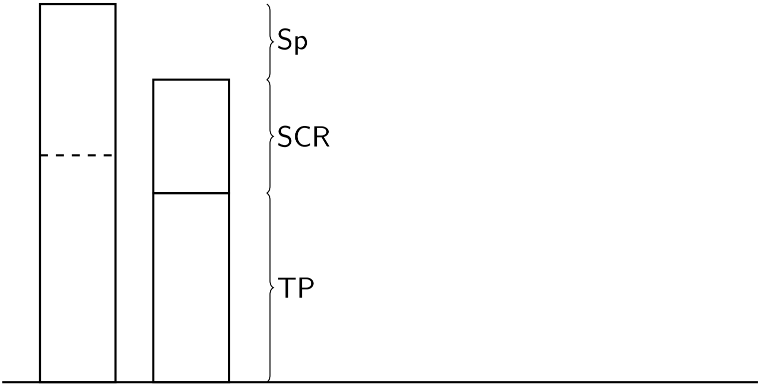
- *What return targets should different assets have?*

Alternative asset allocation



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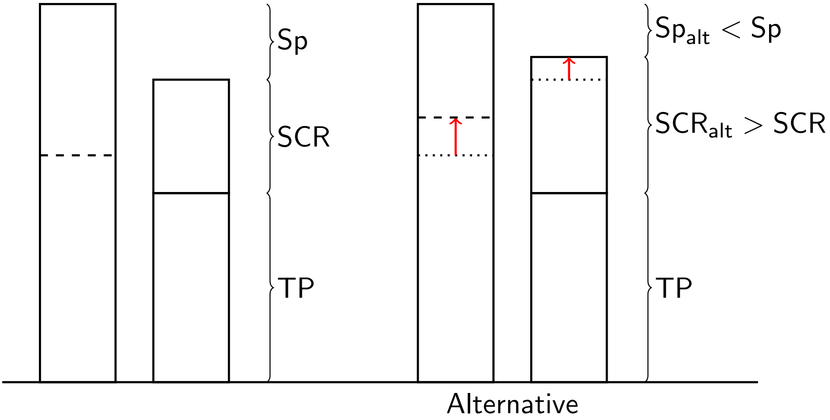

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- Nice-to-have features:
  - **Translation invariance** $R[X - c] = R[X] - c$ for constant c.
  - **Homogeneity** $R[\lambda X] = \lambda R[X]$ for constant $\lambda > 0$.
  - **Monotonicity** $X \leq Y$ a.s. $\Rightarrow R[X] \leq R[Y]$.
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- VaR in general fails to be subadditive.
- Note that SCR is a homogeneous risk measure

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- We want to have a portfolio with m large and r small.
- Assets with large expected value typically increase the risk.
- A natural question is *what return targets should be placed on different asset classes in view of their different costs with regard to the risk capital?*

Homogeneous functions and the Euler capital allocation

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- Note that r, the risk capital, is in general only a bookkeeping fiction and the capital invested has a completely different value!

Motivating the Euler allocation principle

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- This principle is optimal in a certain sense: Let m be the overall expected value of a given portfolio and r its risk. The vector $a = (a_1, \ldots, a_n)$, as a function of the portfolio composition, is *suitable for performance measurement* if $m_i/a_i > (<) m/r$ implies that an infinitesimal increase in asset i increases (decreases) m/r.

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  Theorem (Tasche 1999): The only suitable $a = \nabla r$.

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3. Something based on own risk assessment and utility function...

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- Caring about the surplus means also taking SCR into account.
- My utility could depend on more than the value or the surplus at a single time horizon, and my perception of risk might not be described with a single risk measure.

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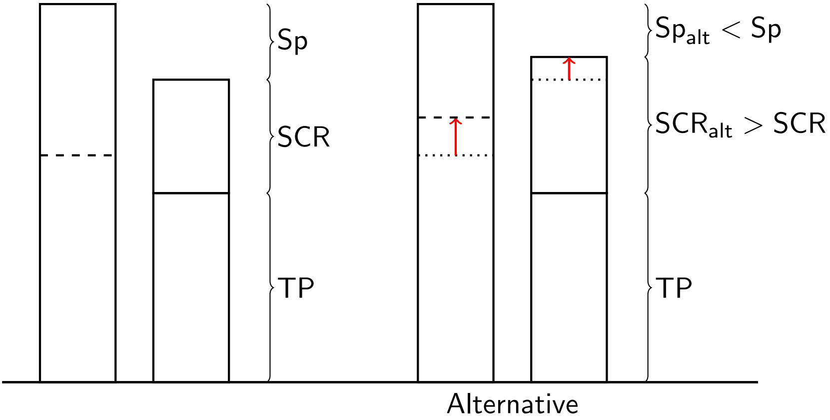

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- One could say that the risk measure used is VaR of surplus minus its expected value, and that the optimal portfolio has expected value for the surplus equal to its VaR.
- We typically want to have fixed weights for the assets, so we may assume the asset portfolio is rebalanced at the horizon. This affects the SCR.

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afa FÖRSÄKRING

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- One could therefore make an Euler decomposition of the VaR.
- One could also make an Euler decomposition of the expected value of the surplus with contributions from the different assets. The decomposition is nontrivial since SCR is nonlinear.

Insights

- The Euler principle is “right”: We want to find expected asset returns that increase the expected value of the surplus at least as much as the VaR increases, so that the probability of negative surplus is kept at the given level.

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- Analytical expressions are messy even in simplified cases.
- Calculation of VaR and its gradient requires simulations.

Thanks for your attention!