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Cohort Effects in Swedish Mortality and Their Effects on Technical Provisions for Longevity Risk

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Abstract

The aim of this survey is to have a close look at Swedish historical mortality and see whether there are any patterns suggestive of cohort effects. Representing and using the Lee-Carter model and the Renshaw-Haberman model, we make mortality forecasts and by using them, calculating, studying and comparing the differences that will appear in technical provisions for longevity risk.

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Preface

This report constitutes a Master's thesis, for the degree of Master of Science in Actuarial Mathematics at Stockholm University.

Acknowledgement

First and foremost, I would like to give my sincere thanks to my supervisor Erik Alm, general Manager at Hanover Life Re Sweden for his support and guidance and for giving me the opportunity to write this thesis.

I would also like to thank the rest of the staff at Hannover Life Re Sweden to have contributed several good ideas and for providing me a friendly environment for my time there. A special thanks to Christian Salmeron for helping me with the statistical programming.

Finally, I would also like to thank my supervisor at Stockholm's University, Erland Ekheden for his good inputs.

CONTENTS

1. Summary

The purpose of this report is to have a close look into the mortality changes in Sweden during the last centuries and see if there are any hints of birth cohort effects. Then we will compare the Swedish mortality data with data for United Kingdom, where there are clear signs of cohort effects for people born around 1930.

Cohort studies are in statistics, a study on a group of individuals with a specific shared experience within a certain time period. A birth cohort is the most common example, i.e. a group of people who are born on a day or during a particular period. A select cohort is a birth cohort characterized by greater rates of mortality improvement than previous and following generations. It is well known that age-period-cohort modeling is problematic, since the three factors are constrained by the relationship; cohort = period - age, (Renshaw, Haberman, 2005).

Cohort life expectancy is the expected lifetime for individuals born in a certain year. If we like to calculate the expected lifetime for individuals in a certain year we have to follow the whole cohort of individuals that were born that year, and therefore we can't know the exactly expected lifetime until more than 100 years afterwards.

We begin by discussing the methods we are using, the Lee-Carter method and after that the Renshaw-Haberman method. The Renshaw-Haberman model is an extension to the first one, but with an extra parameter; a parameter depending on year of birth. These two methods will then be compared and for the insurance point of view, we will also look into how to calculate the technical provisions, and compare the pros and cons.

2. Introduction

Longevity has been a big subject the last decades, especially for the insurance companies who have to have a good knowledge of people's lifespan. Mortality rates are reflections of the evolution in our society and since we have seen improvements for a long time it is important to know how old the insured's might be. During the last century there have been big improvements, particularly in research and medical care, which has led to that people in general live longer. At year 1900 the life expectancy at birth for women was 54 and today (read 2009) that number is about 83. Corresponding figures for men are 51 and 79.*

In figure 2 we can see the general improvement of mortality for women aged 60-95 and how it changed during the 20th Century. During the first decade, the majority of women passed away when they were around 75 years old while the peak today is around the age of 87. The tonality goes from orange to red where the first yellow is year 1900 and the last red is 2007.

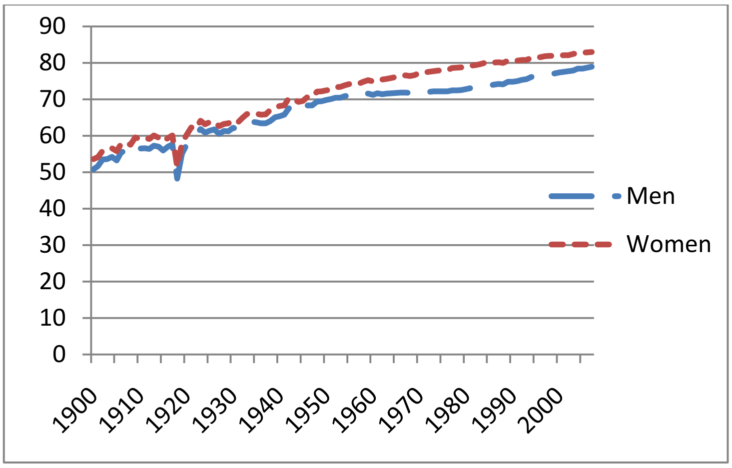

Figure 1. Life expectancy at birth in Sweden, 1900-2007, men and women

* Statistiska Centralbyrån, www.scb.se

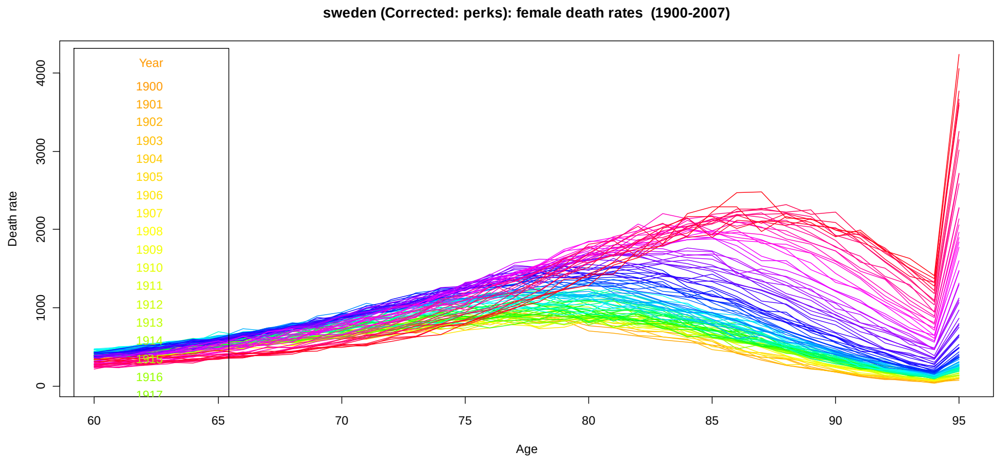

Figure 2. Death rates 1900-2007, women

Certain changes in mortality due to events that affects all age groups that year (periodic effects), and other changes in mortality are caused by events in persons birth/birth decades, the effects of which the group carries the effects throughout life (cohort effects).

In the U.K. population there are clear signs of birth cohort effects. The generations born between 1925 and 1945 (centered on the generation born in 1931) have experienced more rapid improvement than earlier and later generations. General mortality improvement describes the process of bringing historical mortality experience up to the current era. It is not known what has caused these patterns and different scientists have various theories about the causes. For instance, there are differences between the cohort born before 1931 and the cohort born after, e.g. differences in smoking between generations, better nutrition during and after World War II, different sized litters of birth, this implies that people born in a small batch have had less competition when they became older, benefits from the National Health Service that was introduced in the late 1940s, and these generations have been involved in medical research success that has contributed to lower death rates of children and young adults die, (Gallop, 2008).

We shall in this thesis particularly investigate whether a similar pattern can be found on Swedish data by making calculations using the Lee-Carter method, which is being explained in section 4.2, and an extended model with cohort effects. In addition sections, corresponding figures for Danish data can be found in appendix A. In order to see how the Danish data are different from Swedish.

3. The cohort effect

A cohort effect is a variation in health status due to different casual factors that each age cohort in a population is exposed to as environment and society are changing. A cohort study is a study on a group of individuals with at least one particular shared experience within a certain time period. The most common example is a "birth cohort" i.e. the individuals in the group are born in the same year or during the same time period. In a prospective study, which is the focus of this thesis, we are following a group of individuals over time, looking for patterns of effects or outcome due to differences within the group. On the other hand, for a retrospective study data is collected based on certain outcome from past records. In any case, a cohort study (prospective or retrospective) can be said to be the last link in the chain to confirm a link between disease and exposure.

Intuitively, one would think that the changes in mortality would be a response of that year's events. This is most obvious in case of some form of catastrophes such as war and epidemics. One might also assume that antibiotics and medical achievements would affect mortality improvement.

As mentioned in section 2, United Kingdom has seen big mortality improvements for people born in the 1930's. The following tables, table 1 and 2, can be found in the paper "Longevity in the 21st century", (Willets et al, 2004), and it relates to male and female mortality rates in England and Wales. Each of the blocks represents a decade and, for each decade, average annualized mortality improvement rates are shown for five-year age bands.

Age group	1960's	1970's	1980's	1990's
25-29	**1.3%**	0.1%	0.4%	-1.0%
30-34	**1.5%**	1.5%	-0.6%	-0.9%
35-39	**1.5%**	**1.0%**	0.2%	1.0%
40-44	-0.2%	**2.2%**	2.2%	0.6%
45-49	-0.1%	**1.8%**	**2.4%**	1.1%
50-54	0.0%	0.6%	**3.2%**	2.5%
55-59	0.9%	1.1%	**3.1%**	**2.4%**
60-64	0.6%	0.9%	1.7%	**3.2%**
65-69	0.0%	1.4%	1.8%	**3.1%**
70-74	0.0%	1.1%	1.5%	1.9%

Table 1. Average annual rate of mortality improvement, for the England and Wales male population, stratified by age group and mortality decade

Age group	1960's	1970's	1980's	1990's
25-29	**1.5%**	0.6%	2.6%	0.2%
30-34	**2.1%**	1.3%	0.9%	0.6%
35-39	**1.6%**	**1.6%**	1.2%	0.8%
40-44	0.2%	**2.0%**	1.9%	0.4%
45-49	0.4%	**1.8%**	**2.4%**	1.0%
50-54	-0.1%	0.3%	**2.8%**	1.6%
55-59	0.2%	0.5%	**2.1%**	**2.1%**
60-64	1.0%	0.2%	0.6%	**2.8%**
65-69	1.0%	0.8%	0.8%	**2.4%**
70-74	1.1%	1.3%	0.8%	0.9%

Table 2. Average annual rate of mortality improvement, for the England and Wales female population, stratified by age group and mortality decade

One percent of mortality improvement is equal to one percent decrease in mortality. As we can see in tables, the generations born in the 1930's and 1940's give some proof for the argument about a cohort effect. As mentioned earlier there are a number of possible explanations for why the improvement for this cohort has occurred.

4. The data and methods

4.1 The Historical Data

The main source for the data that is used in this study is taken from the human mortality database.

The Human Mortality Database (HMD) contains detailed mortality data for national populations, such as live birth counts, death counts, population size, exposure to risk, death rates and life tables. The database began in the year 2000 and was launched in May 2002. It is a collaborative project between the research teams in the Department of Demography at the University of California, Berkeley (USA) and the Max Planck Institute for Demographic Research (MPIDR) in Rostock (Germany). It gets financial support from the National Institute on Aging (USA) and receives technical advice and assistance from many different of international collaborations. When it is complete, it will contain original life tables for around 35-40 countries. The database is free and available for everyone at www.mortality.org.

At HMD these different rates for the Swedish population are available for the years 1751 to 2007. All the data is organized by sex, age and time, and the population size is given for one year-, five years and ten years intervals.

4.1.1 Swedish mortality data

For the last 300 years, every parish in Sweden has held a continuously updated register of its population. From 1686 it was mandatory for each parish to record baptisms, burials, marriages, divorces, migration and population registers. What makes the Swedish register so special is the fact that it was updated continuously. We can in the population register see, just how many people who lived in a particular parish at a specific time. But a disadvantage is that for 1751-1860 the initial mortality and population estimates were only available in 5-year age groups, and this has been recalculated to single year of age with a method that for the more interested can be found in the Methods Protocol at www.mortality.org. Because of this, the data prior 1861 should be used with extra caution due to problems of data quality.

When we operate the Lee-Carter method we use the ages 20-95 for the years 1900-2007. Why we choose not to use the early and really old ages is because they are not really relevant in this study and the cause of the development of mortality is different for low ages. Another reason is that some of the death rates for these ages are not defined, which means that we would need to do some smoothing to get well defined values.

We use the death-rates of Swedish mortality data for 1x1 intervals, where the first number refers to the age interval, and the second number refers to the time interval.

4.1.2 United Kingdom mortality data

The HMD data for United Kingdom covers the period after 1922. During the war (1939-1945), this series comprises only the civilian population. The demographic data for UK is collected from three different areas, England and Wales, Scotland, and Northern Ireland. The estimates are based on the sum of these regions. We use the death-rates of UK mortality data for 1x1 intervals.

4.1.3 Danish mortality data

The Danish population data is rich and date back to the seventeenth century. In the Danish parishes it became compulsory to registering births, deaths and marriages in the 1640's but data availability varies greatly by parish. For some parishes' data is available dating back to the 1670's but for the majority data is available dating back to the 1750's and this may be due to that earlier records may have been destroyed by fire, mice or insects. The Danish data for 1916 and later is of superior quality than those for earlier periods. This is because for the years prior 1916, data on deaths is only available for five-year age groups. We use the death-rates of Danish mortality data for 1x1 intervals and the years 1900-2008. We will not immerse ourselves in Danish mortality statistics, but only show some similarities with the Swedish data set in the appendix.

4.2 The Lee-Carter model

The twentieth century has seen big improvements in mortality rates. The average longevity in Sweden has risen with about 30 years since year 1900. There are many ways to forecast mortality but the most common one is the Lee-Carter model. In 1992, Ronald D. Lee and Lawrence R. Carter published a new method for long-run mortality forecasts. They developed their study basically on U.S mortality data but nowadays the method is being applied in many different countries all over the world. While many methods assume an upper limit in age, the Lee-Carter method allows age-specific death rates to decline exponentially without limit. One disadvantage of the Lee-Carter approach that John Kingdom (2008) described well is that the estimated coefficients remain constant within the projection period. Hence ages that experienced relatively high mortality improvement in the past and thus have high b_x estimates will face relatively high projected future improvements. Likewise, ages which have experienced lower improvements in the past (e.g. ages greater then 80) will have low projected improvements. If mortality improvements are set to accelerate, this approach will underestimate life expectancy and hence will undervalue annuity products.

The Lee-Carter model shows that the mortality rate is dependent on both a person's age and calendar year.

$$\log(\mathrm{q}_{\mathrm{x,t}}) = m_{xt} = a_x + b_x k_t + \varepsilon_{x,t} \qquad (1)$$

In the model a_x is an age dependant term independent of time, k_t is the mortality index factor which depends on calendar year, and b_x is the age dependent term which measures response speed and age in the mortality rate of change in the mortality index factor.

Interpretation of parameters

$q_{x,t}$	Denotes the central mortality rate for age x at time t
$m_{x,t}$	Describes the logarithmically transformed age-specific central rate of death
a_x	The average of m_{xt} over time t, describing the general pattern of mortality at different ages
k_t	A time-trend index of the general mortality level, describing the general level of mortality at different times. k_t captures the most important trend in death rates at all ages. Since the mortality is a decreasing factor, we can expect this index to decrease.
b_x	Deviations from the age when k_t varies. b_x is an age-specific constant which describe the relative speed of mortality changes at each age, when k_t is changing. The model allows for both positive and negative values of b_x. A negative value of b_x shows us that the mortality rate for a specific age is rising with increasing time. However, in practice, this *usually* does not matter in the long run (Lee and Carter, 1992). When the model is adjusted over a period that is long enough, b_x *mostly* has the same characteristics (Lee and Miller, 2001), with some exceptions, for instance for some European and Central countries (Scherp, 2007).
$\varepsilon_{x,t}$	The error term, including systematic as well as purely random deviations.

4.3 The Renshaw-Haberman cohort model

The Renshaw-Haberman model is an extended version of the Lee-Carter model with an extra parameter. In 2006 Arthur Renshaw and Peter Haberman introduced one of the first stochastic models for population mortality with a cohort effect.

$$\log(q_{x,t}) = m_{x,t} = a_x + b_x^{(1)} k_t + b_x^{(0)} \gamma_{t-x} \qquad (2)$$

where a_x is the main age profile of mortality, the $b_x^{(1)}$ and $b_x^{(0)}$ parameters measure the corresponding interactions with age, k_t is a random period effect and γ_{t-x} is a random cohort effect that is a function of the year of birth, $t-x$. As in the Lee-Carter model we use a few restrictions to be able to estimate the parameters;

$$k_t = 0\,, \sum_x b_x^{(0)} = 1\,;\ \gamma_{t-x} = 0\,, \sum_x b_x^{(1)} = 1 \qquad (3)$$

To obtain effective starting values for k_t and γ_{t-x} , we use $b_x^{(0)} = b_x^{(1)} = 1$.

In the full age-period-cohort model (2), when both the year and the cohort effects are included, the a_x parameter is not adjusted during the iterative process as we will see later on.

5. Fitting and applying the Lee-Carter model

5.1 Parameter estimation for the Lee-Carter model

Basically, what we want to do is to estimate the parameters, a_x, b_x and k_t. To this end we use the weighted least squares solution to (1). The parameterization is unchanged under either of the transformations

$$\{a_x, b_x, k_t\} \quad \longrightarrow \quad \{a_x, b_x/c\,, ck_t\}$$

$$\{a_x, b_x, k_t\} \quad \longrightarrow \quad \{a_x - \ cb_x, b_x, k_t + \ c\}$$

for any constant c.

The estimation of a_x , which minimizes the sum of squares of the error term $\left(S = \sum_{xt} \varepsilon_{x,t}^2\right)$, is the average of $m_{x,t}$.

$$a_x = \frac{1}{T} \sum_t m_{x,t} \tag{4}$$

Where T is the total number of calendar years. Thereafter the difference matrix is formed as $z_{x,t} = m_{x,t} - a_x$ and it satisfies $\sum_t z_{x,t} = 0$ for all x.

To achieve a unique solution, the following restrictions are used

$$\sum_t k_t = 0 \ and \ \sum_x (b_x)^2 = 1 \tag{5}$$

$$Q = \sum_{x,t} (k_t b_x - z_{x,t})^2$$

To find the values that minimize Q we introduce the Lagrange´s multipliers α and β and minimize

$$R = Q - \alpha \sum_t k_t - \beta \sum_x b_x^2$$

Differentiation on R with respect of x and t give us

$$\frac{dR}{dk_t} = 2 \sum_x b_x \left(k_t b_x - z_{x,t} \right) - \alpha \qquad \text{for every t}$$

$$\frac{dR}{db_x} = 2 \sum_t k_t \left(b_x k_t - z_{x,t} \right) - 2\beta \qquad \text{for every x}$$

If the derivates are set equal to zero, we obtain

$$\frac{\alpha}{2} = k_t \sum_x b_x^2 - \sum_x b_x z_{x,t} \tag{6}$$

If we add these sums with respect to t we get that α = 0 and we can solve for k_t and b_x from the system of equations

$$k_t = \sum_x b_x z_{x,t} \tag{7}$$

$$b_x = \frac{\sum_t k_t \, z_{x,t}}{\sqrt{\sum_x \left(\sum_t k_t \, z_{x,t}\right)^2}} \qquad (8)$$

The equations (7) and (8) cannot be solved explicitly and we cannot use ordinary regression. On the other hand it is easy to iterate to reach a solution. We start with $b_x = 1/\sqrt{m}$ is independently of x, where m is the number of ages that are observed. We use our values of b_x and input these into (7), which gives us a value on k_t, and then we use these to update b_x in (8). Then we repeat this cycle. The iteration converges surprisingly rapidly. After less than ten iterations we have a solution with high precision, (Bahr, 2006).

5.2 Adapt the parameters in the Lee-Carter model using Excel

Changes in mortality over time are usually analyzed by calculating rates of mortality improvement. We start by fitting the parameters manually in Excel by using the method described in section 5.1. Figures 3-5 show the three parameters in the Lee-Carter model for ages 20-95 for Sweden during 1900-2007. See also figures A-C in appendix A for another period.
As we can see in figure 4 and as well as in figure B, the estimated mortality time trends are quite similar for males and females. Initially the curve is relatively flat except that it deviates right before year 1920 and it gets pretty steep between 1945 and 1955. The rise in the curve in the late 1910s depends on the high mortality that followed the Spanish flu, when approximately 38 000 Swedes died. Because of this, there are also curves showing the parameter estimates without year 1918, see figure 6 and 7.

Then we have a decline in the figure which may due to the disease tuberculosis. As late as in the 1930s, nearly 10 000 people in Sweden died each year as a result of getting infected by the bacterial. During this period, the public health got better, especially in the health care and the management of producing a vaccine against tuberculosis led to that all newborn in the 1940s to 1975 were vaccinated. This in turn, led to fewer deaths overall and lower infant mortality, see www.epiwebb.se.

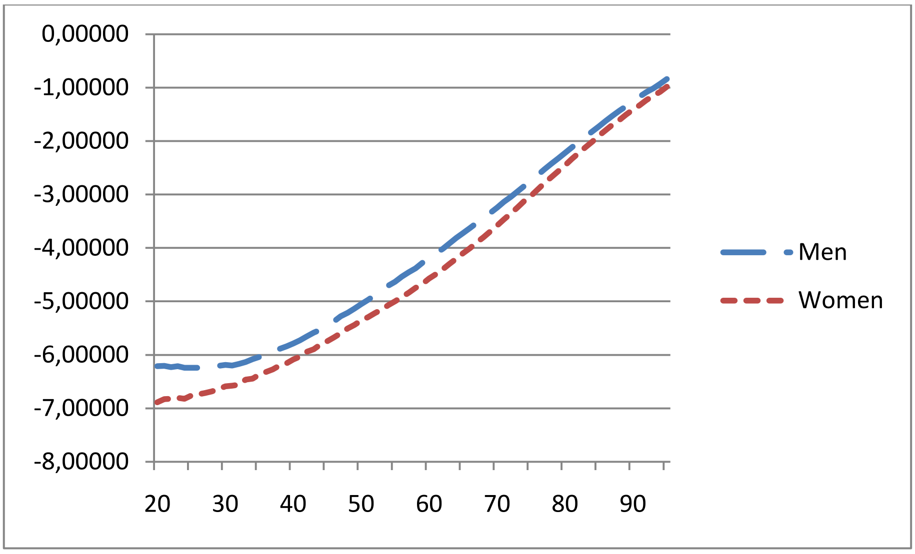

Figure 3. General pattern of Swedish mortality a_x, men and women

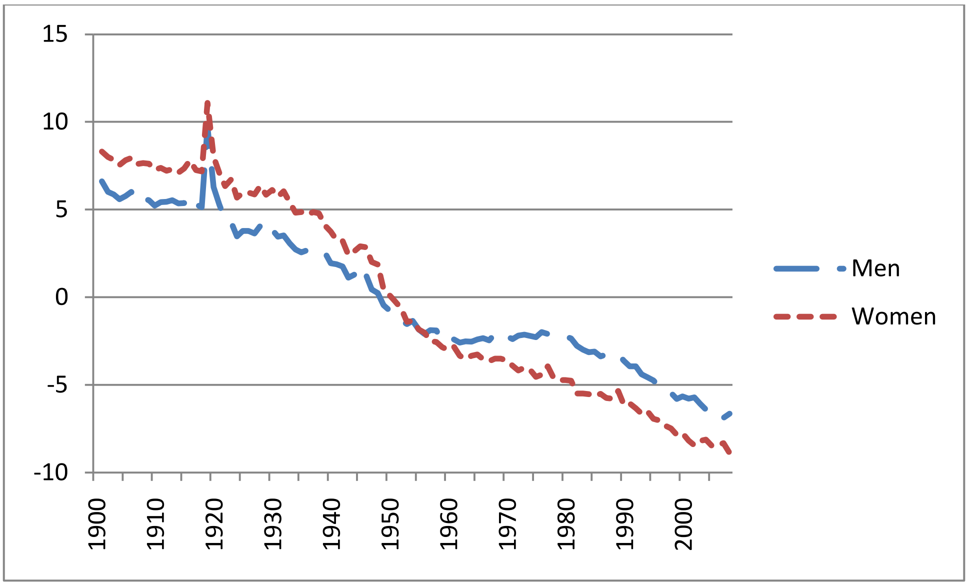

Figure 4. The mortality trend k_t, for Swedish men and women 1900-2007

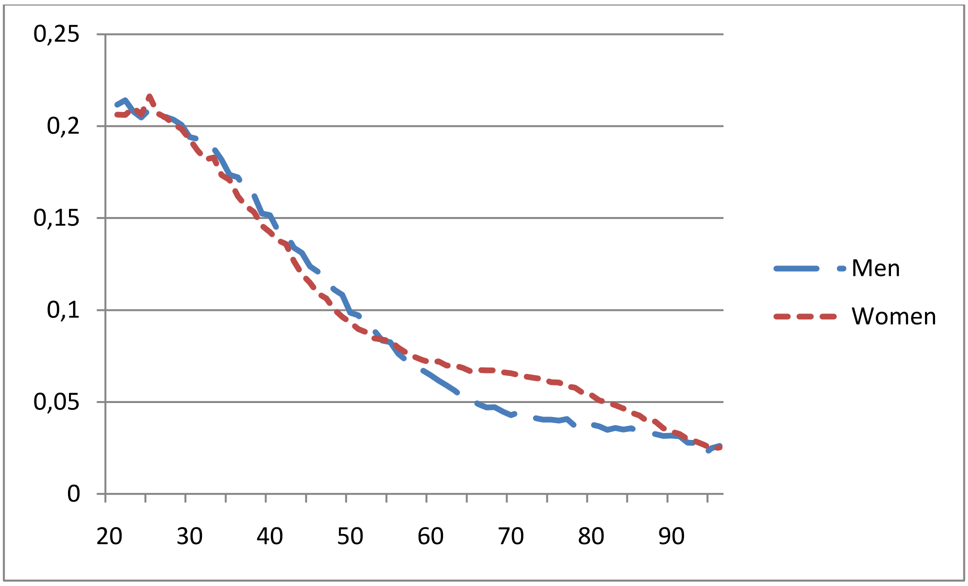

Figure 5. The age-specific constant b_x, for Swedish men and women 1900-2007

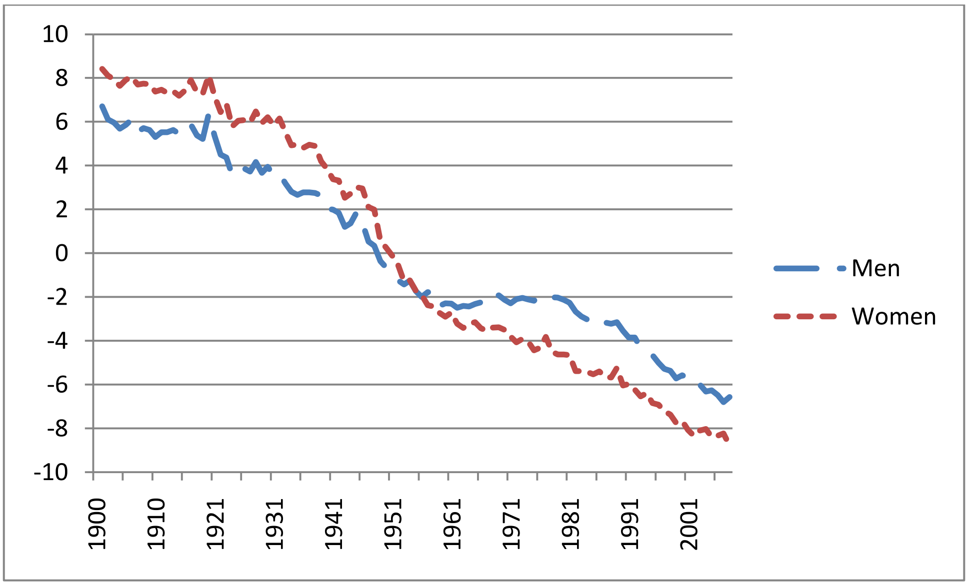

Figure 6. The mortality trend k_t, for Swedish men and women 1900-2007, without year 1918

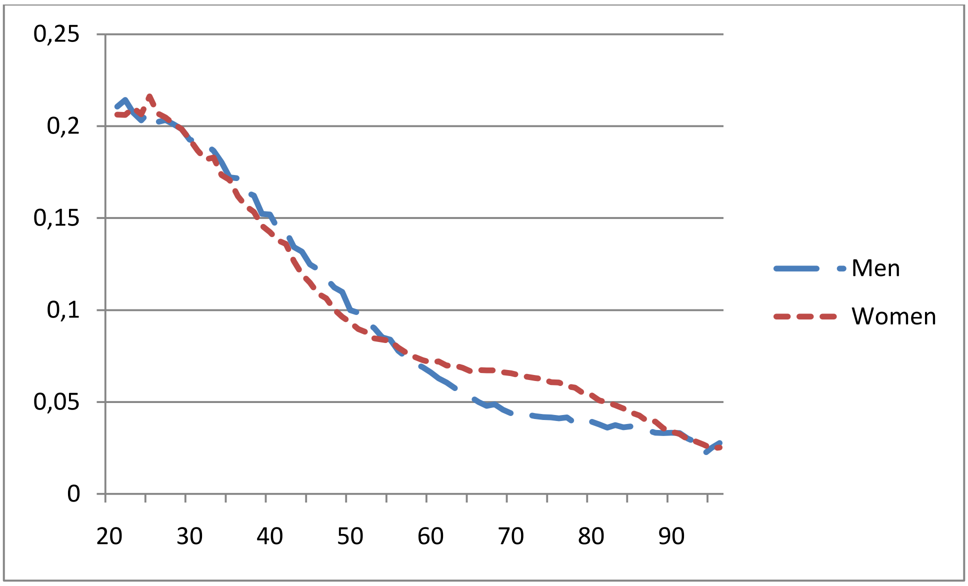

Figure 7. The age-specific constant b_x, for Swedish men and women 1900-2007, without year 1918

The corresponding figures for United Kingdom can be found in Appendix A; figures D-F.

5.3 Patterns of mortality improvement over age and time

Differentiation of (1) with respect on t

$$r_{x,t} = -\frac{d}{dt}\log(a_x + b_x k_t) = -b_x k'_t$$

We can see that the rate of mortality improvement in a given year is a product of the b_x schedule and the negative derivative of the k_t. If k_t is a linear function - as for a random walk with linear drift process - then k'_t is constant and $r_{x,t}$ is also constant over time implying that each age-specific death rate on average declines at its own rate (Andreev and Vaupel, 2005).

A pattern of mortality improvement means that we see reduced mortality for certain age groups during certain periods. In the following figures (figure 8-11) we can see the deviations in the Lee-Carter model using data for Sweden and United Kingdom. Different colors are used for different percentages, the lower the percentage the fewer deaths, i.e. mortality improvement. If we compare at the corresponding figures for Sweden and United Kingdom we can see some big differences. In the UK figures (figures 10 and 11), there are clear signs of mortality improvements for the cohort born around 1931. In the figures for Swedish data we can see some kind of patterns but not as clear as for United Kingdom.

The vertical line that appears just before 1945 in the Swedish figures (figure 8 and 9), is an indirect effect of the tuberculosis vaccine and the following mortality improvement a consequence of it.

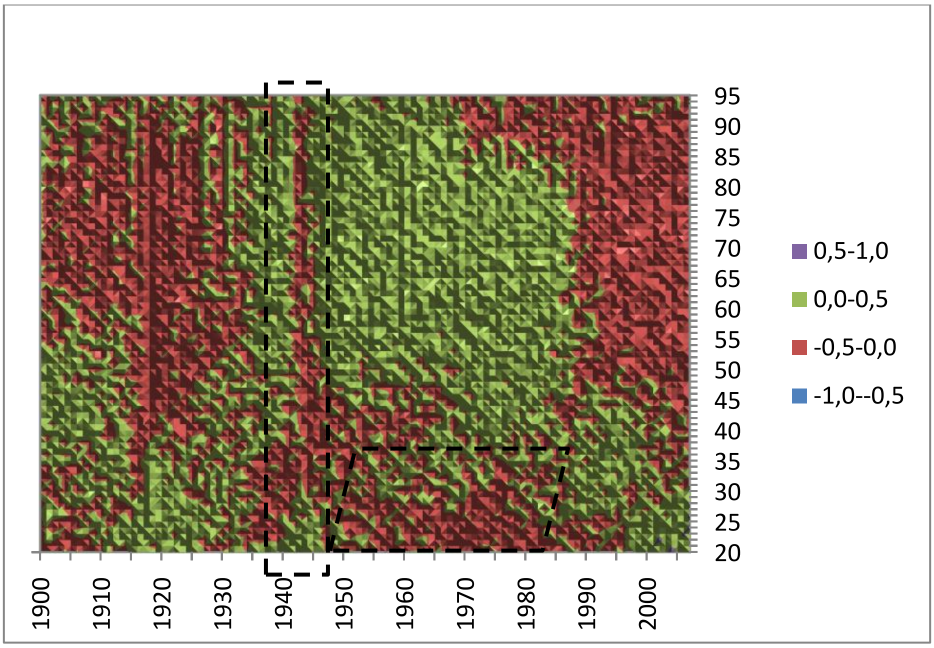

Figure 8. Patterns of mortality rate, Sweden, men 1900-2007

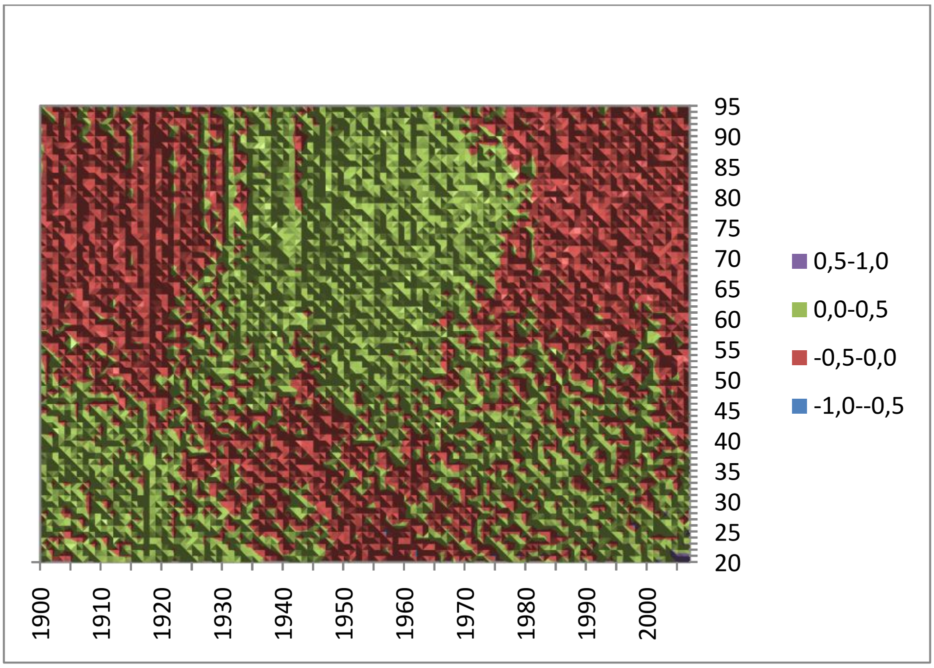

Figure 9. Patterns of mortality rate, Sweden, women 1900-2007

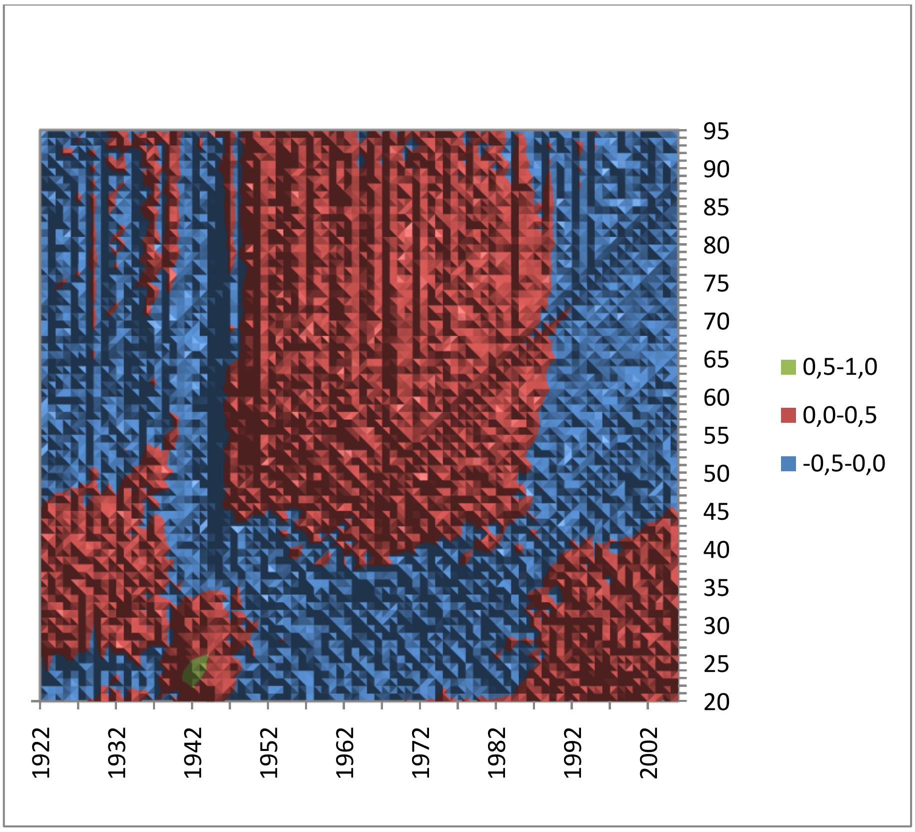

Figure 10. Patterns of mortality rate, UK, men 1922-2006

If we look at the ages 20-35 for the years around 1942 we can see some differences in colors. These differences in mortality are because of the high amount of young men who died during the war. This rubs off on the generations that are not in war. Due to large losses, it results in fewer babies in the next generation.

After contact with Mr. Adrian Gallop[*] we concluded that one possible reason for the diagonal line that we can see in both male and female figures for United Kingdom might be that the there was a rapid change in the birth rate/number of births during the years 1919 and 1920 (and possibly 1918 also). Effectively it means that using the mid-year population estimates as the exposed to risk for these cohorts is not as reliable as for other birth years as the births for these cohorts were not spread evenly over those years and hence may lead to a significant under or over count for those years compared to calculations on a daily basis, if we had that data available.

[*] A. Gallop, Government's Actuary Department, UK, personal contact

If we look at figure 11 there are some patterns which indicate that women in their 50's (year 2002) have had mortality improvement. It is too early to say anything about this, but it could be a good idea to follow up on this particular group. Similar patterns can be found for the same age and gender in Denmark.

Figure 11. Patterns of mortality rate, UK, women, 1922-2006

The corresponding figures G and H for Danish data can be found in appendix A. Those figures are quite interesting compared to Swedish data. It is possible to see mortality improvement for both men and women but at different time periods. For men, there is an improvement for the generation born around 1940 and for the women it started a little bit earlier. There also seem to be some kind of pattern for women that were in there 50's in 2000.

6. Fitting and applying the Renshaw-Haberman model

6.1 Data

When we apply the Renshaw-Haberman model we use the same datasets as earlier. We choose a specific time interval and a specific age to see if there are any signs of cohort effects for those generations.

We use the death-rates and exposure of risk of Swedish, English and Danish mortality data for 1x1 intervals, where the first number refers to the age interval, and the second number refers to the time interval.

6.2 Parameter estimation for the Renshaw-Haberman model

We let the random variable $d_{x,t}$ denote the number of deaths in a population at age x and time t.

We can estimate the central mortality rate $q_{x,t}$ as

$$\hat{q}_{x,t} = \frac{d_{x,t}}{e_{x,t}}$$

where $d_{x,t}$ represent the number of deaths and $e_{x,t}$ represent the matching central exposure for any given subgroup. We let $z = t - x$ define the combination of age and period, i.e. the cohort year. To get the best estimates we approach a seven-step method, (Butt and Haberman, 2009).

1. To estimate the parameters in the Renshaw-Haberman model we use a pre-programmed software for R named LifeMetrics. As in the Lee-Carter model we start to estimate the fixed age effects, but here we use the singular value decomposition (SVD) method to find the least squares solution.

$$a_x = \frac{1}{T}\sum_t m_{x,t}$$

2. After that we will try to get appropriate initial values

$$b_x^{(1)} = b_x^{(0)} = \frac{1}{k}$$

Estimate the simplified period-cohort predictor, with the constraints that $b_x^{(1,0)} = 1$ to get initial values for k_t and γ_z.
⟶ calculate the adapted values $\hat{y}\left(\hat{a}_x\,,\ \hat{b}_x^{(1)}\,,\ \hat{b}_x^{(0)},\ \hat{k}_t\,,\ \hat{\gamma}_z\right)$
⟶ calculate the deviance $\left(y_{x,t}, \hat{y}_{x,t}\right)$.

3. We continue by updating the parameter $\gamma_{x,t}$

$$\hat{\gamma}_z = \hat{\gamma}_z + \frac{\sum_x 2\omega(y - \hat{y})}{\sum_x 2\omega\left(\hat{b}_x^{(1)}\right)^2 \hat{y}}$$

where ω is either 0 for every empty data cells and 1 for every non-empty data cell.

- shift the updated parameter such that $\hat{\gamma}_z = \hat{\gamma}_z - \hat{\gamma}_1$;
⟶ calculate the adapted values $\hat{y}\left(\hat{a}_x\,,\ \hat{b}_x^{(1)}\,,\ b_x^{(0)},\ \hat{k}_t\,,\ \hat{\gamma}_z\right)$
⟶ calculate deviance $\left(y_{x,t}, \hat{y}_{x,t}\right)$.

4. Update parameter $\hat{b}_x^{(1)}$

$$\hat{b}_x^{(1)} = \hat{b}_x^{(1)} + \frac{\sum_t 2\omega(y - \hat{y})}{\sum_t 2\omega\,\hat{\gamma}_z^2 \hat{y}}$$

⟶ calculate the adapted values $\hat{y}\left(\hat{a}_x\,,\ \hat{b}_x^{(1)}\,,\ b_x^{(0)},\ \hat{k}_t\,,\ \hat{\gamma}_z\right)$
⟶ calculate deviance $\left(y_{x,t}, \hat{y}_{x,t}\right)$.

5. Update parameter $\hat{k}_t$

$$\hat{k}_t = \hat{k}_t + \frac{\sum_x 2\omega(y-\hat{y})}{\sum_x 2\omega \left(\hat{b}_x^{(0)}\right)^2 \hat{y}}$$

- shift the updated parameter such that $\hat{k}_t = \hat{k}_t - \hat{k}_1$;
⟶ calculate the adapted values $\hat{y}\left(\hat{a}_x\,,\ \hat{b}_x^{(1)}\,,\ b_x^{(0)},\ \hat{k}_t\,,\ \hat{\gamma}_z\right)$
⟶ calculate deviance $\left(y_{x,t}, \hat{y}_{x,t}\right)$.

6. Update parameter $\hat{b}_x^{(0)}$

$$\hat{b}_x^{(0)} = \hat{b}_x^{(0)} + \frac{\sum_t 2\omega(y-\hat{y})}{\sum_t 2\omega\, \hat{k}_t^2 \hat{y}}$$

⟶ calculate the adapted values $\hat{y}\left(\hat{a}_x\,,\ \hat{b}_x^{(1)}\,,\ b_x^{(0)},\ \hat{k}_t\,,\ \hat{\gamma}_z\right)$
⟶ calculate deviance $\left(y_{x,t}, \hat{y}_{x,t}\right)$.

7. Control the divergent convergence

$$\Delta D = D - D_u$$

where D is the deviance from step **3** and D_u is the updated deviance at step **6**.

- if $\Delta D > 1\ \boldsymbol{x}\ 10^{(-6)}$ $\Longrightarrow$ go to step 3.
- stop iterate process when $\Delta D \approx 0$ and take the adapted parameters as the ML estimates to the observed data.
- Alternatively, stop if $\Delta D < 0$ for 5 updating cycles in a row and consider using other starting values or declare the iterations non-convergent.

8. When convergence is achieved, rescale the new interaction parameters $\hat{b}_x^{(1)}, \hat{b}_x^{(0)}, \hat{k}_t$ *and* $\hat{\gamma}_z$:

$$\hat{b}_x^{(1)} = \frac{\hat{b}_x^{(1)}}{\sum_x \hat{b}_x^{(1)}}\ , \hat{b}_x^{(0)} = \frac{\hat{b}_x^{(0)}}{\sum_x \hat{b}_x^{(0)}}\ ;\ \hat{k}_t = \hat{k}_t\ x \left(\sum_x \hat{b}_x^{(0)}\right)$$

in order to satisfy the age-period Lee-Carter model constraints $\sum_x b_x^{(1)} = \sum_x b_x^{(0)} = 1 \; and \; \sum_t k_t = 0$.

6.3 Adapt the parameters in the Renshaw-Haberman model using R

The main reason for all those calculations is of course to predict future mortality, i.e. forecasting life. To get even better estimates and to forecast mortality we continue by using the statistical program R. It is not possible to get good parameter values for the Renshaw-Haberman method in Excel since we have to iterate so many times.

Before we have a look at the parameters we make residual plots for the Renshaw-Haberman method for the different ages and periods to see if any patterns are revealed.

As before we begin by looking at data for Sweden for the ages 20-95 during the period 1900-2007. The deeper the colour, the stronger are the rates of improvement, i.e. lower mortality. The corresponding figures for Swedish and British men are to be found in figures I and J in appendix A.

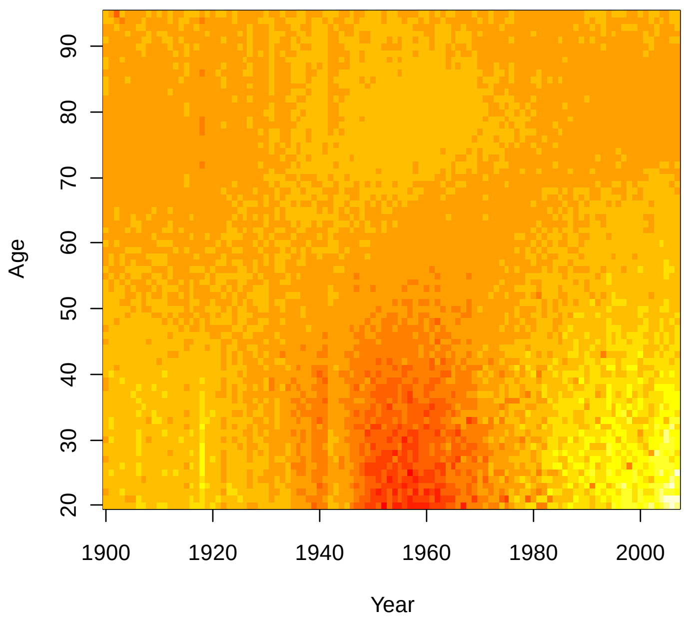

Figure 12. Women, Sweden, 1900-2007

Having a closer look at figure 12 for ages 50-95, we can spot that there is some sort of mortality improvement for Swedish women around 50 years old, born 1900-1910. This can be compared to the patterns that are to be found in the British and Danish data sets of figures 14 and K.

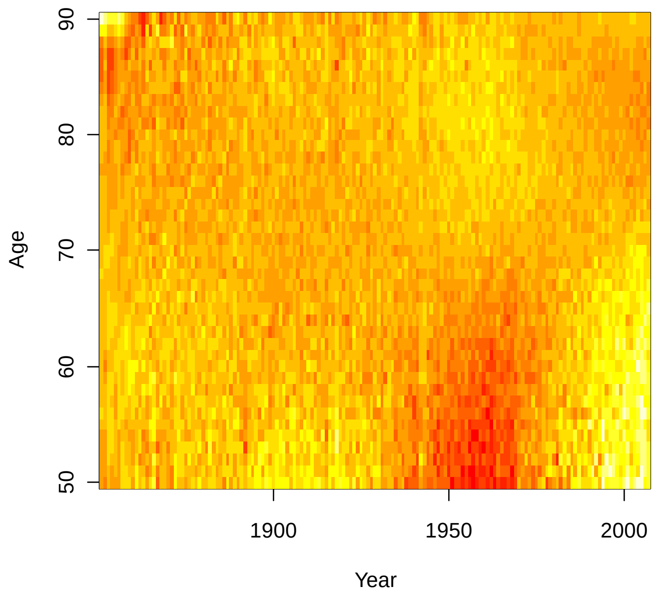

Figure 13. Ages 50-90, Swedish women, 1850-2007

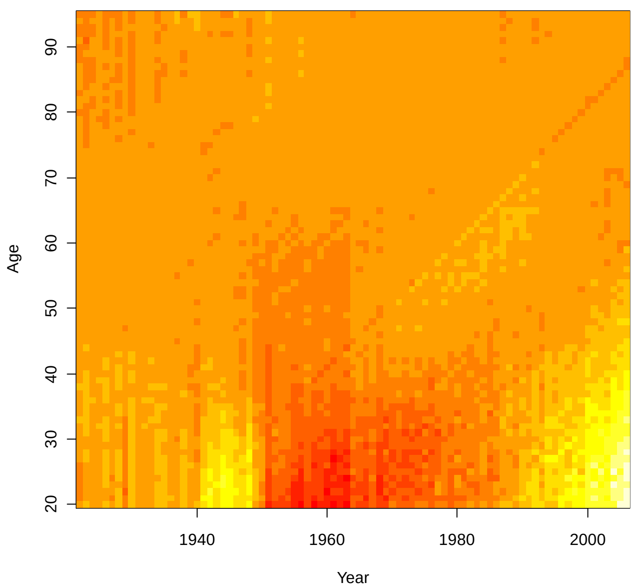

Figure 14. United Kingdom 1922-2006, women

Figure 15 shows parameter values in the Renshaw-Haberman model for Swedish women, the corresponding figures for men are to be found in figure M of appendix A. As we can see there is an extra parameter that depends on time of birth. In section 9 we will, with help of these values, calculate the reserve for a hypothetical individual insurance company.

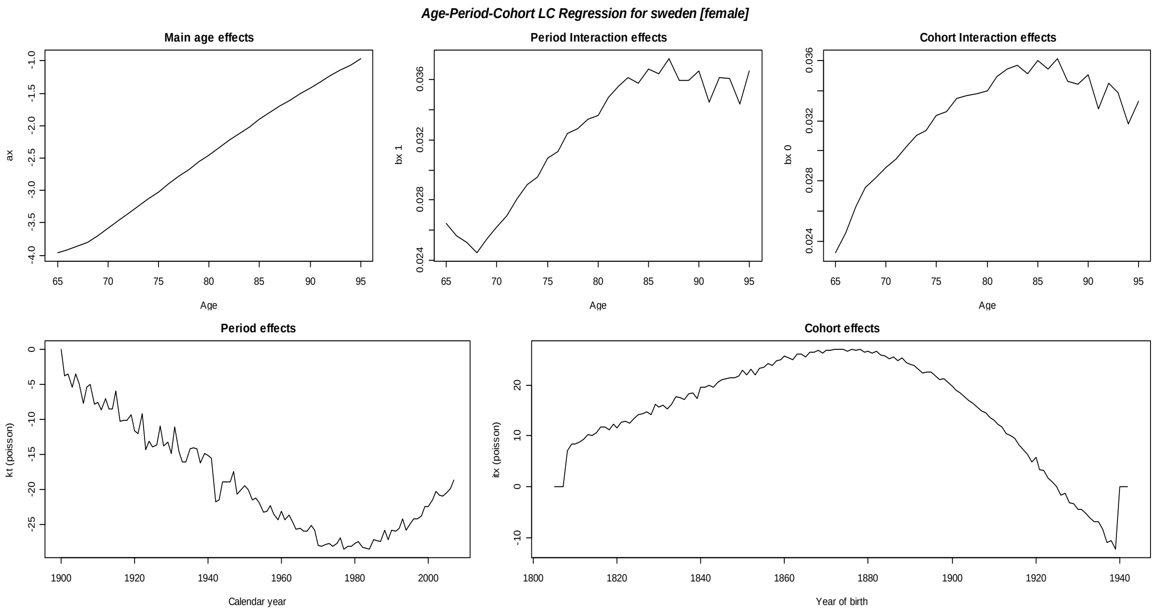

Figure 15. Age-period regression for Sweden, women

The parameter values are 0 at the beginning and the end due to the parameter restriction in the adjustment. It is to get an unambiguous alignment.

7. Reserving with Lee-Carter

In all kind of business, buyers and sellers have to meet the specific requirements so that both parties will be satisfied. In insurance, this means that if an individual is insured, he/she expects to get compensated in case of loss, damage or injury. To be prepared for forthcoming and unpredictable disbursements each company makes technical provisions. In life insurance, incoming and outgoing cash flows are taking place at different times and the outgoing payments will take place over a long period of time in the future.

7.1 Data

For simplicity we calculate the reserves by looking at two groups of 1000 people at a time, men and women separately. The inception we use here is 2007, because that is the latest year from which we have information for the Swedish population. The populations we will use is one group of people that retires in 2007 and another group that will retire in 10 years ahead of 2007, i.e. in 2017. We use the general retirement age in Sweden which is 65. Every year, each individual in the group of 1000 is expecting an amount of money. Therefore, it is important to know what age the people in the test groups will reach.

7.2 Forecasting

When the data is adapted to the model, in the first case, the Lee-Carter model, we already have values of the vectors a_x, b_x and historic values of k_t, so future mortality rates are derived by projecting the mortality index k_t. The easiest way do to this is to use linear regression. Simple linear regression assumes that a straight line can be fitted to the data and the regression is $y = kx + m$.

We look at three different time periods, 1900-2007, 1960-2007 and 1980-2007 to find out how the outcome differs depending on the forecasts we get. The longer forecast we would like to do, the further back in time should we go for retrieving data. We have made a forecast of 40 years and predicted that mortality is about 40% for age

96-105 and 100% for age 106 and older, i.e. the entire test group has passed away.

In figure 16, we used historical data of the time trend index of generally mortality, k_t. We perform linear regression and assume that mortality will follow the same curve the following 40 years. By using different time periods we get different predictions of future mortality. In the first figure we get a clear view of the mortality during the last century. The different colours show the different time periods we chose for our upcoming calculations and the three-drawn lines show the linear regression for each time period.

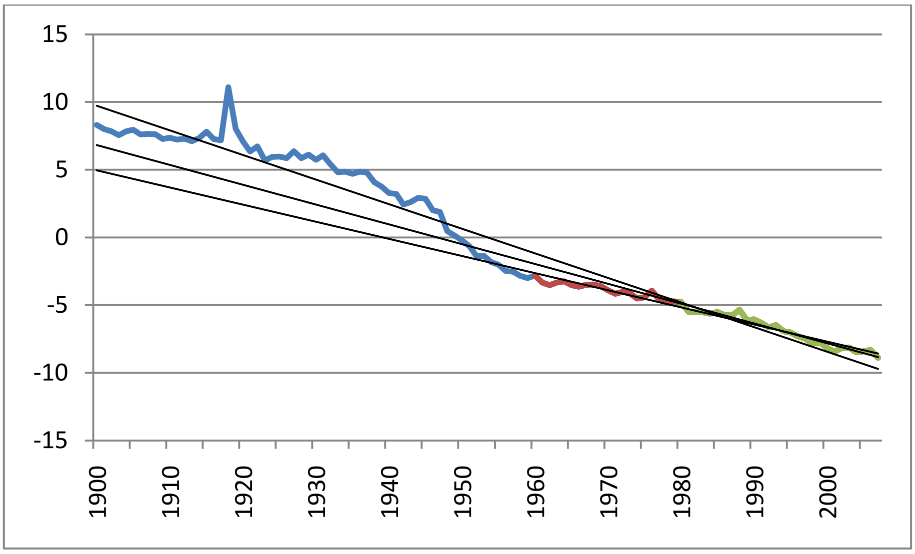

Figure 16. Historical values of k_t, 1900-2007, women

In figures 17-19 we see the different regressions we get from each time period. Note that there are different dates on the x-axes of the figures.

When we look at the entire time interval, 1900-2007, we get a negative slope that decreases with 0,1819 per year, see figure 17.

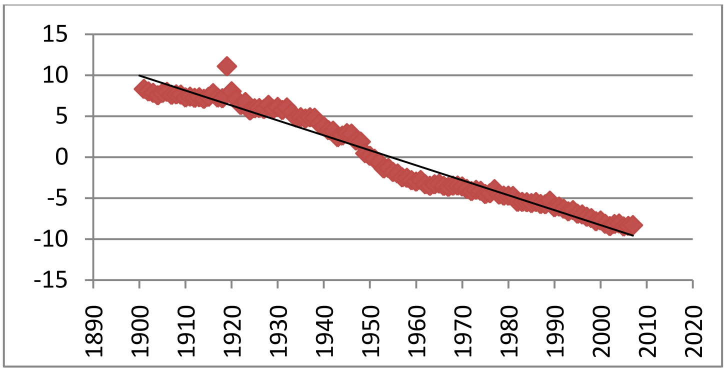

Figure 17. Historical values of k_t, 1900-2007, women

By using the time period 1960-2007 we get a negative slope of 0,1266 per year, see figure 18 and by using the time period 1980-2007 we get a negative of 0,1464 per year, see figure 19.

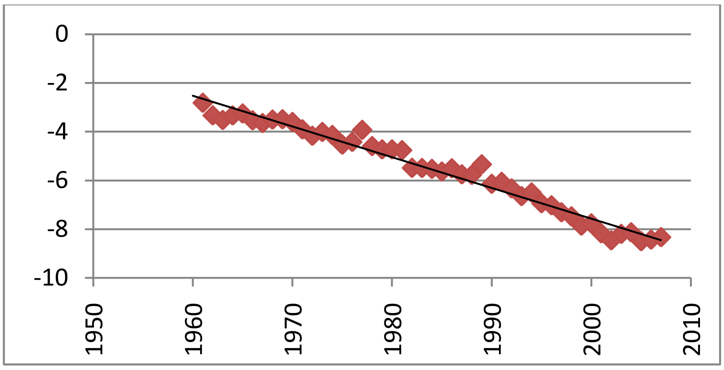

Figure 18. Historical values of k_t, 1960-2007, women

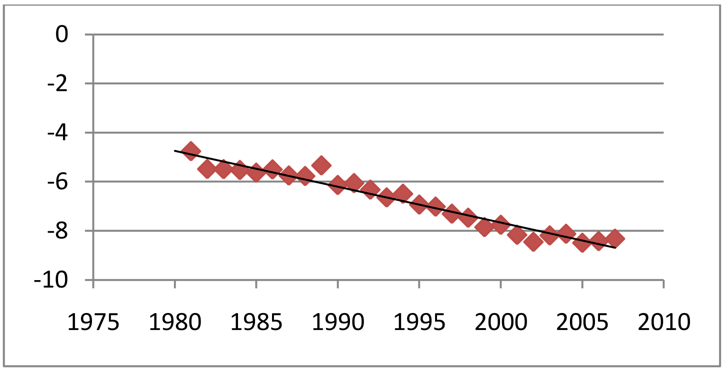

Figure 19. Historical values of k_t, 1980-2007

7.3 Prospective reserving

The reserve for life insurance can be calculated either prospectively or retrospectively. We will get the same results if the assumptions are the same when calculating the reserve and calculating the premium. The prospectively estimated reserve corresponds to the present value of expected future pension benefits minus the present value of expected future premiums which are being paid during the contract period.

If we use the Lee-Carter model to predict the mortality index factor, we have a reasonable image of what the mortality will be in the future. We get

$$l(x+1) = l(x)(1 - q_{x,t}) \tag{9}$$

where $l(x+1)$ denotes the fraction of the cohort alive at age x and $q_{x,t}$ is obtained by taking the exponential function of (1).

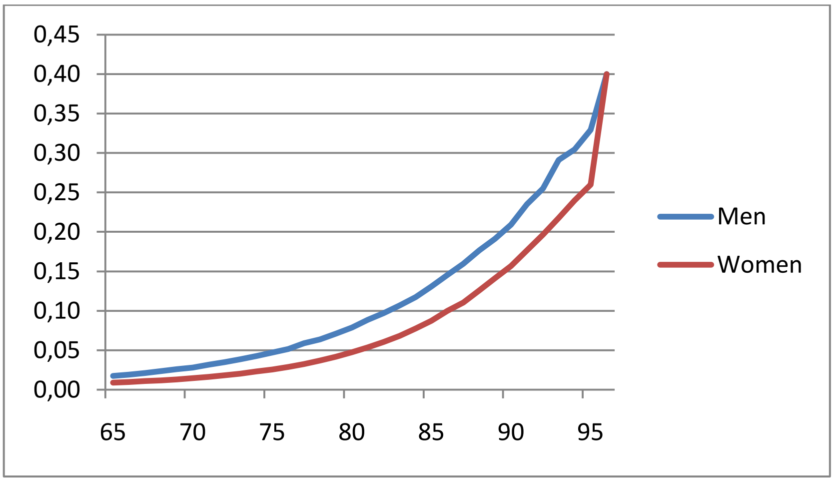

Figure 20. One-year death probabilities, $q_{x,t}$, by calendar year 1900-2007, men and women, permille

It is well known that women normally have a lower mortality rate than men. In the figure above we see how the yearly expected mortality looks for men and women when calculated on 1900-2007. As mentioned in section 2 the life expectancy at birth for women is around 83 years and around 79 years for men. When talking about one-year death probabilities we mean how the death probability changes for a given individual when he/she is getting older.

8. Reserve calculations for Lee-Carter

8.1 Zero interest rate with generation retiring 2007

Immediate annuity

One simple way to calculate the reserve is to calculate with zero interest rate, i.e. the annuities are fixed and we assume that there is no return on our funds (except maybe for expenses). This means that we will put away exactly the amount of money that will be needed in payments. Or we could use a fixed interest, which is the same thing as having the same interest rate over the whole period.

$$r_n = r_0(1 + in)$$

where, r_0 is the interest capital and i is the rate of interest and r_n is the capital that we need to put aside. In our case we will assume that the constant interest is 2 percent per year.

In order to estimate the life expectancy when not using any interest at all, we just have to calculate how many payments we will have to make for the chosen group of people. We do this by summing years of life beyond 65 in the test group and then divide the number of people in the test group. This will give us the average life expectancy at age 65.

For the three time-periods we have get different life expectancies and this will make a difference in future payments.

To derive the life expectancy we have used (Andersson, 2005) and we refer to this book for more details.

Let T_{65} be a non-negative random variable that represents the remaining life expectancy for an individual aged 65. The distribution function is then defined as

$$F_{65}(t) = P(T_{65} \leq t)\ , where\ t \geq 0$$

The function l_x is then defined as the survivor function

$$l_{65}(t) = 1 - F_{65}(t) = P(T_{65} > t), where\ t > 0$$

Simplification gives us

$$P(T_{65} > t) = \frac{l_0(65 + t)}{l_0(65)}$$

which can be written as

$$l_{65}(t) = \frac{l_0(65 + t)}{l_0(65)}$$

The life expectancy at age 65 is

$$e_{65} = E[T_{65}] = \int_0^\infty \big(1 - F_{65}(t)\big)dt = \int_0^\infty \frac{l(65 + t)}{l(65)}$$

We get the following results. Please note to that this is life expectancy given an attained age, in this case an age of 65 and hence is not the same life expectancies treated in section 2.

	1900-2007	1960-2007	1980-2007
Men	16,48	16,39	16,61
Women	20,27	20,03	20,12

Table 3. Life expectancies at age 65 for men and women based of different periods of estimation

If we look at the results for the three different predictions, the differences may be relatively small, but in a large insurance company the group of annuitants is very large and for each person one year corresponds to quite a large amount of money.

8.2 Compound interest with generation retiring 2007

The compound interest

$$r_n = r_0(1+i)^n \tag{10}$$

The capital grows exponentially and we can use the term

$$\log(1 + \mathrm{i}) = \xi$$

for the interest intensity.

In order to find out the amount of money we need to put away for upcoming payments with 2 percent interest we need to use a discount factor. The discount factor we get by discounting according to

$$r_0 = r_n(1+i)^{-n}$$

$$\log(1+i) = \xi$$

we get the interest intensity. The discount factor d id defined as

$$e^{-\xi} = \frac{1}{1+i} = d$$

We assume that all payments from policyholders are made in January 2007 and all the disbursements are made in December. That provides the reserve an additional year with interest. If we look at the results in table 4, we see the total amount of money that is to be needed to fulfill the commitments to the policyholders. If we then look at the results in table 5, we see how large the technical provision needs to be. If we subtract the results in table 4 with the results in table 5 we see how much the company would save on reserving with 2 percent interest.

We get the following results

	Historic period used		
Reserve	**1900-2007**	**1960-2007**	**1980-2007**
Men	16 482,78	16 417,72	16 609,78
Women	20 273,99	20 028,67	20 116,36

Table 4. Reserving with zero interest per 1000 policyholders, whole life annuity

	Historic period used		
Reserve	**1900-2007**	**1960-2007**	**1980-2007**
Men	13 479,59	13 437,02	13 562,53
Women	16 090,25	15 935,30	15 990,74

Table 5. Reserving with 2% interest per 1000 policyholders, whole life annuity

If we assume that we looking at the whole period, i.e. 1900-2007 and then calculate the differences between that period and the other two periods, and continue with comparing 1960-2007 with 1980-2007 we get the following table (table %). In table 6 we compute the reserve at 2007 with 2 percent interest rate based on estimates for a historic period 1900-2007 and then calculate the difference between this reserve and those based on the same interest rates but period 1960-2007 and 1980-2007. As we can see, the time period differences are within $\pm$ 1 percent but if we compare the two tables (table 4 and 5) we get that the effect of discounting is much larger. There is a decrease of about 20 percent whichever period or gender that is chosen.

	1900-2007	1960-2007	1980-2007
Men	13 479,59	-42,57 (-0,316%)	+82,94 (+0,615%)
Women	16 090,25	-154,95 (-0,963%)	-99,51 (-0,618%)
Men		13 437,02	+125,51 (+0,934%)
Women		15 935,30	+55,44 (+0,348%)

Table 6. Differences between the three periods

8.3 Generation retiring 2017 with zero- and compound interest

We use the same calculations as before, except for another group of people. This time the people are 55 years old at 2007. A single premium is paid in January and the annuity payments will start at 2017.

	Historic period used		
Reserve	**1900-2007**	**1960-2007**	**1980-2007**
Men	15 284,16	15 159,19	15 528,19
Women	19 878,36	19 440,76	19 597,49

Table 7. Reserving with zero interest per 1000 policyholders, life annuity deferred by 10 years

	Historic period used		
Reserve	**1900-2007**	**1960-2007**	**1980-2007**
Men	10 221,97	10 151,77	10 358,46
Women	12 878,89	12 644,94	12 728,87

Table 8. Reserving with 2% interest per 1000 policyholders, life annuity deferred by 10 years

In table 9 we can see the differences between the three time periods. Depending on what historic period we use we get different possible reserves. There are effects in the time periods, depending on which two periods we choose between, mostly of them in the interval $\pm$ 1,5 percent, which can make a big difference in payments for the insurance company if they have many policyholders and many commitments.

To see how much the company would save on reserving with 2 percent interest compared to zero interest, we subtract those of table 7 with the results in table 8. We note again that the effects caused by the periods are small compared with the effects from discounting, where there are reductions higher than 33 percent whichever time period we choose.

	1900-2007	**1960-2007**	**1980-2007**
Men	10 221,92	-70,15 (-0,686%)	+136,54 (+1,336%)
Women	12 878,89	-233,95 (-1,817%)	-150,02 (-1,165%)
Men		10 151,77	+206,69 (+2,036%)
Women		12 644,94	+83,93 (0,664%)

Table 9. Differences between the three periods, deferred life annuity

To find out what difference it makes if the policyholders make their payment when they are 55 compared to when they are 65 we compare table 5 and table 8. As we can see it takes less money in the technical provision, the earlier the policies are made. If we assume that the premiums are the same for the policyholders, then the insurance company can save between 19,96 and 24,37 percent on the extra ten years depending on what time period and gender they based their calculations on.

8.4 Variable interest

In reality, nowadays it is getting more common to use variable interest, which makes it more difficult to predict how large the technical provisions need to be. The individual insurer cannot itself affect the interest since it is controlled by the banks and the government.

$$r_n = r_0\left(1 + \int_0^n i(t)dt\right) \tag{11}$$

For practical reasons we will not make any calculations with variable interest.

9. Reserving with Renshaw-Haberman

9.1 Forecasting

When calculate the reserve with the Renshaw-Haberman method we assume the same conditions as in section 7.2. In figures 21 and 22, we used historical data of the time trend index of generally mortality, k_t. We perform linear regression and assume that mortality will follow the same curve the following 40 years. By using different time periods we get different predictions of future mortality. The different colours show the different time periods we chose for our upcoming calculations and the three-drawn lines show the linear regression for each time period. As we can see they differ from the k-values in the Lee-Carter model.

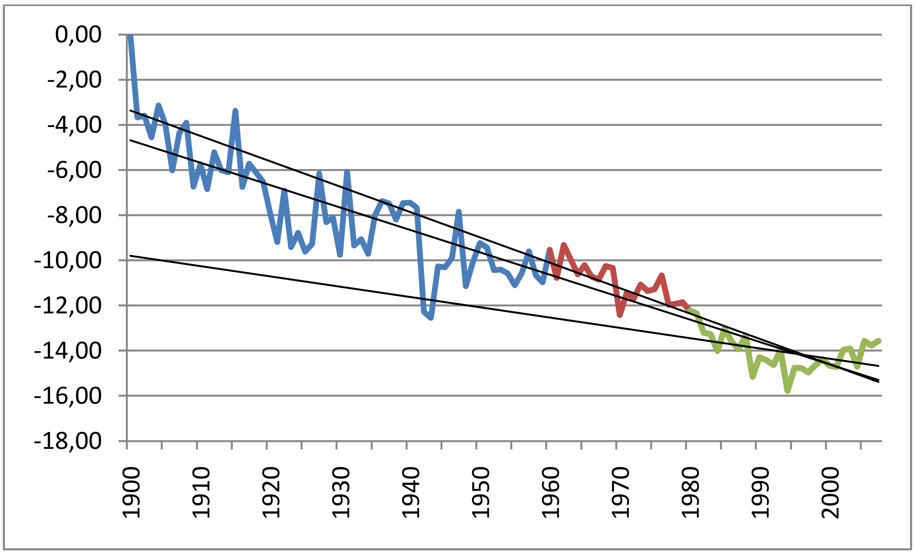

Figure 21. Historical values of k_t, 1900-2007, men

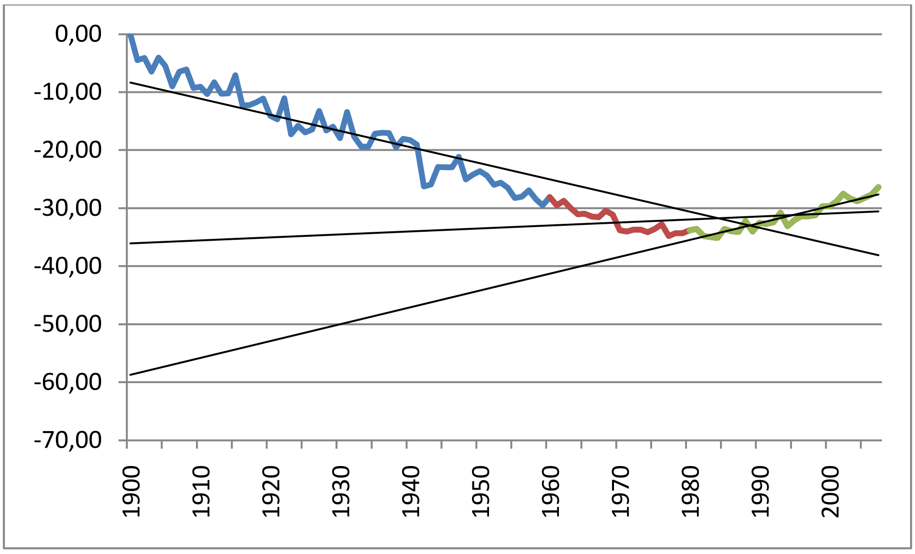

Figure 22. Historical values of k_t, 1900-2007, women

In figure 22 it is easy to see how wrong the predictions might be if we only look at a short time of historical mortality. The curve is decreasing all the way from year 1900 to the 1980 and then it starts increasing. In section 10 we will see how this affects the reserving. In the following three figures we see the different regressions we get from each time period. Note that there are different years on the x-axes.

When we look at the entire time interval, 1900-2007, we get a negative slope of 0,2781 per year, as seen in figure 23.

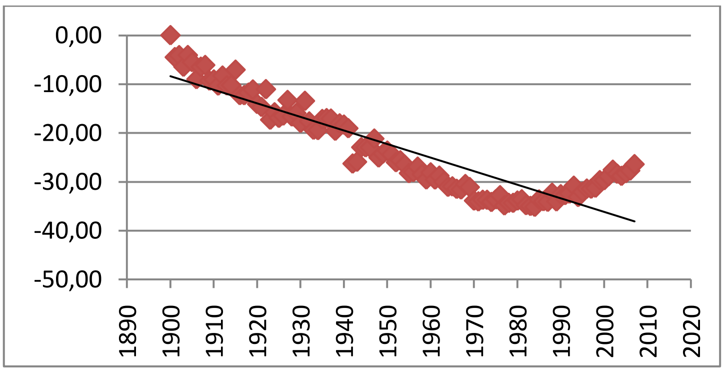

Figure 23. Historical values of k_t, 1900-2007

By using the time period 1960-2007 we get a positive slope of 0,0518 per year, as shown in figure 24 and by using the time period 1900-

2007 we get a positive slope that increase with 0,2905 per year, as shown in figure 25.

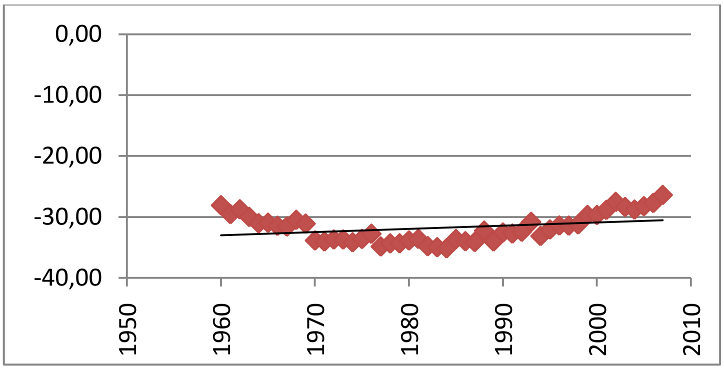

Figure 24. Historical values of k_t, 1960-2007

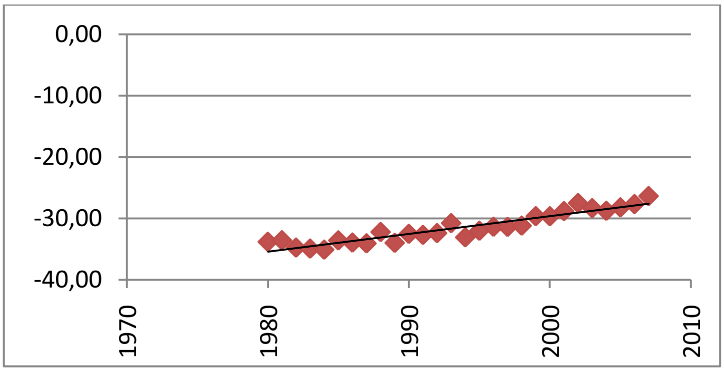

Figure 25. Historical values of k_t, 1980-2007

10. Reserve calculations for Renshaw-Haberman

10.1 Generation retiring 2007

To calculate the reserve using the Renshaw-Haberman method we use the same derivation as in section 8.1 and 8.2.

We assume that all payments from policyholders are made in January 2007 and all the disbursements are made in December. That provides the reserve an additional year with interest. If we look at the results in table 10, we see the total amount of money that is to be needed to fulfill the commitments to the policyholders. And if we look at the results in table 11, we see how large the technical provision needs to be. If we subtract the results in table 10 with those of table 11 we see how much the company would save on reserving with 2 percent interest.

Historic period used			
Reserve	**1900-2007**	**1960-2007**	**1980-2007**
Men	15 516,81	15 559,46	15 379,24
Women	18 425,42	17 445,65	16 783,44

Table 10. Reserving with zero interest per 1000 policyholders, whole life annuity

Historic period used			
Reserve	**1900-2007**	**1960-2007**	**1980-2007**
Men	12 785,40	12 812,84	12 696,66
Women	14 831,83	14 213,52	13 787,01

Table 11. Reserving with 2% interest per 1000 policyholders, whole life annuity

As earlier, we assume that we start with the whole period, i.e. 1900-2007 and then calculate the differences in reserving between that period and the other two periods and continue with comparing 1960-2007 with 1980-2007. In table 12 we see the differences between the three different time periods with 2 percent interest and whole life annuity starting at 2007. Depending on the periods the differences vary between -5,3 and 0,2 percent, a substantially larger span than for the Lee-Carter method. Five percent is quite a big difference but the effect from discounting is still a lot more important. By using 2 percent interest we can save around 17 percent for males and around 22 percent for women depending on the chosen period.

	1900-2007	1960-2007	1980-2007
Men	12 785,40	+27,44 (+0,215%)	-88,74 (-0,694%)
Women	14 831,83	-618,31 (-4,169%)	-1044,82 (-7,044%)
Men		12 812,84	-116,18 (-0,907%)
Women		17 235,04	-426,51 (-3,001%)

Table 12. Differences between the three periods

10.2 Generation retiring 2017

A group of 55 year olds are buying a single premium in January 2007. That gives us a period of ten years before the first payment is made.

We get the following results:

	Historic period used		
Reserve	**1900-2007**	**1960-2007**	**1980-2007**
Men	17 389,37	17 842,32	18 395,16
Women	19 535,34	18 639,37	18 002,04

Table 13. Reserving with zero interest per 1000 policyholders, life annuity deferred by 10 years

	Historic period used		
Reserve	**1900-2007**	**1960-2007**	**1980-2007**
Men	11 402,29	11 645,34	11 940,19
Women	12 643,97	12 173,63	11 835,60

Table 14. Reserving with 2% interest per 1000 policyholders, life annuity deferred by 10 years

In order to find out what difference it makes if the policyholders make their payment when they are 55 and 65 we compare table 10 with table 13 and then table 11 with table 14. When looking at whole life annuity with zero interest the technical provision is larger for the 55 year olds. At first thought, one would think that it would be cheaper for that group since some individuals will die before payments begin. Especially for then men hence they have greater mortality in this age then women. But this generation will also live longer and that effect is stronger, which means that the technical provision increases. In the other comparison we get the same conclusion as before; it takes less money in technical provision the earlier the policies are made. If we make the same assumptions as before, i.e. that the price the insured is paying is independent of the start of payment, then the insurance company can then save 5,96 to 10,82 percent for men and 28,91 to 29,70 percent for women depending on the time period used for parameter estimation.

	1900-2007	1960-2007	1980-2007
Men	11 402,29	+243,05 (+2,132%)	+537,90 (+4,717%)
Women	12 643,97	-470,34 (-3,720%)	-808,37 (-6,393%)
Men		11 645,34	+294,85 (+2,532%)
Women		12 173,63	-338,03 (-2,777%)

Table 15. Differences between the three periods, deferred life annuity

In table 15 we see the differences between the three time periods, calculating with 2 percent interest. Depending on what historic period we use we obtain different reserves. The effects of time period are larger for the Renshaw-Haberman method than for the Lee-Carter method. With deferred whole life annuity the effects are between -6,4 and +4,7 percent. If we subtract the results in table 13 with those of table 14 we see how much the company would save on reserving with 2 percent interest compared to zero interest. We notice once again that the effects caused by time period are much smaller than those caused by discounting, where there is reduction is around 35 percent whichever timer period or gender we choose.

Compared to reserving with the Lee-Carter model we get much bigger differences when reserving with the Renshaw-Haberman model. Especially for women since their mortality time trend index curve looks very different depending on what time period we focus on. The time index curve decreases until around 1980 but then starts increasing. There are differences in technical provisions up to 6 percent and that may cause huge miscalculations. Consequently, the differences in reserving can be very large depending on the historical data we choose to use.

11. Comparison of results between the different methods

Women	1900-2007	1960-2007	1980-2007
Lee-Carter		-0,963%	-0,618%
Renshaw-Haberman		-3,092%	-5,335%
Lee-Carter			0,348%
Renhaw-Haberman			-2,314%

Table 16. Differences for generation retiring 2007 per 1000 policyholders, women

Women	1900-2007	1960-2007	1980-2007
Lee-Carter		-1,817%	-1,165%
Renshaw-Haberman		-3,720%	-6,393%
Lee-Carter			0,664%
Renhaw-Haberman			-2,777%

Table 17. Differences for generation retiring 2017 per 1000 policyholders, women

Comparing the results of technical provisions using the different models (table 16 and 17) we can see that the differences between periods are much larger for the Renshaw-Haberman model than for the Lee-Carter model. This is true for the immediate as well as for the deferred annuity. On the other hand, the Renshaw-Haberman and Lee-Carter models yield similar differences in reserving between zero and two percent interest rate, as shown in tables 18 and 19.

So what model is the best one to apply? My opinion is that if there are not any clear cohorts with mortality improvement it is easier to stick to the simpler method, i.e. the Lee-Carter model. To use an extra parameter gives us more complicated calculations and it complicates interpretation of the results. The corresponding tables for men can be found in appendix B.

Women	**1900-2007**	**1960-2007**	**1980-2007**
Lee-Carter	20,64%	20,44%	20,51%
Renshaw-Haberman	19,50%	18,53%	17,85%

Table 18. Differences between zero- and 2 percent interest per 1000 policyholders, whole life annuity, women

Women	**1900-2007**	**1960-2007**	**1980-2007**
Lee-Carter	35,21%	34,96%	35,05%
Renshaw-Haberman	35,28%	34,69%	34,25%

Table 19. Differences between zero- and 2 percent interest per 1000 policyholders, deferred whole life annuity, women

12. Conclusions

Differences in mortality have always existed and always will. The complicated thing is to predict it.

What are the consequences of a reduced mortality? That depends on when the reduction starts showing. If it is only one generation that makes *extra* mortality improvement, there is a risk that these individuals have to fight more for their survival. Compare for example a baby boom, i.e. a time period during which many more babies are born. Everyone needs kindergarten places, primary education and jobs in the future. But the advantage of continuing mortality improvement from birth is that the society in the meantime has more time to acclimatize. It makes it more complicated if the improvements are shown later, for example when the members of the generation are around 40 years old and continue having improvements for the rest of their lives.

In this survey, we have seen many patterns of mortality improvement. But when looking at mortality improvement it is difficult to decide whether it is just normal improvement or if it is large enough to be called a cohort effect. The reason that mortality improvement has been so great the last thousand years depends largely on medical progress and improved standard of living. The general improvement should soon come to a standstill. So in the future, it may be easier to distinguish cohort effects from usual mortality improvement.

My conclusion is that it is possible to find more evidence of birth cohort effects if we have lower requirements about what an effect is. It is in the researcher's requirements and accuracy. *"There is nothing like looking, if you want to find something. You certainly usually find something, if you look, but it is not always quite the something you were after"* (J.R.R Tolkien). In many ways, there are good patterns, but I do not think that there are any patterns of mortality improvement that last long enough time to be called a birth cohort effect.

Either way, it is really important for insurers to have a reasonable good overview of what future mortality will look like. The most difficult about this is to make a decision about how many years of historical data one should use to get the most successful prediction. A good rule of thumb is that the longer forecast we would like to do, the further back in time should we go for data retrieval. But this depends of course on what the historical data look like. If there are many specific events in history that make mortality change much over short periods, it is a good idea to make some form of smoothing to get a better result.

What effects will a mortality improvement have on the insurers? A predicted improvement should not have any effect on the individual company, since they have had the option to prepare themselves. The technical provisions would be made larger in order to cover future payments. But if not, it could mean a major loss of money, with further payments. A difference in a few percent in our computations of future payments can make a huge difference for the insurer.

The patterns of mortality improvement that were seen for women in United Kingdom and Denmark could be interesting to follow up in a few years to find out if they are still making progress. However, it may be too late to make any changes for insurers since this generation has already passed the age of 60 and begun to retire.

To calculate the technical provision we have used two different models, the Lee-Carter model and the Renshaw-Haberman model. Since we only applied the models for reserving on Swedish data, we can't draw any firm conclusions. The Renshaw-Haberman model is an extended version of Lee-Carter with an extra parameter. It can give inconclusive results if we make the wrong forecasts. But possibly the big differences in our results between the Renshaw-Haberman and Lee-Carter models may be due to the Swedish time trend index k_t. An important result is that regardless of which model we decided to use, and in all different scenarios that we set up; the effect of discounting was a lot larger than the time period effect. The effect of discounting was also very similar for the two models, while the effect of chosen historical time period differed more. That gives us the implication, that as sooner the insurers begin with reserving, the better.

13. Appendix A.

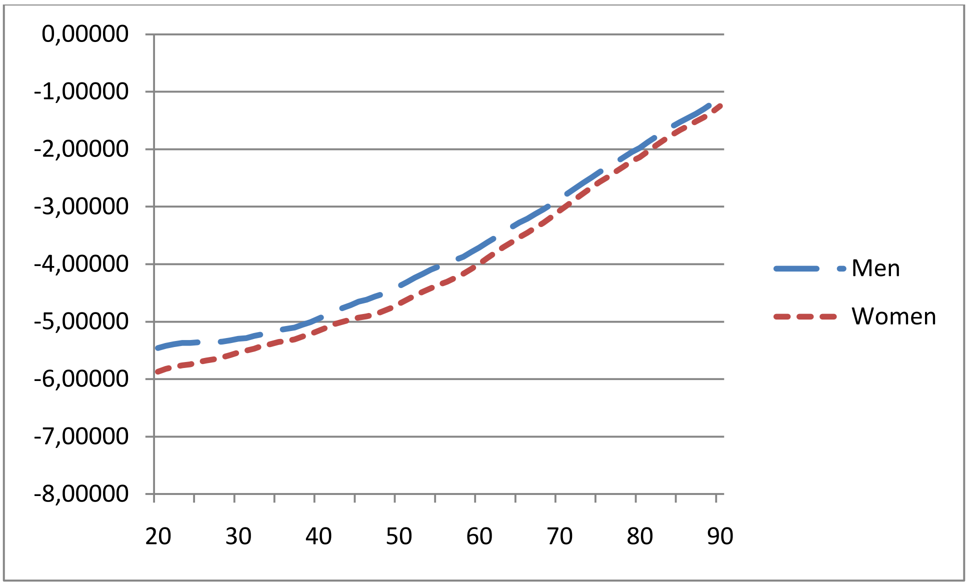

Figure A. General pattern of mortality a_x, Sweden, 1751-2007, men and women

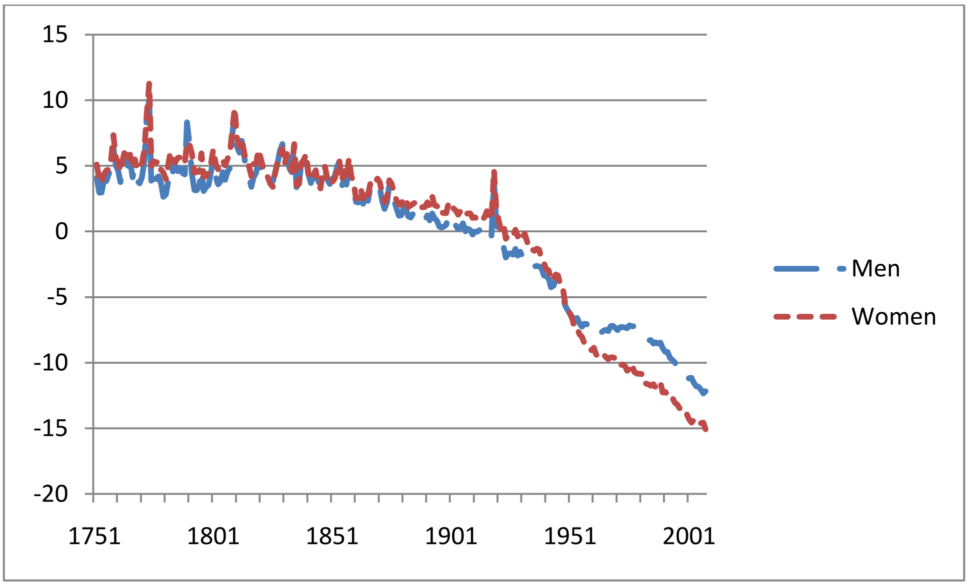

Figure B. The mortality trend k_t, Sweden, 1751-2007, men and women

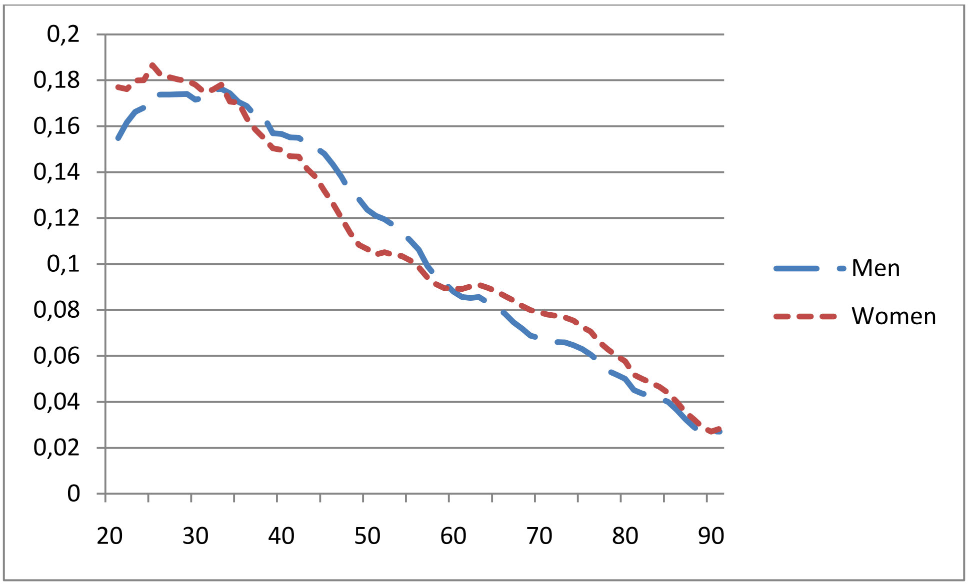

Figure C. Age-specific constant b_x, Sweden, 1751-2007, men and women

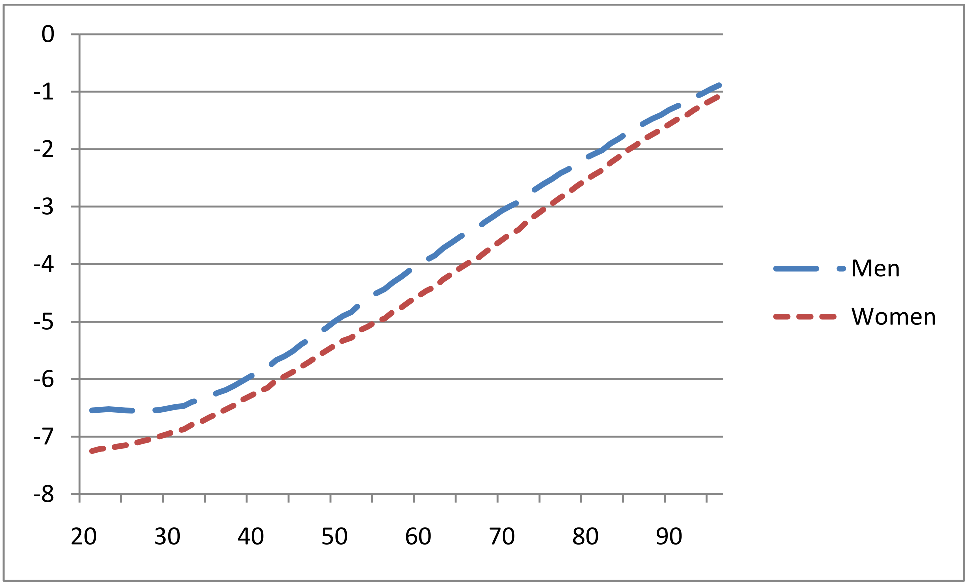

Figure D. General pattern of mortality a_x, United Kingdom, 1922-2006, men and women

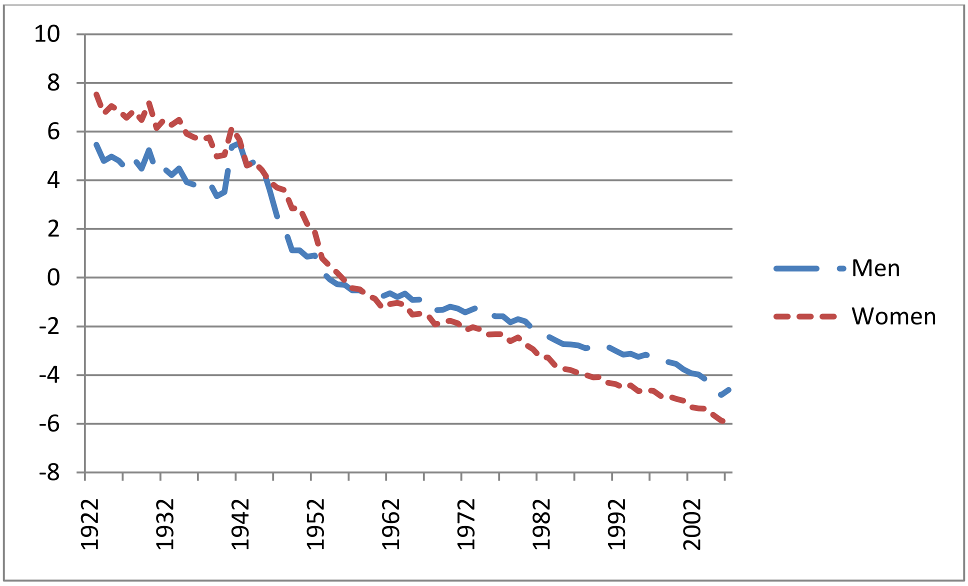

Figure E. The mortality trend k_t, United Kingdom, 1922-2006, men and women

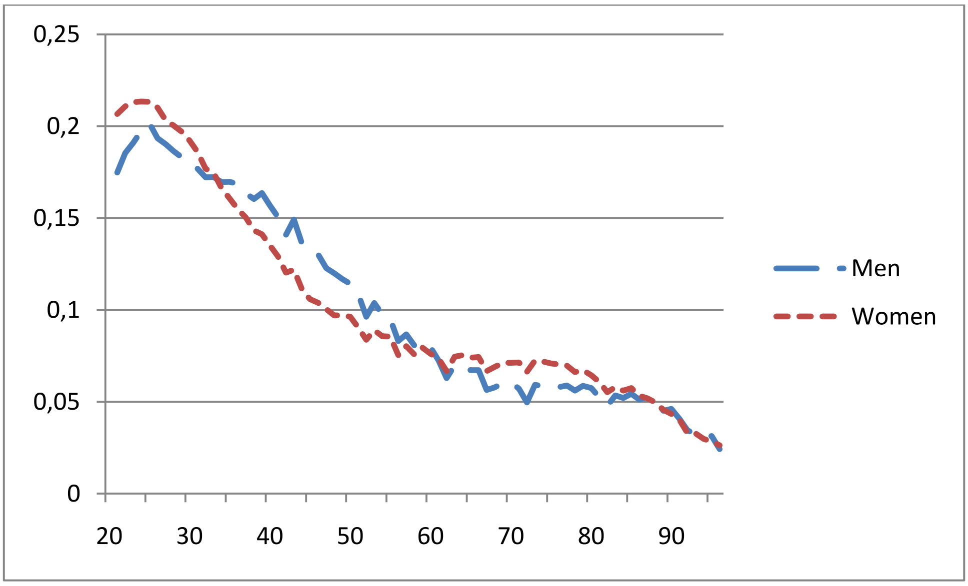

Figure F. age-specific constant b_x, United Kingdom, 1922-2006, men and women

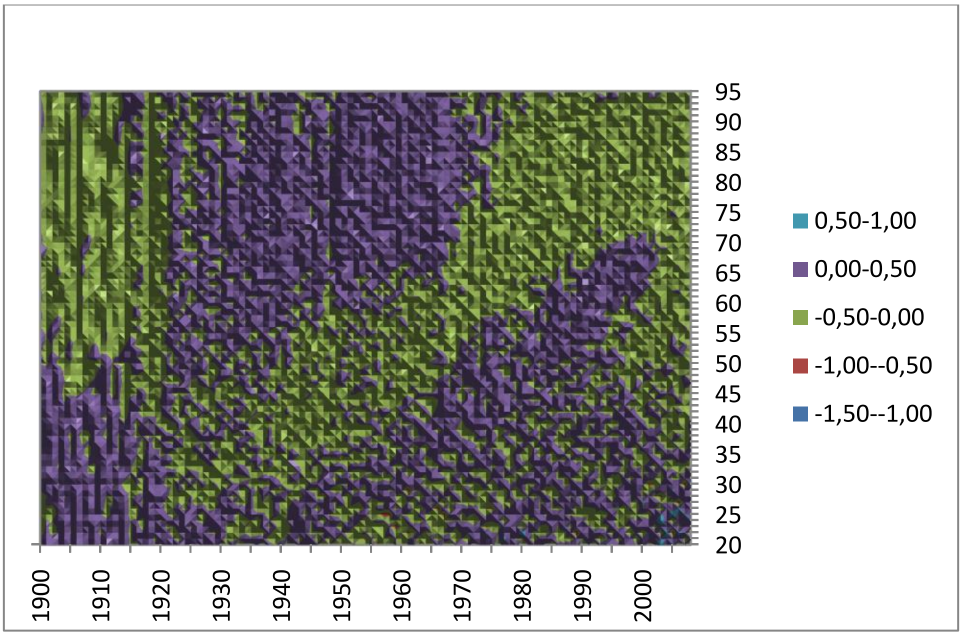

Figure G. Mortality pattern, Denmark, women, 1900-2008

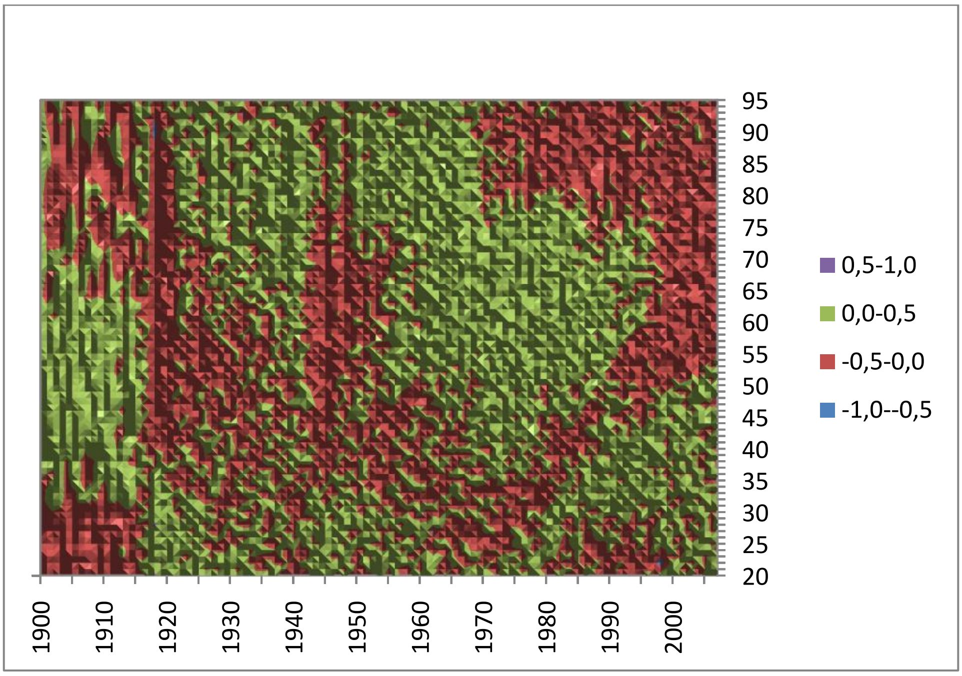

Figure H. Mortality pattern, Denmark, men, 1900-2008

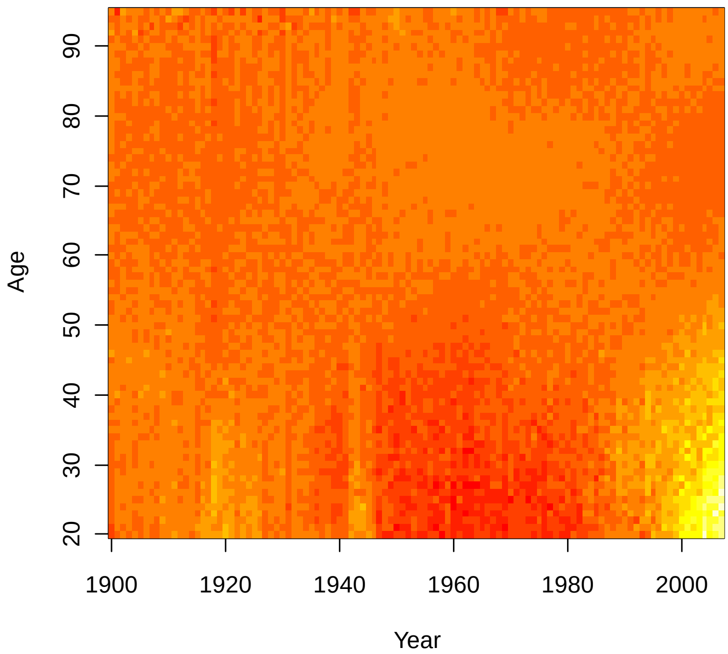

Figure I. Men Sweden, 1900-2007

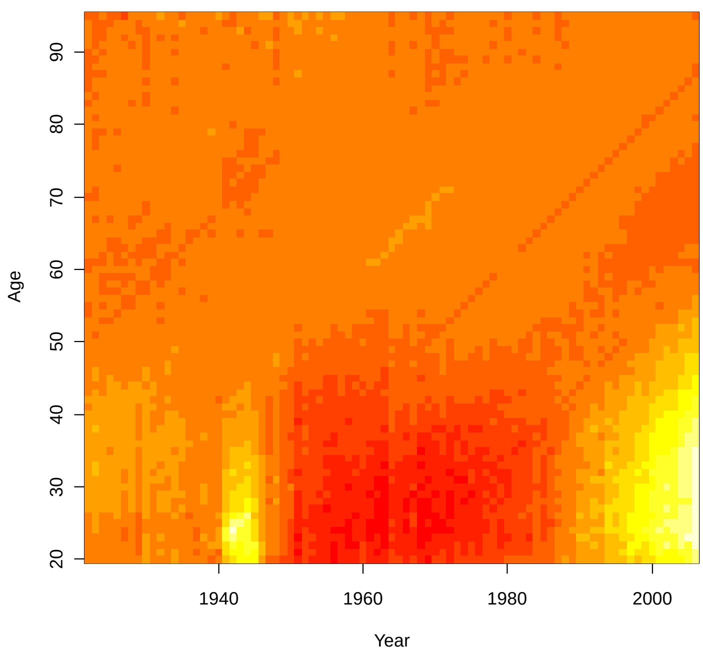

Figure J. Men, UK, 1922-2007

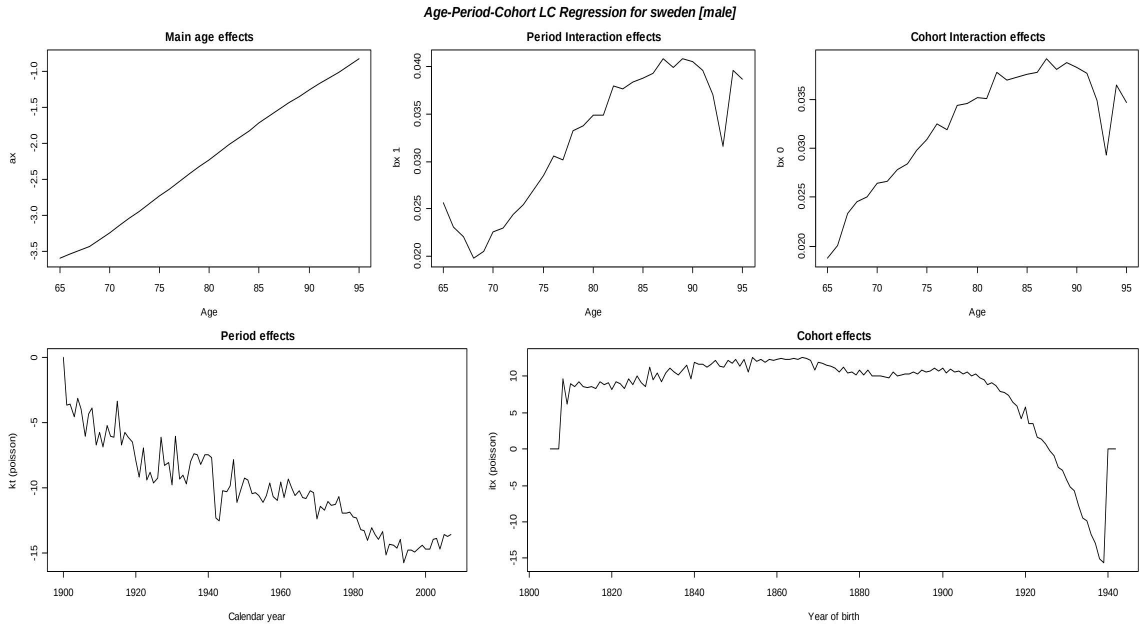

Figure K. Age-period regression for Sweden, male

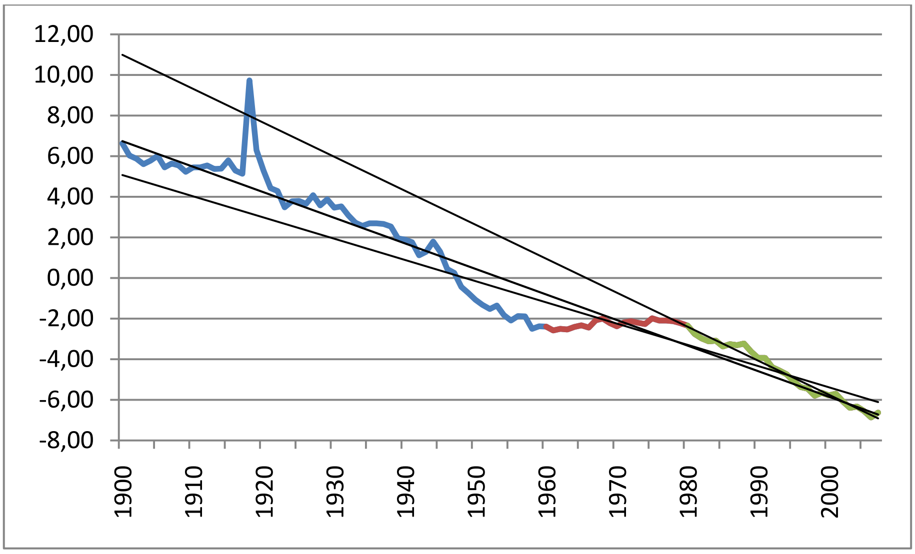

Figure L. Historical values of k_t, 1900-2007, men

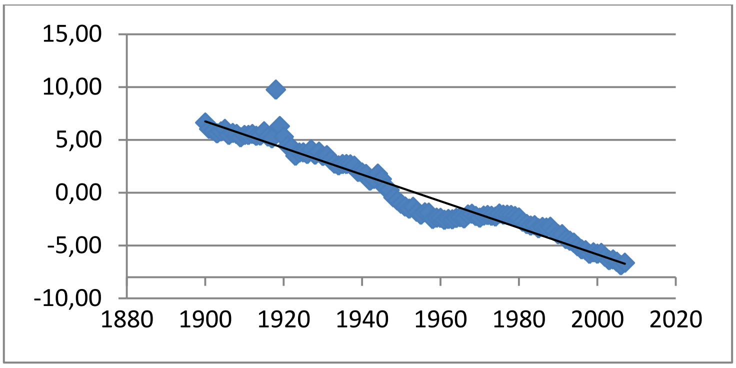

Figure M. Historical values of k_t, 1900-2007, decrease with 0,1259 per year

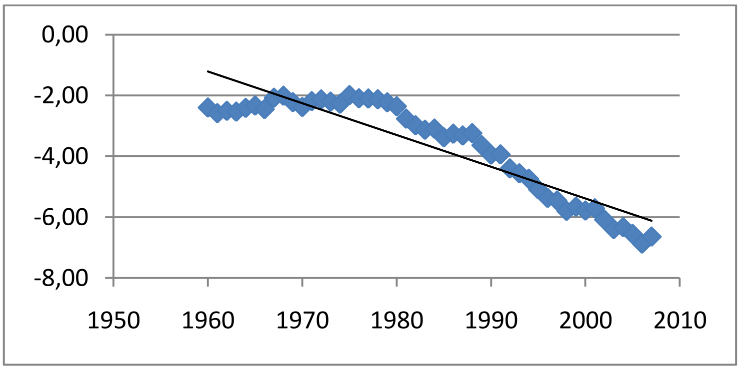

Figure N. Historical values of k_t, 1960-2007, decrease with 1,1032 per year

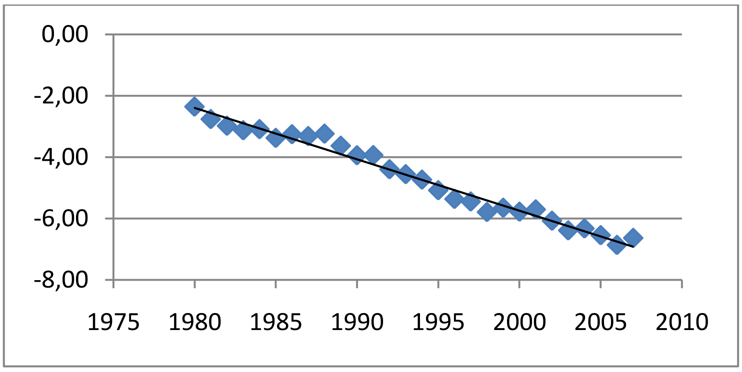

Figure O. Historical values of k_t, 1980-2007, decrease with 0,9777 per year

14. Appendix B.

Men	1900-2007	1960-2007	1980-2007
Lee-Carter		-0,423%	0,615%
Renshaw-Haberman		0,215%	-0,694%
Lee-Carter			1,043%
Renhaw-Haberman			-0,907%

Table AA. Differences for generation retiring 2007, men

Men	1900-2007	1960-2007	1980-2007
Lee-Carter		-0,686%	1,336%
Renshaw-Haberman		2,132%	4,717%
Lee-Carter			2,036%
Renhaw-Haberman			2,532%

Table BB. Differences for generation retiring 2017, men

Men	1900-2007	1960-2007	1980-2007
Lee-Carter	18,22%	18,10%	18,35%
Renshaw-Haberman	17,60%	17,65%	17,44%

Table CC. Differences between zero- and 2 percent interest, whole life annuity, men

Men	1900-2007	1960-2007	1980-2007
Lee-Carter	33,12%	33,03%	33,29%
Renshaw-Haberman	34,43%	34,73%	35,09%

Table DD. Differences between zero- and 2 percent interest, deferred whole life annuity, men

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