A remark on small-sample properties of logistic regression in three-point designs

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Abstract

Nottingham and Birch [1] recently alleged that the maximum likelihood estimator $\hat{\beta}$ of the steepness parameter in a logistic regression model for quantal dose-response data could be seriously underestimated. They performed a simulation study, investigating in particular a small-sample three-point design with a large spacing between the doses, and concluded that the steepness had a negative expected error of 25% of its true value. The present work demonstrates that this allegation was not correct. In their design there is a high probability (almost 70%) of outcomes with an infinitely high $\hat{\beta}$. Nottingham and Birch seem to have neglected these outcomes, and since the remaining 30% represent the lower part of the distribution for $\hat{\beta}$, it is not surprising that these appear to underestimate β, when regarded alone. Here it will instead be pointed out and illustrated by exact calculations in some examples that the asymptotic normal distribution fits quite well even with small samples, except in the upper tail when the design has a substantial probability of infinite outcomes of $\hat{\beta}$.

Key words: Tolerance threshold distribution, Maximum likelihood estimator, D-optimal design, Normal approximation.