Quasi Equilibrium Approximations of the Fixation Index under Neutrality: The Finite and Infinite Island Models

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April 30, 2012

Abstract

The fixation index $F_{ST}$ and the coefficient of gene differentiation $G_{ST}$ are analyzed for the finite island model under short time spans, ignoring mutations. Dividing the reproduction cycle into the three steps gamete formation, fertilization, and migration we develop a new approach for computing quasi equilibrium formulas for $F_{ST}$ (and $G_{ST}$). Our formulas generalize earlier ones and reveal that the equilibrium value of $F_{ST}$ is influenced not only by the migration rate and local effective population size, $N_e$, but also the local census size $N$, particularly so when the migration rate is high. The order of migration and fertilization is found to have a smaller effect on $F_{ST}$. A major advantage compared to previous approaches is that stochastic allele frequency of migrants is easily accommodated, thereby avoiding $F_{ST}$ to be underestimated for large migration rates.

Key words: Effective population size, fixation index, fertilization, gamete formation, island model, migration, quasi equilibrium, reproduction.

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